$$\overset{a}{A} \xrightarrow{f} \overset{b}{B}_{\text{co domain}}$$

domain

$f(a) = b$ $\quad f(a_1) = f(a_2)$

$\Rightarrow a_1 = a_2$

$\overset{m}{|d|} \le \overset{n}{|\text{co-domain}|}$

functions

one - one function (Injective)

$\boxed{\text{on-to function (Surjective)}}$ for every b $\exists a$: $b = f(a)$ $\quad |d| \ge |\text{co-domain}|$

1-1 & Onto function (bijective)

$|d = |\text{co-domain}|$

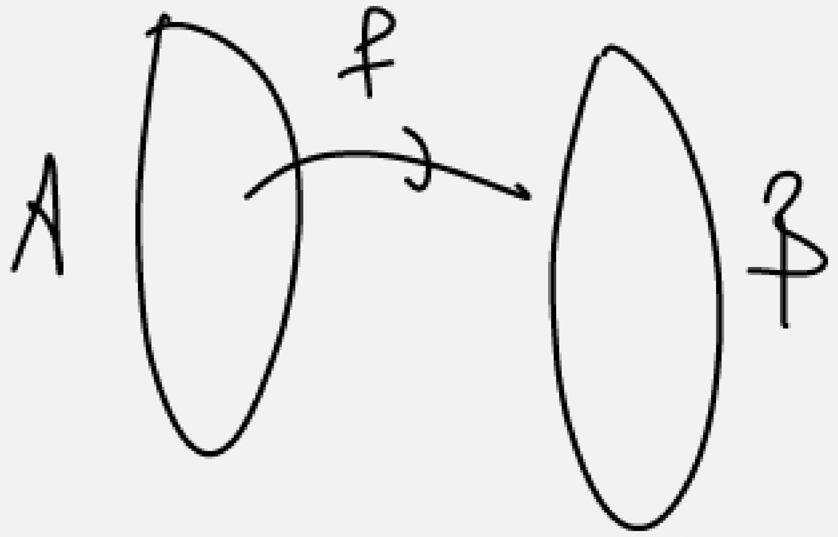
2 $\quad$ 3

3 $\quad$ 4 $\quad |n|$

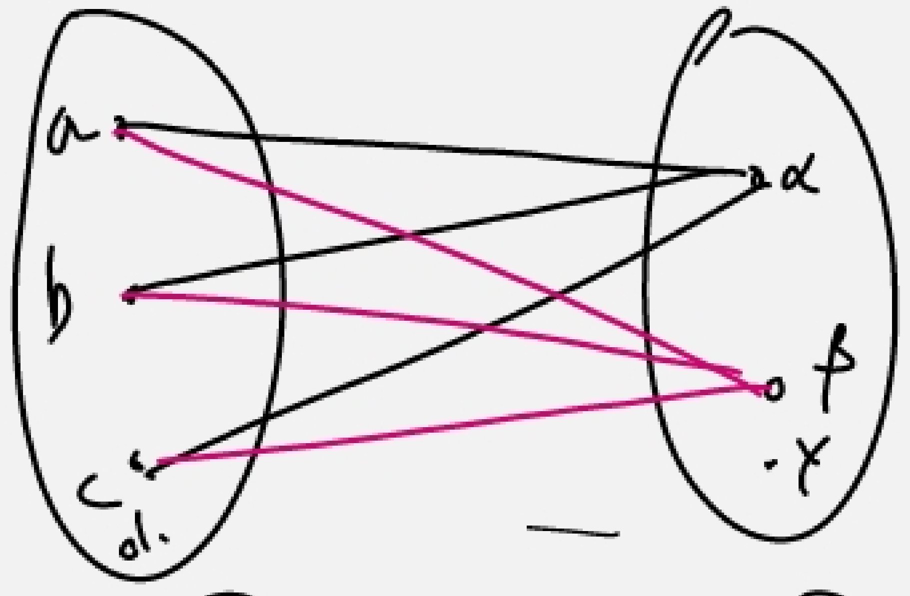
$|m|$

$y = x^2$

$f: \mathbb{R} \to \mathbb{R}$ as function

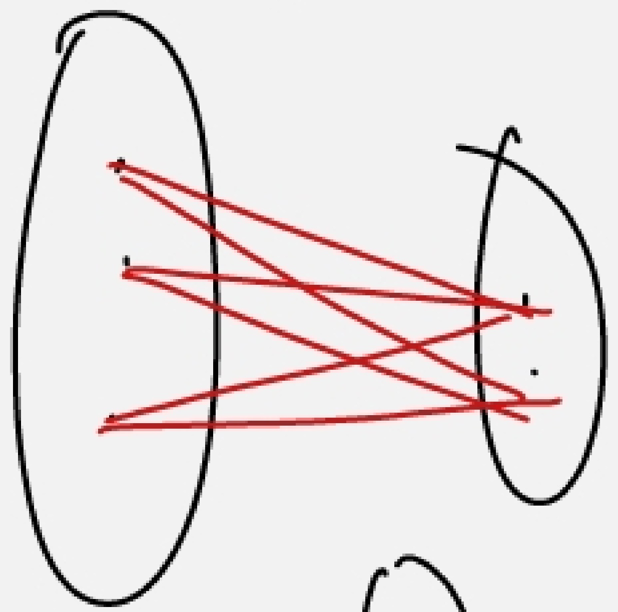


A $\quad$ B

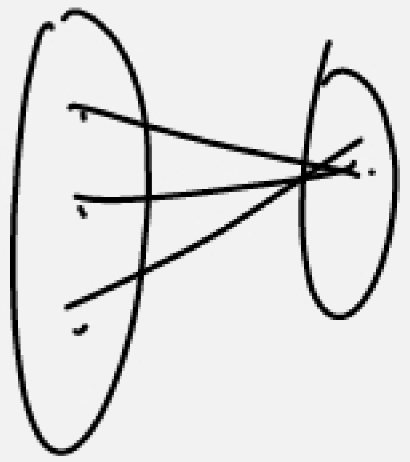


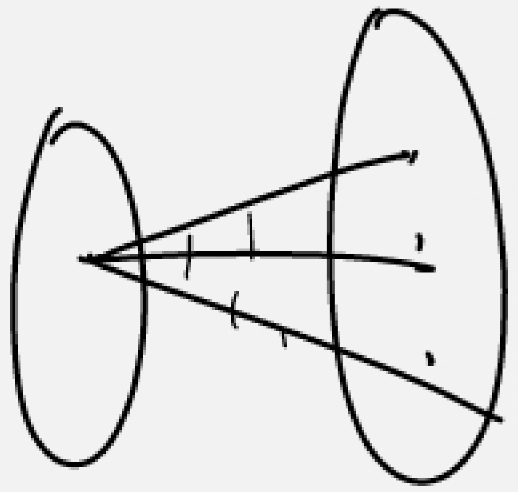
3 $\quad$ 2

$|m|$ 8 $\quad |n|$ $\quad n^m$



a	b	c
α	α	α
α	α	β
α	β	α

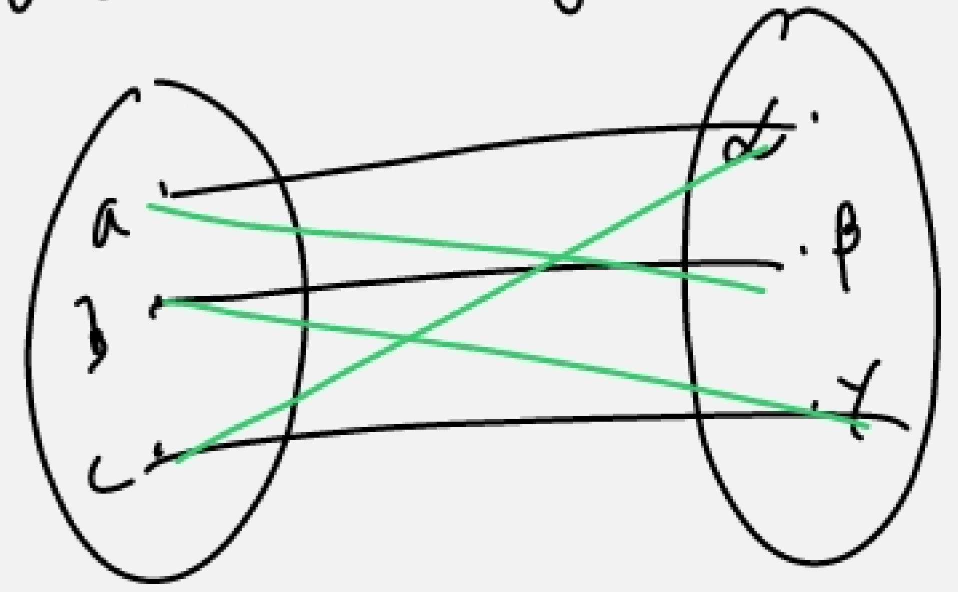




$|m| \to |n|$

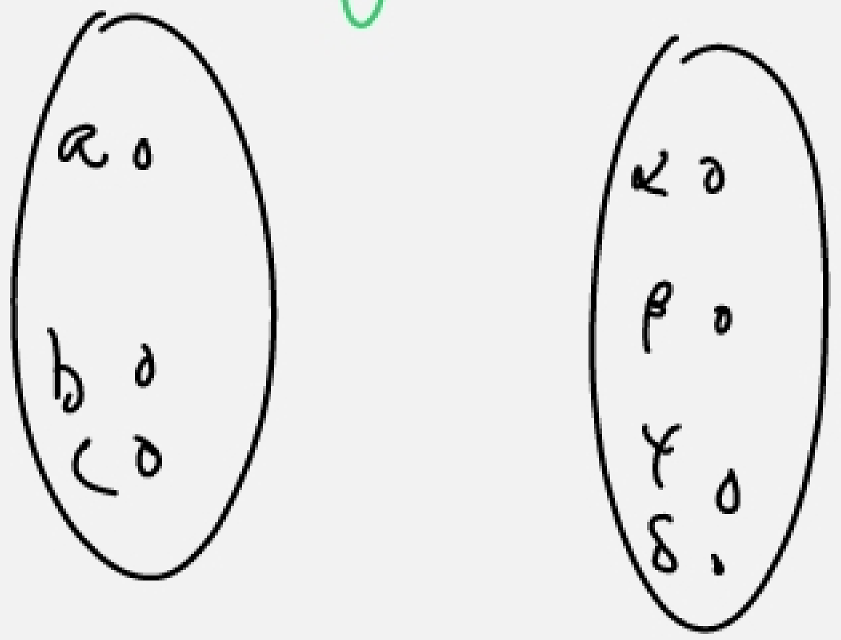
n^m function.

① how many 1-1

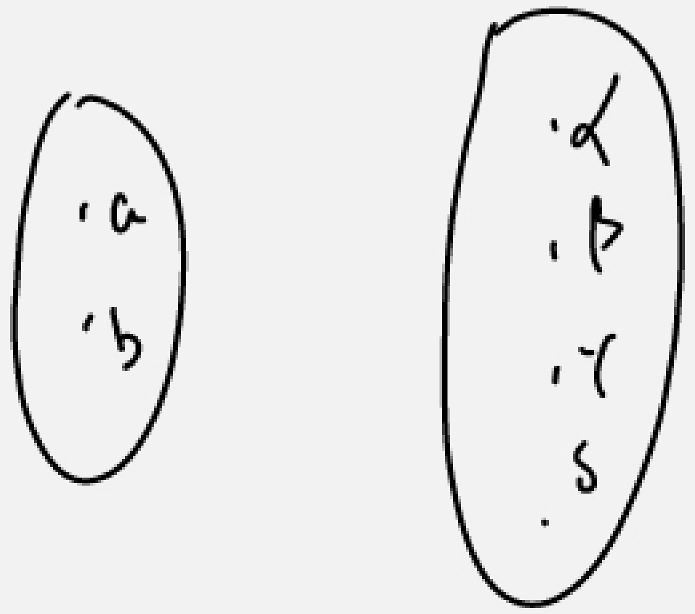


$|m|$ 3 $|n|$ 3

total 1-1 function 3!



