

ITI9200 – Category theory 2024

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20/01/2024

What is this Category
Theory about

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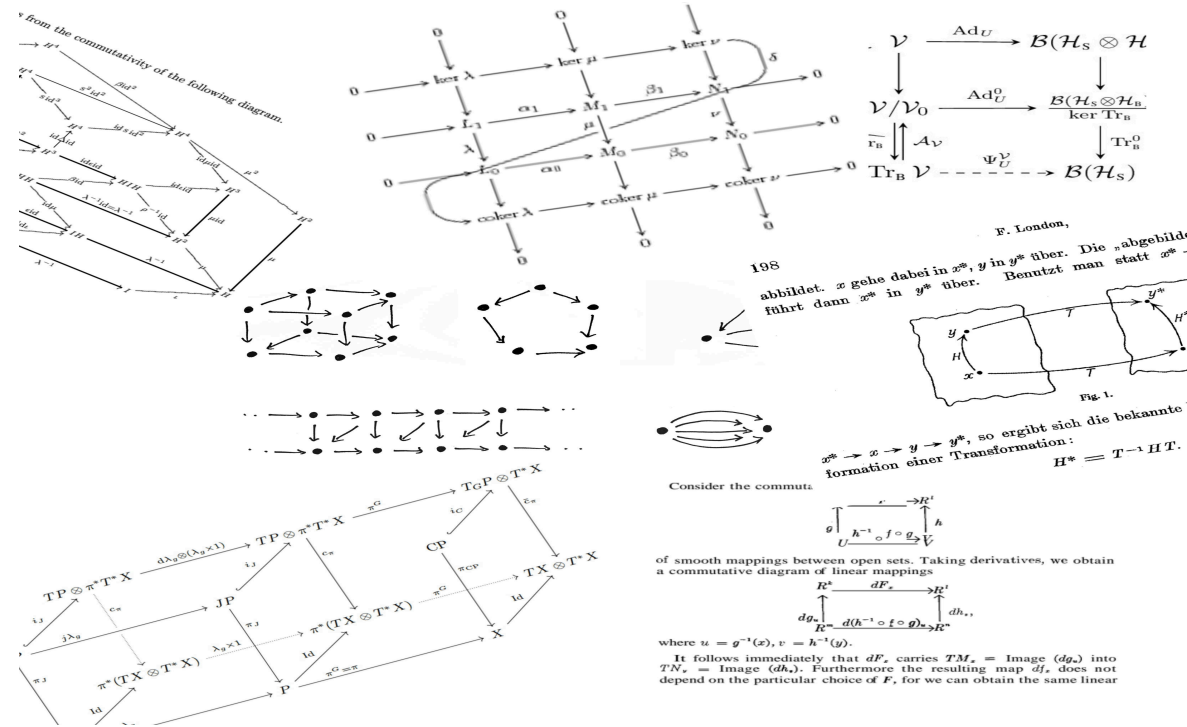
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Category theory can be seen as a theory of *systems* and *processes*. (Hu-Vicary, 2021)

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Information about a problem is presented as a **diagram** (an oriented graph of a very special kind):



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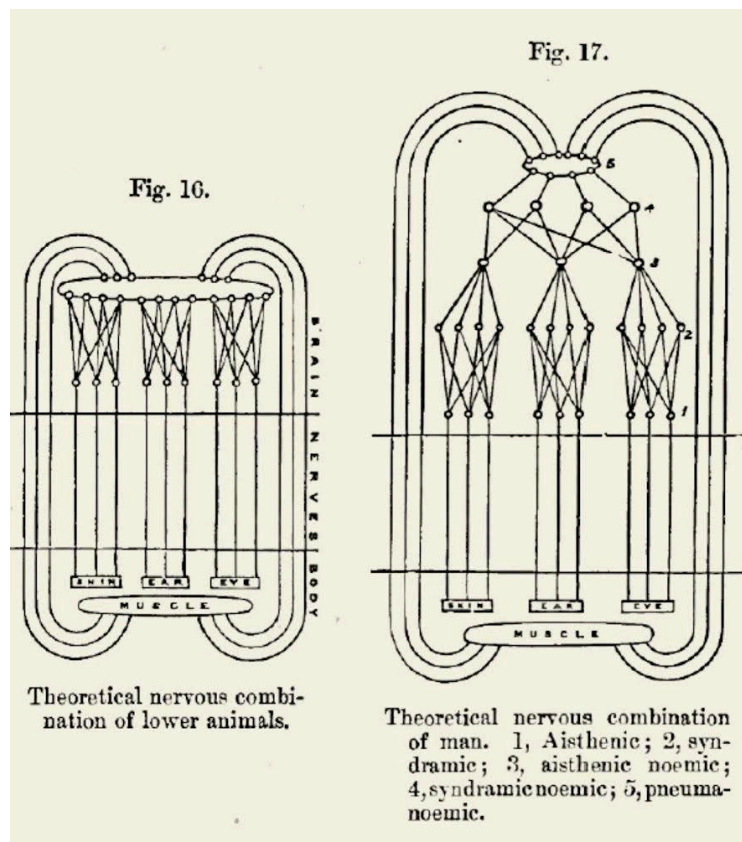
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Category theory is a science of relations between pieces of a complex structure, apt to unravel the emergent properties of the latter.

