- Ph.D. at SISSA - Trieste 🇮🇹

  Stable homotopy theory, ∞-categories, derived AG

- University of Western Ontario 🇨🇦

  ∞-categories, derivators

- Masaryk University 🇨🇿

  Accessible categories, derivators, 2-categories

- Max Planck Inst. für Math. 🇩🇪

  2-categories, derivators, applied category theory

- Centro de Matemàtica da Universidade de Coimbra 🇵🇹

  2-categories; finishing my first book

- Tallinna Tehnikaülikooli 🇪🇪

  2-categories; functorial semantics; categorical probability theory and its applications

STABLE HOMOTOPY THEORY

∞-categories: a thickening of the notion of category, suitable for homotopy-invariant mathematics (algebraic geometry, algebraic topology type theory).

Turns out some parts of Mathematics are easier if stated in these terms:

- homological algebra : the scary part of algebraic topology

  higher algebra: the linear algebra of ∞-categories
- 1-topos theory : a synthetic type theory,

  ∞-topos theory: a synthetic homotopy theory of homotopy types

A stable ∞-category is an ∞-category

with all finite limits and colimits such that a square is cartesian iff cocartesian

The homotopy category of a stable ∞-cat is always triangulated

The correspondence sending an abelian A into its derived category has a nice and clear universal property

Stable, rational, p-adic, … homotopy theory are pieces of commutative algebra of ∞-categories

A t-structure on D triangulated is a pair of triangulated subcategories of D such that every object X lies in a sequence

$$\dots$$

[FL14] : On stable ∞-cats a t-structure is a factorization system (E,M)

- such that E and M are 3-for-2 classes
- thus the category of E-cofibrant objects is coreflective
- and the category of M-fibrant objects is reflective
- cof/fib replacement are neg/pos truncation

(Algebraic) geometry reduces to a piece of categorical algebra.

Plan: redo t-structures

[FL15] On the set of t-structures there is a natural choice of Z-action (Z = the integers); study Z-equivariant monotone maps from a poset J to TS(C).

Bridgeland stability manifolds, Postnikov towers on ∞-toposes

[FL15b] Every stratified manifold (X,s) generates a pair of t-structure that can be glued together

recollements, stratified schemes, representation of algebras

Project: algebraic geometry is easier, with factorization systems.

Rational and p-adic homotopy   rational and p-adic geometry number-theoretic factorization systems

Every PhD finishes with two conjectures…

$$\{J\colon \mathbb{R} \to \mathit{TS}(\mathcal{C}) \mid J \text{ is Sorgenfrey continuous}\}$$

is an interesting set [FL-PhD]

$$\{J\colon \text{Spec}(\mathbb{Z}) \to \mathit{TS}(\mathcal{C}) \mid J \text{ is Zariski continuous}\}$$

Blakers-Massey in char p is a thm about factorization systems

Fact: Bord(n) is the free (∞,n)-symmoncat on the point

monoidal functors $Z : \mathrm{Bord}(n) \to \mathrm{Vect}$ are completely classified and used in QFT

Morse theory is the theory of suitable factorization systems on Bord(n)

critical points of a Morse function

correspond to

critical values of a certain ”slicing” $J : R \to \mathrm{FS}(\mathrm{Bord}(n))$

disegno del toro

DERIVATORS

A derivator is a strict 2-functor

$$\mathbb{D} : \textbf{Cat}^{\mathrm{op}} \to \textbf{CAT}$$

satisfying sheafy conditions. They form the 2-category Der.

They subsume most of ∞-category theory; in particular, their stable homotopy.

LV17 : A t-structure on a stable derivator is still a certain kind of factorization system*
*And FS are still algebras for the "squaring" 2-monad $(\_)^2 : A$ ⊠ A^2 (see [KT93])

Lor18 : reflective subderivators correspond to reflective factorization systems, and to algebras for idempotent monads
(the "formal theory of monads" still holds in Der, a monad T : D → D is just defined objectwise)

There is a Yoneda structure on the 2-category of derivators

If this is true

- comprehensive account of various notions of accessible and locally presentable derivator using [DLL18]; categorical logic for derivators; derivator toposes?
- a convincing form of adjoint functor theorem for derivators: useful, innit?
- profunctors between derivators; fibered derivators; operads in derivator theory; applications in representation theory, stable homotopy, ...

FORMAL CATEGORY THEORY

2-CATEGORIES IN FUNCTIONAL PROGRAMMING

2-CATEGORIES IN PROBABILITY THEORY