$|m| \to |n| \quad (|m| \leq |n|)$



a	b
γ	δ
β	γ

2 out of 4

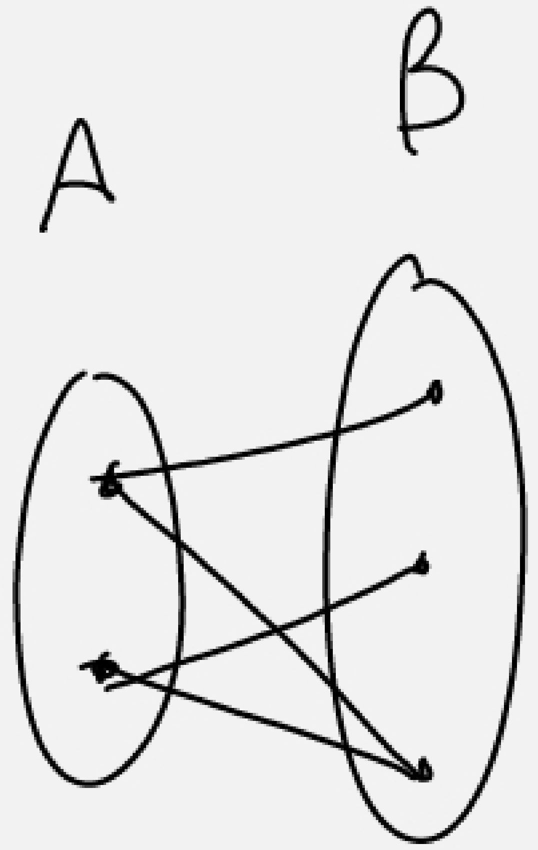
^nP_m

2 of 3

a	b
α	β
α	γ
β	γ
β	α
γ	β
γ	α

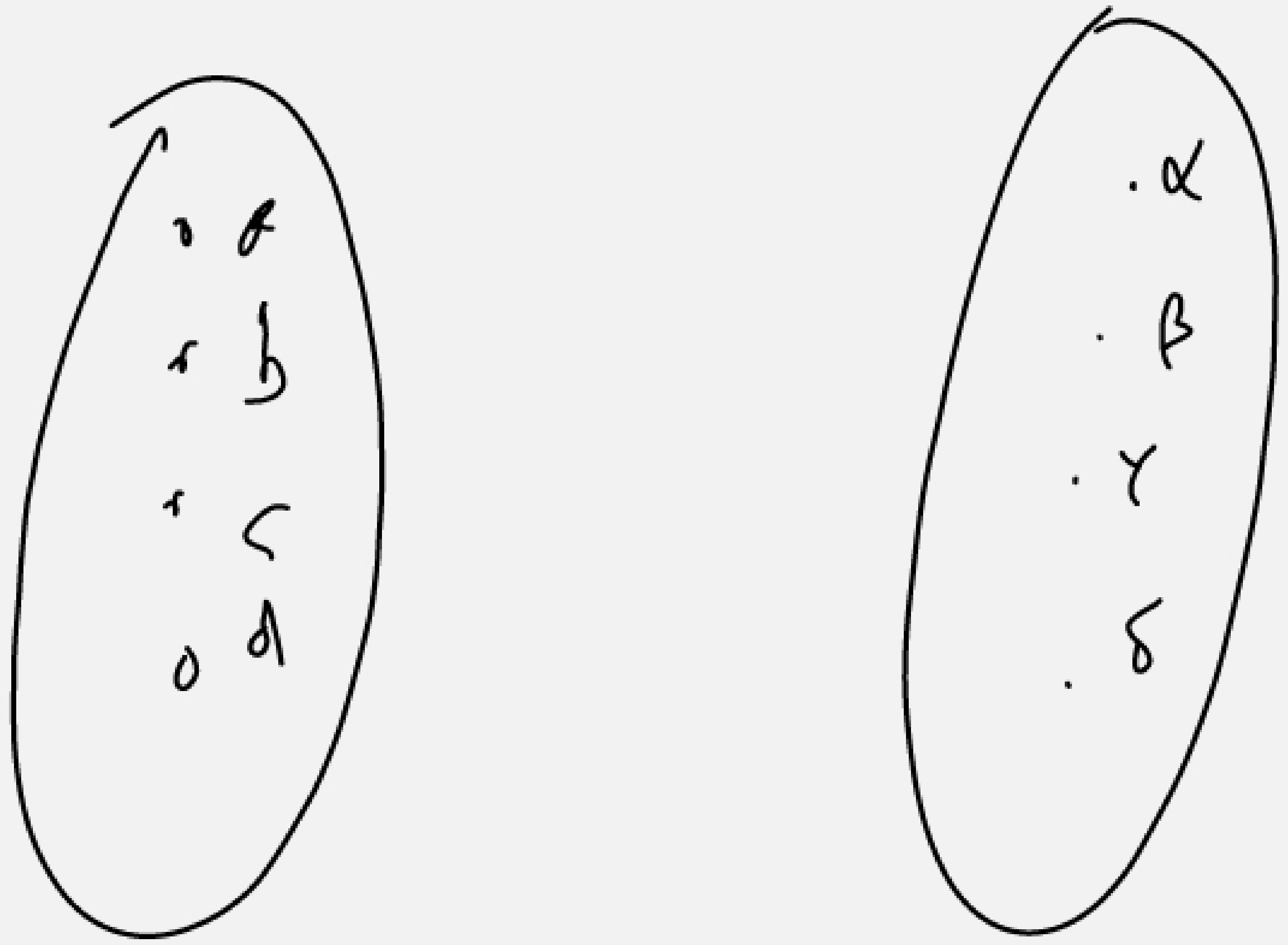
a	b	c
α	β	γ

3 of 4



no. of 1-1 + onto functions

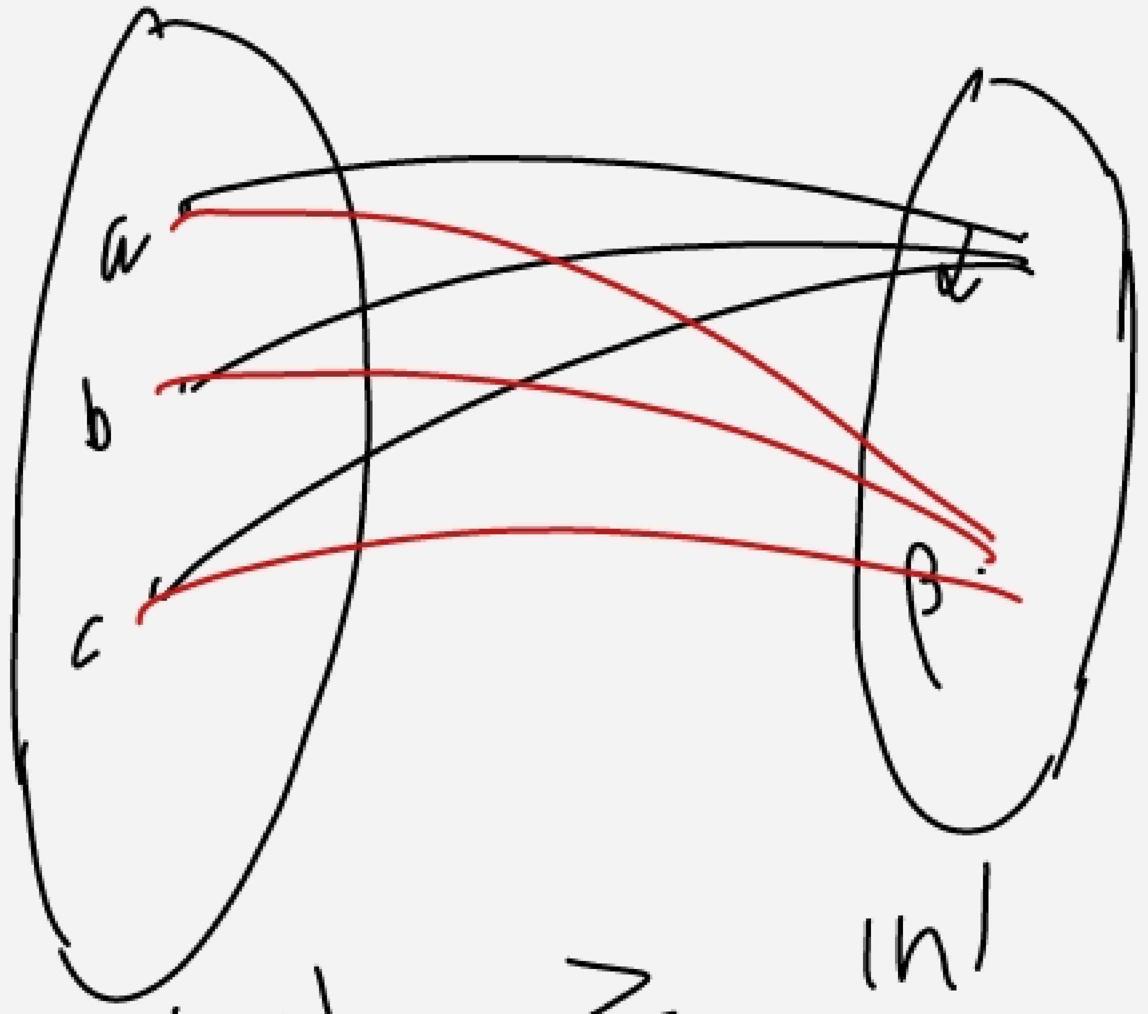
$m = n$



a	b	c	d
α	β	γ	δ
α	β	δ	γ

$4!$

$n!$ OR $m!$



$|m| \geq |n|$

$8 - 2$

$= 6$

a	b	c
α	α	α
α	α	β
α	β	α
α	β	β
β	α	α
β	α	β
β	β	α
β	β	β

How many positive integers not exceeding 1000 are divisible by 3 or 7? [3 or 7 or 11]

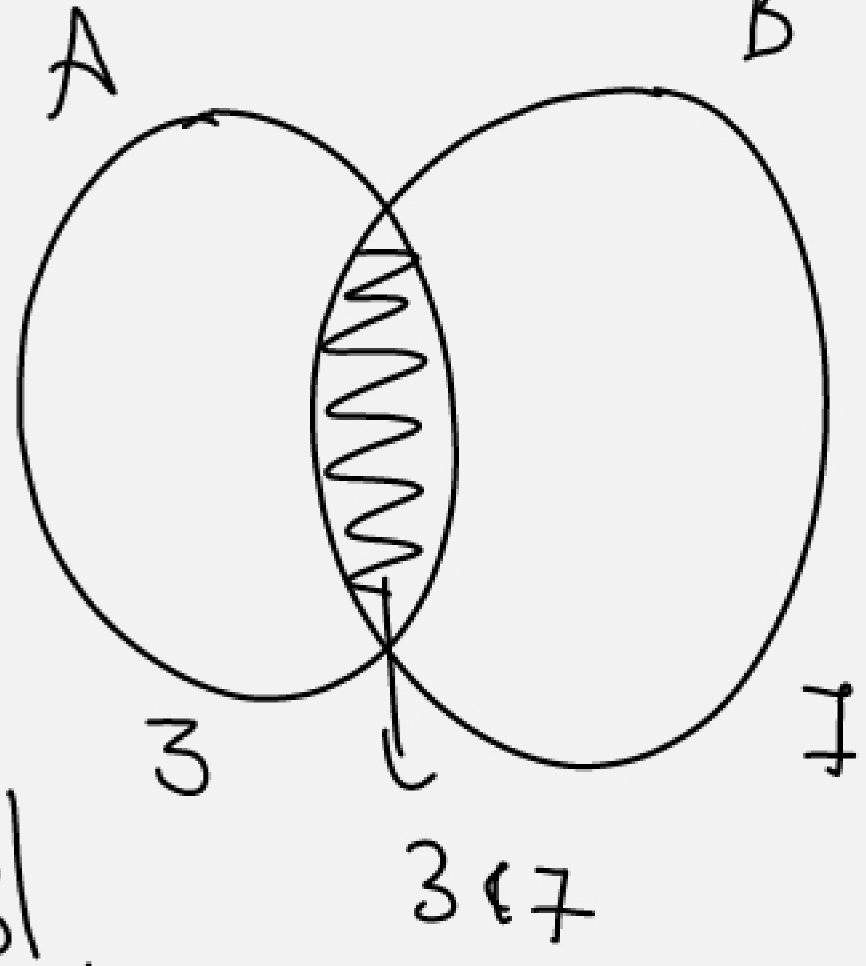
$476 + 4 = 480$

$|A \cap B| \left\lfloor \frac{1000}{3 \cdot 7} \right\rfloor = 47$

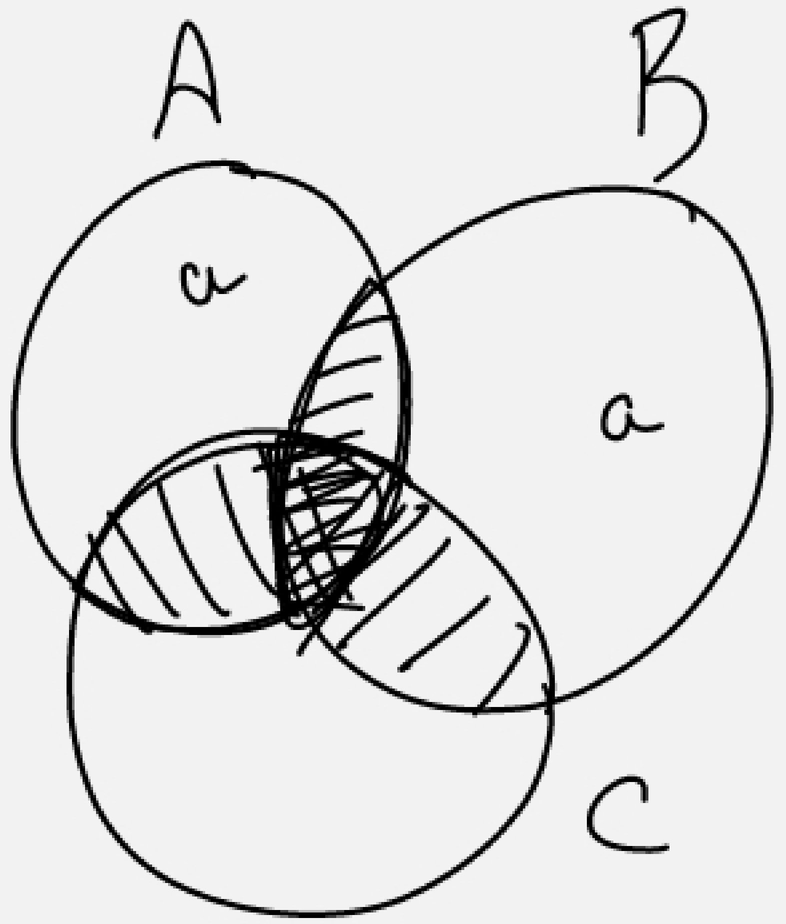
$|A| \left\lfloor \frac{1000}{3} \right\rfloor = 333$

