From Fields to Trees

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UAI 2004

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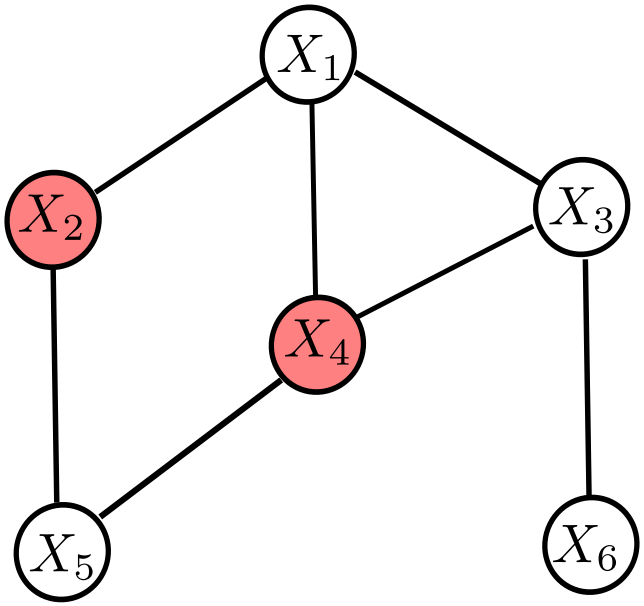
Why ?



Estimate $\mathbb{E}[h(X_2, X_4)]$

MCMC Samples: $X_1, X_2, \ldots X_N$

Empirical Estimator

$$\mu_0 = \frac{1}{N} \sum_{i=1}^{N} h(X_2^{(i)}, X_4^{(i)})$$

Rao-Blackwellised Estimator

$$\mu_{rb} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}[h(X_2^{(i)}, X_4^{(i)}) | X_{1,3,5}^{(i)}]$$

Liu, Kong, Wong, 1994

Independent Samples: RB estimator is better (known).

MCMC Samples: RB estimator is better (LKW, 1994)

$L(\pi) = \{t : \Omega \to \mathbb{R}; \mathbb{E}_\pi = 0, \mathbb{V}_\pi(t(X)) < \infty\}$ with i/p $< t, s >= \text{Covar}_\pi(t(X), s(X))$ forms a Hilbert Space.

$F : L(\pi) \to L(\pi)$, such that $[Ft](\cdot) = \mathbb{E}[t(X_1)|X_0 = \cdot]$ is an operator with $\|F\| \leq 1$.

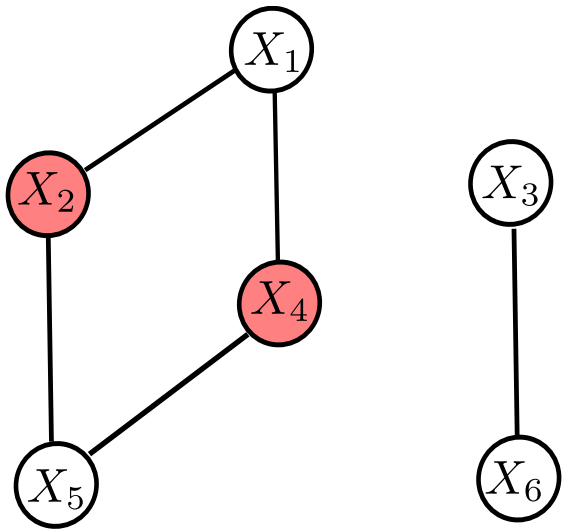
How ?

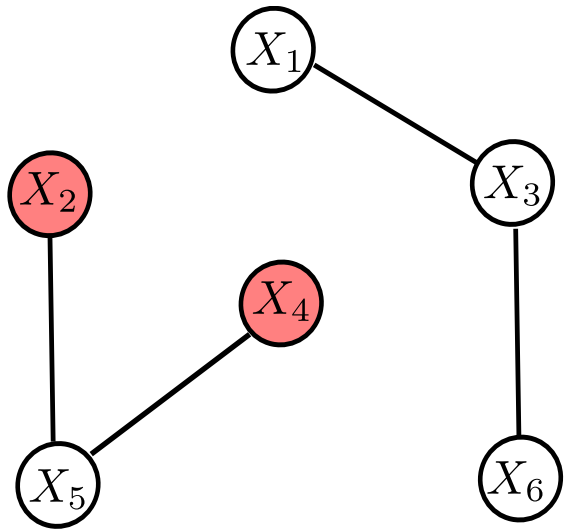
$$|\mathbb{E}_n h(X) - \mathbb{E}_\pi h(X)| \leq C \, \|F\|^n \, \|h\| \quad \text{(Liu, 2001).}$$

