$S_1 \quad S_1 \subseteq S_2 \quad S_2$

$S_1 \supseteq S_2$

$$a \in \overset{a}{A} \cup (\overset{a}{B} \underbrace{\cap}_{a} \overset{a}{C}) = (\overbrace{A \cup \overset{a}{B}}^{a}) \cap (\overbrace{A \cup \overset{a}{C}}^{a}) = a$$

$a \quad a$

$$A \cup (A \cap B) \overset{?}{=} A$$

$$A \cap (A \cup B) = A$$

let $a \in (A \cup B) \cap (A \cup C)$

$a \in (A \cup B)$ and $a \in (A \cup C)$

$a \in A$ or $a \in B$ and $a \in A$ or $a \in C$

$a \in A$ or $a \in B$ and $a \in C$

$A \cup (B \cap C)$

$a \in A \cup (B \cap C)$

$a \in A$ + $a \in (B \cap C)$

$a \in (B \cap C)$

$a \in B$

$a \in C$

$a \in (A \cup B) \cap (A \cup C)$

If $a \in A$ then $a \in (A \cup B)$

If $a \in A$ then $a \in (A \cup C)$

For $i = 1, 2, \ldots$

$A_i = \{i, i+1, i+2, \ldots\}$

~~$A_n = \{$~~ $A_1 = \{1, 2 \ldots\}$ $A_2 = \{2, 3, \ldots\}$

$A_{10} = \{10, \ldots\}$

Compute $\bigcup_{i=1}^{n} A_i = \{1, 2, 3, \ldots\}$

$\bigcap_{i=1}^{n} A_i = A_n$

$A_i = \{1, 2, \ldots i\}$

$\bigcup_{i=1}^{n} = \{1, 2, \ldots n\} = A_n$

$\bigcap_{i=1}^{n} = \{\cancel{A_i}$ $A_i = \{1\}$

$A_1 \cap A_2 = \{2, 3 \ldots\}$

$A_1 \cap A_2 \cap A_3 = A_3$

Cartesian Product

$A = \{a_1 \dots, a_n\}$ ordered n tuple

$B = \{b_1 \dots b_n\}$

$B \times A = \{(b,a) \mid b_1 \in B, a \in A\}$

(b,a)

$\{1, a, 6, f\}$

$\{a, f, 1, 6\}$

$A = B \Rightarrow a_1 = b_1 - a_n = b_n$

The cartesian product of $A \times B = \{(a,b) \mid (a \in A, b \in B\}$

$A = \{a, b, c\}$ how will elements of $(A \times B)$ look like

$B = \{\alpha, \beta, \gamma\}$ $\{(a,\alpha), (a,\beta), (a,\gamma), (b,\alpha) \dots\}$

$A \times B \overset{?}{\neq} B \times A$

$A = \{1, 2\}$

$A^2 = \{(1,1), (1,2), (2,1), (2,2)\}$

$(a, b) \in \underbrace{A \times B}$

$\{0, 1, 2, 3\}$

$(a,b) : a \leq b$

$(0,0), (0,1), (0,2)\ (0,3)$

$(1,2)\ (1,3)$

$p = 2x$

$p^2 = 4x^2 = 2q^2$

$p^2 = 2q^2 = 4 \cdot ?$

Prove that $\sqrt{2}$ is irrational

M M
Tu Tu
W W
Th Th
F F
S S
Su Su

Take a set of 15 days atleast 3 must fall on the same day

$2 \times 7 = 14$

· Statement: $\sqrt{2}$ is irrational

Challenge the Statement

$\boxed{p^2 = 2q^2}$

multiple of 4

$\frac{4}{6}$ $\quad \sqrt{2} = p/q$

$\frac{60}{75}$

$\boxed{2 = \frac{p^2}{q^2}}$

$\boxed{2q^2 = p^2} \Rightarrow$ p is even

p^2 even

$2q^2$

$p = 2 * X$

$12/15$ $\quad 4/5$ $\quad$ q is even then p is even

(1) If p is even, p^2 is a multiple of 4.