$|B| \left\lfloor \frac{1000}{7} \right\rfloor = 142$

428



$|A \cup B| = |A| + |B| - |A \cap B|$



$|A \cup B \cup C|$

$= |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C|$

$\boxed{+ |A \cap B \cap C|}$

Principle of Inclusion-Exclusion

$|A + B + C + D|$

$(1+1)^n$ $\quad {}^nC_0 = 1$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0 \qquad (1+(-1))^n$$

$$|A_1 \cup A_2 \ldots \cup A_n| \qquad {}^nC_0 = 1$$

$$= \sum_{1 \le i \le n}^{n} |A_i| - \sum_{1 < i < j < n}^{n} |A_i \cap A_j|$$

$$+ \sum_{1 \le i < j < k \le n}^{n} |A_i \cap A_j \cap A_k| - $$

$$+ (-1)^{n+1} |A_1 \cap A_2 \ldots \cap A_n|$$

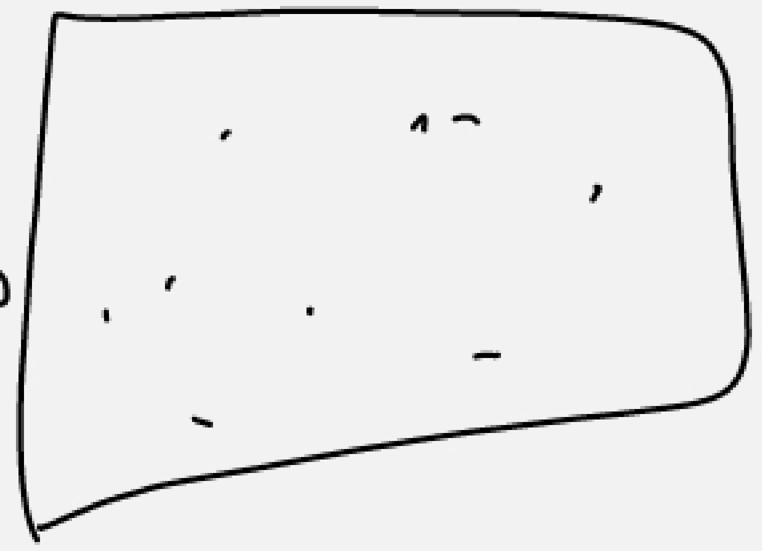
'a' is a member of r sets $\quad 1 \le r \le n$

This 'a' is counted r times in $\sum |A_i|$ $\quad {}^rC_1 \qquad {}^rC_m$ times for a summation involving 'm' sets of A_i

Counted ${}^rC_2$ times in $|A_i \cap A_j|$

$${}^rC_1 - {}^rC_2 + {}^rC_3 + \ldots (-1)^{r+1}\, {}^rC_r \stackrel{?}{=} 1$$

$$\left\{ \begin{array}{l} {}^nC_0 - {}^nC_1 + {}^nC_2 + \ldots = 0 \\ \hline {}^nC_0 - [\quad \text{count} \quad] = 0 \end{array} \right.$$

78


$x_i \geq 0$

$$x_1 + x_2 + x_3 = 11$$

$$78 = \binom{13}{2} = \binom{13}{11}$$

$x_1 \leq 3,\ x_2 \leq 4,\ x_3 \leq 6$

$$\binom{n+r-1}{n} = \binom{n+r-1}{r-1}$$

$\underset{P_1}{x_1 \geq 4},\ \underset{P_2}{x_2 \geq 5},\ \underset{P_3}{x_3 \geq 7}$

9 1 1

$x_1 \geq 4,\ x_2 \geq 0,\ x_3 \geq 0$

$\underbrace{[4+y_1]}_{x_1} + x_2 + x_3 = 11$; $y_1 + x_2 + x_3 = 7$ $\binom{9}{2}$

50 as a sum of 4 (positive) integers?

$x_1 + x_2 + x_3 + x_4 = 50$

$x_i \geq 0$ $\binom{53}{3}$ $x_1 + x_2 = 1$

$x_i > 0$ $\binom{49}{3}$ $x_2 - x_1 = 6$

$2x_1 - x_2 = 19$

$y_1 + 1 + y_2 + 1 + y_3 + 1 + y_4 + 1 = 50$

$y_1 + y_2 + y_3 + y_4 = 46$ $x_1 + x_2 = 1$

$y_i \geq 0$

no of onto functions

$$x_1 + x_2 + x_3 = 11$$

$$\binom{13}{2} = 78$$

$x_1 \le 3$, $x_2 \le 4$, $x_3 \le 6$

$x_1 \ge 4$ $\binom{9}{2}$

$x_2 \ge 5$ $\binom{8}{2}$

$$x_1 + y_2 + 5 + x_3 = 11 \quad x_1 + y_2 + x_3 = 6$$

$x_3 \ge 7$ $\binom{6}{2}$ $x_1 + x_2 + (y_3 + 7) = 11$ $x_1 + x_2 + y_3 = 4$

$x_1 \ge 4$, $x_2 \ge 5$ —

$$(y_1 + 4) + (y_2 + 5) + x_3 = 11$$

$$y_1 + y_2 + x_3 = 2$$

$x_2 \ge 5$, $x_3 \ge 7$ (0)

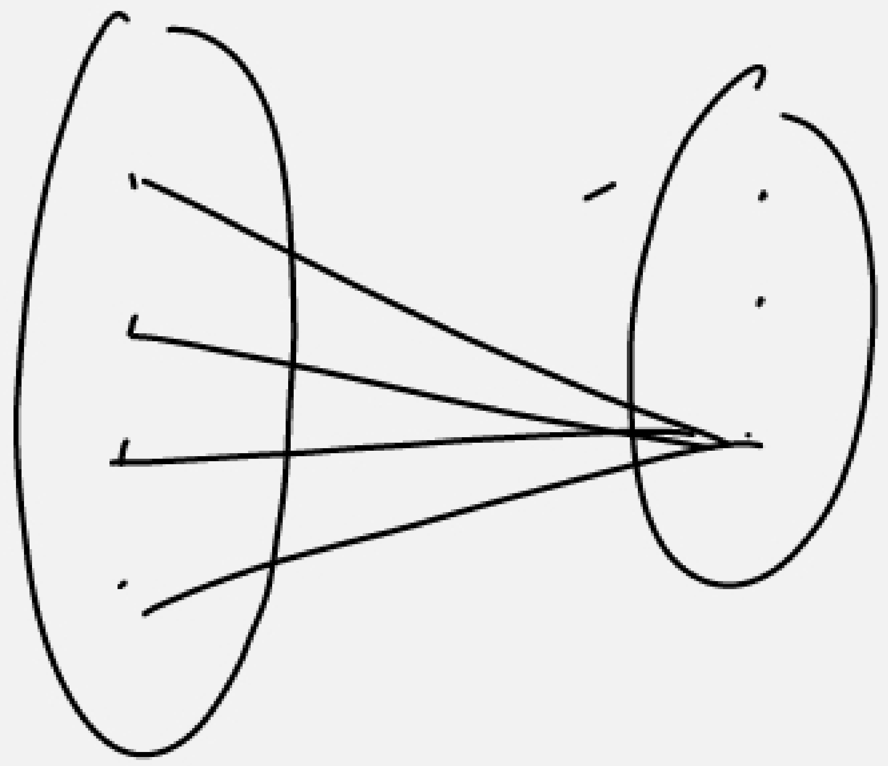
$x_1 \ge 4$, $x_3 \ge 7$ $\binom{2}{2}$

$$78 - \binom{9}{2} - \binom{8}{2} - \binom{6}{2} + \binom{4}{2} + 0 + \binom{2}{2} - 0 = 6$$

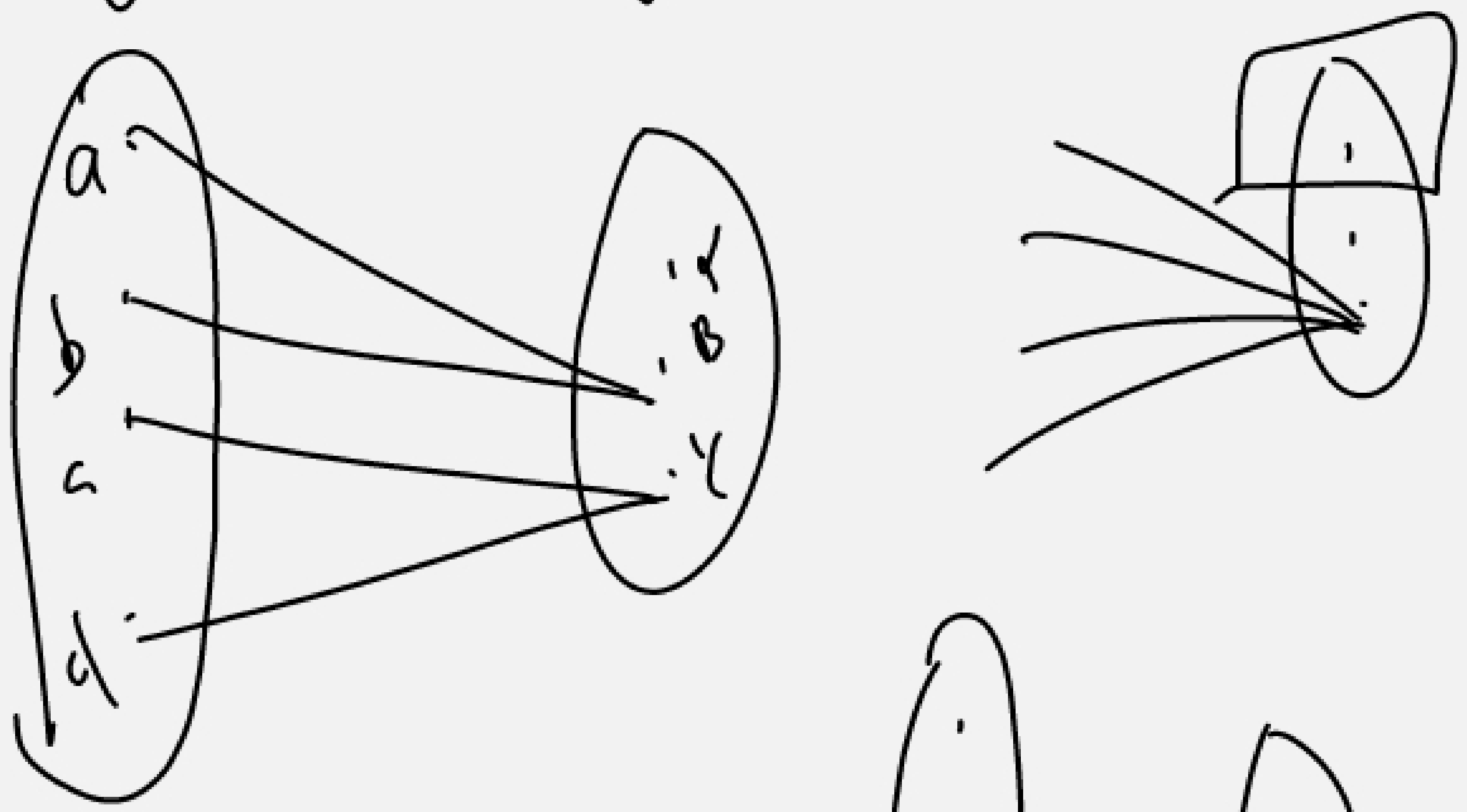
$x_1 \ge 4$, $x_2 \ge 5$, $x_3 \ge 7$ (0)

$$x_1 + x_2 + x_3 + x_4 = \cancel{2}\,20$$

$0 \le x_i \le \cancel{5}\ 10$



no. of onto function:



