Improving Mixing times in Gibbs Sampling

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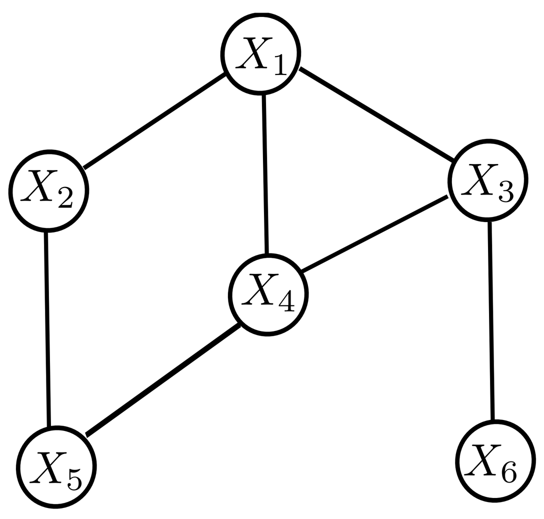
10-701 Introduction to Machine Learning, Course Project

Summary

Improve mixing time in Gibbs Sampling via Tree Blocks that group correlated variables together.

Introduction



Given: Graphical Model with known Parameters.

Goal: Estimate $\mathbb{E}[h(X)]$
E.g.: $1_A(x)$, $\sum_i X_i$

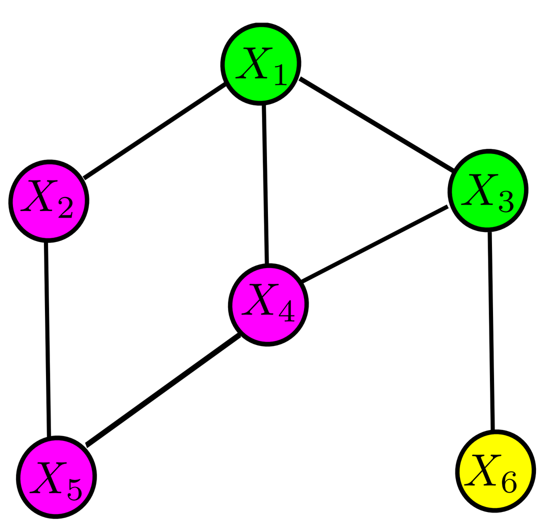
Generate Samples: $X^{(1)}, X^{(2)}, \dots X^{(N)}$

Empirical Estimator

$$\mu_0 = \frac{1}{N}\sum_{i=1}^{N} h(X^{(i)})$$

Gibbs Sampling–one way to generate samples.

