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Application of Fourier transformation to signal processing implied by ARIMA model

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***Abstract*—When developing a method of modeling for future forecast, it is important to determine the noise in data in order to optimize the quality of a model, which may be considered a stage of the process of signal processing. In the paper, the Fourier transformation method has been used to establish better results. It is proposed to use the data previously transformed with the help of Fourier transform in the ARMA model. A developed approach is based on data on exchange rates in the Euro area throughout the last 22 years. The methods described were applied in the software environment RStudio. The results may be used in forecasting exchange currency rates applying the use of Fourier transformations – one more way of improving the forecast.**

Keywords— signal processing, Fourier transformation, ARIMA.

I. Introduction

A constantly increasing amount of information in the world makes it necessary to boost the accuracy of its analysis. One of the widely-used tools of modeling is ARIMA models (autoregressive–moving-average models), which utilize the historical data of only one variable to predict its values for future periods. The application is possible for forecasting the required quantity of goods for the next period built upon the dataset for previous timespans, forecasting sales and interpreting seasonal changes in sales, predicting financial and economic indicators, etc. However, in order to make the best use of it, the first step is to normalize the data. Since the data must be stationary, it must be adjusted to such a form. An effective way to get rid of seasonality is the use of Fourier signal processing. Signal processing enables us to analyze the perception of the approximation of moving averages in the differencing operator of the ARIMA model [1-5].

The paper focuses on the possibility of using Fourier series for ARIMA models on the example of economic indicators. Future exchange rates not only depend on the features of the past data but also on the past predicted values. So ARIMA has been chosen as the required instrument which owns such a feature. The number of delays on the feedback side is suitably selected to provide the best possible prediction performance. The training sample is selected from the past time series depending on the number of days ahead the exchange rate to be predicted.

II. Preliminaries

We note that the history of currency exchange rate measurements consists of two parts: the superposition of seasonal trends and some statistical deviation (noise) from the trends. In the ARIMA model, noisy data is distributed according to a standard normal distribution. However, the data often have seasonal effects that are not part of the statistical noise. For example, a change in currency exchange rates during vacations or holidays is a recurring seasonal effect, not a statistical outlier. To capture such effects by ARIMA model, some Fourier modes should be removed, as defined in [4-9] that successfully illustrated the point of approximation by imitation the signal processing methods. Meantime, ours differs in the research subject as well as comparing modelled signal (with Fourier transformation) to the original one. The Fourier signal is collected for all requested bases and orders and removed from the original signal. It is probable that the remaining signal has an arbitrary distribution, while the ARIMA algorithm expects standard normally distributed (“white”) noise. To standardize the noise, we perform a transformation of variable through a standard normal distribution:

$$y = \varphi^{-1}[f(x - F)], \tag{1}$$

where φ is a standard normal distribution cumulative distribution function, is the empirical cumulative distribution of the Fourier-reduced signal, is the detrended residual signal, and y is the standardized residual signal.

After removing the Fourier trends and converting the data into following a standard normal distribution, the residual signal can be statistically captured by the ARIMA model. The autoregressive (AR) model is given as

$$y_t = \sum_{i}^{P} \varphi_i y_{t-1} + \varepsilon_t \tag{2}$$

where y is the whitened, Fourier-reduced signal; P is the maximum number of AR lag terms; is the linear extrapolation term; and is random Gaussian noise. Furthermore, a moving average (MA) is added to the autoregressive model to yield the ARIMA,

The necessary steps for ARIMA modeling are to consider the order estimate of the model and the coefficient estimate. The first step is to determine the correct order of the model (i.e., determine the orders of the AR and MA polynomials). To model the average component of a financial time serie, we apply the autoregressive moving average process. The ARIMA model is a linear combination of the autoregression model along with the moving average model [5, 10-13]. The ARIMA(p, d, q) order model is calculated using the formula

$$y_t = \sum_{i}^{P} \varphi_i y_{t-1} + \varepsilon_t + \sum_{j}^{Q} \theta_j \varepsilon_{t-j}, \tag{3}$$

where Q is the maximum number of terms in the moving average and θ is a moving weight associated with the moving average lag term. The terms φ and θ are fitted to maximize a likelihood function. It is noteworthy that some tuning of Fourier periods is often required to result in a signal that is stationary; the ARIMA model is only valid for stationary data, or data whose joint probability distribution do not change when shifted in time. A stationarity check is useful in determining if sufficient signal has been captured through Fourier detrending.