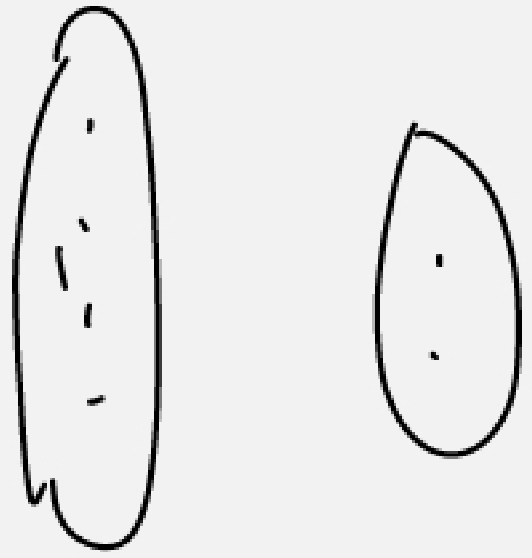
3^4

$- 3 \cdot 2^4$

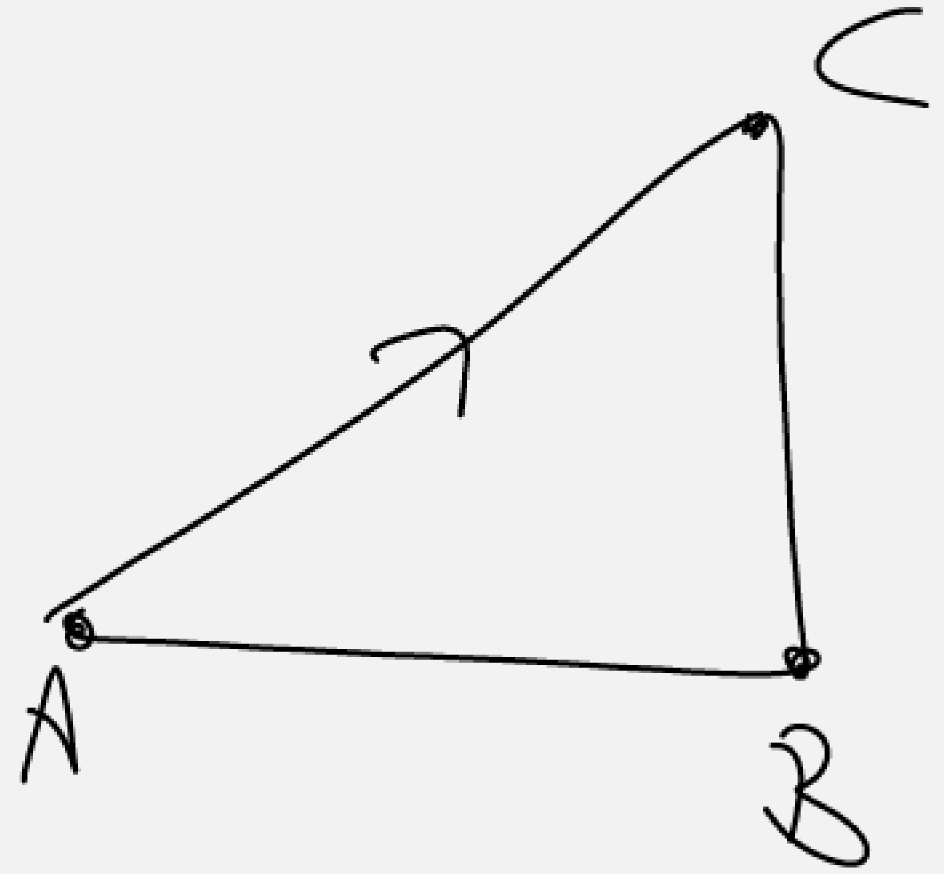
$+ 3 \cdot 1^4$

$=$

$\underline{P(\alpha)}$



Student: what is application of Pythagorous theorm



How many positive integers from 1 - 2000 are not divisible by 5, 7, 11.

$|S| = 2000$

$N(P_1)$: nos. divisible by 5 $\frac{2000}{5\ 4} = 400$

$N(P_2)$ 7 $\left\lfloor \frac{2000}{6\ 7} \right\rfloor = 285$

LCM of (4, 6)

$N(P_3)$ 25 $\left\lfloor \frac{2000}{11\ 9} \right\rfloor = 181$

$N(P_1, P_2) = \frac{2000}{35} = 57$; $N(P_2 P_3) = 2000 / 77$ 6, 9 $N(P_1 P_3) = 2000 / 55$ 4, 9 36

$N(P_1 P_2 P_3) = \frac{2000}{385} = 5$

$\bar{N} = N - [N(P_1) + N(P_2) + N(P_3)]$

$+ [N(P_i, P_j)]$

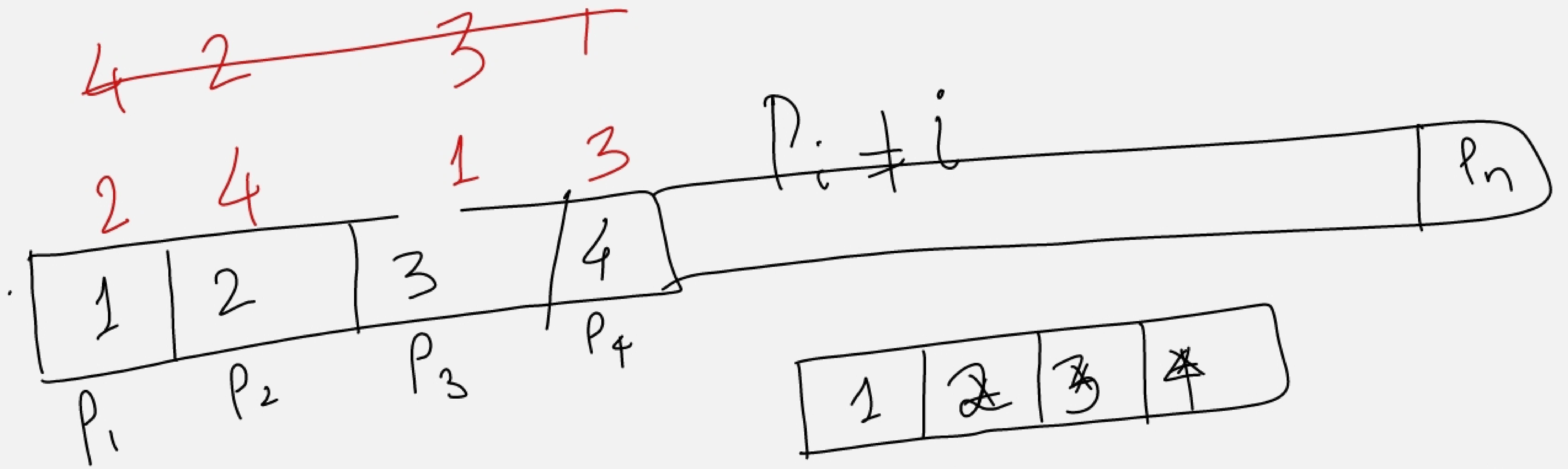
$- N(P_1 P_2 P_3)$

$\frac{2000}{2\ 2\ 2\ 3}$

4, 6, 9.

$\frac{2000}{4}$ $\frac{2000}{6}$

$LCM \frac{2000}{(4, 6)}$



$|m| \to |n|$

n^m

$4!$

$|S| = 24$

$$\binom{4}{0}4! - \binom{4}{1}3! + \binom{4}{2}2! - \binom{4}{3}1! + \binom{4}{4}0!$$

$N(P_1)$ $\quad P_1 \Leftrightarrow 1$ $\quad 3!$

$N(P_2)$ $\quad 3!$

$N(P_3)$ $\quad 3!$ $\quad N(P_4) = 3$

$S = 26$

$\boxed{|S| = 26!}$

ONE, QUIZ, PART

$|26!| - (24! + 23! + 23!)$

$+ (21! + 20! + 21!)$

$- (18!)$

AUNEBC . . .

24!

$N(P_1)$ → no. of words which contain ONE

QUIZ

PART

$\underbrace{\text{ONE}}_{1} \cdot \underbrace{\text{ABCDFG} \ldots \quad \text{Z}}_{23}$

$\underbrace{\text{QUIZ}}_{1} \left[\underbrace{\text{A} \ldots \quad \text{Y}}_{22}\right]$

23!

23!

$N(P_2)$

$N(P_3)$

$N(P_1 P_2)$

$N(P_2 P_3)$

OQNUEIZ

$\underbrace{\text{ONE}}_{1} \; \underbrace{\text{QUIZ}}_{1} \; \underbrace{\quad\quad\quad}_{19}$

$19 + 1 + 1 = 21!$

$\underbrace{\text{QUIZ}}_{1} \; \underbrace{\text{PART}}_{1} \; \underbrace{\quad\quad\quad}_{18}$ $N(P_1 P_2)$

$20!$

$= 21!$

$N(P_1 P_2 P_3) = 18!$

$\underbrace{\text{ONE}}_{1} \; \underbrace{\text{PART}}_{1} \; \underbrace{\text{QUIZ}}_{1} \; \underline{\quad 15 \quad}$

floor $\lfloor x \rfloor$

Celing $\lceil x \rceil$

$\lfloor 1.2 \rfloor = 1$

$\lceil 2.1 \rceil = 3$ $\quad \lfloor 2.1 \rfloor = 2$

$\lceil 1.2 \rceil = 2$

$\lfloor -\frac{1}{2} \rfloor = -1$

$\lceil -\frac{1}{2} \rceil = 0$

floor function:

$x \in \mathbb{R}$. $\quad f(x) \to \mathbb{Z}$.

it assigns the largest integer $\leq x$

Celing function:

assigns the Small integer $\geq x$

$\lfloor x+n \rfloor = \lfloor x \rfloor + n$

$\lceil x+y \rceil \overset{?}{=} \lceil x \rceil + \lceil y \rceil$

$\lfloor x \rfloor = m$

$m \leq x < m+1$

$m+n \leq \lfloor x+n \rfloor < m+1+n$

$\lfloor x+n \rfloor = m+n = \lfloor x \rfloor + n$

$\lceil x+y \rceil \stackrel{?}{=} \lceil x \rceil + \lceil y \rceil \quad x = \frac{1}{2},\ y = \frac{1}{2}$

$1 \neq 2$

$\lfloor x \rfloor = n$ iff $n \le x < n+1$

$\lfloor x \rfloor = n$ iff $x-1 < n \le x$

$\lceil x \rceil = n$ iff $n-1 < x \le n.$

iff $x \le n < x+1$

$x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$

$\lfloor x+n \rfloor = \lfloor x \rfloor + n$

$\lceil x+n \rceil = \lceil x \rceil + n$

$\lfloor x \rfloor = n$

$\lfloor x + \frac{1}{2} \rfloor = n+1$

$\lfloor x \rfloor = \lfloor n \rfloor$

$x = n + \varepsilon$

$\boxed{0 \le \varepsilon < \frac{1}{2}}$

$\boxed{\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor}$

$2x = 2n + 2\varepsilon \quad 4 \quad \lfloor 2x \rfloor = 2n. \left\{ \lfloor x \rfloor = n \right.$

$\lfloor x + \frac{1}{2} \rfloor = \lfloor n + \varepsilon + \frac{1}{2} \rfloor = n$

$\frac{1}{2} \le \varepsilon < 1$

$\lfloor 2x \rfloor = \lfloor 2n + 2\varepsilon \rfloor = \lfloor 2n+1 + 2\varepsilon - 1 \rfloor = 2n+1$

$\lfloor x + \frac{1}{2} \rfloor = \lfloor n + \frac{1}{2} + \varepsilon \rfloor = \lfloor n+1 + \varepsilon - \frac{1}{2} \rfloor = \lfloor n+1 \rfloor$

