Convolutional Neural Networks using separable filters

Petrescu Viviana
EPFL
viviana.petrescu@epfl.ch

Abstract

CNN are the state-of-the-art machine learning techniques which achieved best results on various computer vision tasks ranging from the large scale ImageNet object recognition challenge to segmentation in bio medical imaging. This technical report presents initial results on using CNN with separable filters for speeding up the testing time.

1 Introduction

Although proven to be very powerful, CNN are much slower for both training and testing than their counter parts SVM or Random Forests. In the forward pass, the computational complexity of evaluating one image of size $W \times H$ with J filters of size $d_1 \times d_2$ is $O(WHJd_1d_2)$.

Although improving the training time would be very beneficial since it would allow for more configurations to be tried and for larger networks, increasing effort has been put also into speeding up only testing time, since training can be done offline.

2 Speeding up CNN

By far the most common approach for speeding up CNNs is to run them on the GPU, making use of the paralelism nature of the algorithm. Alternatively or combined, FFT can be used for the convolutional operations (for both training and testing)[5]. For testing, a significant speedup can be obtained in hardware by using FGPAs [4]. The major drawback of FGPAs is that they are harder to program for people who do not work in the field.

In [2] they prove speedup of CNNs for two different schemes using separable filters, one of which is similar with our approach but uses a different optimization algorithm for obtaining the separable filters. If the $2D$ filters are decomposed into a set of separable $1D$ filters of rank K, the complexity per image becomes $O(WH(J + d_1 + d_2))$. Thus, we obtain a speedup if $K << \frac{Jd_1d_2}{J+d_1+d_2}$.

3 Separable filters

In our approach, the set of filters χ for one convolutional layer is approximated as a set of separable filters, as shown in Fig1.

We can obtain an approximation by minimizing the equation:

$$\underset{a,b,c}{\text{minimize}} \quad \|\chi - \sum_{r=1}^{r=R} a_r \circ b_r \circ c_r\|_2^2$$

We optimize the above equation using a Matlab implementation of the Canonical Polyadic decomposition (a generalization of SVD for tensors) with non linear conjugate gradient method. The

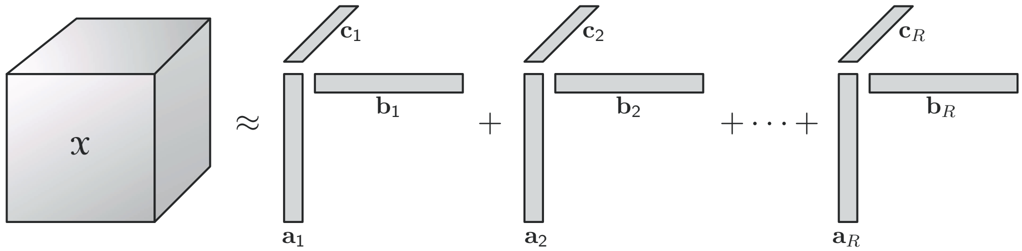


Figure 1: Tensor decomposition

Tensor size	Typical rank
$I \times J \times K$ with $JK \leq I$ (very tall)	JK
$I \times J \times K$ with $JK - J < I < JK$ (tall)	I
$I \times J \times K$ with $I = JK - J$ (compact)	$I, I+1$

Table 1: Typical rank for certain tensor shapes

optimization framework requires as input the rank R, which is almost never known. When the rank used for decomposition is not the theoretical one, the decomposition is not very accurate and the algorithm needs to be run with different initialization parameter until it converges. What is known is an upper bound on the theoretical rank R for a general tensor $3D$ tensor $\chi \in \mathbb{R}^{I\times J\times K}$, given by:

$$\text{rank}(\chi) \leq \quad min\{IJ, JK, KI\}$$

Besides the theoretical rank, there is also the notion of typical rank, which is any rank that appears with probability greater than 0, or that it is most common [3]. Some of the known typical ranks are shown in Table 1. Looking at the first row of Table 1, we conclude that for a 'very tall' set of filters like the ones present in large CNNs, we should expect the rank to be equal with the product of the two kernel dimensions.

4 Experiments

We ran the experiments in Python, using the Theano library. We varied the rank for every convolutional layer and compared the separable version with the non separable one. We recorded the time, the change in performance and how well the separable filters approximate the $2D$ filters. We experimented on the MNIST and on the Mitocondria Striatum datasets.