Detrending is primarily required to eliminate trend effects, so the clear effect is observed explicitly; it allows to see clear effect of the variables on the resultant. In our case the data is exposed to high seasonality, so we need to perform deseasonalize non-stationary data [14]. ARMA, might not provide the detrended basis as some economic values (e.g., rate of inflation, exchange rate) share commonly the second-degree differencing.

Future exchange rates not only depend on the features of the past data but also on the past predicted values. So ARIMA has been chosen as an attribute to such a feature. The number of delays on the feedback side is suitably selected to provide the best possible prediction performance. The training sample is selected from the past time series depending on the number of days ahead the exchange rate to be predicted. The predicted exchange rate is compared with the training sample to produce the error or mismatch value. The effectiveness of the results of ARIMA models with the application/non-application of Fourier series in forecasting the exchange rate for 20 years was also investigated [14-20].

Noteworthy, the signal processing along with the application of multiple kernels in Fourier transformations were applicable for another fields, which led to significant results as well [21-23].

III. Main Results

The paper centers around application of Fourier signal processing transform on the example of the exchange rate in the Euro area (19 countries) in the period from December 2001 to June 2023 [17-21]. Subsequently, the data is used to build the ARIMA model and forecast the exchange rate from July to December 2023. The combination of such methods normalizes the data to a form that is acceptable for use in the forecasting model [16-20]. The implementation was carried out in the Rstudio software environment [24, 25].

Historical data show dynamics with a trend. There are several autocorrelations that are significantly non-zero. The time series is non-random and has seasonality. There is high degree of autocorrelation between values in PACF plot and geometric decay in ACF plot (Figure 1).

For the time series we applied Fourier signal processing transformation. The results of Frequency Spectrum, Real, and Imaginary Part of Fourier Transformation of the exchange rate in each period (Figure 2) were used to prepare data for further use.

Based on preliminary data analysis and comparison of different types of models, it was found that model ARIMA(2,1,3) describes the data with the highest accuracy (Table 1). The model shows that an exchange rate in period t has two lag observations (t-1, t-2), the first degree of differencing and the third size of the moving average window. However, we still need remember that the data might be prone to selection bias [25-30].

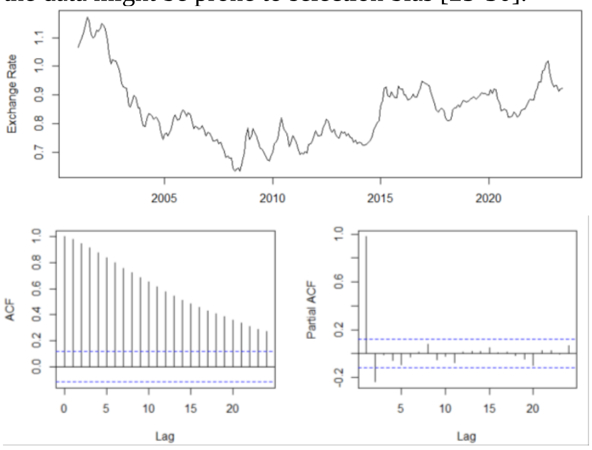

Figure 1. Historical data on the exchange rate of the USD/EUR currency in the European area, with the highlight of ACF and PACF.