$1, 2, 4, 8, 16 \ldots$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16}$

Geometric progression

$a_1 \quad a_2 \quad a_3 \quad a_n = r a_{n-1}$

$a, \; ar, \; ar^2 \ldots$

Sequences

$a_1 = 1$

$r = 4$

$a_2 = r a_1$

$a_3 = r a_2 = r^2 a_1$

$\boxed{a_n = r a_{n-1}}$

$a_n =$

$\boxed{n! = n(n-1)!}$

Arithmetic progression

$2 \quad 4 \quad 6 \quad 8 \ldots$

$a_0 = 1$

$a_1 = 2$

$\underbrace{a+b, \; a+2b}_{b}, \; \underbrace{a+2b, \; a+3b}_{b}, \; \underbrace{a+3b, \ldots}_{b}$

$n=0 \quad n=1 \quad n=2 \quad n=\ldots$

$a_n = a_{n-1} - a_{n-2}$

$a_2 = 2 - 1 = 1$

$a_3 = a_2 - a_1 = -1$

$x(0) = 1$

$\sum_{n=1}^{10}$

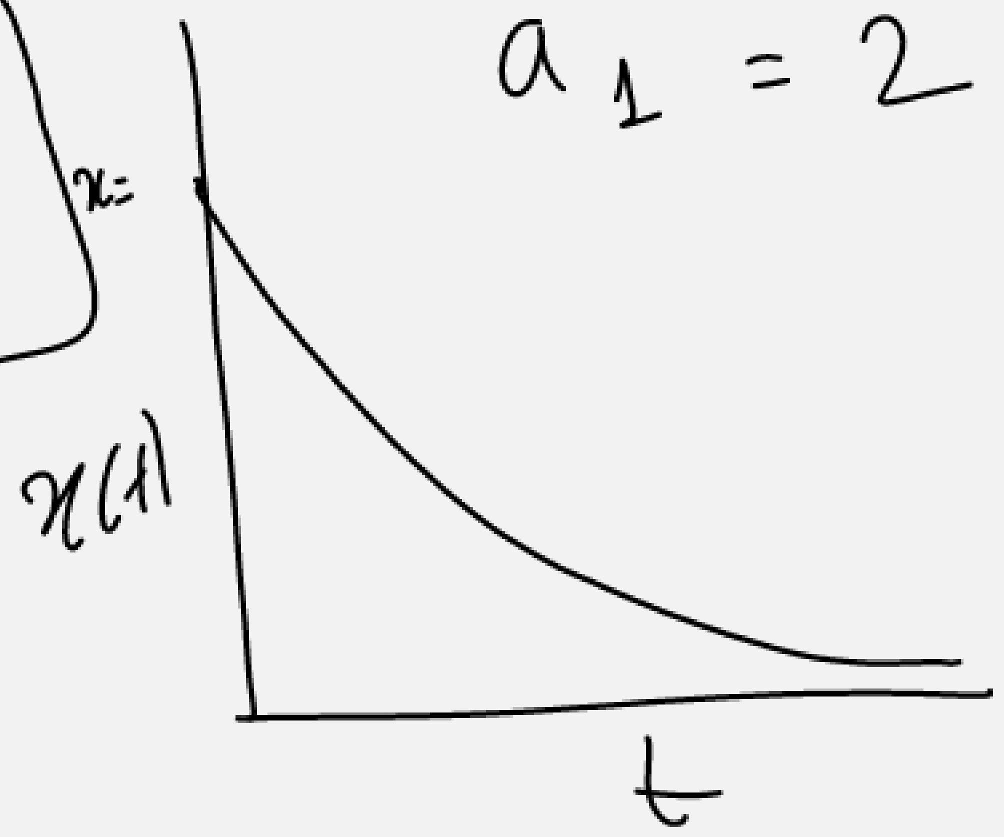
$x(1) = e^{-1} \cdot 1$

$1 + 2 + \ldots + 10$

$\frac{10(10+1)}{2} = 55$

$\boxed{\frac{dx}{dt} = -x}$

$x(t) = e^{-t} x(0) = e^{-t} \cdot 1$



recurrence relations

$a_n = r\, a_{n-1}$

$a_0 = \frac{1}{4}$

$r = 4.$

$a_1 = 4 \cdot a_0$

$a_2 = 4 \cdot a_1 = 4^2 \cdot a_0$

$a_3 = 4 \cdot a_2 = 4^3 \cdot a_0$

$\vdots$

$a_n = 4 \cdot a_{n-1} \dots = 4^{(n)} \cdot a_0$

$A_4 = 1.1 A_3$

$= (1.1)^3 A_1$

Sequence

$$\boxed{a_n = r^n \cdot a_0}$$

$X(t) = e^{-at} \cdot X(0)$

Compound

$M_1 = 100$

$M_2 = 110 = (1.1)100$

$M_3 \quad (1.1)110 = 121$

$M_4 = (1.1) \cdot 121$

$= 133.1$

$\frac{dx}{dt} = -aX \qquad X(0)$

Simple Interest

$M_1 \quad 100\$ \quad 10\%$

$M_2 = 110$

$M_3 = 120$

$A_n = (1.1) A_{n-1}$

$A_n = (1.1)^{n-1} A_1$

$A_2 = 1.1 A_1$

$A_3 = 1.1 \cdot A_2 = (1.1)^2 A_1$

$a_n = a_{n-1} + a_{n-2}$

$0, 3, 9, 18, 30, 45, \ldots$

$a_2 - a_1 = 3$

$a_3 - a_2 = 6$

$a_4 - a_3 = 9$

$a_5 - a_4 = 12$

$a_n = 3(n-1) + a_{n-1}$ $(a_1 + a_2\ a)$

$n \qquad 2^n$

$a_n = a_{n-1} + 3n$

$(a_0, a_1, \ldots)$

$P\left(1 + \frac{r}{100}\right)^n$

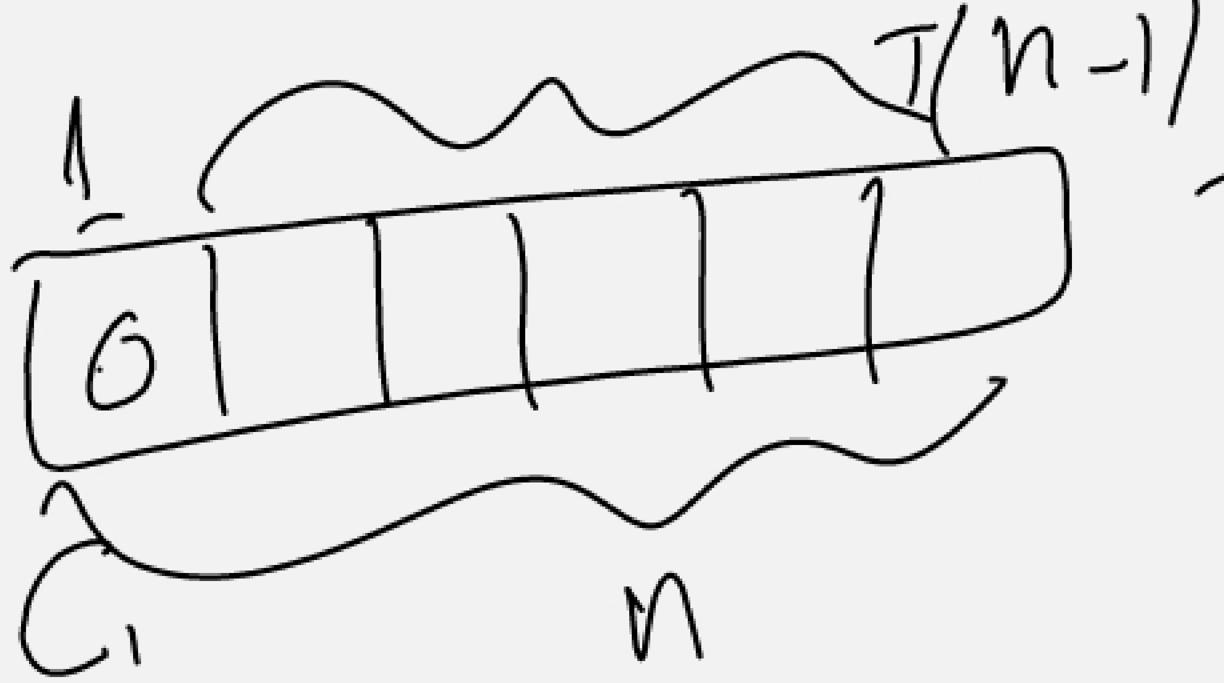
$a_n = \frac{n}{2} \cdot n +$

$a_n = f(a_{n-1}, a_{n-2})$

a_n

$1, 2, 3, 5, 8, 13, 21, \ldots$

$\boxed{a_n = a_{n-1} + a_{n-2}}$



$T(n)$

C_1

$T(1) = 2$

$T(n) = 2^{n-1} T(1)$

$= 2^{n-1} \cdot 2$

$= 2^n$

C_1

$C_1 \; C_2$

$C_1 \; C_2 \; \boxed{C_3}$

$C_1 \; C_2 \; C_3 \; \boxed{C_4 \; C_5}$

$C_1 \; C_2 \; C_3 \; C_4 \; C_5 \; \boxed{C_6 \; C_7 \; C_8}$

$n - K = 1$

$K = n - 1$

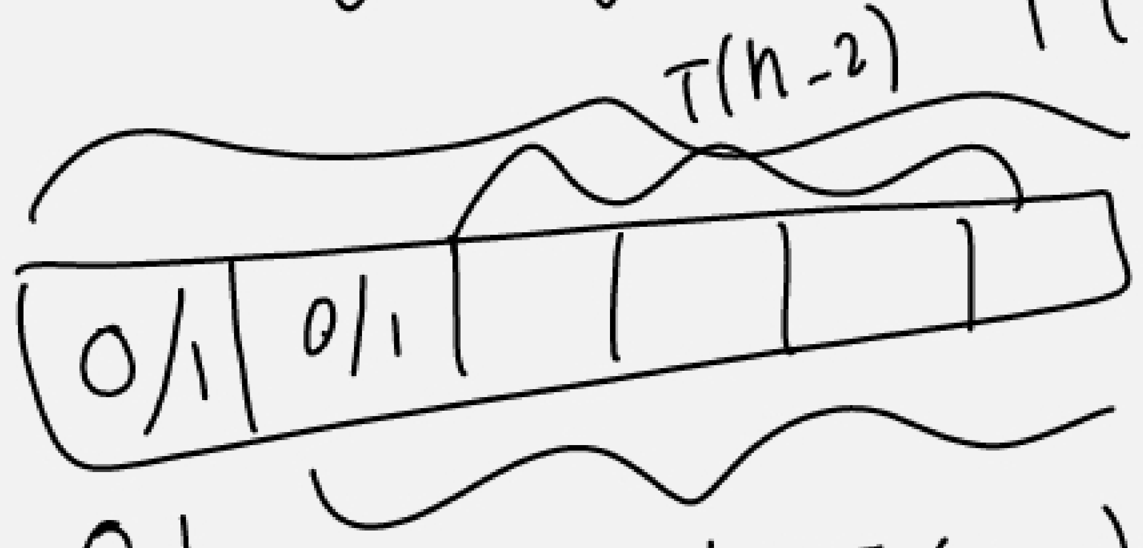
$$C_n = \underbrace{C_{n-1}}_{\text{no of pairs in prev. month}} + \underbrace{C_{n-2}}_{\text{new born,}}$$

$T(n) \quad T(n-1)$

2^n

How many binary digit strings are possible of length n.

