

$$a \in A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = a$$

$S_1$        $S_1 \subseteq S_2$        $S_2$   
 $S_1 \supseteq S_2$

let  $a \in (A \cup B) \cap (A \cup C)$

$a \in (A \cup B)$  and  $a \in (A \cup C)$

$a \in A$  or  $a \in B$  and  $a \in A$  or  $a \in C$

$a \in A$  or  $a \in B$  and  $a \in C$

$A \cup (B \cap C)$

$$\left\{ \begin{array}{l} a \in A \cup (B \cap C) \\ a \in A \rightarrow a \in (B \cap C) \\ \hline a \in (B \cap C) \\ a \in B \\ a \in C \\ \hline \text{if } a \in A \text{ then } a \in (A \cup B) \\ \text{if } a \in A \text{ then } a \in (A \cup C) \end{array} \right.$$

$A \cup (A \cap B) = A$   
 $A \cap (A \cup B) = A$   
 $a \in (A \cup B) \cap (A \cup C)$

For  $i = 1, 2, \dots$

$$A_i = \{i, i+1, i+2, \dots\}$$

~~$A_n = \{$~~   $A_1 = \{1, 2, \dots\}$   $A_2 = \{2, 3, \dots\}$   
 $A_{10} = \{10, 11, \dots\}$

Compute

$$\bigcup_{i=1}^n A_i = \{1, 2, 3, \dots\}$$

$$\bigcap_{i=1}^n A_i = A_n$$

$$A_i = \{1, 2, \dots, i\}$$

$$\bigcup_{i=1}^n \{1, 2, \dots, i\} = A_n$$

$$\bigcap_{i=1}^n A_i = \{1\}$$

$$A_1 \cap A_2 = \{2, 3, \dots\}$$

$$A_1 \cap A_2 \cap A_3 = A_3$$



# Cartesian Product

$$A = \{a_1, \dots, a_n\} \quad \text{ordered n tuple}$$

$$B = \{b_1, \dots, b_n\}$$

$$B \times A = \{(b, a) \mid b \in B, a \in A\}$$

$$A = B \Rightarrow a_1 = b_1 - a_n = b_n$$

$$\begin{aligned} &\{1, a, b, f\} \\ &\{a, f, 1, b\} \end{aligned}$$

$$\text{The Cartesian product of } A \times B = \{(a, b) \mid (a \in A, b \in B)\}$$

$$A = \{a, b, c\} \quad \text{how will elements of } (A \times B) \text{ look like}$$

$$B = \{\alpha, \beta, \gamma\} \quad \{(a, \alpha), (a, \beta), (a, \gamma), (b, \alpha) \dots\}$$

$$\nabla A \times B \neq B \times A$$

$$A = \{1, 2\}$$

$$A^2 = \{ \underset{\substack{| \\ |}}{(1,1)}, (1,2), (2,1), (2,2) \}$$

$$(a, b) \in \underline{A \times B}$$

$$\{0, 1, 2, 3\}$$

$$(a, b) : a \leq b$$

$$(0,0), (0,1), (0,2), (0,3)$$

$$(1,2), (1,3)$$



$$p = 2^x$$

$$p^2 = 4x^2 = 2q^2$$

$$p^2 = 2q^2 - 4 \cdot 9$$

Prove that  $\sqrt{2}$  is irrational

Statement:  $\sqrt{2}$  is rational

Challenge the Statement

$$\sqrt{2} = \frac{p}{q}$$

$$2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2$$

$p^2$  is even  
 $p$  is even

$$p = 2 \cdot x$$

$q$  is even then  $p$  is even

MM  
 TT  
 WW  
 TT  
 FF  
 SS  
 SS

Take a set of 15 days  
 at least 3 must fall on the  
 same day  $2 \times 7 = 14$

① If  $p$  is even,  $p^2$  is a multiple of 4.

②  $p^2$  is even, multiple of 4

$$4 \cdot \text{Even} = p^2$$

③  $2 \cdot \text{Even} = q^2 \Rightarrow q$  is also even

$\sqrt{2}$  is irrational

$\sqrt{2}$  is rational  $\rightarrow$

$$\sqrt{2} = \frac{p}{q}$$

$$2 = \frac{p^2}{q^2}$$

$$p^2 = 2q^2 \Rightarrow p \text{ is even}$$

Square of any even number is a

multiple of 4.  $2q^2 = 4 \cdot x$  (contradiction)  
 $q^2 = 2 \cdot x \Rightarrow q$  is even

If  $\underline{3n+2}$  is odd then  $n$  is odd.

$n$  is NOT odd.  $\Rightarrow n$  is even

$$n = 2K$$

$$3 \cdot 2K + 2 = 2(3K + 1)$$

$\Rightarrow 3n+2$  is even

$$\underbrace{3n+2}_{\text{(odd)}} \Rightarrow \underline{n \text{ is odd}}$$

even

$N$  is even

MATHEMATICAL  
LOGIC



DM is <sup>NOT</sup> an interesting course

$p$  Statement  
Proposition

negation  $\neg p$   
OR operation

P, Q	OR	M
S/W or H/W		
Two simple statements		
0	0	0
0	1	1
1	0	1
1	1	1

P	Q	M	S
0	0	0	0
			1
			1
			1
			1

Q:  $6+4 = 3$  (F-0)  
 $\neg Q$ :  $6+4 \neq 3$  (T-1)

$$\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3}$$

Power set  $2^3$

and.

SW + HW

P	Q	S
0	0	0
0	1	0
1	0	1
1	1	1

$(P \text{ AND } Q) \text{ OR NOT } P$

P	Q	$P \wedge Q$	$\neg P$	$(P \wedge Q) \vee \neg P$
0	0	0	1	1
0	1	0	1	1
1	0	0	0	0
1	1	1	0	1

Truth table



P	Q	$\neg P$	$\neg Q$	$\neg Q \rightarrow \neg P$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

Inverse

$$\neg P \rightarrow \neg Q$$

CONTRADICTORY  
 $H \rightarrow E$

If NOT Q  
 false then  
 NOT P

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$$

Prove  $P \rightarrow Q$   
 or  $\neg Q \rightarrow \neg P$   
 Logical Implication

$$P \rightarrow Q$$

$$E \rightarrow S$$

E	S	$E \rightarrow S$
0	0	1
0	1	1
1	0	0
1	1	1

When is a CONDITIONAL Statement

$$P \rightarrow Q \text{ false? } \neg Q \rightarrow \neg P$$

When  $P = F$  &  $Q = T$

If  $3n+2$  is odd then  $n$  is odd

$$H \rightarrow E$$

not eating  $\rightarrow$  not hungry

H	E	$H \rightarrow E$	$\neg E$	$\neg H$	$\neg E \rightarrow \neg H$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1