Categorical Ontology I$\frac{1}{2}$: Erkennen

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Sezione tecnica con solo robe di CT.

3. Theories and models

Qui esploitiamo il linguaggio introdotto nelle precedenti due sezioni;

Definition 3.1 (Theory). A *theory* $\mathcal{L}$ is the syntactic category $\mathcal{T}_L$ (cf. []) of a first-order, finitely axiomatisable language L.

Definition 3.2. A *world* is a large category $\mathcal{W}$; a *universe* is a world that, as a category, admits all small colimits.

Given a theory $\mathcal{L}$ and a world $\mathcal{W}$, a $\mathcal{L}$-*canvas* of $\mathcal{W}$ is a functor

$$\mathcal{L} \xrightarrow{\phi} \mathcal{W}.$$

A canvas $\phi : \mathcal{L} \to \mathcal{W}$ is a *science* if ϕ is a dense functor.

Remark 3.3. The NR paradigm exposed in **??** now entails that

- If $\mathcal{W}$ is a world, we obtain a *representation* functor
$$\mathcal{W} \longrightarrow [\mathcal{L}^{\mathrm{op}}, \mathsf{Set}];$$
this means: given a canvas ϕ of the world, the latter leaves an image on the canvas.
- If $\mathcal{W}$ is a universe, we obtain a NR-adjunction
$$\mathcal{W} \rightleftarrows [\mathcal{L}^{\mathrm{op}}, \mathsf{Set}];$$
this means: if $\mathcal{W}$ is sufficiently expressive, then models of the theory that explains $\mathcal{W}$ through ϕ can be used to acquire a two-way knowledge. Phenomena have a theoretical counterpart in $[\mathcal{L}^{\mathrm{op}}, \mathsf{Set}]$ via the nerve; theoretical objects strive to describe phenomena via their realisation.
- If an $\mathcal{L}$-canvas $\phi : \mathcal{L} \to \mathcal{W}$ is a science, 'the world' is a full subcategory of the modes in which 'language' can create interpretation.

The terminology is chosen to inspire the following idea in the reader: science strives to define *theories* that allow for the creation of representations of the world; said representations are descriptive when there is dialectic opposition between world and models; when such representation is faithful, we have reduced 'the world' to a piece of the models created to represent it.

The tongue-in-cheek here is, la scienza (nel senso usuale) non è una scienza (nel senso della definizione **??**), se non in potenza; i tentativi di generare pensiero scientifico sono i tentativi di

- Riconoscere un mondo $\mathcal{W}$ come un oggetto sufficientemente espressivo da contenere fenomeni e informazione;
- Creare un linguaggio L, sufficientemente 'compatto', la cui categoria sintattica permette di rappresentare nel mondo;
- Ottenere una aggiunzione tra $\mathcal{W}$ e modelli del mondo $[\mathcal{L}^{\mathrm{op}}, \mathsf{Set}]$ ottenuti dal linguaggio $\mathcal{L}$, per generare modelli a partire da fenomeni, e per prevedere fenomeni a partire da modelli;
- Ottenere che 'il linguaggio sia un sottospazio denso del mondo', con ciò intendendo che l'aggiunzione del punto precedente è sufficientemente well-behaved da descrivere il mondo come un frammento delle rappresentazioni semantiche del linguaggio L.

Evidentemente, la tensione qui è tra due opposte qualità che L deve avere: non deve essere troppo esteso, per essere trattabile; non deve essere troppo ristretto, per parlare di "tutto" il mondo che si prefigge di descrivere.

4. The tension between observational and theoretical

All based on the proportion

truth values : proposition = section : presheaf

The tension between observational and theoretical can be faithfully represented through profunctor theory;

Definition 4.1. Let $\mathcal{T}, \mathcal{O}$ be two categories, respectively the *theoretical* and the *observational* one. A $(1,1)$-*ary Ramsey map* is a profunctor $\mathfrak{k} : \mathcal{T} \nrightarrow \mathcal{O}$ (maybe ops have to be added for the sake of convention).

There is nothing, in their mere syntactical presentation, that allows to tell the observational and the theoretical category apart; justify with the self-involution of Prof.

A limitation of the above definition is that in practice all sorts of configurations are possible:

- una singola O si lascia descrivere da due T, e non meno
- una stesso T descrive due O diverse
- etc
- etc

Thus we have to admit multiple arguments in domain and codomain.

5. Ramseyfication and beyond: generalised profunctors

We can generalise the definition above to encompass Ramsey sentences:

Definition 5.1. Let $\mathcal{T}, \mathcal{O}$ be two categories; a *Ramsey map*, or a (n,m)-*ary Ramsey map* is a profunctor $\mathfrak{K} : \mathcal{T}^n \nrightarrow \mathcal{O}^m$

The set $\mathfrak{k}(\underline{T}, \underline{O})$ represents the type of proofs that the observational tuple $\underline{O}$ admits a description in terms of the theoretical tuple $\underline{T}$.

This formalism allows to speak about particular worlds, obtained as presheaf categories over observational $\mathcal{O}$; if $\mathcal{T}, \mathcal{O}$ is a theoretic pair, we can instantiate **??** above in the particular case where $\mathcal{W} = [\mathcal{O}^{\text{op}}, \mathsf{Set}]$; observe that $\mathcal{W}$ is a universe! We can thus address a certain number of questions, arising from the canonical adjunction obtained by virtue of **??** and **??**:

$$[(\mathcal{O}^m)^{\text{op}}, \mathsf{Set}] \rightleftarrows [(\mathcal{T}^n)^{\text{op}}, \mathsf{Set}];$$

Vale la pena notare che siccome il triangolo

$$\begin{tikzcd}
(\mathcal{O}^m)^{\text{op}} & & [(\mathcal{T}^n)^{\text{op}}, \mathsf{Set}] \\
& [(\mathcal{O}^m)^{\text{op}}, \mathsf{Set}] &
\end{tikzcd}$$

pseudocommuta, allora la composizione $L \circ y$ fa esattamente (mate di) $\mathfrak{K}$. Ciò significa: gli $\mathcal{O}$-modelli, interpretati nei $\mathcal{T}$-modelli, hanno rappresentazioni corrispondenti ai termini osservazionali interpretati nei $\mathcal{T}$-modelli; ovvero, la rappresentazione è 'coerente' sui generatori dei modelli osservazionali, ovvero...

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