ITI9200 – Category theory

Fosco Loregian

February 2026

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«Category theory can be seen as a theory of *systems* and *processes.*» (Hu-Vicary, 2021)

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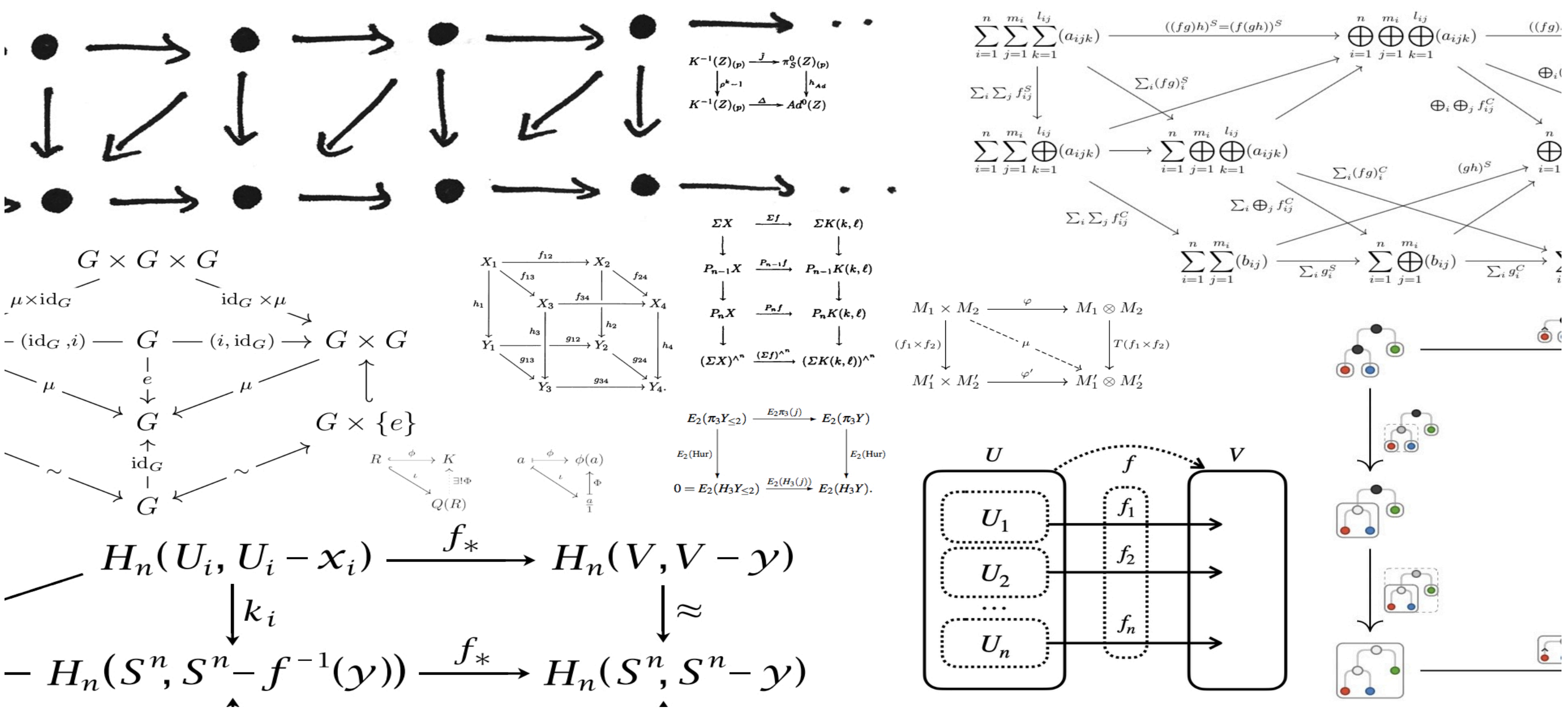
Information about a problem is presented as a *diagram* (an oriented graph of a very special kind):



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- **identities** exist $1_X : X \to X$ so that $\frac{f:X\to Y \quad 1_Y:Y\to Y}{f:X\to Y} \qquad \frac{1_Y:Y\to Y \quad g:Y\to Z}{g:Y\to Z}$

«When this (skrt: इदं, *idaṃ*) exists, that comes to be.
With the arising (skrt: उप्पाद, *utpāda*) of this, that arises.
When this does not exist, that does not come to be.
With the cessation (skrt: निरोध, *nirodha*) of this, that ceases.»

—Samyutta Nikaya 12.61. 3rd century BC?