(2) p^2 is even, multiple of 4

$4 \cdot \text{Even} = \cancel{2}\, q^2$

(3) $2 \cdot \text{Even} = q^2 \Rightarrow q$ is also even

If p is even $\Rightarrow$ q even

$\sqrt{2}$ is irrational

$\sqrt{2}$ is rational $\rightarrow$

$\sqrt{2} = p/\underline{q}$

$2 = \frac{p^2}{q^2}$

$p^2 = 2\underline{q}^2 \Rightarrow p$ is even

Square of any even number is a multiple of 4. $2q^2 = 4.X$ (CONTRADICTION)

$q^2 = 2.X \Rightarrow q$ is even

If $\underbrace{3n+2}$ is odd then n is odd.

n is NOT odd. $\Rightarrow n$ is even

$n = 2K$

$3.2K + 2 = 2(3K+1)$

$\Rightarrow 3n+2$ is even

$\underset{\text{(odd)}}{3n+2} \Rightarrow \underline{n \text{ is odd}}$

even

N is even

MATHEMATICAL LOGIC

DM is NOT / an interesting course

p Statement

Proposition

negation $\neg p$

$\bar{p}$

OR operation

P, Q ORM

S/W or H/W

Two simple statements

P	Q	S
0	0	0
0	1	1
1	0	1
1	1	1

P	Q	M	S
0	0	0	0

Q: $6+4 = 3$ (F-0)

$\neg Q$: $6+4 \neq 3$ (T-1)

$\binom{3}{0}+\binom{3}{1}+\binom{3}{2}+\binom{3}{3}$

Power set 2^3

and.

SW + HW

P	Q	S
0	0	0
0	1	0
1	0	0
1	1	1

(p AND q) OR NOT p

OR $p \vee q$

AND $p \wedge q$

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \vee \neg p$
0	0	0	1	1
0	1	0	1	1
1	0	0	0	0
1	1	1	0	1

Truth table

p	q	¬p	¬q	¬q → ¬p
0	0			1
0	1			1
1	0			0
1	1			1

¬p → ¬q

Inverse

CONTRAPOSITIVE

when is a CONDITIONAL Statement

$P \to q$ false? | $\neg q \to \neg p$ false

When p Q = F & P = T

If 3n+2 is odd then n is odd

Q

If NOT Q then NOT P

H → E

Prove P → Q

for ¬Q → ¬P

Logical Implication

P → Q

E → S

E	S	E → S
0	0	1
0	1	1
1	0	0
1	1	1

$P \wedge (q \vee r)$

$= (P \wedge q) \vee (q \vee r)$

$H \to E$

not eating $\to$ not hungry

H	E	$H \to E$	$\neg E$	$\neg H$	$\neg E \to \neg H$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

$\neg q \rightarrow \neg p$

If $3n+2$ is odd $\rightarrow$ n is odd.

$p \rightarrow q$

A B

p	q	$p \rightarrow q$	
0	0	1	1
0	1	1	1
1	0	0	0 $\rightarrow$ NOT POSSIBLE
1	1	1	1 POSSIBLE