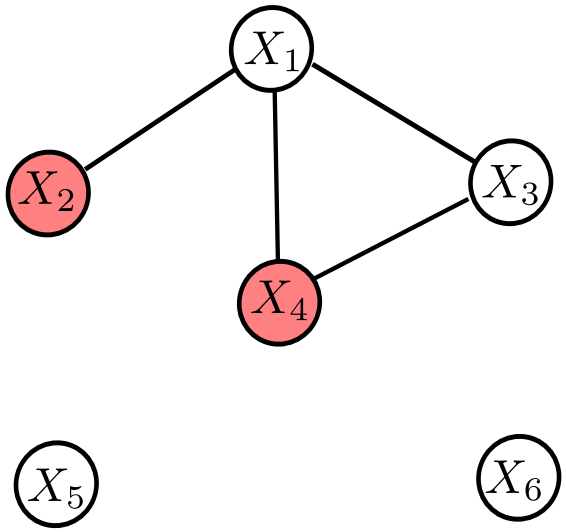
Partition the Graph so that $\|F\|$ is small !

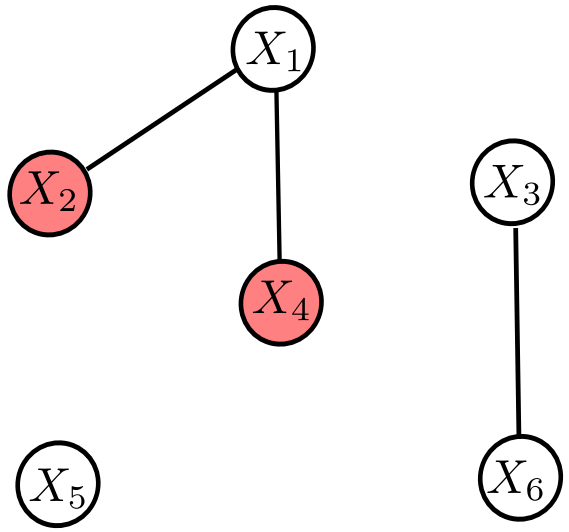
Not sure how this can be done ☹.

How to Partition ?



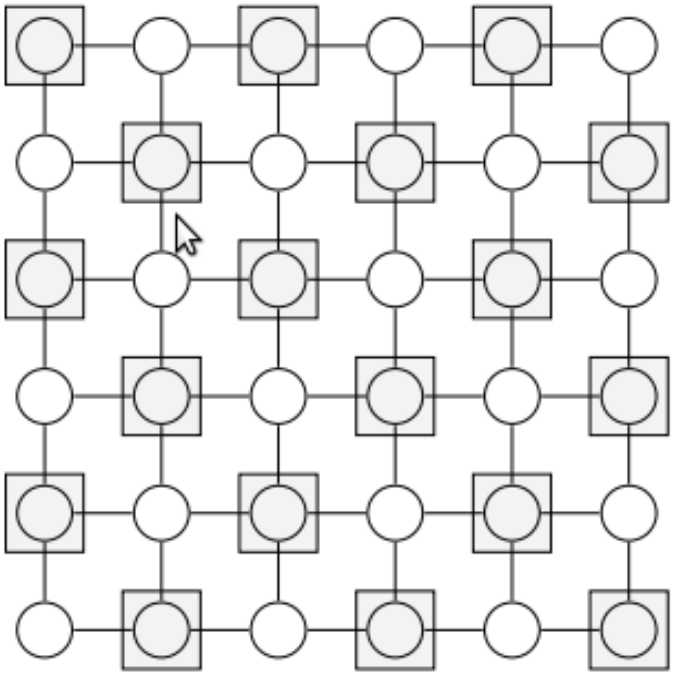




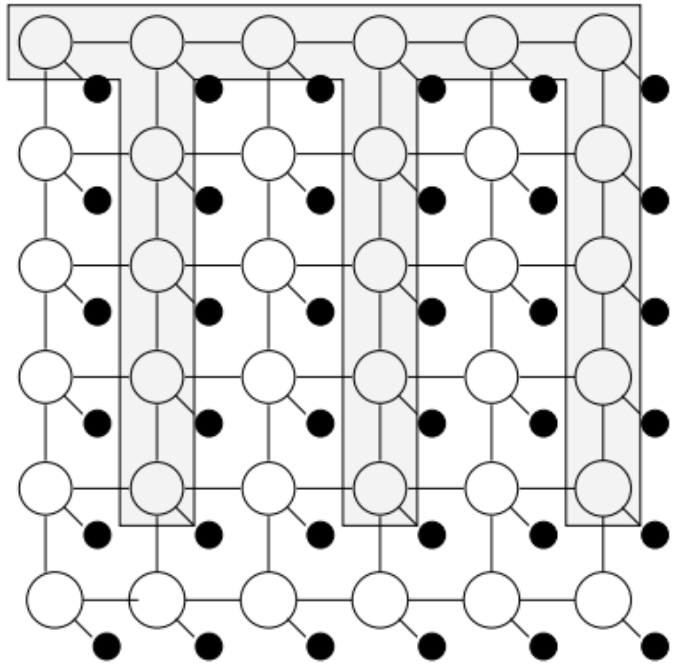


Best Tree Partition ? (Also not easy).