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The Buddhist doctrine of *pratītyasamutpāda* (skrt: प्रतीत्यसमुत्पाद, roughly: *co-dependent origination*) states that all phenomena (skrt: धर्म, *dharma*) arise *dependent upon other phenomena.*

In each atom the Buddhas of all times
Appear, according to inclinations;
While their essential nature neither comes nor goes,
By their vow power they pervade the worlds.

—Buddhāvataṃsaka Sūtra, 7:I, Bk 4 *circa* 4th century CE

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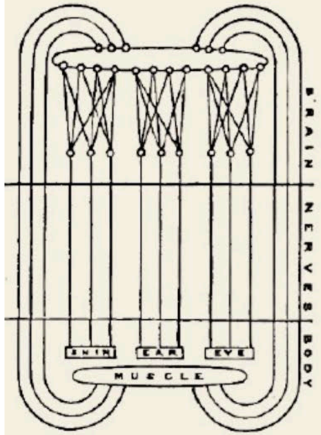
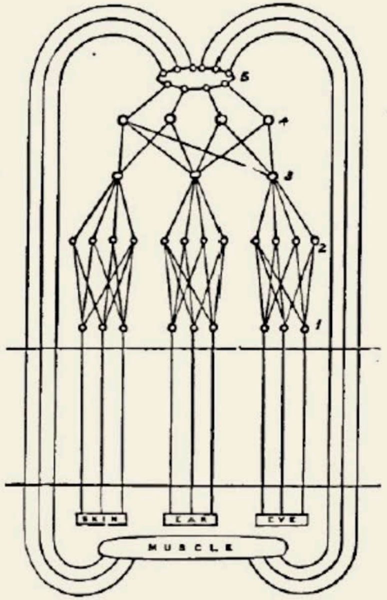
In Vedic mythology, when Indra dreams the world, He builds it as a spiderweb or network, with each crossing adorned with a jewel. Every *dharma* is a node in this network, and *the surface of each jewel reflects every other*, so that every thing that exists implies all the others.

Fig. 16.



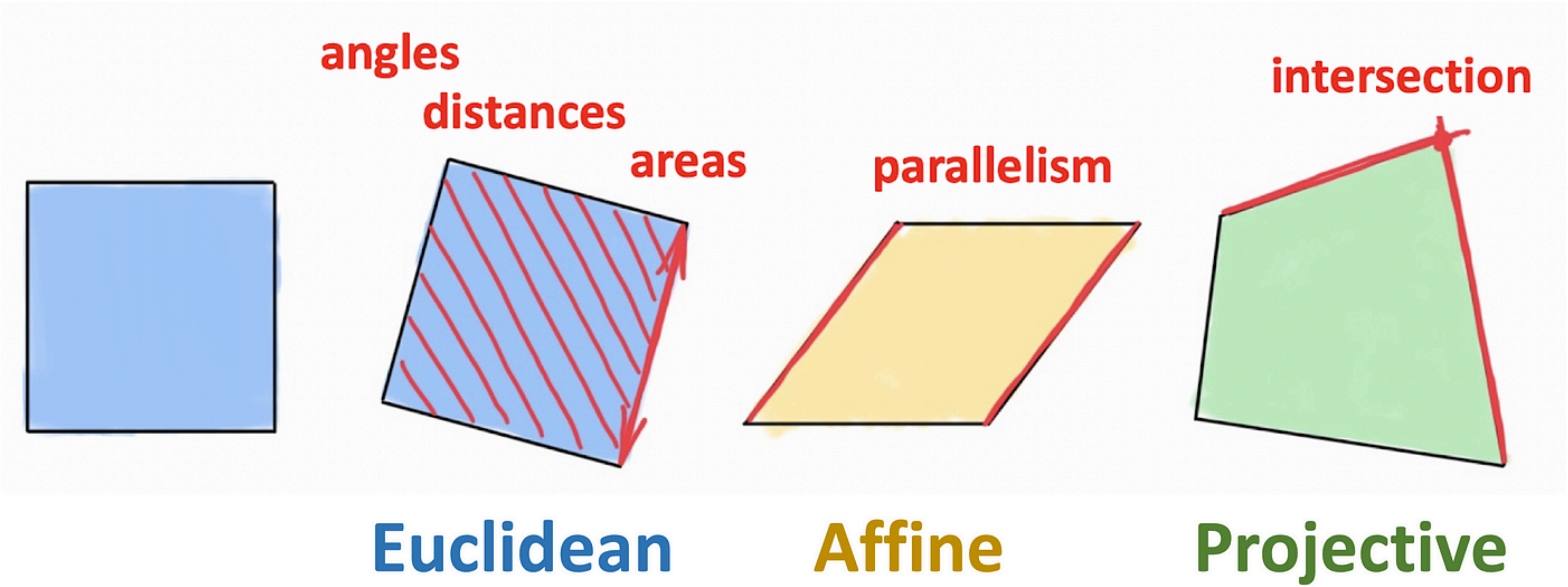
Theoretical nervous combination of lower animals.

