
Fosco Loregian 

May 11, 2020

- Ph.D. at SISSA - Trieste 




Stable homotopy theory, ∞ -categories, derived AG





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- Tallinna Tehnikaülikooli 
2-categories; functorial semantics; categorical probability theory and its applications

STABLE HOMOTOPY THEORY

∞ -categories: a thickening of the notion of category, suitable for homotopy-invariant mathematics (algebraic geometry, algebraic topology, type theory).

Turns out some parts of Mathematics are easier if stated in these terms:

- homological algebra : the scary part of algebraic topology
higher algebra: the linear algebra of ∞ -categories
- 1-topos theory : a synthetic type theory,
 ∞ -topos theory: a synthetic homotopy theory of homotopy types

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- with all **finite limits and colimits**
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Stable, rational, p -adic, ... homotopy theory become pieces of the commutative algebra of ∞ -categories.

A t-structure on a triangulated \mathcal{D} is a pair of triangulated subcategories of \mathcal{D} such that every object X lies in a sequence


$$X_{\leq} \rightarrow X \rightarrow X_{\geq} \rightarrow X_{\leq}[1]$$

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(Algebraic) geometry reduces to a piece of categorical algebra.

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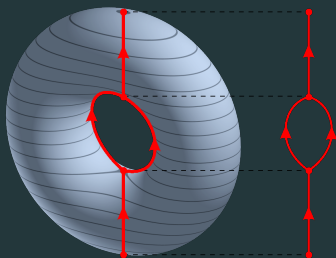
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Blakers-Massey in positive characteristic is a theorem about factorization systems.

Morse theory as a theory of FS



Fact: $\text{Bord}(n)$ is the free (∞, n) -symmoncat on the point.

Monoidal
functors $Z : \text{Bord}(n) \rightarrow \text{Vect}$
are completely classified.

Morse theory is the theory of suitable factorization systems on $\text{Bord}(n)$.

critical points of a Morse function correspond to

critical values [L-PhD, Ch.7] of a certain slicing $J : \mathbb{R} \rightarrow \text{FS}(\text{Bord}(n))$.

DERIVATORS

A **derivator** is a strict 2-functor

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satisfying stacky conditions. They form the 2-category **Der**.


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[Lor18 formal theory of monads [S80] still holds in **Der**, a monad $T : D \rightarrow D$ is just defined objectwise)

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If this is true

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- comprehensive account of various notions of **accessible** and **locally presentable** derivator using the theory of locally presentable objects in a Yoneda structure, done in [DLL18]; categorical logic for derivators (see Prest's treatment of **definability** for module categories); derivator topos theory?

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- a convincing form of **adjoint functor theorem** for derivators; existence of a six-operation calculus for mappings of derivators. 2-categorical account of Grothendieck duality complicated diagrams...

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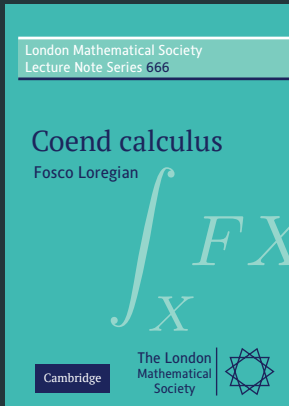
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- a convincing form of **adjoint functor theorem** for derivators; existence of a six-operation calculus for mappings of derivators. 2-categorical account of Grothendieck duality complicated diagrams...
- **profunctors** between derivators; fibered derivators; **operads** in derivator theory; applications in representation theory of algebras, stable homotopy, ...

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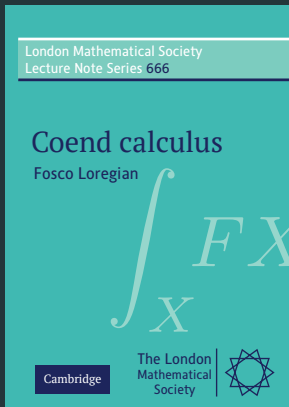
COENDS AND DG-STUFF

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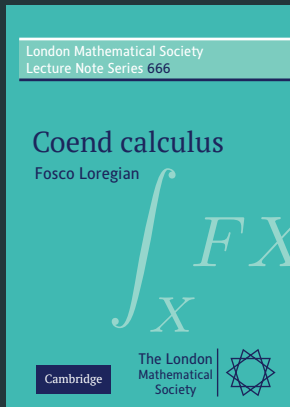
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- The book is being extensively cited (45 citations on Scholar May 11, 2020)

In [L20, 7.2.2]

For example: if \mathcal{A} is any dg-category its identity profunctor $\mathcal{A} \rightsquigarrow \mathcal{A}$ is a functor $\mathcal{A}^{\text{op}} \boxtimes \mathcal{A} \rightarrow \text{Ch}(\mathbb{Z})$, so that the coherent end

$$\oint_{\mathcal{A}} \mathcal{A}(A, A) \tag{7.82}$$

i.e. the object of derived natural transformations of the identity functor $\text{id}_{\mathcal{A}}$, recovers the *Hochschild complex* of \mathcal{A} . Then, if \mathcal{A} is an associative algebra regarded as a one-object dg-category concentrated in degree zero, the object $H^n(\int_* A)$ is the *Hochschild cohomology* of A , understood in the classical sense of, say, [Pie82, Ch. 11].

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Applications to Kuznetsov-Lunts **categorical resolutions of singularities**: a smooth DG-category is a \mathcal{D} such that its identity profunctor $h : \mathcal{D} \rightsquigarrow \mathcal{D}$ is a perfect object (read as: a variety is smooth if the diagonal map $\Delta : X \rightarrow X \times X$ is smooth)

TEACHING AND ORGANIZATIONAL ACTIVITIES

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- 2020 **Category theory** course Teacher @taltech. Mentoring activity for MSc students interested in category theory in CS.

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- Reviewer for zbMath and AMS.

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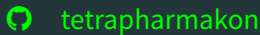
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- more in detail, “2-semantics” of algebraic theories: profunctorial PROPs and theories, categorical algebra of cartesian bicategories...

- Reach me out at [my web page](#):



- I'm quite open about my projects:



- Taltech has an extremely active [ongoing CT seminar](#):



Reach us out!