Layer	Type	Maps and neurons	Kernel size
0	input	1 map of 28x28	
1	convolutional	20 maps of 24x24	5x5
2	max pooling	20 maps of 12x12	
3	convolutional	50 maps of 8x8	5x5
4	max pooling	50 maps of 4x4	
5	fully conntected	500	
6	fully conntected	2 neurons	

Table 2: Small CNN for MNIST

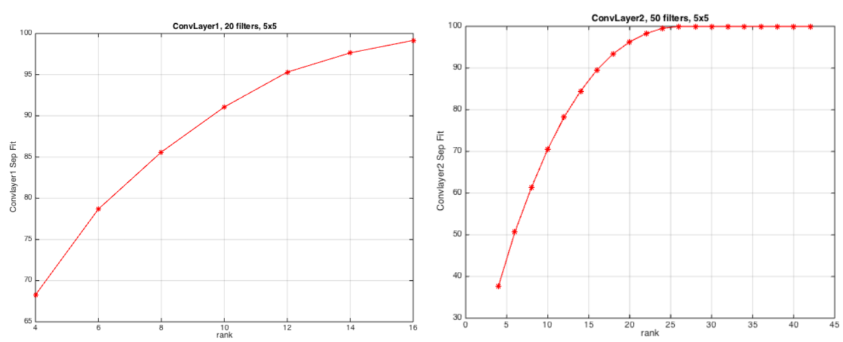
4.1 MNIST

MNIST consists of a curated set of black and white images (28x28) depicting handwritten digits. The dataset is split into 60.000 training samples, 10.000 validation and 10.000 testing samples. For this set, the approach of [1] using CNNs is the best winning model with a 0.23 error rate (equivalent with 23 out of 10.000 digits not recognized correctly). We start our experience with the reference Theano model for MNIST which achieves 0.82 error rate. Its configuration is shown in Fig2

4.1.1 CNN Model 1

We decompose the filter map from the first convolutional layer. From the theoretical point of view, according to Table 1, we have a compact tensor $R^{20\times5\times5}$ with a typical rank of 20 or 21, so we expect a very good aproximation for a rank in that range. Fig2a shows how well is the approximation with varying rank from 4 to 16 in steps of 2 on the x axis and the fit on the y axis (100 corresponds to perfect fit). As expected, the fit is almost perfect for high ranks and decreases afterwards.

Using separable filters, we obtain a theoretical speedup for convolutional layer 1 if $K << \frac{Jd_1d_2}{J+d_1+d_2} = \frac{20\times5\times5}{20+5+5} = 16.66$. Fig3a shows the time improvements using separable filters. The blue line represents the time per layer using non separable filters. In our implementation, the separable filters give a speedup for any rank, as the red curve is below the blue one. We note that theoretically using non separable filters (blue line) and using separable filters with rank 16 should have given the same running time. The reason for the difference is due to implementation details and it is a language specific artifact.