Fig. 17.



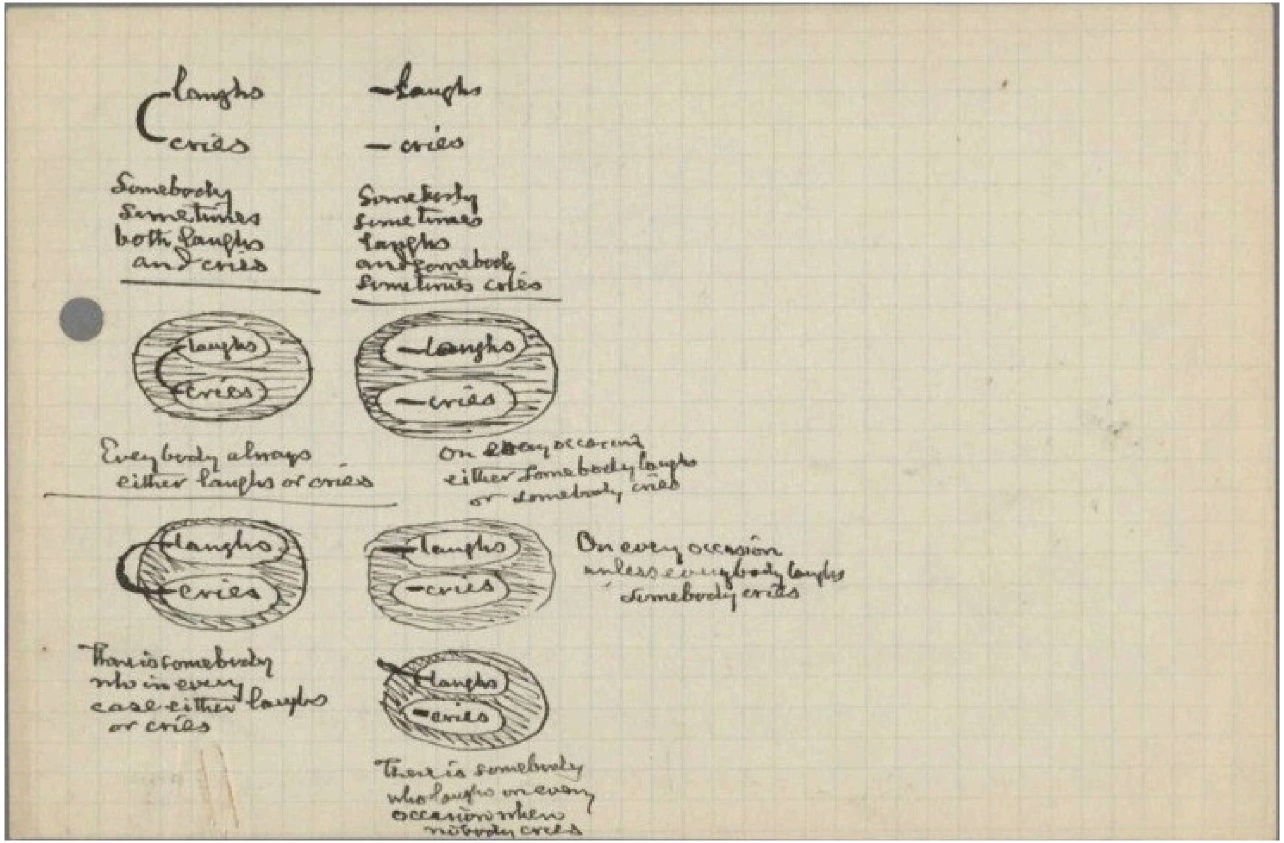
Theoretical nervous combination of man. 1, Aisthenic; 2, syndramic; 3, aisthenic noemic; 4, syndramic noemic; 5, pneumanoemic.

Alfred Smee (1818–1877). *Instinct and Reason Deduced from Electrobiology*, 1850.