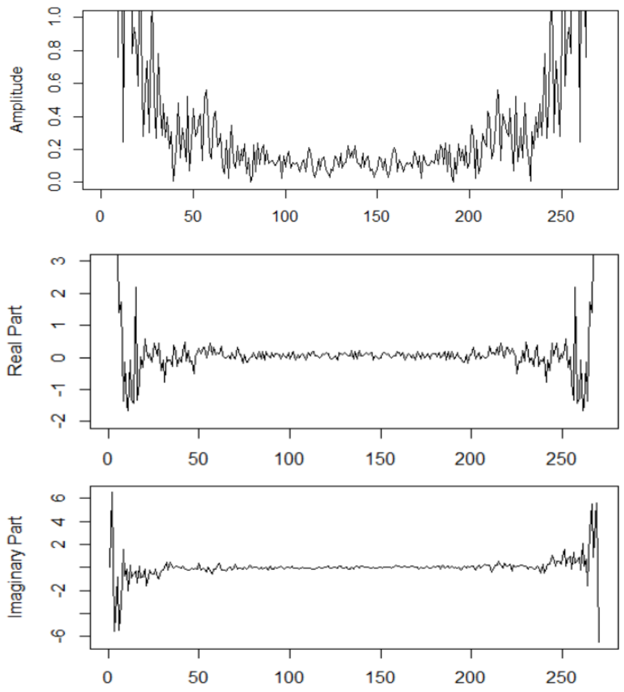

Figure 2. Spectrum; Real, and imaginary part of Fourier Transform

Based on the model output, the exchange rate forecast for the last 6 months of 2023 was made. It assumed that the model showcases the coefficients much closer to reality, considering the Fourier transformation utilized before. Signal processing allowed us to recognize these in case of ARIMA's moving averages. Thus, manipulating with kernel of Fourier analysis as well as ARIMA moving averages boosts up the results intensively.

Table 1. Input parameters of the ARIMA model

ARIMA(2,1,3)							
Coeffi-cients:	ar1	ar2	ma1	ma2	ma3	sar1	sma1
	-0.666	0.294	1.022	-0.012	-0.988	0.143	-0.239
s.e.						0.677	0.666

The following results show that with a probability of 95%, the values predicted by the model correspond to the real values for this period (Table 2). Therefore, our model has displayed sufficient accuracy to claim that Fourier transformation along with signal processing enhances the predicting power of the model [31-34].

To test the validity of selected frequency, one might perform Monte Carlo simulation. In this circumstance the statistics can be seen by interpretation of F or Student's distribution. In other words, the coefficients might be closer to real ones (F-ARIMA model improves the model in such way) at expense of their significance [36-38].

Table 2. Forecast of currency exchange rate in the period from July 2023 to December 2023 according to model ARIMA(2,1,3)

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95	Real
Jul 23	0.9209	0.8991	0.9427	0.8876	0.9542	0.9044
Aug 23	0.9183	0.8816	0.9550	0.8621	0.9744	0.9167
Sep 23	0.9182	0.8705	0.9660	0.8452	0.9912	0.9360
Oct 23	0.9151	0.8587	0.9715	0.8289	1.0013	0.9468
Nov 23	0.9204	0.8562	0.9846	0.8223	1.0119	0.9254
Dec 23	0.9219	0.8510	0.9928	0.8135	1.0303	0.9173

The main reason for F-ARIMA research is to observe the conditional volatility that presents in data. By that, we imply that ARCH testing might not even be required as the predictability may have already achieved its maximum by maximizing MLE likelihood function. Therefore, it is often argued that Fourier analysis brings ARIMA to a much higher level of accuracy thus predicting power.

IV. Conclusions

The paper depicts the application of Fourier series for improving the forecast established by the ARIMA model. As can be seen, the improvement appears to be highly useful as the real exchange rate was within the range of predicted one. High accuracy arises from the data being polished from noise disturbance that whereas disrupts significance but still may bring additional details to the data. To have the noisy disturbance conveying the meaningfulness, we have removed Fourier nodes and allowed for the ARIMA algorithm to transform noise to standard normal distribution. Signal processing improved the F-ARIMA model better point forecasts which were compared with original ones. However, it is argued that despite the coefficients are to be more informative, their significance might suffer because of the Fourier extrapolation made before. Worth noting, Signal processing has also exhibited its applicability for other economic variables, but in case of enough differencing. In our case, we have done so by employing the ARIMA model. Moreover, the integration of system analysis techniques could further enhance the forecasting model by considering the interdependencies and feedback loops present in complex economic systems. By incorporating system dynamics and feedback mechanisms into the modeling process, we can achieve a more comprehensive understanding of the underlying dynamics driving exchange rate fluctuations.

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