0 1

$x \notin A$, but $x \in B$

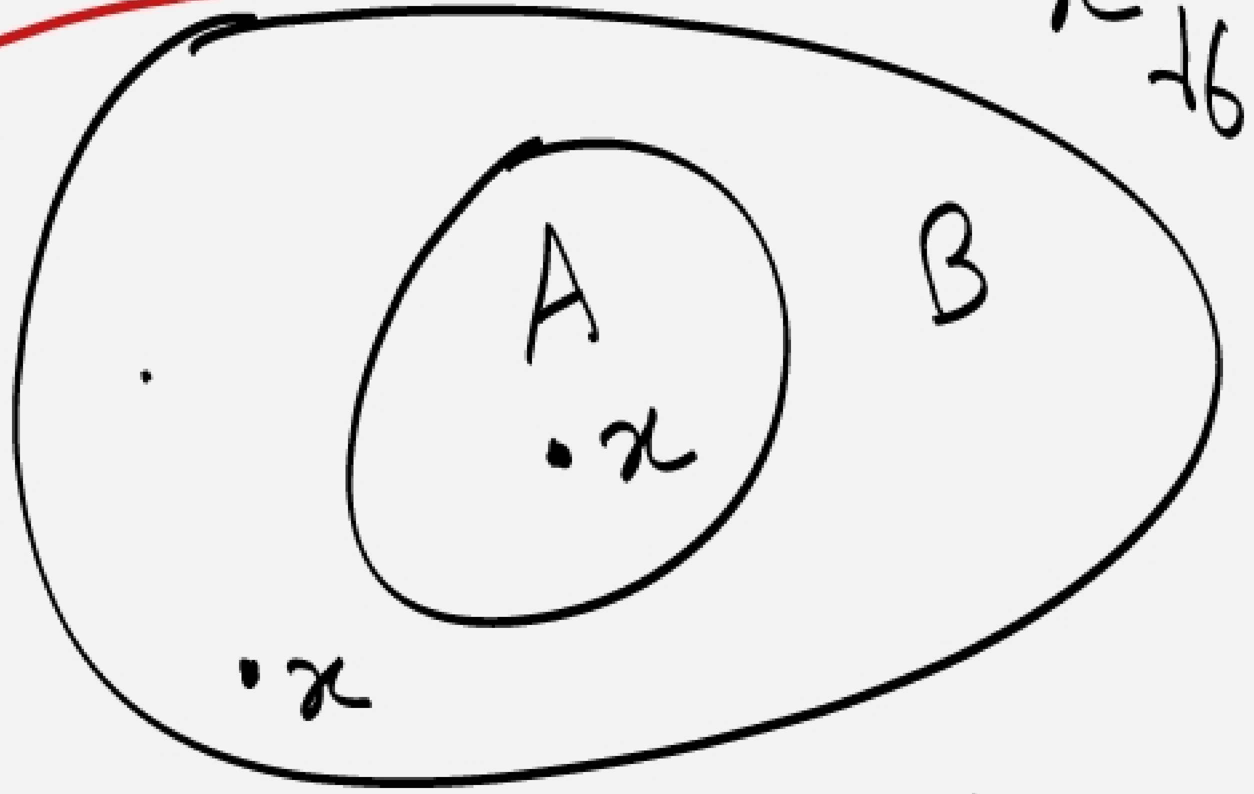
$x \in A, x \notin B$

If $x \in A$ then $x \in B$

$\rightarrow$ If $x \in A \rightarrow x \in B$

$\cdot x$

If $x \in B \xrightarrow{?} x \in A$



$A \subset B$

If $x \notin A$, $x \notin B$

0 0

$\neg q \to \neg p$ Contrapositive $p \to q$

$\neg q \to \neg p$

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

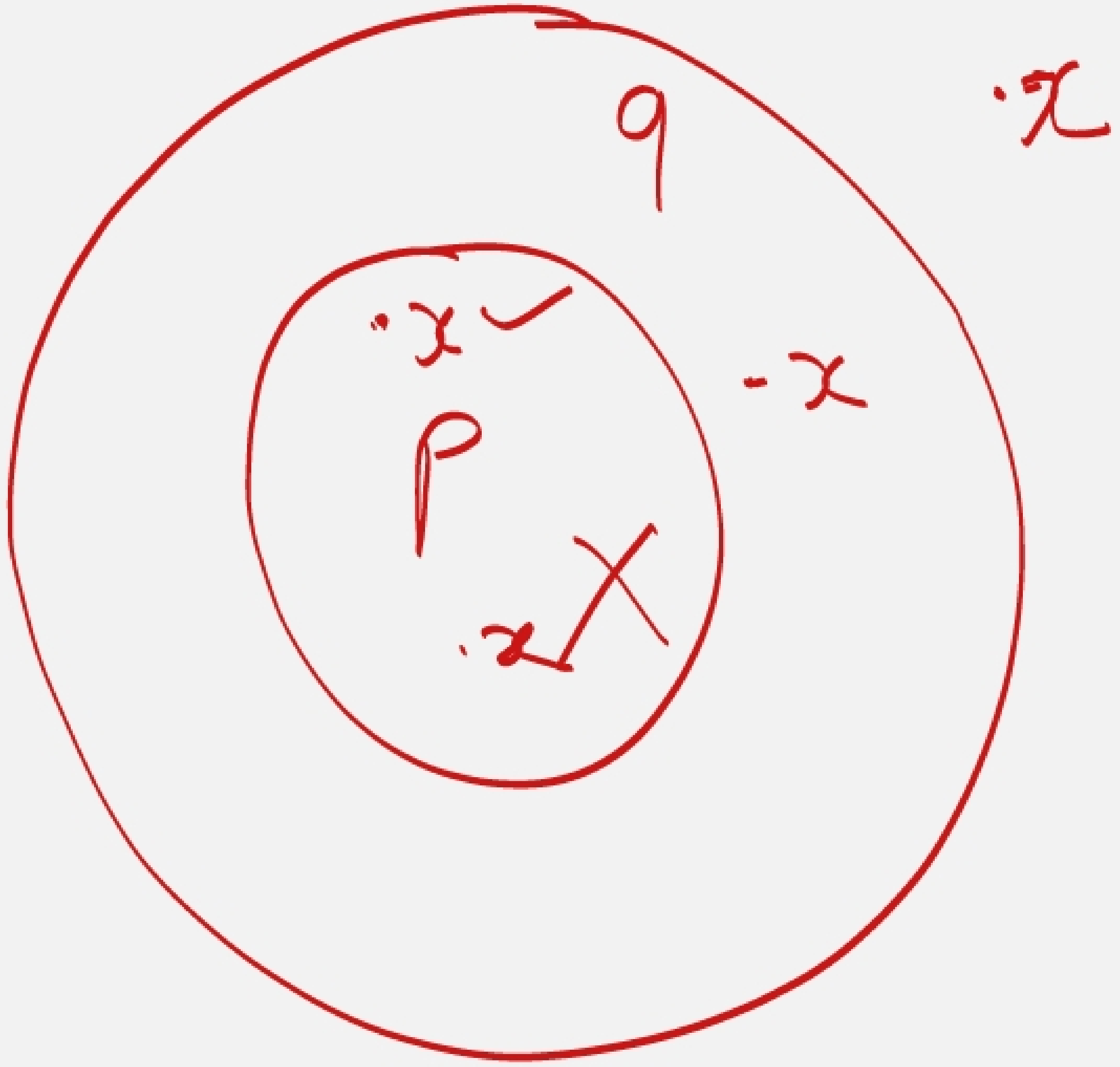
$\overset{p}{x \in A} \Rightarrow \overset{q}{x \in B}$

$\underset{\neg q}{x \notin B} \Rightarrow \underset{\neg p}{x \notin A}$

If $x \in A \to x \in B$

If $x \in B \to x \in A$ $A = B$

$A = B$



p q

$3n+2$ is odd $\to$ n is odd

$\neg q \Rightarrow \neg p$

n is even $\Rightarrow$ $3n+2$ is also even

$p \to q$

$q \to p$

AND

p	q	$p \to q$	$q \to p$	$p \to q \wedge q \to p$	
0	0	1	1	1	
0	1	1	$\boxed{0}$	0	IMPOSSIBLE
1	0	$\boxed{0}$	1	0	IMPOSSIBLE
1	1	1	1	1	

$p \longleftrightarrow q$ ~~Ae~~ Sufficient

If n is even $\Rightarrow$ n is divisible by 2.