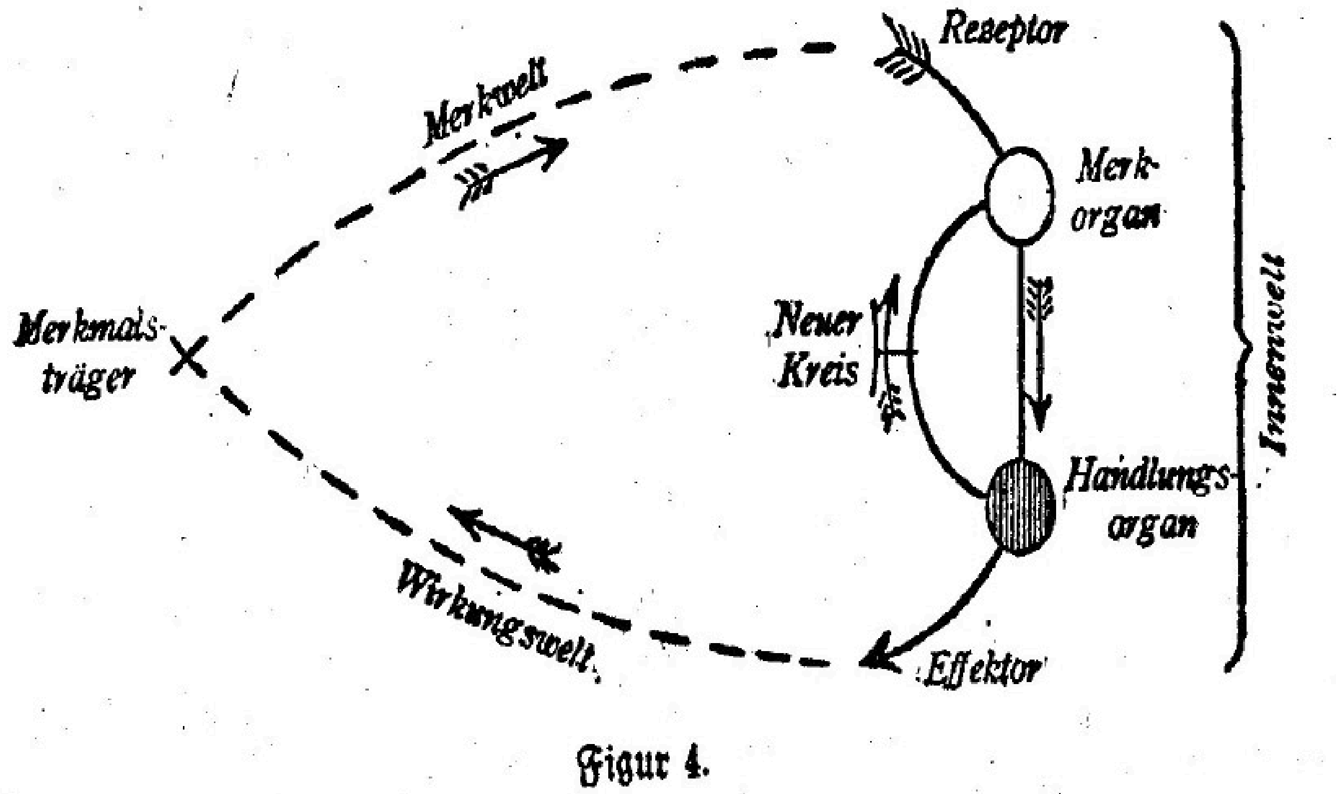


$$\mathrm{Iso} \leq \mathrm{Aff} \leq \mathrm{PGL}$$

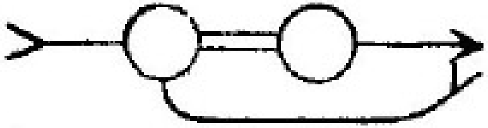
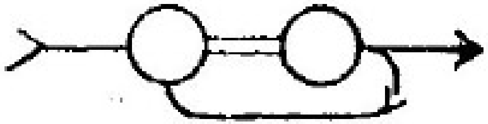
Schematization of the *Erlangen program*, F. Klein, 1872



C.S. Peirce, *Prolegomena to an Apology of Pragmaticism*, 1906



Jacob J. von Uexküll. “Early Scheme for a circular Feedback Circle” from *Theoretische Biologie* 1920.