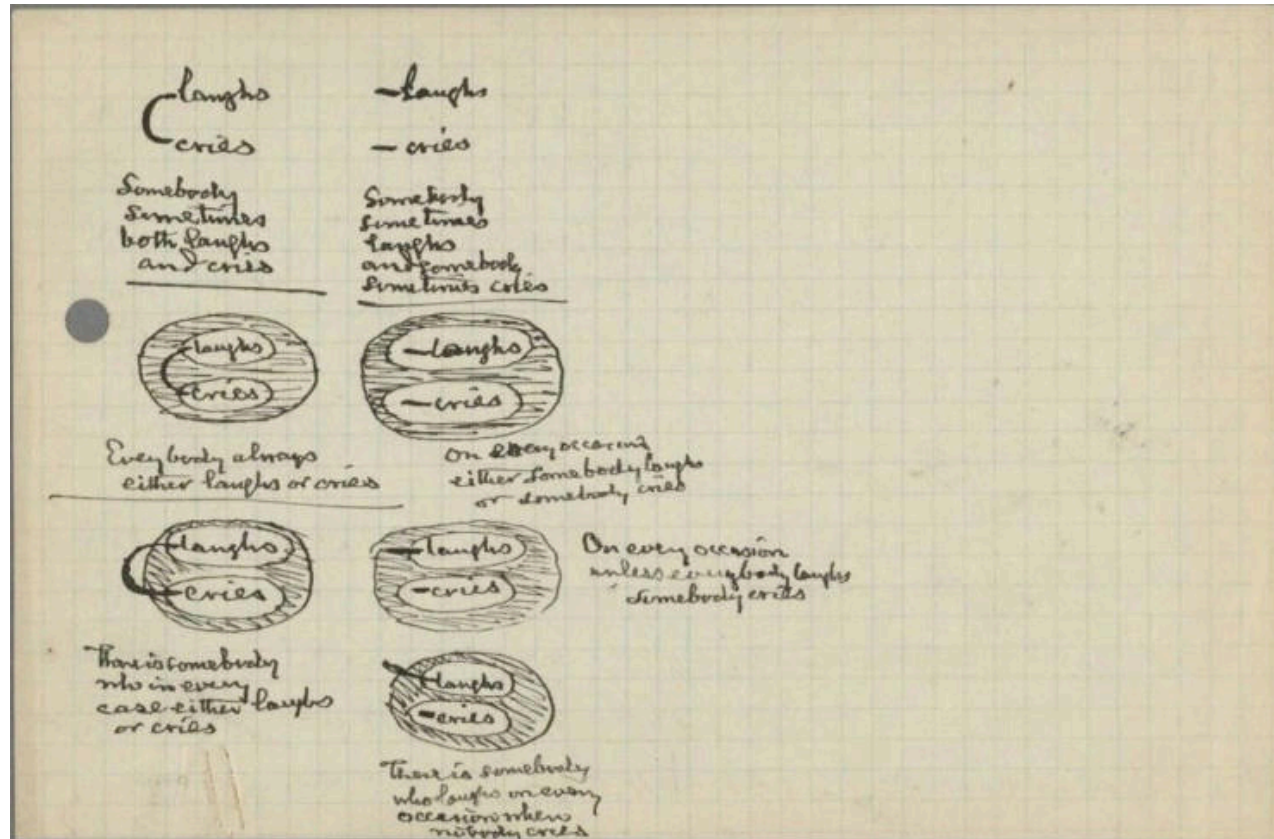
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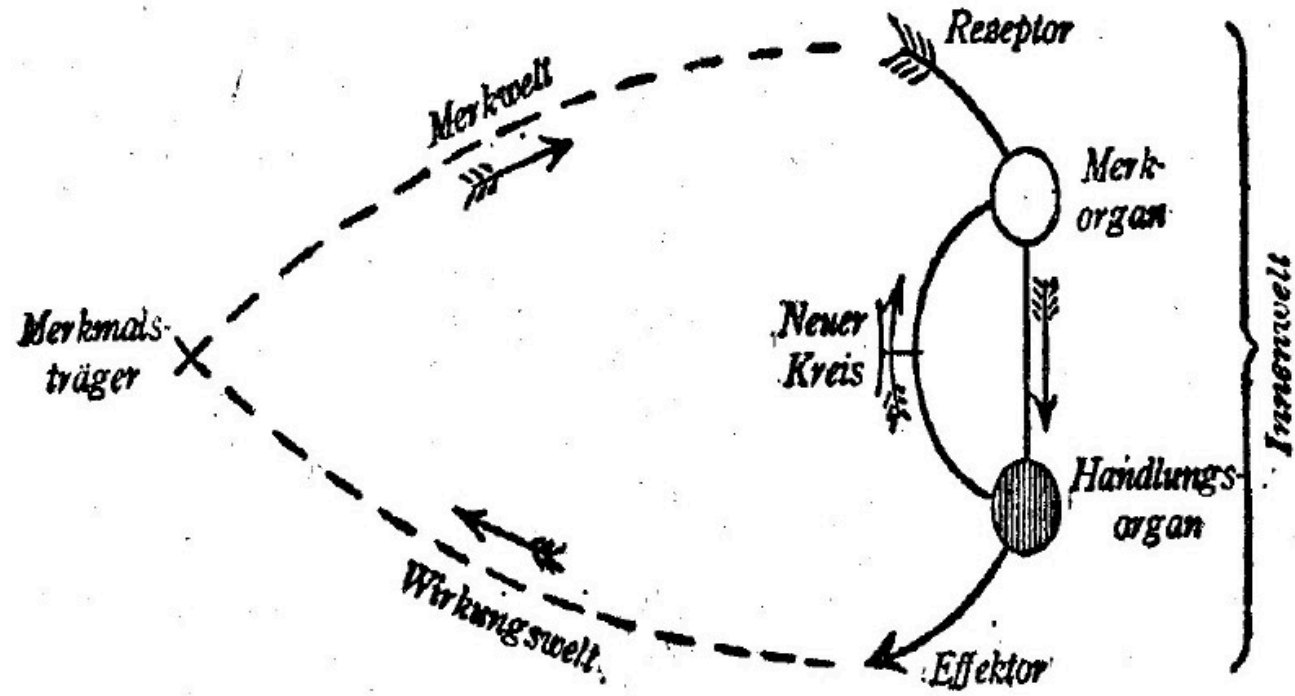
All the following things relate to Category Theory or to the particular philosophy that generated it.