$T(n)$



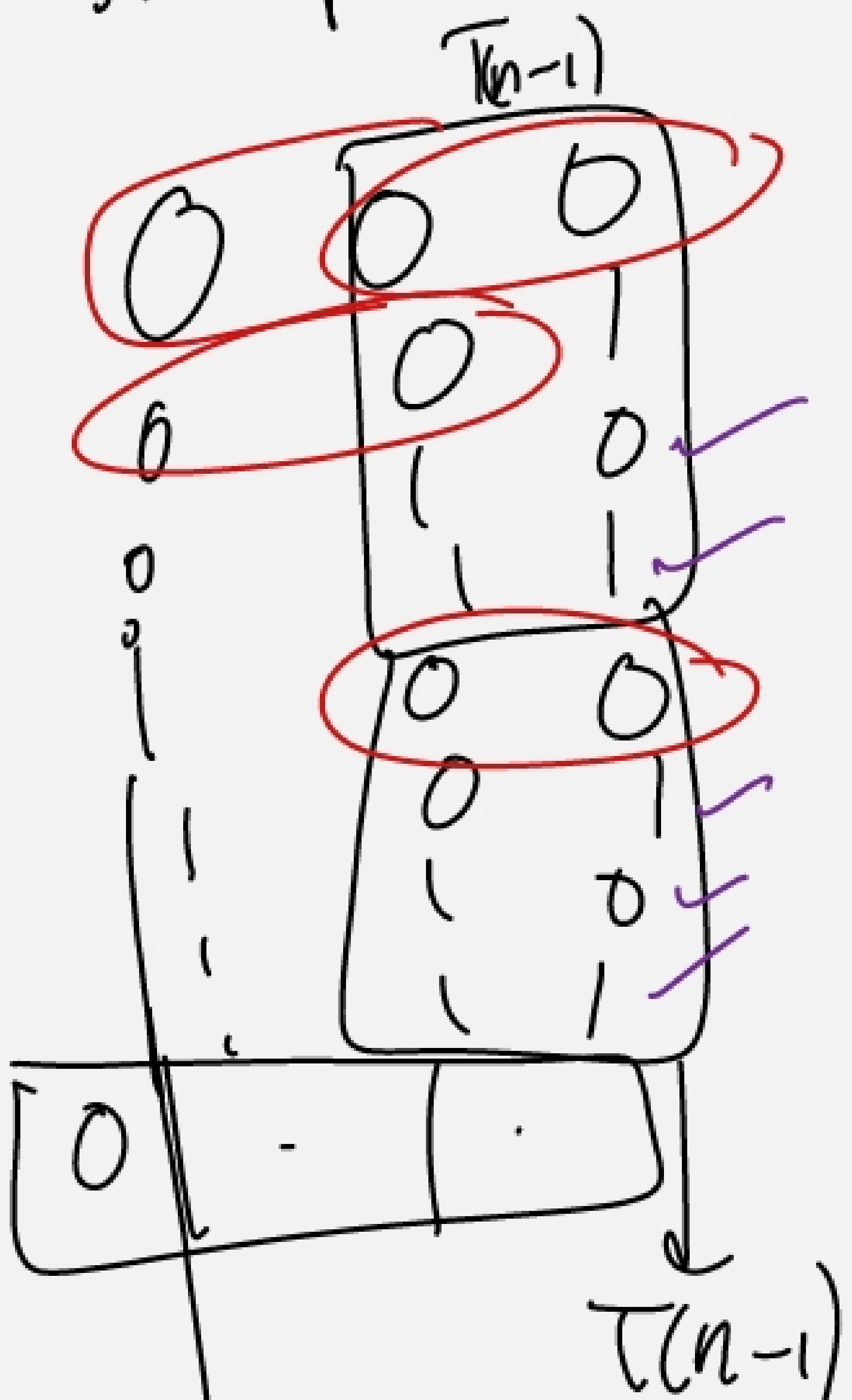
$$\boxed{T(n) = T(n-1) + T(n-1)}$$

$= 2\,T(n-1)$

$= 2 \cdot 2\,[T(n-2)] = 2^2\,T(n-2)$

$= 2^K\,T(n-K)$

$= 2^{n-1}\,T(1)$



$T(n-1) \quad T(1)$

$T(1)$

0

1

T(n) T(n-1)

T(1) 0
1

T(2) = 3

How many n-digit strings are possible without consecutive zeros

4 consecutive 1's

T (n)

$T(n-1) = T(n-2) + T(n-?)$

0	0
0	1
1	0
1	1

0	1					
1	0/1					

$T_n = \boxed{T(n-1) + T(n-2)}$

$T(n-1) + T(n-?)$

$T(3) = 5$ $T(4) = 8$

$T(1) = 2$
$T(2) = 3$

1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\frac{dx}{dt} = -x$$

$x(0) = 0$

$x'(0) = 1$

$$\boxed{\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 0}$$

Solve for $x(t)$

$$\boxed{x(t) = Ae^{\lambda_1 t} + Be^{+\lambda_2 t}}$$

$\lambda^2 - 2\lambda - 3 = 0$; $\lambda_1 = 3$

$\lambda_2 = -1$

$$\boxed{x(t) = \frac{1}{4}e^{3t} - \frac{1}{4}e^{-t}}$$

$$-3(x) = \left(\frac{1}{4}e^{3t} - \frac{1}{4}e^{-t}\right)$$

$$-2\left(\frac{dx}{dt}\right) = \left(\frac{3}{4}e^{3t} + \frac{1}{4}e^{-t}\right)$$

$$\frac{d^2x}{dt^2} = \frac{9}{4}e^{3t} - \frac{1}{4}e^{t}$$

$x(0) = \frac{1}{4} - \frac{1}{4} = 0$

$x'(0) = \frac{3}{4} + \frac{1}{4} = 1$

$x(0) = e^{-0}$ $\quad$ ($x(t) = e^{-t}$ a solution: x at $t=0$ $= 1$

$$x(t) = Ae^{3t} + Be^{-t}$$

$0 = A + B$

$1 = 3A - B$

$A = \frac{1}{4}$; $B = -\frac{1}{4}$

$$x'(t) = 3Ae^{3t} - Be^{-t}$$

Recurrence relation:-

10 % (Compound) Interest

$$\boxed{a_n = (1.1)\, a_{n-1}}$$

$a_0 = 1000$

$a_1 = 1.1\, a_0$

$a_2 = 1.1\, a_1 = (1.1)^2 a_0$

$a_{14} = (1.1)\, a_{13}$

$= (1.1)(1.1)\, a_{12}$

$a^n = (1.1)^n a_0 = (1.1)^n \cdot 1000$

$a_{14} \;\; (1.1)^{14} \cdot 1000$

$a_n = (1.1)^n a_0$

Solution to the recurrence relation:-

$C_n = C_{n-1} + C_{n-2}$

$$a_n = c_1 a_{n-1} + c_2 a_{n-2}$$

Solve $\left(x^2 - c_1 x - c_2 = 0 \right)$

x_1, x_2 (roots of quad eq)

$$a_n = \alpha_1 x_1^n + \alpha_2 x_2^n$$

0, 1, 1, 2, 3, 5, 8, 13, 21, . . .

$$a_n - c_1 a_{n-1} - c_2 a_{n-2} = 0$$

$$a_n = 3a_{n-1} - 2a_{n-2}. \quad \begin{cases} a_0 = 1 \\ a_1 = 2. \end{cases}$$

$$x^2 - 3x + 2 = 0$$

$$x_1 = 2 \quad x_2 = 1.$$

$$a_n = \alpha_1 (2)^n + \alpha_2 (1)^n.$$

$$a_0 = 1 = \alpha_1 + \alpha_2$$

$$a_1 = 2 = 2\alpha_1 + \alpha_2$$

$$\alpha_1 = 1, \quad \alpha_2 = 0 \qquad a_n = (2)^n.$$

$$a_n = a_{n-1} + a_{n-2}$$

$$a_n = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$a_0 = 0$

$a_1 = 1$

$$x^2 - x - 1 = 0$$

$$x_1 = \frac{1+\sqrt{5}}{2} \qquad x_2 = \frac{1-\sqrt{5}}{2}$$

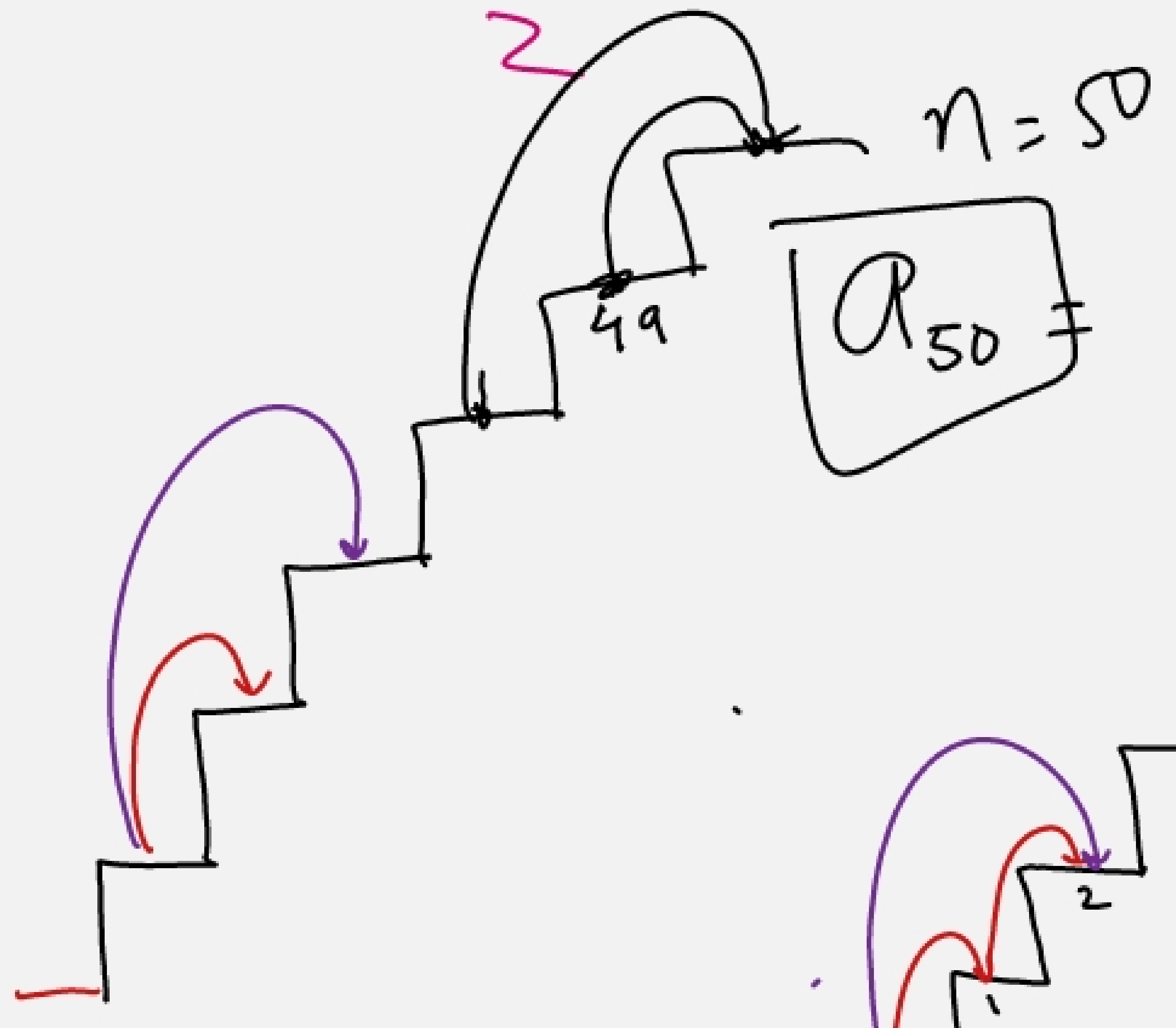
$$\alpha_1 + \alpha_2 = 0$$

$$\frac{\alpha_1 + \alpha_2}{2} + \frac{\sqrt{5}\alpha_1}{2} - \frac{\sqrt{5}\alpha_2}{2} = 1$$

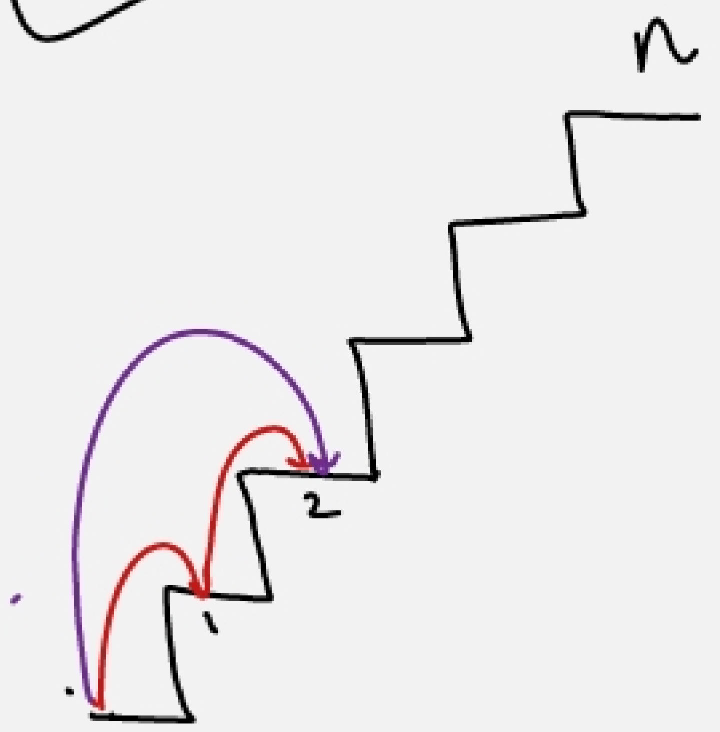
$$a_n = a_{n-1} + a_{n-2}$$

$$\alpha_1 = \frac{1}{\sqrt{5}} \qquad \alpha_2 = -\frac{1}{\sqrt{5}}$$

a_n



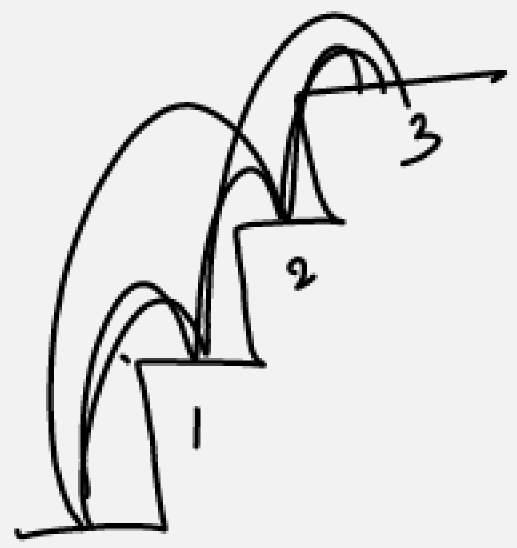
$$\boxed{a_{50}} = a_{49} + a_{48}$$



$$a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\boxed{\begin{aligned} a_1 &= 1 \\ a_2 &= 2 \end{aligned}}$$