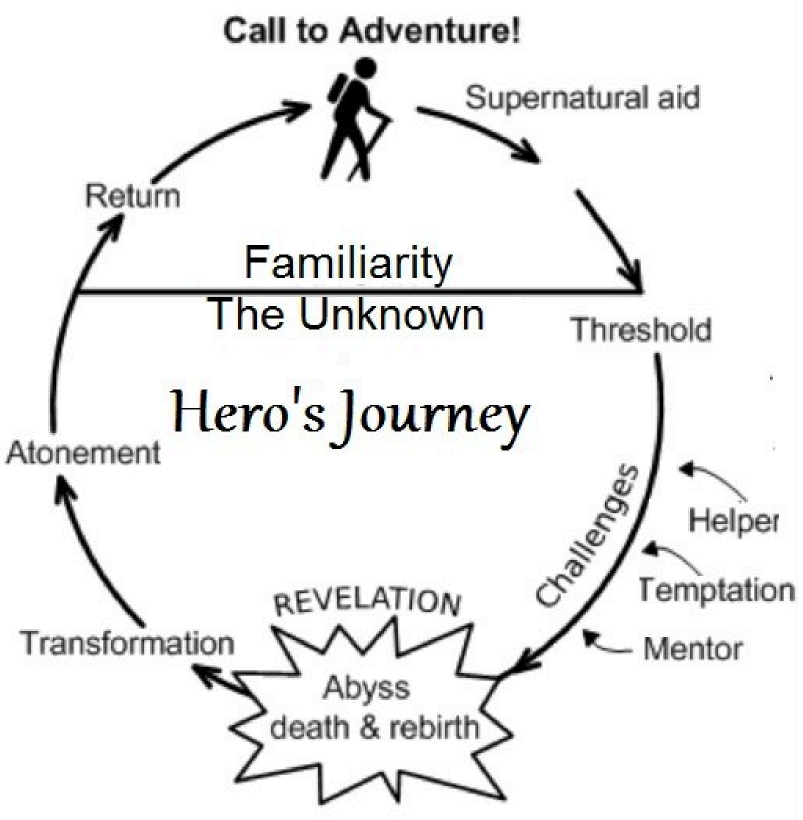
sind zwei Fälle zu unterscheiden: entweder wir
Effektorenmuskeln durch besondere sensible Nerve
beifolgende Schema zeigt.  Oder es
torischen Nerven übertragene Erregung durch
Rezeptoren zum Teil aufgefangen und dem N
 Diese Rezeptoren bilden das zentr
Helmholtz, das anatomisch noch völlig im Dunkeln

Jacob J. von Uexküll. “Zirkuläre Schemen” from *Theoretische Biologie* 1920. Diagrammatic description of double feedback system of autonomic nerves in the brain.

to another, the correlate of the one has the relation Q to the correlate of the other, and *vice versa*. A figure will make this clearer. Let x and y be two terms having the relation P. Then there are to be two terms z, w, such that x has the relation S to z, y has the relation S to w, and z has the relation Q to w. If this happens with every pair of terms such as x

$$\begin{tikzcd} x \arrow[r, "P"] \arrow[d, "S"'] & y \arrow[d, "S"] \\ z \arrow[r, "Q"'] & w \end{tikzcd}$$

B. Russell, *Introduction to Mathematical Philosophy*, 1919 (p 54).



J. J. Campbell, *The Hero with a Thousand Faces*, 1949. (mythogenesis of heros' tales and folk stories)

Category theory starts with the observation that many properties of mathematical systems can be unified and simplified by a presentation with diagrams of arrows. Each arrow $f : X \to Y$ represents a function; that is, a set X, a set Y, and a rule $x \mapsto fx$ which assigns to each element $x \in X$ an element $fx \in Y$; whenever possible we write fx and not $f(x)$, omitting unnecessary parentheses. A typical diagram of sets and functions is

$$\begin{tikzcd} & Y \arrow[dr, "g"] & \\ X \arrow[ur, "f"] \arrow[rr, "h"'] & & Z \end{tikzcd};$$

