







Fosco Loregian



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Past & present positions

- Ph.D. at SISSA - Trieste IT  (but advisor in Rome, [D. Fiorenza](#))
Stable homotopy theory, ∞ -categories, derived AG
- University of Western Ontario - London CA 
 ∞ -categories, derivators
- Masaryk University - Brno CZ 
Accessible categories, derivators, 2-categories
- Max Planck Inst. für Math. - Bonn D 
2-categories, derivators, applied category theory
- Centro de Matemática - Coimbra PT 
2-categories; finishing my first book
- Tallinna Tehnikaülikooli - Tallinn EE 
2-categories; functorial semantics; categorical probability theory and its applications

STABLE HOMOTOPY THEORY

∞ -categories: a thickening of the notion of category, suitable for homotopy-coherent mathematics (math.AG, math.AT, math.LO, cs.PL...).


A **stable ∞ -category** is an ∞ -category

- with all **finite limits and colimits**
- such that a square is **cartesian iff cocartesian**
- The homotopy category of a stable ∞ -cat is always triangulated.
- Sending an abelian category \mathcal{A} into its derived category has a nice and clear **universal property** stated in terms of the heart of a canonical t -structure.
- Stable, rational, p -adic, ... homotopy theory become pieces of the commutative algebra of ∞ -categories.

Each PhD starts with a question

A **t-structure** on a triangulated \mathcal{D} is a pair of triangulated subcategories of \mathcal{D} such that every object X lies in a sequence

$$X_{\leq} \rightarrow X \rightarrow X_{\geq} \rightarrow X_{\leq}[1]$$

[FL14 \infty-categories a t-structure is a factorization system (E, M)

- such that E and M are 3-for-2 classes
- thus the category of E -cofibrant objects is coreflective
- and the category of M -fibrant objects is reflective
- cof/fib replacement = \pm truncation

Plan: redo t -structures (w/ Domenico)

- [FLM15

Plan: redo t -structures (w/ Domenico)

- The set of slicings on a stable ∞ -category has a metrizable topology
- The space

$$\{J: \mathbb{R} \rightarrow TS(\mathcal{C}) \mid J \text{ is Sorgenfrey continuous}\}$$

is an interesting set [L-PhD]. Bridgeland: demistified.

Conjecture

Study

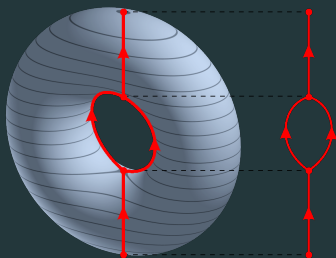
$$\{J: \operatorname{Spec}(\mathbb{Z}) \rightarrow TS(\mathcal{D}(X_p)) \mid J \text{ is Zariski continuous}\}$$

(X_p a variety in positive characteristic) to get something about motivic t -structure.

Conjecture

Blakers-Massey in positive characteristic is a theorem about factorization systems.

Todo: Morse theory is a theory of FS



Bord(n) is the free (∞, n) -symmoncat on the point.

Tensor functors
 $Z : \text{Bord}(n) \rightarrow \text{Vect}$
are completely classified.

Morse theory is the theory of suitable factorization systems on $\text{Bord}(n)$.

critical points of a Morse function correspond to

critical values [L-PhD, Ch.7] of a certain slicing $J : \mathbb{R} \rightarrow FS(\text{Bord}(n))$.

DERIVATORS


The formal category theory of derivators

A **derivator** is a strict 2-functor


$$\mathbb{D} : \mathbf{Cat}^{\mathrm{op}} \rightarrow \mathbf{CAT}$$

satisfying stacky conditions. They form the 2-category **Der**.

They subsume most of ∞ -category theory; in particular, their stable homotopy.