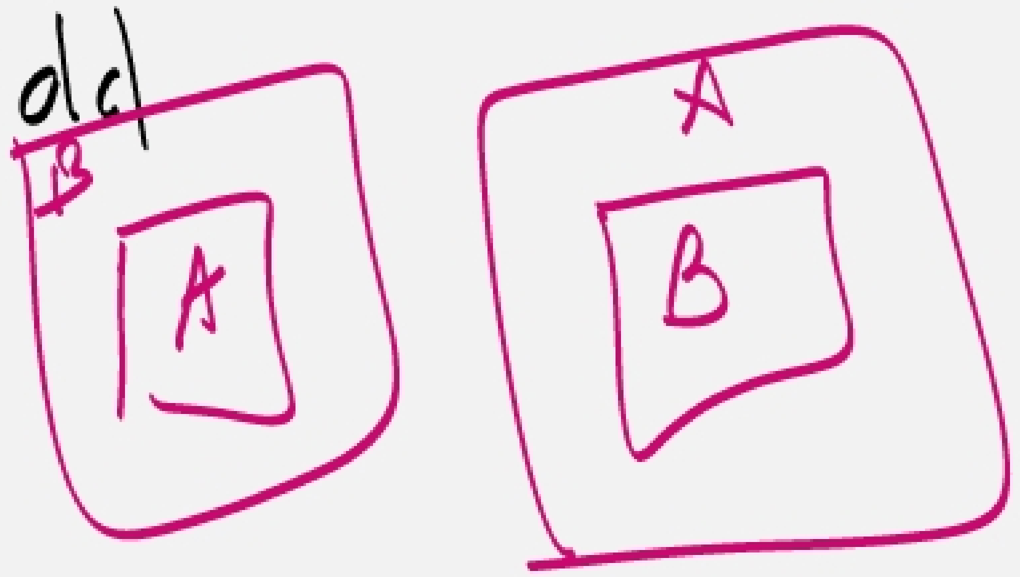
If n is div by 2 $\Rightarrow$ n is even

$q \longrightarrow p$

$p \longleftrightarrow q$

n is odd

$x \in A, x \notin B$



$p \to \neg p$

p	$\neg p$	$p \to \neg p$
0	1	1
1	0	0

$p \to q$

Converse $q \to p$

Contrapositive $\neg q \to \neg p$

Inverse: $\neg p \to \neg q$

Tautology	Logical Equivalance
converse	

$p \to p \vee q$

1
1
1
1

TAUTOLOGY

ALWAYS TRUE

$\neg(p \to q) \wedge q$ ALWAYS FALSE

$\neg(p \to q) \wedge q$
0
0
0
0

CONTRADICTION

A is Logically eq. to B

(A) $p \to q$	(B) $\neg p \vee q$
1	1
1	1
0	0
1	1

$(p \wedge \neg q) \vee (\neg p \wedge q)$

$\vee$	$\wedge$	p	q	
0	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	1	1	1	0

XOR

one of P & E is a good player

E is good

$(p \wedge \neg q) \vee (\neg p \wedge q)$

$p \wedge q = q \wedge p$

$p \wedge (q \wedge r) = (p \wedge q) \wedge r$ (associative)

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ [distributive over $\wedge, \vee$

$p \vee p = p$, $p \wedge p = p$

$p \vee \neg p = 1$ (True)

$p \wedge \neg p = 0$ (F)