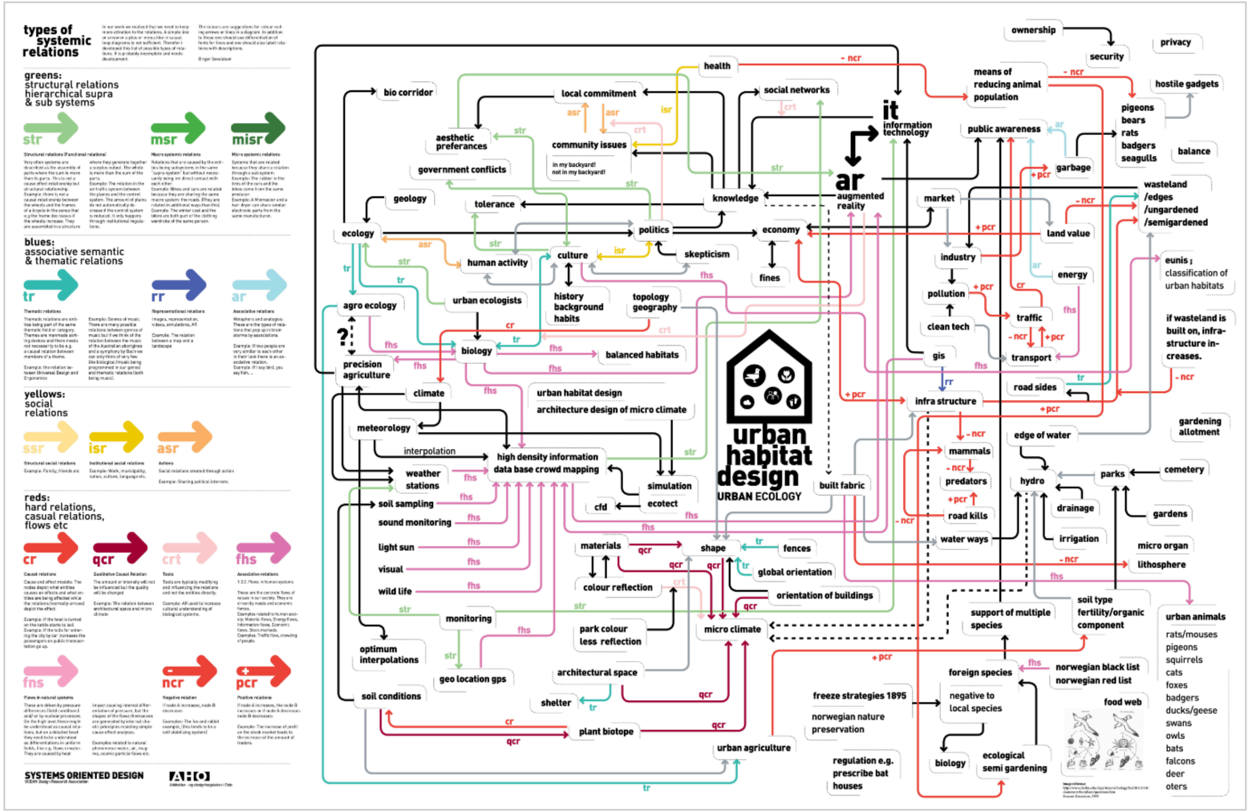
S. Mac Lane, *Categories for the working Mathematician.* (1971)

A snippet of `haskell` code (*circa* 1990)

```
compose n = do
  n1 <- f n
  n2 <- g n1
  n3 <- h n2
  return n3
```

composing two *partial functions*.

A certain construct (a "monad") encapsulates the fact that a transformation like $\mathrm{head} : \mathrm{List}(A) \to A$ might fail (what's the head of an empty list?)



Relational gigamapping, from https://systemsorienteddesign.net/gigamap-relational/

There are some useful rules of thumb for how to gigamap. These rules have emerged through years of experience producing such maps and instructing students and professionals in gigamapping.

> Within the final and true world image everything is related to everything, and nothing can be discarded a priori as being unimportant. – Fritz Zwingy 1969

The concept of *gigamapping*, in System Oriented Design https://systemsorienteddesign.net

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- models of a fixed theory/theories in logic, and
- elementary particles as representations of an abstract group in physics, and
- many other things (getting there in a second)

are all instances of categorical constructs called *functors*, i.e. mappings *between categories* that preserve relations between objects.

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- there is a fundamental difference between the product $V \times W$ (product of vector spaces, where states are *separable*) and $V \otimes W$ (states are *entangled*).
- Universal objects solve *optimization problems*: given initial data such-and-such build a device such-and-such that satisfies all requirements such-and-such.

Using category theory in physics

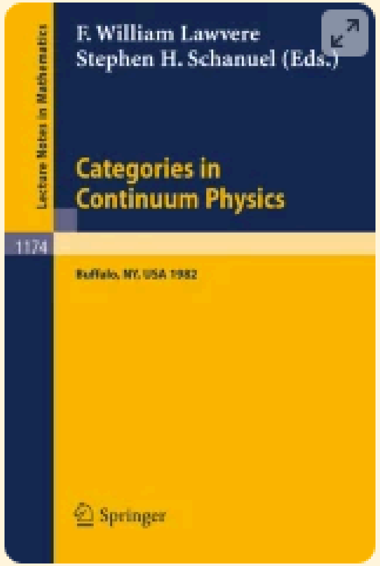
Differential cohomology in a cohesive infinity-topos