[LV17 


FS are still **strict 2-algebras** for the "squaring" 2-monad $(_)^2$

[Lor18 

(the **formal theory of monads** [S80] still holds in **Der**)

The formal category theory of derivators

There is a **Yoneda structure**¹ on the 2-category of derivators

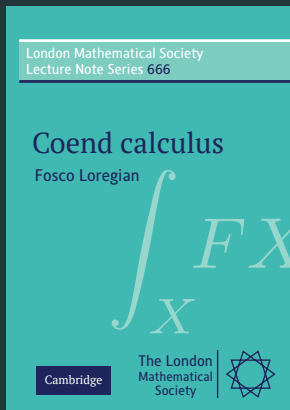
- notions of **accessible** and **locally presentable** derivator using the theory of LPAO in a Yoneda structure, done in [DLL18definability for module categories); derivator **topos theory**?
- **adjoint functor theorems** for derivators; existence of a **six-operation** calculus. 2-categorical account of Grothendieck duality complicated diagrams (without multicategories)
- **profunctors** between derivators; fibered derivators; **operads** in derivator theory; applications in representation theory of algebras, stable homotopy, ...

¹A 2-categorical device to encode the calculus of pointwise Kan extensions.

COEND CALCULUS

Coends

I have written a book [L20] on **coend calculus**, soon to appear under Cambridge LNSs:



- Coends $\int_C T$ are universal objects associated to $T : \mathcal{C}^{op} \times \mathcal{C} \rightarrow \mathcal{D}$, treated as integrals (a “Fubini rule” is valid).
- applications in (monoidal) category theory, algebraic topology, universal algebra, algebraic geometry, categorical logic, representation theory (see ch.7 for an application to **DG-categories**), functional programming...
- The book is being extensively cited (45 citations on Scholar December 11, 2021)

Coends are useful for

- category theory
- functional programming
- algebraic geometry
- algebraic topology
- module/representation theory
- ...

A book about them needed to be written.

CATEGORICAL SEMANTICS OF NETS

A Petri net is a certain mathematical model for distributed systems.

A Petri net is a presentation for a free (commutative) monoidal category.

Study monoidal categories: you will understand Petri nets.

Extensions of Petri nets:

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-
-
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TEACHING AND ORGANIZATIONAL ACTIVITIES

Teaching and...

- 2015 A short course on **model categories** @unipv;
- 2016 “**Elements of Finite Mathematics**” @uwo (mostly statistics to kinesiologists).
- 2016 **Advisor** of a BSc thesis @unibo, “Elementary aspects of adjoint functors”. I enjoyed it!
- 2018 A short course on **2-category theory** @unipd: monoidal and enriched, categories, the calculus of coends and Kan extensions, bicategories, monads...
- 2020 **Category theory** course Teacher @taltech. Mentoring activity for MSc students interested in category theory in CS.


Teaching and...

- advisor for master student's thesis: *M. Roselli*, 'Categorical linguistics'
 - advisor for master student's thesis: T. Massacrier, 'Differential Polynomial endofunctors'
 - advisor for master student's thesis:
 - SIGPLAN's mentor: long-term mentoring program for programming languages researchers.
- frequent PRs to the [agda-categories](#) library

Organisation of conferences

- 2015 and 2019 Attendee and speaker at the **Kan Seminar I**
a webinar on category theory
- and **Applied Category Theory 2019**
(a webinar on applied category theory, from which the paper [MLR⁺20] stemmed)
- 2018 **Organiser** of the 103rd **PSSL**
Peripathetic Seminar on Sheaves and Logic, Brno.
- 2019 and 2020 Among the **organisers** of ItaCa
Italian Category theorists) in Milan and soon on zoom, due to COVID19.
- Reviewer for zbMath and AMS.

Miscellaneous projects

- Homotopy theory and set theory: [DL18 no homotopy category of a model category is “concrete”; what about ∞ -categories?
- Functional programming and type theory:
HoTT, linear types, proof-checkers, categorical algebra in relational database architecture; natural language processing using category theory...
- Categorical logic and foundations of mathematics;
functorial semantics *à la* Lawvere, but sprinkled with operads and multicategories.
- more in detail, “2-semantics” of algebraic theories:
profunctorial PROPs and theories, categorical algebra of cartesian bicategories...

Reach me out at [my web page](#):

