

Random Variable

Random experiment

Sample Space = Set of possible

Outcomes of Random
experiment.

Motivation:- We are not interested

in the whole experiment, but

Value of a numerical quantity

which comes out of random experiment.

Random Variable :- A function which takes input from sample space and maps it to Real Numbers.

OR Real Valued function defined on sample space.

Random experiment.

E.g. Rolling a dice twice.

Q1 What is the probability that sum is 5.

Q2 What is the probability that the smaller of the outcomes is 3.

$$S = \{ (1,1), (1,2), \dots, (1,6), (2,1), \dots (2,6) \\ \dots (6,1), \dots (6,6) \}$$

X — Sum of outcomes of two rolls.

$$X: S \rightarrow \mathbb{R}$$

$$X \in \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$Y =$ Minimum of Outcome of a Roll.

$$Y \in \{1, 2, 3, 4, 5, 6\}$$

X	Events	Probability.
2	$\{(1,1)\}$	$\frac{1}{36}$
3	$\{(1,2), (2,1)\}$	$\frac{2}{36}$
4	$\{(1,3), (2,2), (3,1)\}$	$\frac{3}{36}$
5		$\frac{4}{36}$
...		
12		$\frac{1}{36}$

X	P	X	P
2	$\frac{1}{36}$	9	$\frac{4}{36}$
3	$\frac{2}{36}$	10	$\frac{3}{36}$
4	$\frac{3}{36}$	11	$\frac{2}{36}$
5	$\frac{4}{36}$	12	$\frac{1}{36}$
6	$\frac{5}{36}$		
7	$\frac{6}{36}$		
8	$\frac{5}{36}$		

Y
1
2
3
4
5
6

Events

$\{(1,1), \dots, (1,6), (2,1), (3,1), (4,1), (5,6), (6,1)\}$

$\{(2,2), \dots, (2,6), (3,2), (4,2), (5,2), (6,2)\}$

Probability

$\frac{11}{36}$

$\frac{9}{36}$

$\frac{7}{36}$

$\frac{5}{36}$

$\frac{3}{36}$

$\frac{1}{36}$

$\frac{1}{36}$

Random experiment.

E.g. >

Tossing a Coin three times.

$$S = \{HHH, HHT, HTH, HTT, THT, THH, TTH, TTT\}.$$

Q1 of three tosses, how many times will be heads?

Q2 of three tosses, which toss results in a head first?

X —	No. of heads that appear.	
	X	Y
HHH	3	1
HHT	2	1
HTH	2	1
HTT	1	1
THH	2	2
THT	1	2
TTH	1	3
TTT	0	Nil

$$P(X=0) = \frac{1}{8}$$

$$P(X=1) = \frac{3}{8}$$

$$P(X=2) = \frac{3}{8}$$

$$P(X=3) = \frac{1}{8}$$

$$P(Y=1) = \frac{4}{8}$$

$$P(Y=2) = \frac{2}{8}$$

$$P(Y=3) = \frac{1}{8}$$

$$X \in \{0, 1, 2, 3\}$$

Y — Toss in which head appears first.

$$Y \in \{1, 2, 3\}$$

Discrete Random Variable \rightarrow A Random

Variable is Called Discrete Random Variable if it takes only Countably finite or infinite Values.

Apartment Complex data

- There are 12 apartments in a apartment Complex.
- Each floor has 3 apartments:
 - One bedroom, two bedroom, three bedroom.

Apartment NO.	Floor NO.	No. of bedrooms.	Size of apartment	Distance of apartment from left.
1	1	1	900.23	500.23
2	1	2	1175.11	325.3
3	2	3	1785.09	:
4	2	1	900.12	:
5	2	2	:	:
6	2	3	:	:
7	3	2	:	:
8	3	2	:	:
9	3	2	:	:
10	5	1	1786.30	455.37
11	5	2	[900, 1800]	
12	5	3		