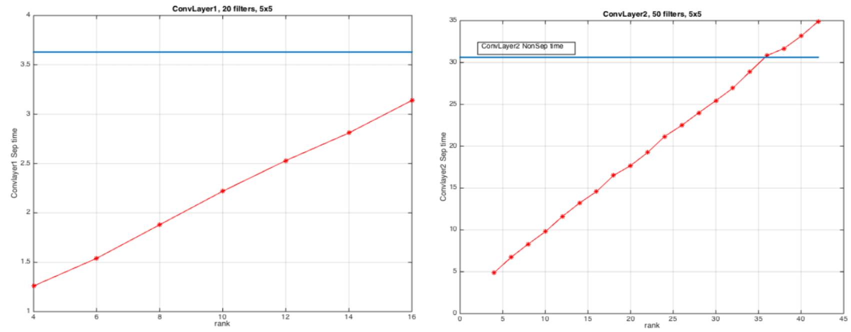


(a) Convolutional layer 1

(b) Convolutional layer 2

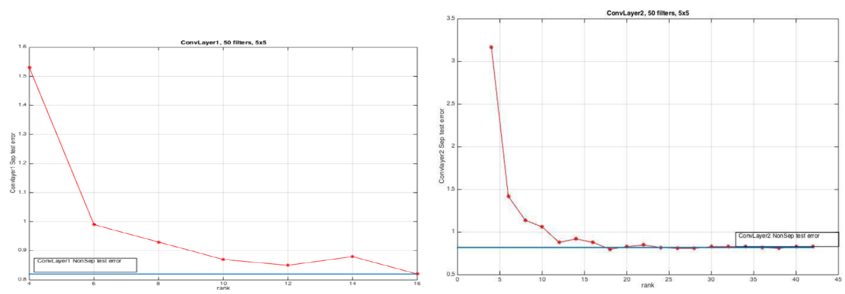
Figure 2: Rank versus Fit

The question that remains is to see how far we can afford to reduce the rank such that we do not lose much accuracy. Fig 4 a shows how the performance of the CNN drops with decreasing rank. With rank 16 the performance is the same, while for rank between 10 and 16 the error rate stays between 0.8 and 0.9 per cent, which is quite low. According to the application, even a rank of 8 or 6 is acceptable, since the drop is not more than 1 per cent.



(a) Convolutional layer 1 (b) Convolutional layer 2

Figure 3: Rank vs Time



(a) Convolutional layer 1 (b)

Figure 4: Convolutional layer 2

In the second experiment, we kept the first layer unchanged and approximated the second convolutional layer. Similarly, according to Table 1, we have a very tall tensor $R^{50\times5\times5}$ with a typical rank of 25. Fig2b shows how well is the approximation with varying rank from 4 to 42 in steps of 2 . As expected, the fit is almost perfect (99.9 fit) for rank higher than the theoretical one of 25.

From the complexity perspective, the separable CNN is faster if we use a rank $K << \frac{Jd_1d_2}{J+d_1+d_2} = 20.83$. In our case, for rank = 20 we obtained almost a 40% speedup. This is due as before to language specific implementation. For rank 10, using separable filters is actually 3 times faster (theoretically 2 times faster). If we keep the rank greater than 10, the recognition never drops more than 1 error rate. For rank 12 the fit is 79 but the error rate is still almost unchanged which means we can easily use rank with relatively bad fit and still obtain good performance. The first layer with separable filters of similar fit of 79 obtain a slitghly lower performance. This might imply that it is more important to approximate well the first layer, which is the building block for the following layers.

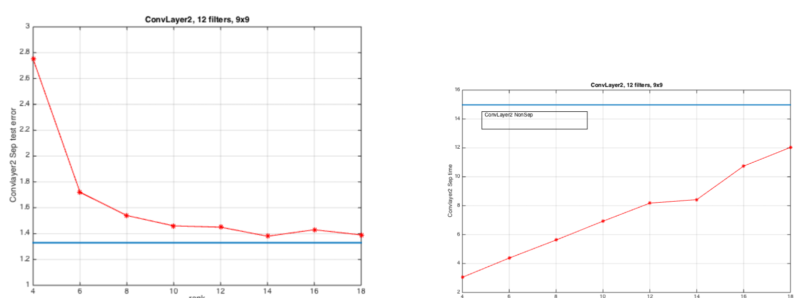
4.1.2 CNN Model 2

In the following experiment, we keep the same number of filters per layers but increase the kernel sizes to 9×9 Fig3.

Here we only approximate convolutional layer 2. The theoretical rank is less than 49 and we obtain speedup if the rank $K << 59.5$ The results can be seen in the figure below

Layer	Type	Maps and neurons	Kernel size
0	input	1 map of 28x28	
1	convolutional	20 maps of 24x24	9x9
2	max pooling	20 maps of 12x12	
3	convolutional	50 maps of 8x8	9x9
4	max pooling	50 maps of 4x4	
5	fully conntected	500	
6	fully conntected	2 neurons	

Table 3: Bigger CNN for MNIST set



(a) Error vs Rank

(b) Rank vs Time

Figure 5: Convolutional layer 2

Layer	Type	Maps and neurons	Kernel size
0	input	1 map of 51x51	
1	convolutional	10 maps of 46x46	6x6
2	max pooling	10 maps of 23x23	
3	convolutional	20 maps of 18x18	6x6
4	max pooling	20 maps of 9x9	
3	convolutional	50 maps of 4x4	6x6
4	max pooling	50 maps of 2x2	
5	fully conntected	100	
6	fully conntected	2 neurons	