Grid Graph

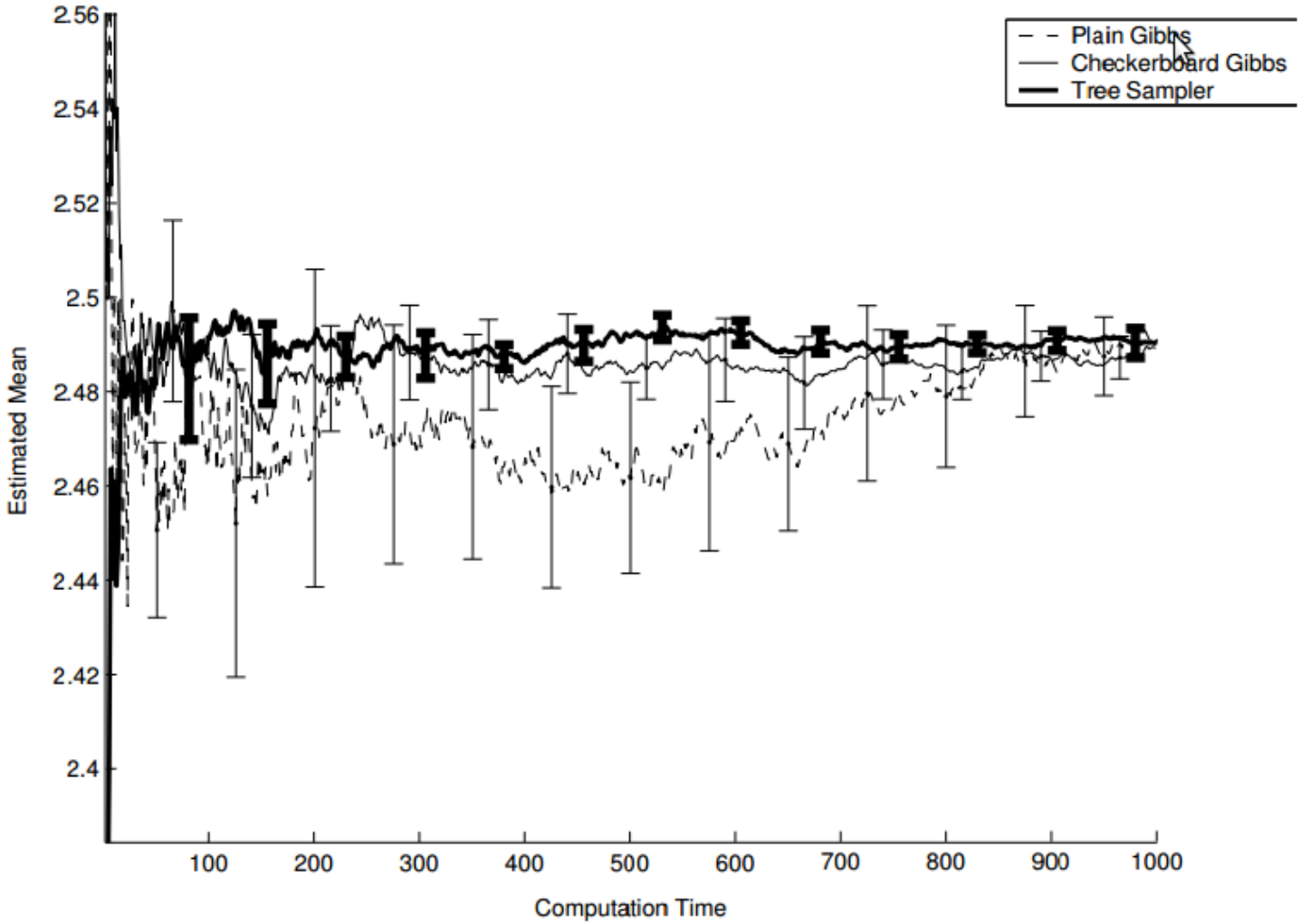


Checkerboard



Two-Trees

Results


Plain Gibbs
Checkerboard Gibbs
Tree Sampler
Estimated Mean
2.56
2.54
2.52
2.5
2.48
2.46
2.44
2.42
2.4
100
200
300
400
500
600
700
800
900
1000
Computation Time