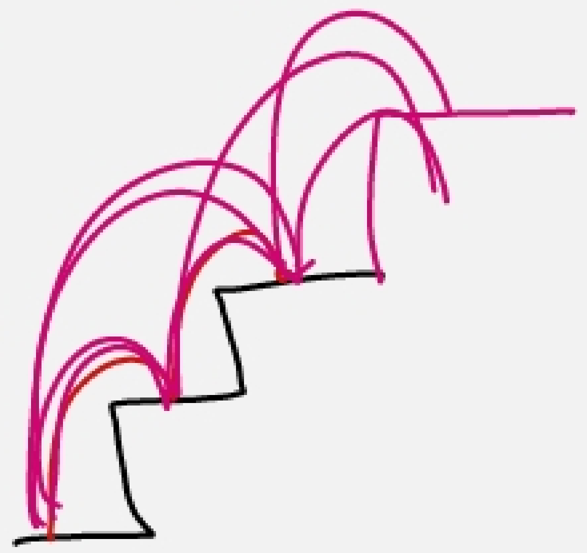
$a_3 = 3$



$$T(n) \overset{?}{\approx} T(n-1)$$

$$T(50) = T(49) + T(48)$$

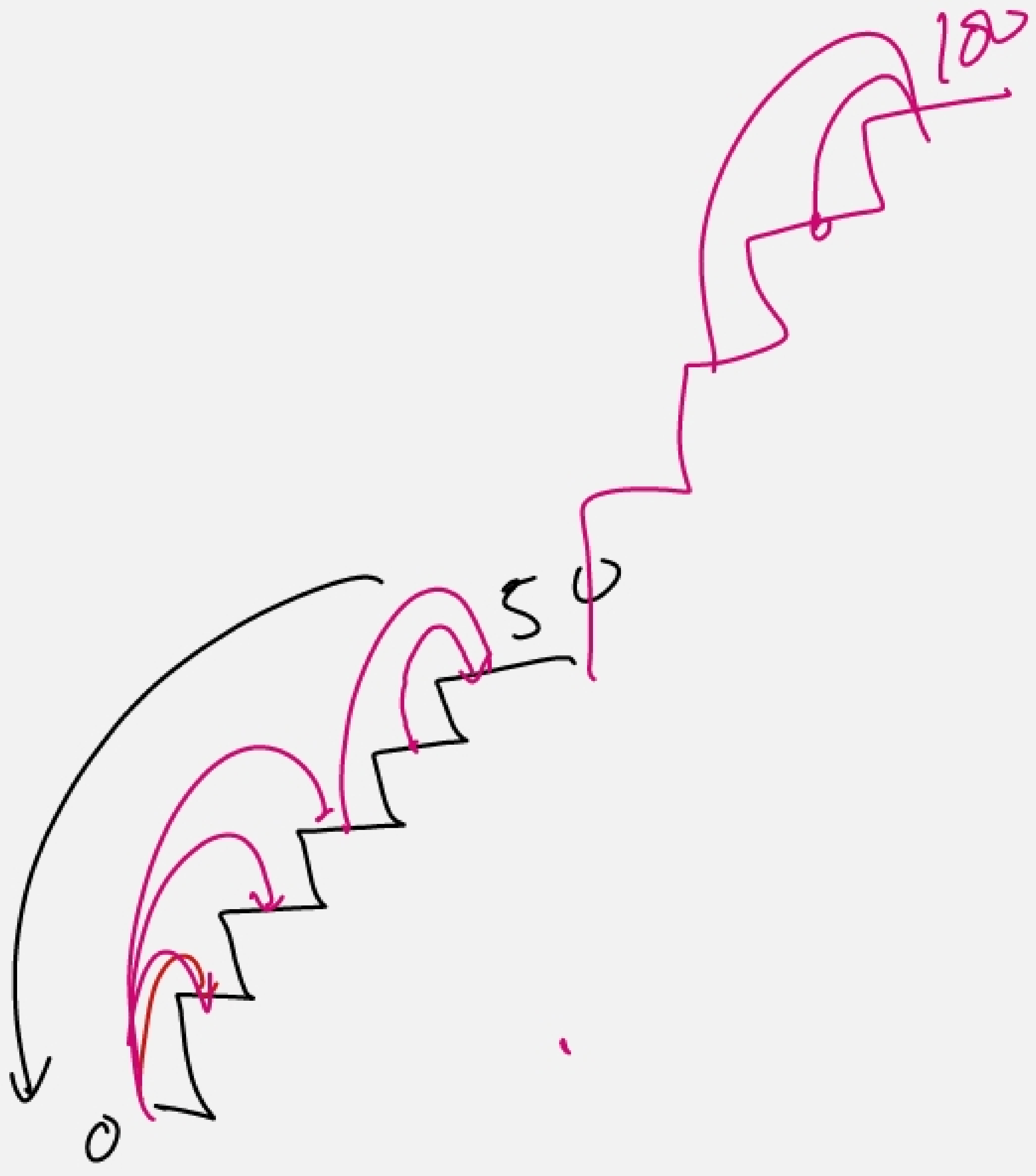
$$T(n) = T(n-1) + T(n-2)$$



$$n! = n \cdot (n-1)!$$

$$T(n) = n \cdot T(n-1)$$

$$T(100) = T(99) + T(98)$$



2^n

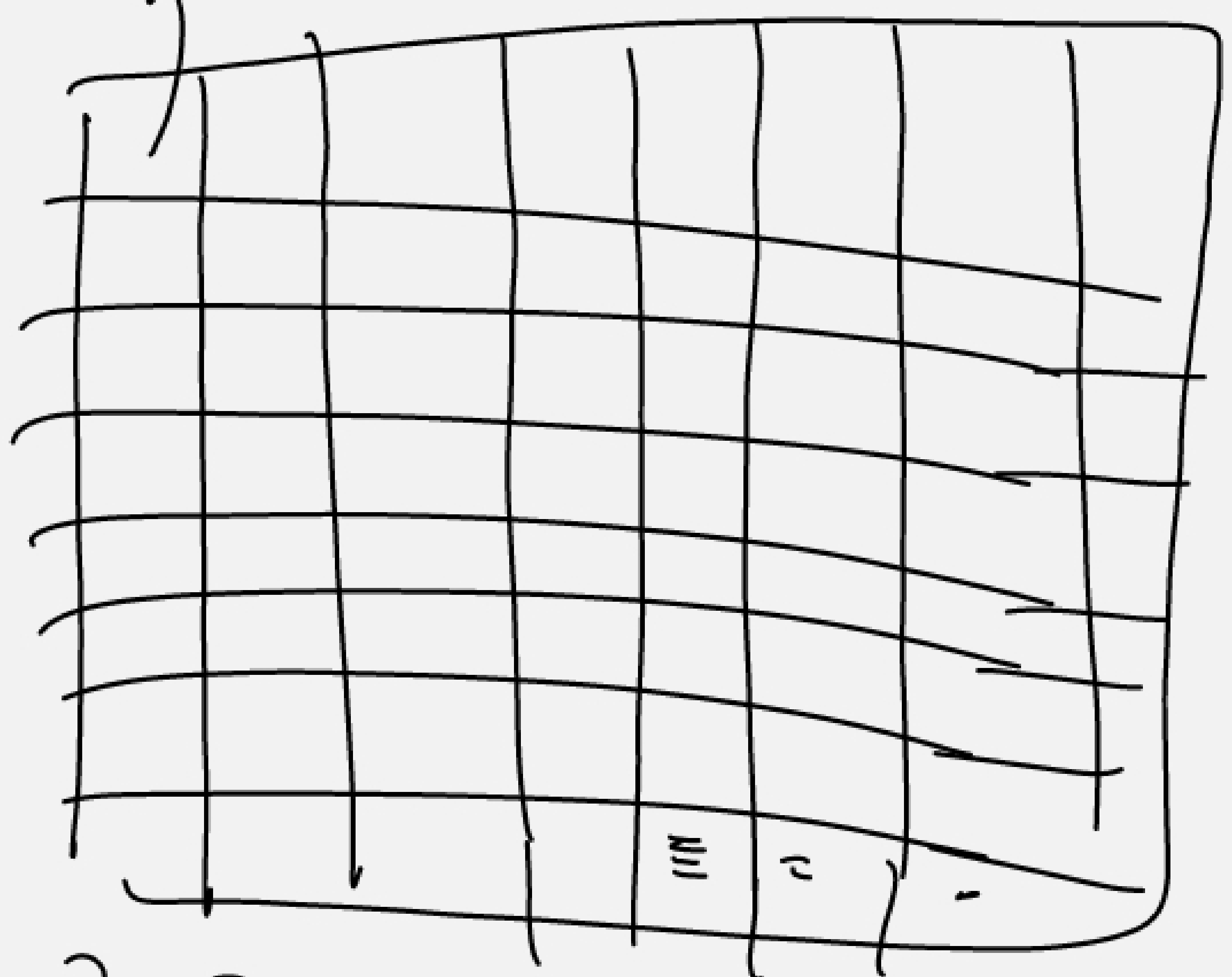
$2n+1$

$2^n - 1$

$T(n) = 1 + T(n-1) + T(n-1)$

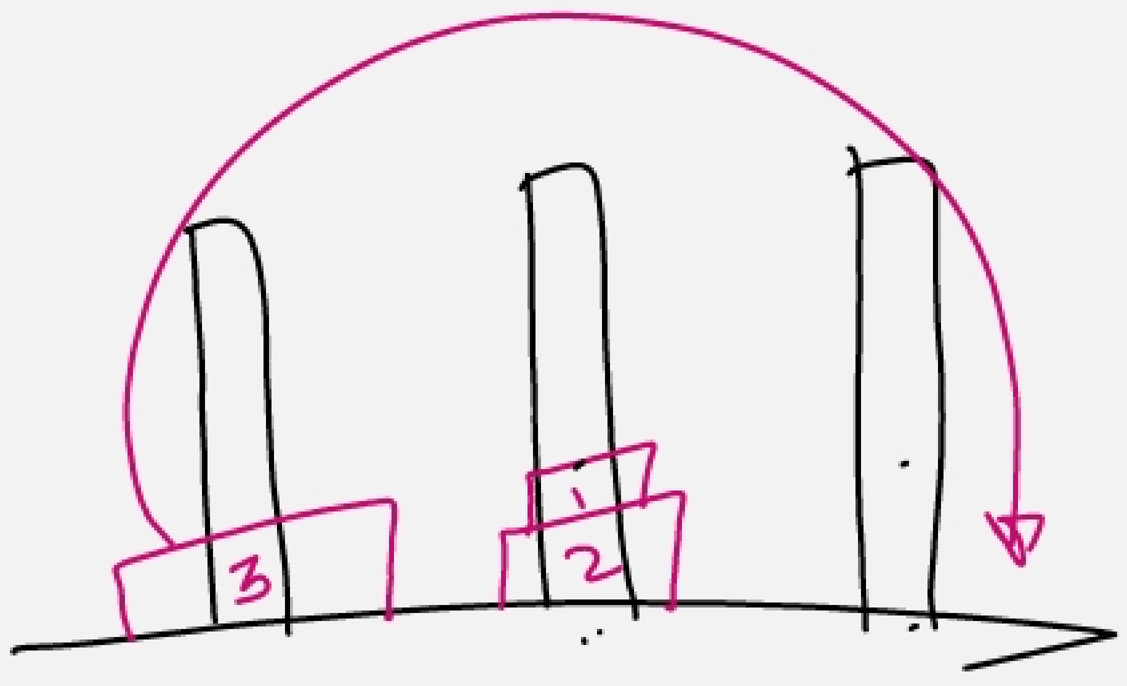
2^{64}

$T(n) = 2T(n-1) + 1$

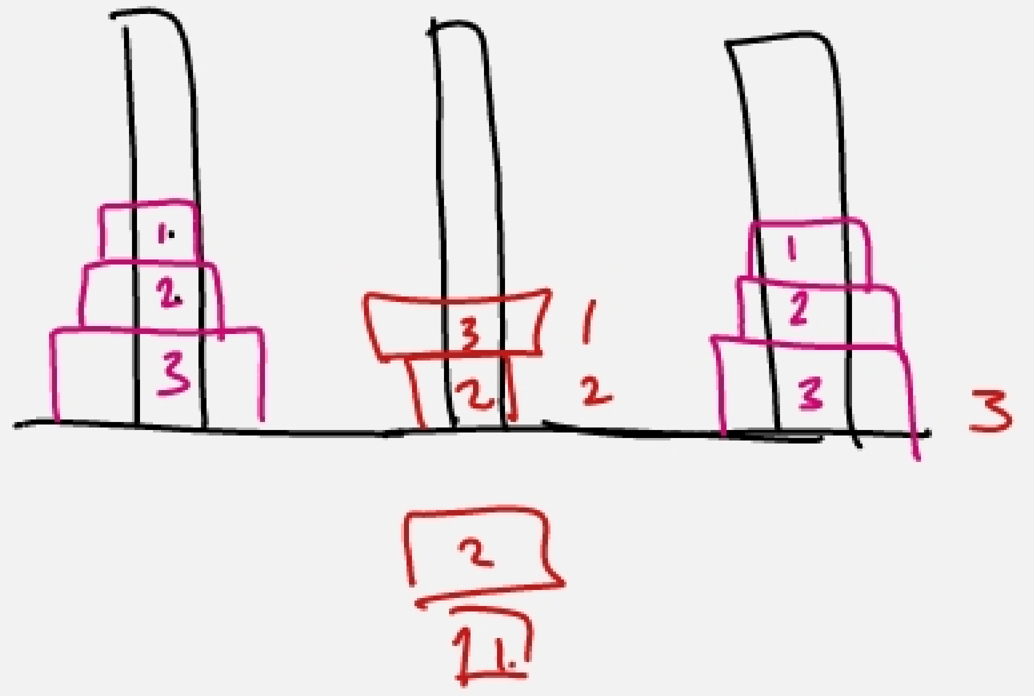


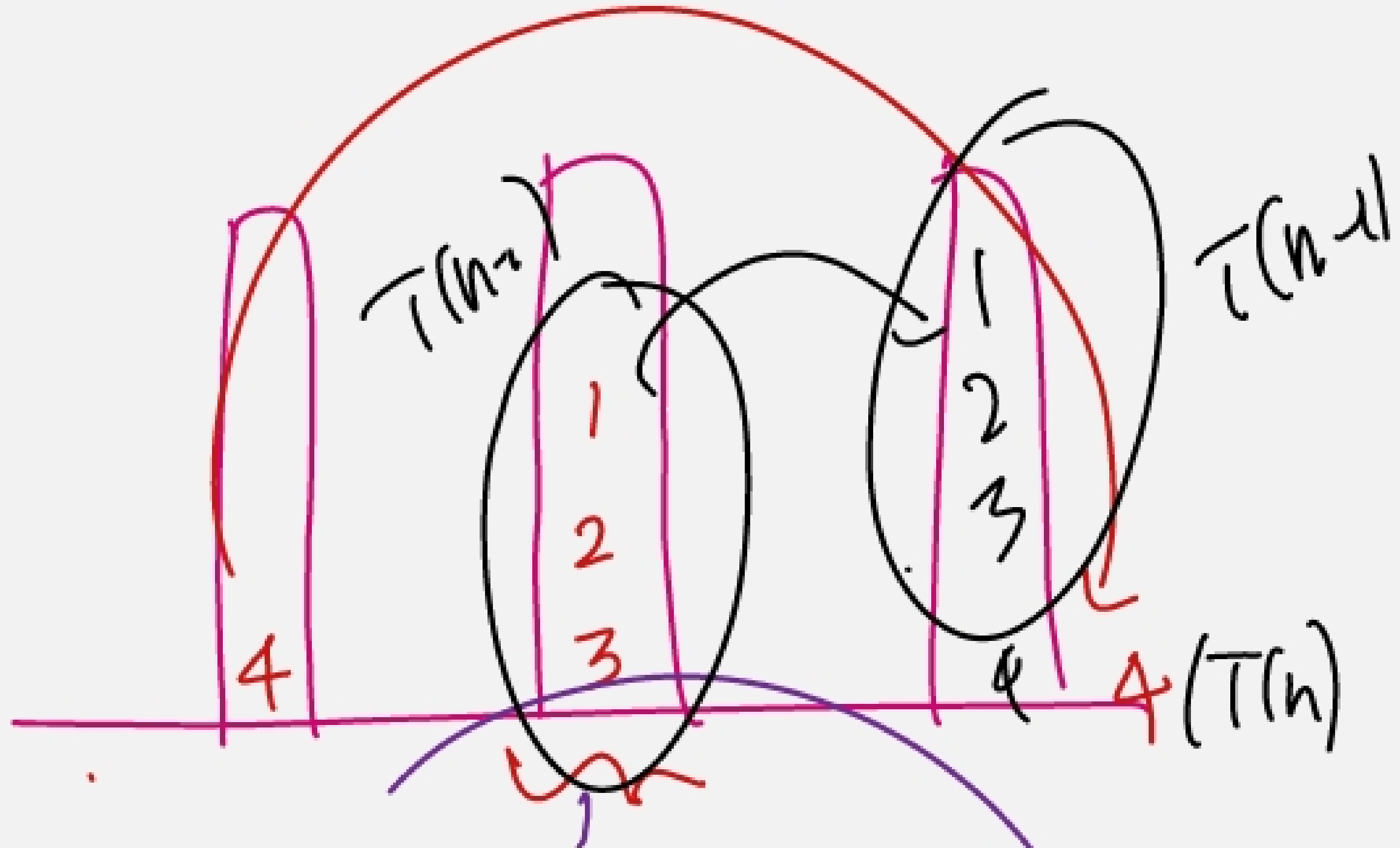
1, 3, 7, 15,