Urs Schreiber

We formulate differential cohomology and Chern-Weil theory -- the theory of connections on fiber bundles and of gauge fields

Using category theory in physics

Home › Conference proceedings



Categories in Continuum Physics

Lectures Given at a Workshop Held at SUNY, Buffalo 1982

Conference proceedings | © 1986

Using category theory in physics

DYNAMICAL SYSTEMS AND CATEGORIES

G. DIMITROV, F. HAIDEN, L. KATZARKOV, M. KONTSEVICH

ABSTRACT. We study questions motivated by results in the classical theory of dynamical systems in the context of triangulated and A_∞-categories. First, entropy is defined for exact endofunctors and computed in a variety of examples. In particular, the classical entropy of a pseudo-Anosov map is recovered from the induced functor on the Fukaya category. Second, the density of the set of phases of a Bridgeland stability condition is studied and a complete answer is given in the case of bounded derived categories of quivers. Certain exceptional pairs in triangulated categories, which we call Kronecker pairs, are used to construct stability conditions with density of phases. Some open questions and further directions are outlined as well.

Using category theory in probability theory

Markov Categories and Entropy

Paolo Perrone

University of Oxford,
Department of Computer Science

Abstract

Markov categories are a novel framework to describe and treat problems in probability and information theory. In this work we combine the categorical formalism with the traditional quantitative notions of entropy, mutual information, and data processing inequalities. We show that several quantitative aspects of information theory can be captured by an enriched version of Markov categories, where the spaces of morphisms are equipped with a divergence or even a metric.

Following standard practices of information theory, we get measures of mutual information by quantifying, with a chosen divergence, how far a joint source is from displaying independence of its components.

More strikingly, Markov categories give a notion of determinism for sources and channels, and we can define entropy exactly by quantifying how far a source or channel is from being deterministic. This recovers Shannon and Rényi entropies, as well as the Gini-Simpson index used in ecology to quantify diversity, and it can be used to give a conceptual definition of generalized entropy.

Using category theory in probability theory

A category-theoretic proof of the ergodic decomposition theorem

Sean Moss and Paolo Perrone

January 18, 2023

Abstract

The ergodic decomposition theorem is a cornerstone result of dynamical systems and ergodic theory. It states that every invariant measure on a dynamical system is a mixture of ergodic ones. Here we formulate and prove the theorem in terms of string diagrams, using the formalism of Markov categories. We recover the usual measure-theoretic statement by instantiating our result in the category of stochastic kernels. Along the

Using category theory in neuroscience

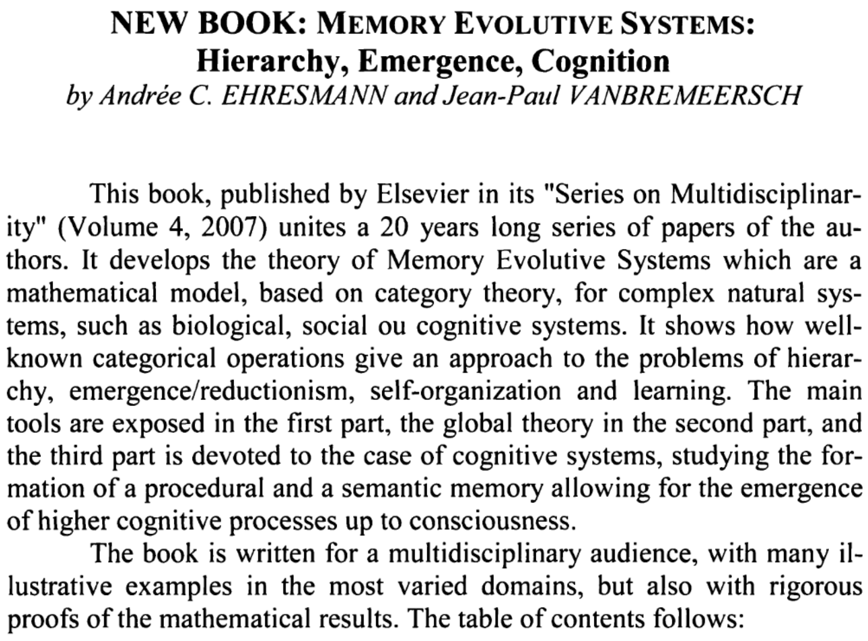
Home > A New Foundation for Representation in Cognitive and Brain Science > Chapter

