

Categorical Ontology I₂¹: Erkennen

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1. Profunctors / Grothendieck construction

Sezione tecnica con solo robe di CT.

2. Nerve and realisations

Sezione tecnica con solo robe di CT.

3. Theories and models

Qui exploitiamo il linguaggio introdotto nelle precedenti due sezioni;

Definition 3.1 (Theory). A *theory* \mathcal{L} is the syntactic category $\mathcal{T}_{\mathcal{L}}$ (cf. []) of a first-order, finitely axiomatisable language L .

Definition 3.2. A *world* is a large category \mathcal{W} ; a *universe* is a world that, as a category, admits all small colimits.

Given a theory \mathcal{L} and a world \mathcal{W} , a \mathcal{L} -*canvas* of \mathcal{W} is a functor

$$\mathcal{L} \xrightarrow{\phi} \mathcal{W}.$$

A canvas $\phi : \mathcal{L} \rightarrow \mathcal{W}$ is a *science* if ϕ is a dense functor.

Remark 3.3. The NR paradigm exposed in ?? now entails that

- If \mathcal{W} is a world, we obtain a *representation* functor

$$\mathcal{W} \longrightarrow [\mathcal{L}^{\text{op}}, \text{Set}];$$

this means: given a canvas ϕ of the world, the latter leaves an image on the canvas.

- If \mathcal{W} is a universe, we obtain a NR-adjunction

$$\mathcal{W} \rightleftarrows [\mathcal{L}^{\text{op}}, \text{Set}];$$

this means: if \mathcal{W} is sufficiently expressive, then models of the theory that explains \mathcal{W} through ϕ can be used to acquire a two-way knowledge. Phenomena have a theoretical counterpart in $[\mathcal{L}^{\text{op}}, \text{Set}]$ via the nerve; theoretical objects strive to describe phenomena via their realisation.

- If an \mathcal{L} -canvas $\phi : \mathcal{L} \rightarrow \mathcal{W}$ is a science, ‘the world’ is a full subcategory of the modes in which ‘language’ can create interpretation.

The terminology is chosen to inspire the following idea in the reader: science strives to define *theories* that allow for the creation of representations of the world; said representations are descriptive when there is dialectic opposition between world and models; when such representation is faithful, we have reduced ‘the world’ to a piece of the models created to represent it.

The tongue-in-cheek here is, la scienza (nel senso usuale) non è una scienza (nel senso della definizione ??), se non in potenza; i tentativi di generare pensiero scientifico sono i tentativi di

- Riconoscere un mondo \mathcal{W} come un oggetto sufficientemente espressivo da contenere fenomeni e informazione;
- Creare un linguaggio L , sufficientemente ‘compatto’, la cui categoria sintattica permette di rappresentare nel mondo;
- Ottenere una agguinzione tra \mathcal{W} e modelli del mondo $[\mathcal{L}^{\text{op}}, \text{Set}]$ ottenuti dal linguaggio \mathcal{L} , per generare modelli a partire da fenomeni, e per prevedere fenomeni a partire da modelli;
- Ottenere che ‘il linguaggio sia un sottospazio denso del mondo’, con ciò intendendo che l’aggiunzione del punto precedente è sufficientemente well-behaved da descrivere il mondo come un frammento delle rappresentazioni semantiche del linguaggio L .

Evidentemente, la tensione qui è tra due opposte qualità che L deve avere: non deve essere troppo esteso, per essere trattabile; non deve essere troppo ristretto, per parlare di “tutto” il mondo che si prefigge di descrivere.

4. The tension between observational and theoretical

All based on the proportion

truth values : proposition = section : presheaf

The tension between observational and theoretical can be faithfully represented through profunctor theory;

Definition 4.1. Let \mathcal{T}, \mathcal{O} be two categories, respectively the *theoretical* and the *observational* one. A $(1, 1)$ -ary *Ramsey map* is a profunctor $\mathfrak{k} : \mathcal{T} \multimap \mathcal{O}$ (maybe ops have to be added for the sake of convention).

There is nothing, in their mere syntactical presentation, that allows to tell the observational and the theoretical category apart; justify with the self-involution of Prof.

The set $\mathfrak{k}(\underline{T}, \underline{O})$ represents the type of proofs that the observational tuple \underline{O} admits a description in terms of the theoretical tuple \underline{T} .

A limitation of the above definition is that in practice all sorts of configurations are possible:

- una singola O si lascia descrivere da due T , e non meno
- una stesso T descrive due O diverse
- etc
- etc

Thus we have to admit multiple arguments in domain and codomain.

5. Ramseyfication and beyond: generalised profunctors

We can generalise the definition above to encompass Ramsey sentences:

Definition 5.1. Let \mathcal{T}, \mathcal{O} be two categories; a *Ramsey map*, or a (n, m) -ary *Ramsey map* is a profunctor $\mathfrak{K} : \mathcal{T}^n \multimap \mathcal{O}^m$

This formalism allows to speak about particular worlds, obtained as presheaf categories over observational \mathcal{O} ; if \mathcal{T}, \mathcal{O} is a theoretic pair, we can instantiate ?? above in the particular case where $\mathcal{W} = [\mathcal{O}^{\text{op}}, \text{Set}]$; observe that \mathcal{W} is a universe! We can thus address a certain number of questions: ...

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