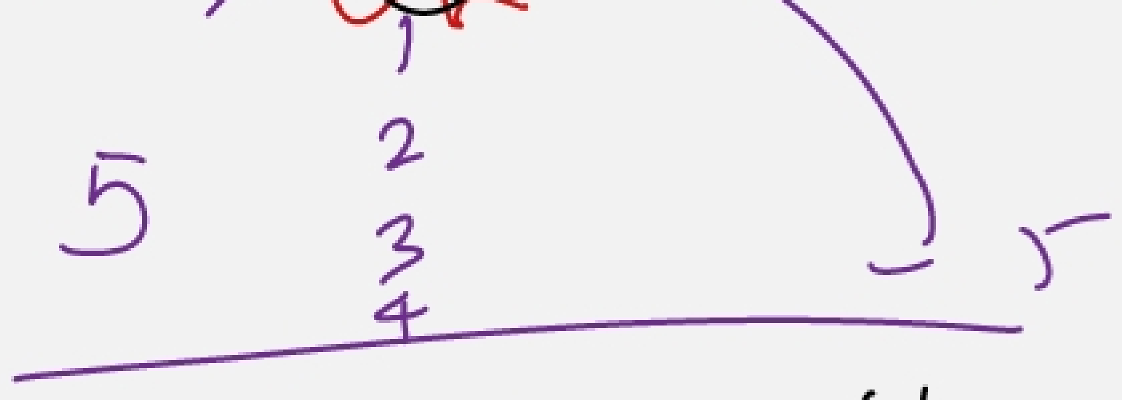
$T(n)$

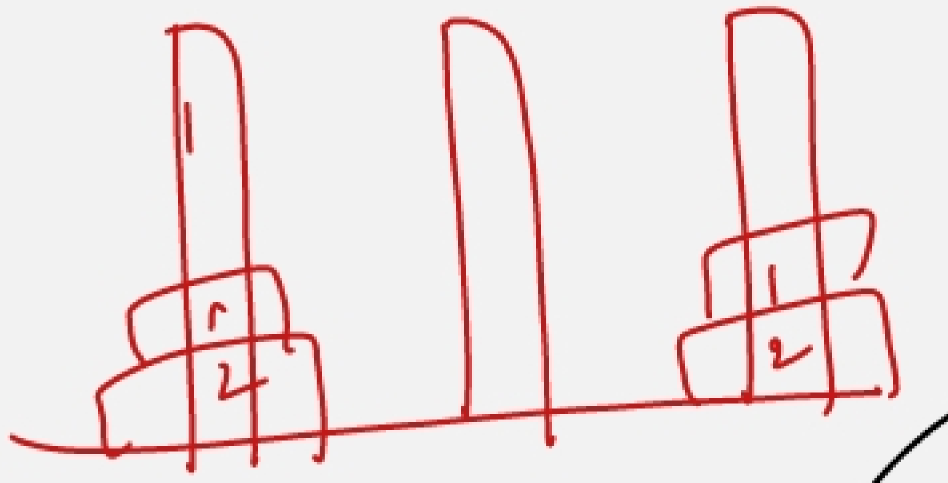


7









$2^{64} - 1$

3

2^{64}

$$T(n) = 2T(n-1) + 1$$

$$= 2\left[2T(n-2) + 1\right] + 1 = 2^2 T(n-2) + 2 + 1$$

$$= 2^2\left[2T(n-3) + 1\right] + 2 + 1 = 2^3 T(n-3) + 2^2 + 2 + 1$$

$T(1) = 1$

$n - K = 1$

$K + 1 = n.$

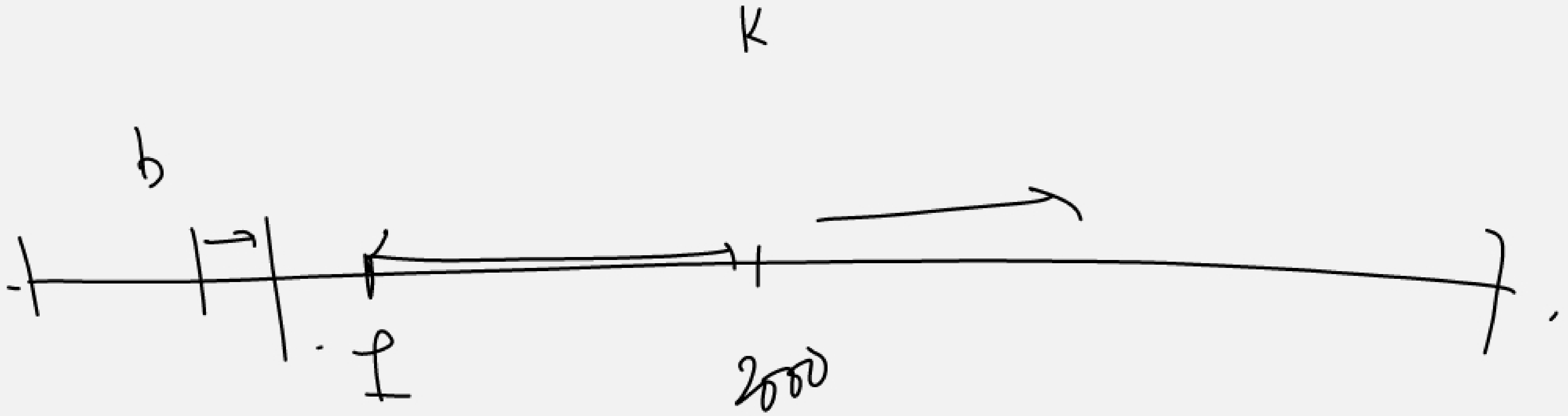
$$T(n) = 2^K T(n-k) \cdots + 2^3 + 2^2 + 2 + 1$$

$$2T(n) = 2^{K+1} T(n-k) + 2 \quad \cdots \quad + 2^3 + 2^2 + 2$$

$$2T(n) - T(n) = 2^{K+1} T(n-k) - 1$$

$$\boxed{T(n) = 2^n - 1}$$

discombombulated



$\log(n)$