Matrices

Columns

$$\underset{m\times n}{\underline{\underline{A}}} = \begin{array}{cl} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & a_{ij} & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} & \begin{array}{l} \leftarrow \\ \leftarrow \text{ Rows} \\ \leftarrow \text{ Row } i \\ \\ \end{array} \\ \uparrow \text{Col } j & \end{array} = \{a_{ij}\}_{m\times n}$$

$$\underset{3\times 5}{\underline{\underline{A}}} = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 2 & 4 & 1 & 5 & 2 \\ 3 & 1 & 2 & 4 & 0 \end{bmatrix}$$

$$\underline{\underline{A}}^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 2 \\ 1 & 5 & 4 \\ 1 & 2 & 0 \end{bmatrix}$$

Transposition

3. Multiplication

$$\{a_{ij}\}_{m\times n} \{b_{jk}\}_{n\times p} = \{c_{ik}\}_{m\times p}$$

$$C_{11} = \begin{array}{ccccc} 1 & 2 & 3 & 1 & 1 \\ 1 & 2 & 1 & 0 & 5 \\ \hline \end{array}$$

$$1+4+3+0+5 = 13$$

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 2 & 4 & 1 & 5 & 2 \\ 3 & 1 & 2 & 4 & 0 \end{bmatrix}_{3\times 5} \qquad \underline{\underline{B}} = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 2 \\ 0 & 1 \\ 5 & 0 \end{bmatrix}_{5\times 2}$$

$$\underset{3\times 2}{C} = \underline{\underline{A}}\,\underline{\underline{B}} = \begin{bmatrix} 13 & 12 \\ 21 & 17 \\ 7 & 18 \end{bmatrix}$$

4) Square matrices $m = n$

Identity matrix $I_m = \begin{bmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & 1 & \\ & & & & 1 \end{bmatrix}$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A_{m \times n} I_n = A_{m \times n}$

$I_m A_{m \times n} = A_{m \times n}$

$$\underset{3 \times 5}{\begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 2 & 4 & 1 & 5 & 2 \\ 3 & 1 & 2 & 4 & 0 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = $$

1 2 3 1 1

Augmented

$\times(-2)$

$$x + 2y + 3z = 1$$
$$2x + y + z = 2$$
$$x + 3y = 3$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 3 & 0 & 3 \end{array}\right]$$

$Eq1 \times (-2) + Eq2$

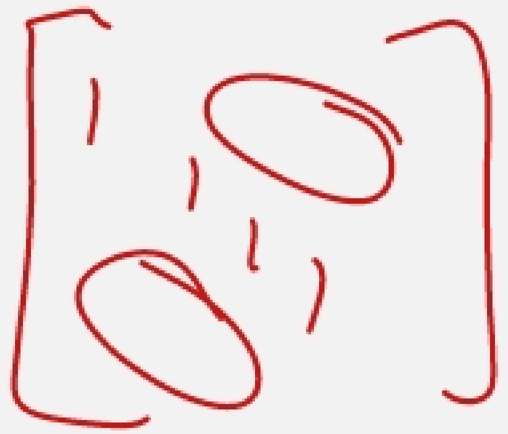
$Eq2\left(\frac{1}{3}\right) + Eq3$

$$x + 2y + 3z = 1$$
$$-3y - 5z = 0 \quad \times \frac{-1}{3}$$
$$y - 3z = 2$$

Gaussian (Elimination) Reduction

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3/-3 & -5/-3 & 0 \\ 0 & 1 & -3 & 2 \end{array}\right]$$

$$x + 2y + 3z = 1$$
$$1y + \frac{5}{3}z = 0$$
$$-\frac{16}{3}z = 2$$



Gauss-Seidel Reduction

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 5/3 & 0 \\ 0 & 0 & -14/3 & 2 \end{array}\right]$$

Augm. matrix

$$2x + 3y = 2 \quad (1)$$
$$y + z = 1 \quad (2)$$
$$x + 2y + z = 3 \quad (3)$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & \frac{1}{2} & 1 & 2 \end{array}\right]$$

$\times -\frac{1}{2}$; add to Row 3

$$\left[\begin{array}{cccc} 1 & 3/2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1/2 & 3/2 \end{array}\right]$$

PIVOT

$$\frac{1}{2}z = \frac{3}{2} \text{ or } z = 3$$

$$CO + 2H_2 = CH_3OH$$

$$\begin{array}{cccc} 32 & -28 & -4 & \\ CH_3OH & - CO & - 2H_2 & = 0 \\ A_1 & A_2 & A_3 & \end{array}$$

$$\begin{bmatrix} 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = 0$$

$$\underline{\nu} = \begin{bmatrix} +1 \\ -1 \\ -2 \end{bmatrix}$$

$$\sum_{j=1}^{S} \nu_j A_j = 0 \longrightarrow \underline{\nu}^T \underline{a} = 0$$

$$\underline{a} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_S \end{bmatrix} \xrightarrow{\text{Repl}} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_S \end{bmatrix} = \underline{m}$$

$$\underline{\nu}^T \underline{m} = 0 \leftarrow \text{mass balance}$$

$$\underline{\nu} = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_S \end{bmatrix}$$

Ex:

$$\left.\begin{aligned} A_1 + A_2 - A_3 - A_4 &= 0 \\ -A_2 + 2A_3 - A_4 &= 0 \end{aligned}\right\}$$

Show that $\frac{1}{2}m_4 < m_3 < 2m_4$

$$m_1 + m_2 - m_3 - m_4 = 0$$

$$-m_2 + 2m_3 - m_4 = 0$$

$$m_1 + m_3 - 2m_4 = 0$$

$$m_1 + m_3 = 2m_4 \qquad m_3 < 2m_4$$