Geometric distribution

Recall:

1. Trials are Bernoulli.
2. Trials are independent and identical.
3. Random Variable, X is the no. of trials required to obtain the first success.

pmf X	1	2	3	....
$P(X=x_i)$	p	$(1-p)p$	$(1-p)^2 p$	....

$0<p<1$

$$P(X=n) = (1-p)^{n-1}p.$$

$$\underline{\sum_{n=1}^{\infty}(1-p)^{n-1}p = 1}$$

$$E[X] = \sum_{n=1}^{\infty} n(1-p)^{n-1} p \quad , \quad 0 < p < 1$$

$(n-1+1)$

$$\sum_{n=1}^{\infty} (n-1+1)(1-p)^{n-1} p = \sum_{n=1}^{\infty} (n-1)(1-p)^{n-1} p + \sum_{n=1}^{\infty} (1-p)^{n-1} p$$

$$= \sum_{n=1}^{\infty} (n-1)(1-p)^{n-1} p + 1$$

$$= \sum_{n=0}^{\infty} n(1-p)^{n} p + 1 = (1-p)\sum_{n=1}^{\infty} n(1-p)^{n-1} p + 1$$

$$E[X] = (1-p)E[X] + 1$$

$$\Rightarrow (1-1+p)E[X] = 1 \Rightarrow \boxed{E[X] = \frac{1}{p}}$$

$E[X^2]$

$$E[X^2] = \sum_{n=1}^{\infty} n^2 (1-p)^{n-1} p = \sum_{n=1}^{\infty} (n-1+1)^2 (1-p)^{n-1} p$$

$$= \sum_{n=1}^{\infty} (n-1)^2 (1-p)^{n-1} p + \sum_{n=1}^{\infty} 2(n-1)(1-p)^{n-1} p + 1$$

$$= \sum_{n=0}^{\infty} n^2 (1-p)^{n} p + \sum_{n=0}^{\infty} 2n(1-p)^{n} p + 1$$

$$E[X^2] = (1-p)E[X^2] + 2(1-p)E[X] + 1$$

$$\Rightarrow (1-1+p)E[X^2] = 2\left(\frac{1-p}{p}\right) + 1 \Rightarrow pE[X^2] = \frac{2(1-p)+p}{p}$$

$$\Rightarrow E[X^2] = \frac{2(1-p)+p}{p^2} = \frac{2-p}{p^2}$$

$$P(X \leq N)$$

$$Var(x) = E[x^2] - (E[x])^2$$

$$= \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}$$

$$\boxed{Var(x) = \frac{1-p}{p^2}}$$

Cdf ✓

$$P(X \leq N) = P(X=1) + P(X=2) + \dots + P(X=N)$$

$$= \sum_{n=1}^{N} P(X=n) = \sum_{n=1}^{N} (1-p)^{n-1} p$$

$$= \sum_{n=0}^{N-1} (1-p)^{n} p$$

$$p\left(\frac{1-(1-p)^N}{1-(1-p)}\right) = 1-(1-p)^N$$

$$\boxed{\begin{aligned} P(X \leq N) &= 1-(1-p)^N \\ P(X > N) &= (1-p)^N \end{aligned}}$$