Alfred Smee. *Instinct and Reason Deduced from Electrobiological*, 1850.

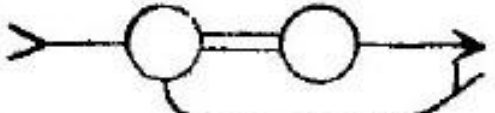


C.S. Peirce, *Prolegomena to an Apology of Pragmaticism*, 1906

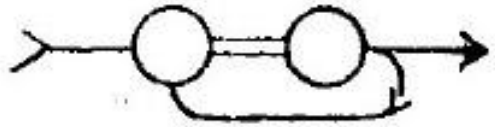


Figur 4.

J. J. von Uexküll. "Early Scheme for a circular Feedback Circle" from *Theoretische Biologie* 1920.

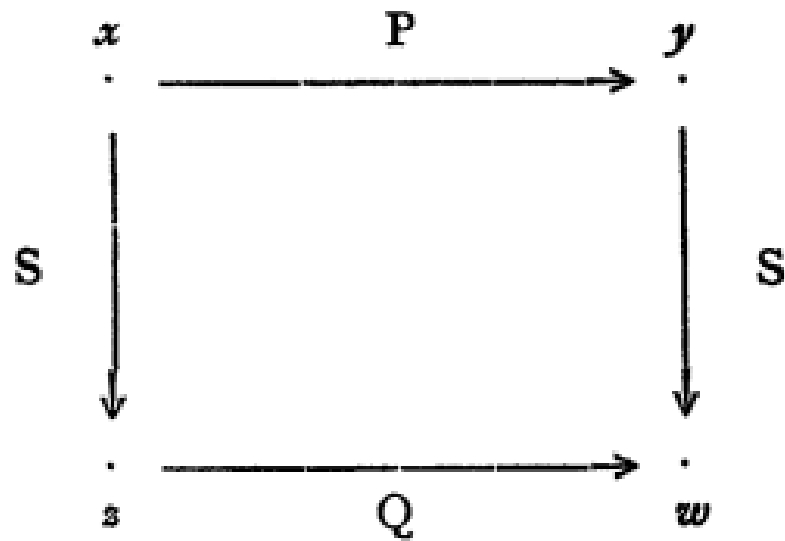
sind zwei Fälle zu unterscheiden: entweder wird Effektoren-muskeln durch besondere sensible Nerven beifolgende Schema zeigt.  Oder es

torischen Nerven übertragene Erregung durch Rezeptoren zum Teil aufgefangen und dem N

 Diese Rezeptoren bilden das zentrale Gehirn, das anatomisch noch völlig im Dunkeln

J. J. von Uexküll. "Zirkuläre Schemen" from *Theoretische Biologie* 1920. Diagrammatic description of double feedback system of autonomic nerves in the brain.

to another, the correlate of the one has the relation Q to the correlate of the other, and *vice versa*. A figure will make this




clearer. Let x and y be two terms having the relation P . Then there are to be two terms z , w , such that x has the relation S to z , y has the relation S to w , and z has the relation Q to w . If this happens with every pair of terms such as x



J. J. Campbell, *The Hero with a Thousand Faces*, 1949. (mythogenesis of heros' tales and folk stories)

There are some useful rules of thumb for how to gigamap. These rules have emerged through years of experience producing such maps and instructing students and professionals in gigamapping.



Within the final and true world image everything is related to everything, and nothing can be discarded a priori as being unimportant. – Fritz Zwicky 1969

The concept of *gigamapping*, in System Oriented Design <https://systemsorienteddesign.net>

«When this (एतद्, *etad*) exists, that comes to be. With the arising (उप्पाड, *uppada*) of this, that arises. When this does not exist, that does not come to be. With the cessation (निरोध, *nirodha*) of this, that ceases.»

—Samyutta Nikaya 12.61.

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The Buddhist doctrine of *pratītyasamutpāda* (skrt: प्रतीत्यसमुत्पाद, roughly: *co-dependent origination*) states that all phenomena (dharma) arise in dependence upon other phenomena.

In each atom the Buddhas of all times
Appear, according to inclinations;
While their essential nature neither comes nor goes,
By their vow power they pervade the worlds.

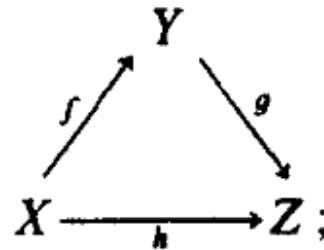
—Buddhāvataṃsaka Sūtra, 7:I, Bk 4

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In Vedic mythology, when Indra dreams the world, he builds it as a spiderweb or network, with each crossing adorned with a jewel. Every *dharma* is a node in this network, and the surface of each jewel reflects every other, so that every thing that exists implies all the others.

Category theory starts with the observation that many properties of mathematical systems can be unified and simplified by a presentation with diagrams of arrows. Each arrow $f : X \rightarrow Y$ represents a function; that is, a set X , a set Y , and a rule $x \mapsto f x$ which assigns to each element $x \in X$ an element $f x \in Y$; whenever possible we write $f x$ and not $f(x)$, omitting unnecessary parentheses. A typical diagram of sets and functions is



S. Mac Lane, *Categories for the working Mathematician*. (1971)

A snippet of haskell code

```
compose n = do  
  n1 <- f n  
  n2 <- g n1  
  n3 <- h n2  
  return n3
```

composing two partial functions.

What is this course about



Our plan is to study a bit of this stuff (mostly Mathematics, and programming, but a bit of Vedic mythology might come back every now and then).

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- third floor of kybi

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Plus TAs or other teachers:

- N. Arkor (<https://arkor.co>)
- A. Laretto (<https://iwilare.com>)
- M. De Pascalis (michele.de@taltech.ee)

On the exam methods:

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The course webpage:

<https://tetrapharmakon.github.io/CourseCategoryTheory.html>



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So tell us:

Who are you?