Inverse $p \wedge (p \vee q)$

$(p \wedge p) \vee (p \wedge q)$

$p \vee (p \wedge q)$

$p \vee T = T$

$p \wedge F = F$

$p \vee (p \wedge q) = p$

$p \wedge (p \vee q) = p$

$p \wedge (p \vee q)$

$p \wedge T = p$

$p \vee F = p$

$(p \wedge T) \vee (p \wedge q)$

$p \wedge (T \vee q)$ Dis

$p \wedge (T)$

p

$p \vee (p \wedge q)$

$= (p \vee p) \wedge (p \vee q)$

$$\neg(\neg p) = p$$

$$\neg(p \wedge q) = \neg p \vee \neg q$$

$$\neg(p \vee q) = \neg p \wedge \neg q$$

$$(A \cap B)^c = A^c \cup B^c$$

$$(A \cup B)^c = A^c \cap B^c$$

De Morgan's laws

$\wedge$

OR $\Leftrightarrow$ UNION

AND $\Leftrightarrow$ Intersecti

$\neg$ $\Leftrightarrow$ Complement

p	q	$\neg(p \wedge q)$	$\neg(p \vee q)$	$\neg p$	$\neg q$
0	0	1	1	1	1
0	1	1	0	1	0
1	0	1	0	0	1
1	1	0	0	0	0

$(\neg p)^{T/1}$

Ex $\underline{(p^{0} \vee q^{1})} \stackrel{?}{=}$ If this statement is true

P is false (0) $p^{F/0}$

$\neg P = T$

Statement $\rightarrow$ Conclusion

$(p^{T} \rightarrow q)^{T}$, & P is True

$(p \rightarrow q^{1})^{T}$

p^{T}

$q \Rightarrow$ True

$(p \rightarrow q)^{T}$

$q \rightarrow$ True [no conclusion / Inference]

0	0	1
0	1	1
1	0	0
1	1	1

$$(p \to q),\ (q \to r) \overset{?}{\Rightarrow} (p \to r)$$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	r	$p \to r$
0	0	0	1	1	1	0	1
0	1	0	1	0	0	1	1
1	0	1	0	1	0	0	0
1	1	1	1	1	1	1	1 ...

when is $p \to r$ false

p	r
1	0

$p \to q$		$q \to r$	
1	0	1	0

Permut.
Combi.
$n!$
Binomial
Multinomial
Catalan number
Set → De Morgan laws
Logic
OR, AND, $\neg$
Inferences

Inferences

$p.$
$p \to q$

$\therefore q.$

$p^0 \vee q^1$
$(\neg p)$

$\therefore q$

F
$p' \vee D$
S

Rain
$R \to$ good weather

$(p \wedge (p \to q)) \to q$

$(\neg q)^0$
$p^0 \to q^0$

$\therefore \neg p$

$\neg q = T$
$q \to$

F — p
F — $D \to S$

$(p \to q) \cdot (q \to r), \quad p \to r$

$p' \to q'$

$q' \to r'$

$\therefore p' \to r$

contradiction

Rain $\to$ grounds are wet

grounds are wet $\to$ no play

Hypothetical Syllogism

Shambhari - IITM Z $\to$ ambitious

ambitious $\to$ wakes up early

wake up early $\to$ Sleep early

assume $(p \to r)^{o}$

$(p' \to q)'$

$q = 1$

$(p \vee q)$

$(q \vee r)$

$p \; q \; \cancel{r} = 0$

$\to$ no conclusion

p'

$(p' \to q')$

$(p' \to (q' \to r'))$

$p^0 \to q^0$

$r' \to (\neg q)'$

r'

p	$\neg p$	q	r	$p \vee q$	$\neg p \vee r$
0	1	0	1	0	1
0	1	1	0	1	1
1	0	0	1	1	1
1	0	1	0	1	0

$p^0 \vee q'$

$(\neg p)' \vee r^0$

r^0

$\therefore q \vee r$

$(\neg p) \vee s^0 = T$

$((\neg p)$ or $s^0) = T$

$(\neg p)' \vee s^0$

$\neg t \vee (s^0$

$(\neg q) \vee r$

$p^0 \vee q^0$

$\therefore r^0 \vee s$

p	$\neg p$	q	r
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1

$(p \vee q \vee t)$ T when is $\neg p \vee r$ [illegible]