Table 4: CNN for Mitocondria set

		Layer 1	Layer 2	Layer 3
NonSep	Kernel Size	10	20	50
	Size	6x6	6x6	6x6
	Speedup rank	16.36	22.5	29.03
	Theoretical rank	16.4	¡36	36
	time	1	2	4
Sep	time	1	2	4
	rank	-	-	-
Sep	time	1	2	4
	rank	-	-	-

Table 5: CNN for Mitocondria set

4.2 Mitocondria Striatum

For the Mitocondria Striatum set, CNN are employed to solve a segmentation problem. Every pixel is classified as being part of the mitocondria or not. A patch of 51x51 surrounding the pixel is extracted from the image and fed as input to a CNN which acts as a binary classifier. For determining the parameters of the CNN we used a set of 100.000 samples for training and 20.000 for validation (containing half positive and half negative samples). After determining the best setup, the network was trained on a larger set of 1 milion samples for training and 200.000 for validation.

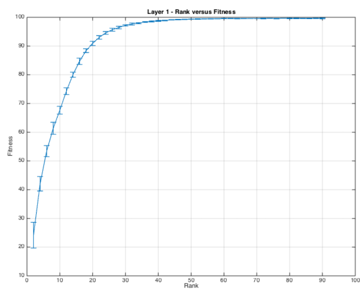
The final test data represented a 3D image volume consisting of 318 slices of size 400x661. Thus, one frame contains 264.400 datasamples that are fed as test input for the CNN, while all frames contain 97 million datasamples.

The best setup obtained using the small training and validation set was the one presented in Fig5. This gave us a VOC error on the first frame of 72. We then trained this net on the big set and obtained an error of 74. We notice that this is not comparable with state of the art methods which achieve results higher than 79 VOC or that use 3D information. The goal of this is to see if we can obtain a speedup by using separable filters.

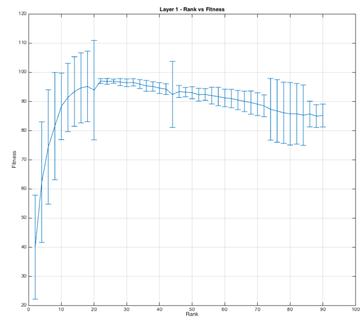
Example of the first layer filters are shown in Fig[ref].

4.3 Pre trained ImageNet CNN

Here we look to see if we could gain something from CNNs of large networks. The network below is the reference net from Alex Kristevski paper (without the split to be run on two GPUs). The net was trained by [cite] here.

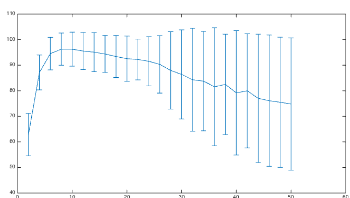


(a)

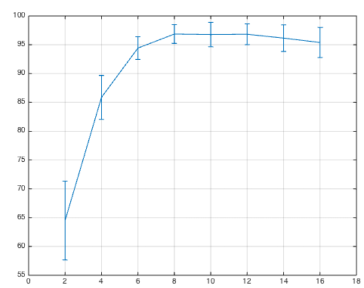


(b)

Figure 6: ImageNet Layers



(a)



(b)

Figure 7: ImageNet Layers

5 Conclusions

Theoretical bounds of the separabe filters choice are proven.

Acknowledgments

We would like to thank Simon Arosi that helped me a lot during this project giving me constant feedback.

References

[1] Dan C. Ciresan, Ueli Meier, and Jürgen Schmidhuber. Multi-column deep neural networks for image classification. *CoRR*, abs/1202.2745, 2012.

[2] M. Jaderberg, A. Vedaldi, and A. Zisserman. Speeding up convolutional neural networks with low rank expansions. In *British Machine Vision Conference*, 2014.

[3] Tamara G. Kolda and Brett W. Bader. Tensor decompositions and applications. *SIAM Review*, 51(3):455–500, September 2009.

[4] Yann LeCun, Koray Kavukcuoglu, and Clément Farabet. Convolutional networks and applications in vision. In *Circuits and Systems (ISCAS), Proceedings of 2010 IEEE International Symposium on*, pages 253–256. IEEE, 2010.

[5] Michaël Mathieu, Mikael Henaff, and Yann LeCun. Fast training of convolutional networks through ffts. *CoRR*, abs/1312.5851, 2013.