

Analisi e Programmazione

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Prefazione

Avvertenze

Questo libro è risultato molto meno rigoroso e preciso di quanto un matematico pretenderebbe da un testo accademico. Si è cercato di esporre gli argomenti tipici di un insegnamento universitario di Analisi I nella maniera più semplice possibile per renderlo fruibile a un pubblico più ampio; il lavoro è stato svolto da uno studente senza la pretesa di sostituire i libri autorevoli e i materiali didattici consigliati nei corsi di studio di matematica. Pertanto il lettore è invitato, se ne ha la pazienza, a confrontare sempre i ragionamenti, le definizioni, i teoremi esposti in Matematica del Continuo al Calcolatore, capitolo dopo capitolo, con quelli presentati dagli autori più competenti in altre opere. A tal proposito nel presente libro è fornita un'ampia bibliografia,

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Capitolo 1

Il campo reale

1.1 Il prodotto cartesiano

Siano A, B sottoinsiemi di numeri reali, non vuoti.

Definizione 1.1.1

Il **prodotto cartesiano** di A e B è l'insieme costituito da tutte le coppie ordinate $\{(a, b), (b, a) : a \in A, b \in B\}$; questo insieme è indicato con $A \times B$:

$$A \times B = \{(a, b) : a \in A, b \in B\}.$$

Qualora A e B coincidano si scrive $A \times A = A^2$.

Si consideri il prodotto cartesiano $\mathbb{R} \times \mathbb{R}$ o \mathbb{R}^2 :

$$\mathbb{R}^2 = \{(a, b) : a, b \in \mathbb{R}\},$$

su questo insieme valgono le operazioni di somma e prodotto tra coppie di numeri reali.

Definizione 1.1.2

Date due coppie di numeri reali $(a, b), (c, d)$ si definisce **somma tra coppie** l'operazione $+$ che produce la sequenza $(a + c, b + d)$:

$$(a, b) + (c, d) = (a + c, b + d).$$

Si definisce **prodotto tra coppie** l'operazione \cdot che produce la sequenza $(a \cdot c - b \cdot d, a \cdot d + b \cdot c)$:

$$(a, b) \cdot (c, d) = (ac - bd, ad + bc).$$

Osservazione 1.1.3

Il simbolo \cdot del prodotto tra coppie può essere omissso se non crea nessun equivoco. Per cui

$$(a, b) \cdot (c, d) = (a \cdot c - b \cdot d, a \cdot d + b \cdot c) = (ac - bd, ad + bc).$$

Osservazione 1.1.4

L'insieme \mathbb{R}^2 è un campo con due operazioni di somma e prodotto, quindi $(\mathbb{R}^2, +, \cdot)$, rispetto alle quali vale la proprietà commutativa e in \mathbb{R}^2 ogni elemento diverso da zero possiede inverso

Capitolo 2

I numeri complessi

- 2.1 Equazioni cubiche e nascita dei numeri immaginari
- 2.2 Il campo dei numeri complessi
- 2.3 Il sottoinsieme dei numeri reali
- 2.4 Forma algebrica di un numero complesso
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Appendice A

Linguaggio di programmazione Fortran

Appendice B

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