Blocked Gibbs Sampling



Current Sample: $X^{(t)}$

$X^{(t+1)}_{1,3} \mid X^{(t)}_{2,4,5,6}$

$X^{(t+1)}_{2,4,5} \mid X^{(t+1)}_{1,3}, X^{(t)}_{6}$

$X^{(t+1)}_{6} \mid X^{(t+1)}_{1,2,3,4,5}$

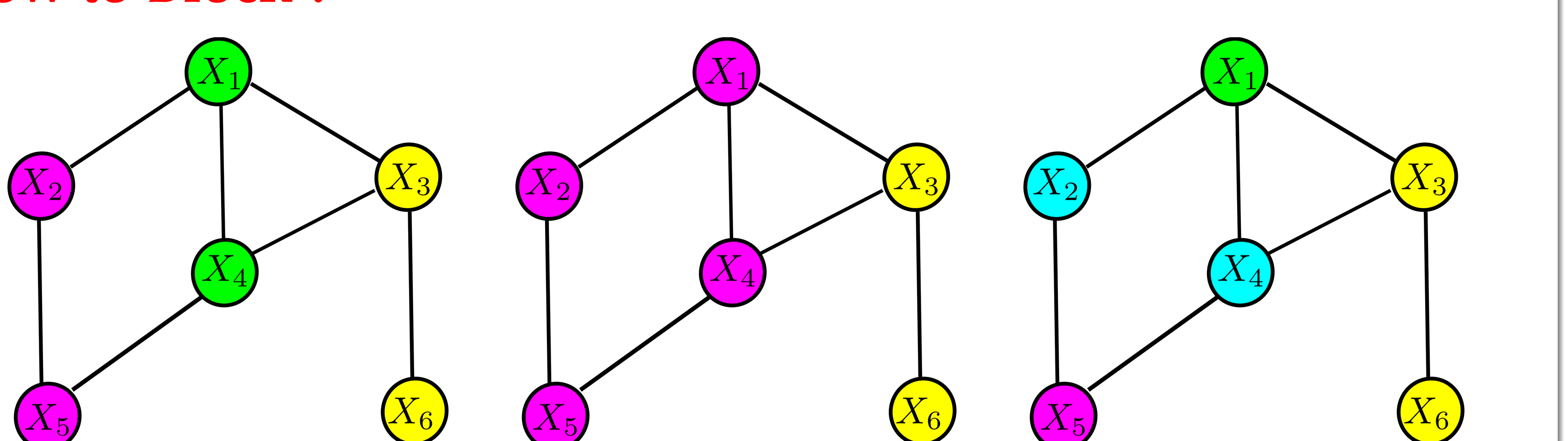
Next Sample: $X^{(t+1)}$

Why ?

Sampling is now more difficult.

BUT, Chain mixes faster $\implies$ better samples.

How to Block ?



Strategy

- We will focus only on **Tree Partitions**.
  - Otherwise Problem is too big.
  - Inference on Trees is easy (Belief Propagation converges in linear time).
- Will consider **correlations** between variables when developing tree blocks.

Why Correlations ?

MC: $X^{(1)} \to X^{(2)} \to \cdots \to X^{(t)} \to X^{(t+1)} \to \dots,$ Eqlbm Distribtuion: π

$L_0(\pi) = \{h : \Omega \to \mathbb{R} :\ \mathbb{E}_\pi h(X) = 0,\ \mathbb{V}_\pi h(X) < \infty\}$, $\langle h, g\rangle = \mathrm{Covar}_\pi(h(X), g(X))$
$(L_0(\pi), \langle\cdot,\cdot\rangle)$ is a Hilbert Space.

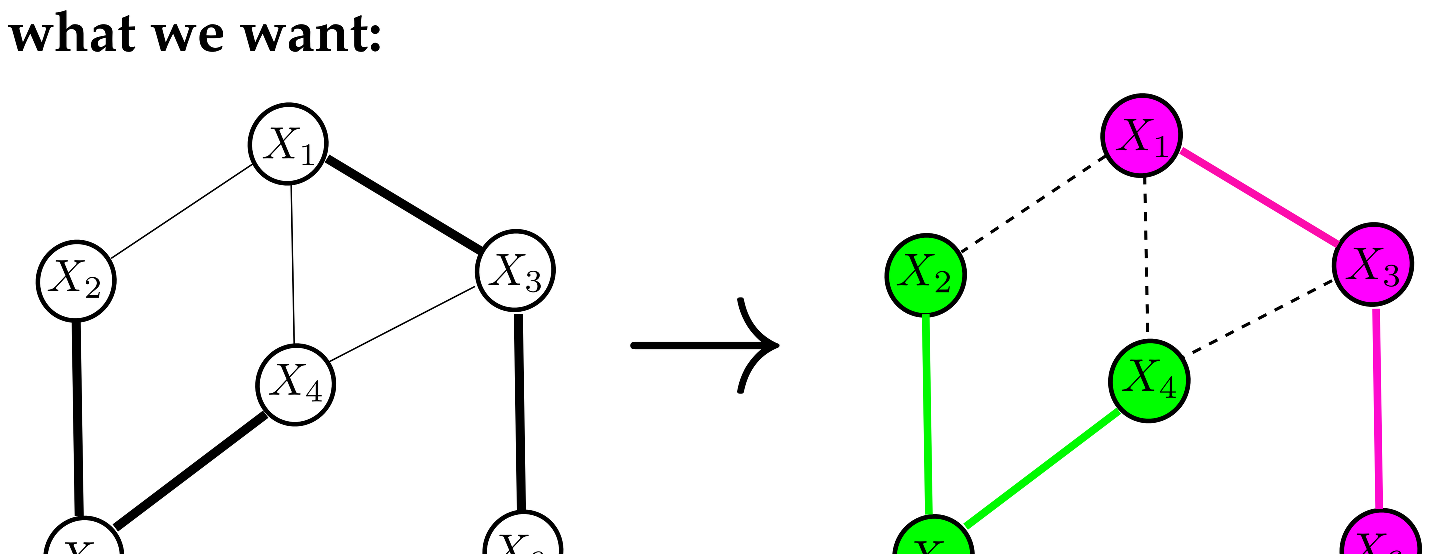
Define $F : L_0(\pi) \to L_0(\pi)$, $[Fh](z) = \mathbb{E}[h(X^{(1)}) | X^{(0)} = z]$