A Theory of Hippocampus Structure and Function Based on Category Theory

Chapter | First Online: 01 January 2013

pp 141–160 | Cite this chapter

Using category theory in evolutive biology



NEW BOOK: MEMORY EVOLUTIVE SYSTEMS:
Hierarchy, Emergence, Cognition
by Andrée C. EHRESMANN and Jean-Paul VANBREMEERSCH

This book, published by Elsevier in its "Series on Multidisciplinarity" (Volume 4, 2007) unites a 20 years long series of papers of the authors. It develops the theory of Memory Evolutive Systems which are a mathematical model, based on category theory, for complex natural systems, such as biological, social ou cognitive systems. It shows how well-known categorical operations give an approach to the problems of hierarchy, emergence/reductionism, self-organization and learning. The main tools are exposed in the first part, the global theory in the second part, and the third part is devoted to the case of cognitive systems, studying the formation of a procedural and a semantic memory allowing for the emergence of higher cognitive processes up to consciousness.

The book is written for a multidisciplinary audience, with many illustrative examples in the most varied domains, but also with rigorous proofs of the mathematical results. The table of contents follows:

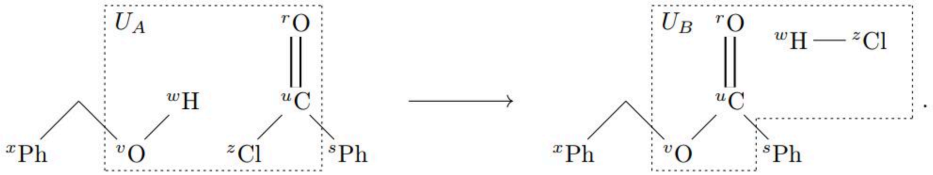
Using category theory in chemistry

A Categorical Model for Retrosynthetic Reaction Analysis

Ella Gale * Leo Lobski † Fabio Zanasi †‡

Abstract

We introduce a mathematical framework for retrosynthetic analysis, an important research method in synthetic chemistry. Our approach represents molecules and their interaction using string diagrams in layered props – a recently introduced categorical model for partial explanations in scientific reasoning. Such principled approach allows one to model features currently not available in automated retrosynthesis tools, such as chirality, reaction environment and protection-deprotection steps.



Using category theory in evolutive biology

NATURAL TRANSFORMATIONS OF ORGANISMIC STRUCTURES

■ ION C. BAIANU
Cavendish Laboratory,
University of Cambridge,
England

The mathematical structures underlying the theories of organismic sets, (M, R)-systems and molecular sets are shown to be transformed naturally within the theory of categories and functors. Their natural transformations allow the comparison of distinct entities, as well as the modelling of dynamics in "organismic" structures.

One is rarely interested in studying animals alone, much more in studying the ecosystem they form.

Entropy and diversity

The axiomatic approach

TOM LEINSTER
University of Edinburgh

$\Rightarrow$ Uses category theory to *quantify biological diversity.*

Using category theory in linguistics

THE MATHEMATICS OF SENTENCE STRUCTURE*

JOACHIM LAMBEK, McGill University

The definitions [of the parts of speech] *are very far from having attained the degree of exactitude found in Euclidean geometry.*

—Otto Jespersen, 1924.

1. Introduction. The aim of this paper is to obtain an effective rule (or algorithm) for distinguishing sentences from nonsentences, which works not only for the formal languages of interest to the mathematical logician, but also for natural languages such as English, or at least for fragments of such languages. An attempt to formulate such an algorithm is implicit in the work of Ajdukiewicz.† His method, later elaborated by Bar-Hillel [2], depends on a kind of arithmetization of the so-called *parts of speech*, here called *syntactic types*.‡

Using category theory in

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- `fosco.loregian@gmail.com`
- I coteach with *Andrea Laretto*, `andrea.laretto@taltech.ee`
- 2nd floor of ICT

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reach out if you have questions.

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- we will introduce the notion of category, functor, natural transformation, see examples of categories;
- we will introduce the notion of *universal property*, see instances of it and its connection with a fundamental result called *Yoneda lemma* (a result which inspired Matthew 7, 16-20: “Ye shall know them by their fruits.”);
- The most fruitful application of functoriality is in the study of *adjoints* between categories, and *monads* on categories.

On the exam methods:

- Solve a few exercises during the course (exercise sheets assigned periodically, approx 3-4 times);
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- Pick a topic from a list (also available at the course webpage), or take inspiration from a book, and give a short presentation;
- Endure a couple of questions from the teacher.

All information will be published on the *course webpage*:

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https://tinyurl.com/iti9200-ttu

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Now, for something not-so-completely-different.