Fact: $|\mathbb{E}^{(n)} h(X) - \mathbb{E}_\pi h(X)| \leqslant C\, \|F\|^n\, \|h\|$

$\|F\| = \sup_{f,g} \mathrm{Corr}(f(X^{(t+1)}), g(X^{(t)}))$

Correlated Variables in different blocks $\implies$ successive samples correlated.

So this is what we want:



Algorithms

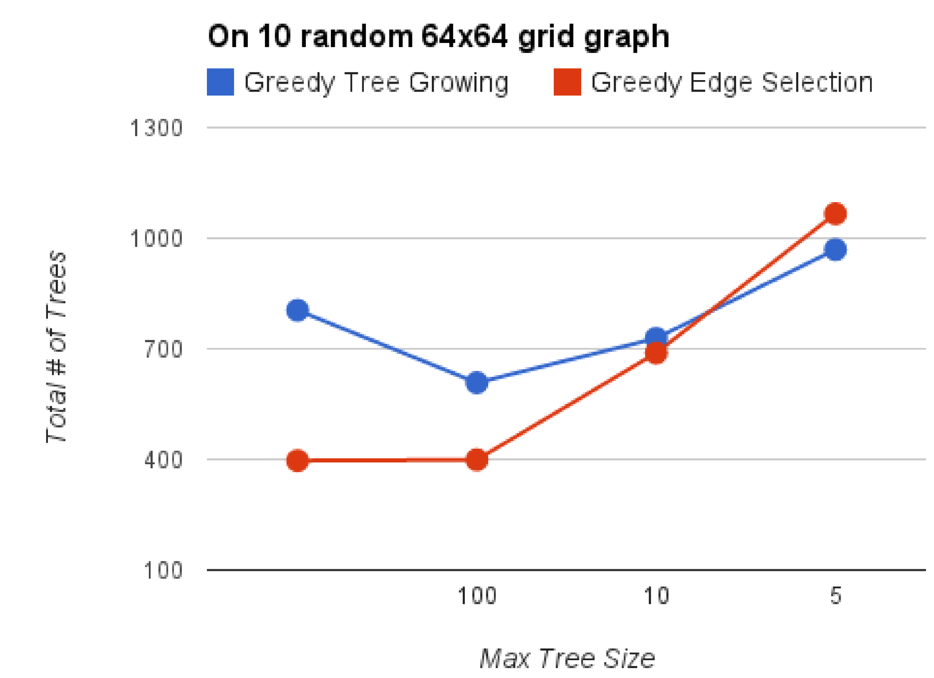
- Baseline: Greedy Tree Growing Algorithm
- Our Algorithm: Greedy Edge Selection Algorithm

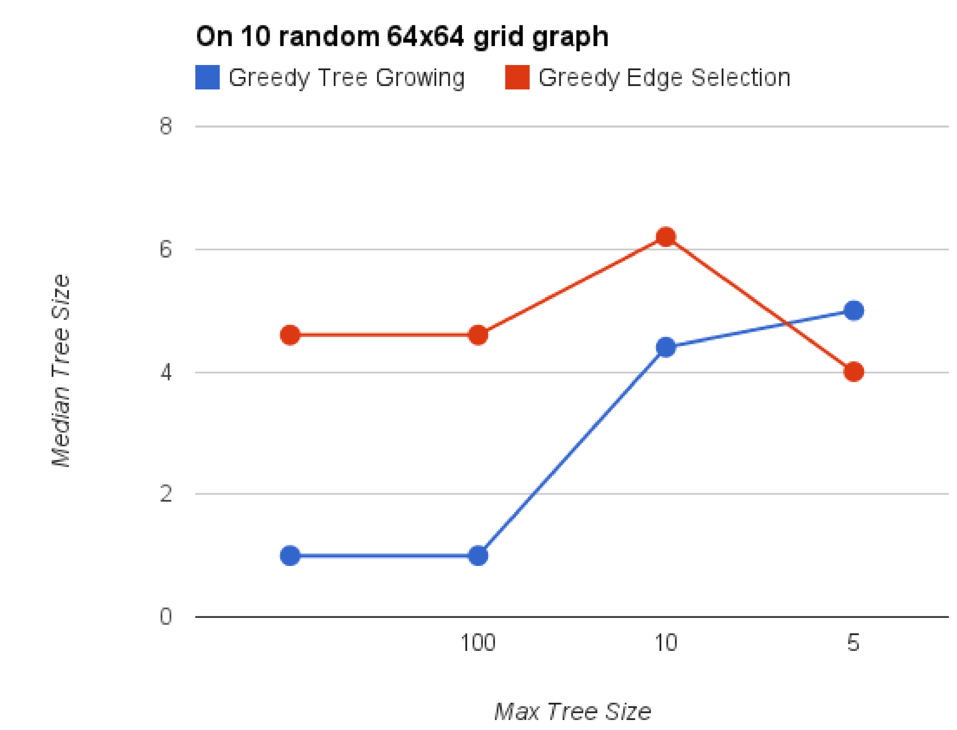
Greedy Edge Selection Algorithm

1. Construct an ordered list of edges, E, with $E[0]$ being the highest weight edge. Edges are vertex pairs (i, j).
2. Initialize an all-zero n-dimensional integer list V of vertex colors. ($V[i]$ is the color of vertex i, and $V[i] = 0$ means that vertex i has not yet been colored.)
3. Initialize n empty vertex sets: $T_1, ..., T_n$ (Logically, T_i is the set of vertices labeled with color i.)
4. Initialize $unusedColor = 1$.
5. For each edge $e = (i, j)$ in E,
   - If $V[i] = V[j] = 0$,
     - Set $V[i] = V[j] = unusedColor$
     - Add i, j to $T_{unusedColor}$
     - Increment $unusedColor$ by 1
   - Else if $V[i] = 0$ and $V[j] \notin getOtherNeighborColors(\{i\}, e)$,
     - Set $V[i] = V[j]$
     - Add i to $T_{V[j]}$
   - Else if $V[j] = 0$ and $V[i] \notin getOtherNeighborColors(\{j\}, e)$,
     - Set $V[j] = V[i]$
     - Add j to $T_{V[i]}$
   - Else if $V[i] \neq 0$ and $V[j] \neq 0$ and $V[i] \notin getOtherNeighborColors(T_j, e)$,
     - For each $k \in T_j$, set $V[k] = V[i]$
     - Set $T_i = T_i \cup T_j$
     - Set $T_j = \emptyset$
   - Otherwise do nothing
6. For each vertex i, if $V[i] = 0$, set $V[i] = unusedColor$, $unusedColor{+}{+}$
7. Output $\{T_j : T_j \neq \emptyset\}$

Greedy Tree Growing Algorithm

1. Initialize $i = 0$ and V to the vertex set.
2. While $V \neq \emptyset$
   - Select $v \in V$
   - Start a new tree T_i and a priority queue Q_i. Add v to Q_i
   - While $Q_i \neq \emptyset$
     - Pop u from Q_i.
     - Initialize $neighborsInT = 0$.
     - For all $v \in T_i$, if $u \in N(v)$, increment $neighborsInT$
     - If $neighborsInT \leqslant 1$,
     - Add u to T_i and remove v from V
     - Add $N(u)$ to Q_i.
3. Return $\{T_i\}$





Greedy Edge Selection Algorithm performs much better when max tree size is not limited.

As max tree size is controlled, Greedy Tree Growing Algorithm caught up.

Lists

- You can make
- lists, that
- allow people to see quickly

Math

Include math within the text is as simple as $1 + 1 = 2$. You can also highlight more important equations like this:

$$\int_0^1 \sin(x) + \cos^2(x) + \alpha x \, dx$$

Pictures

Experiments

Remember to put lots of figures on your poster... Nobody reads anymore!

Conclusion

Much less annoying than PowerPoint. Copy and Paste from your document. Overall, a great idea!

keywords: Gibbs Sampling, Markov Networks, Mixing Times

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