Recap

Alg. Linear systems & matrices

Augmented matrix

Upper diagonal form

$$\begin{array}{l} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \end{array} \left.\begin{array}{r} x + 2y + 3z = 1 \\ 2x + y + z = 2 \\ x + 3y \quad = 3 \\ 4x + 6y + 4z = 6 \end{array}\right\} \quad \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 3 & 0 & 3 \\ 4 & 6 & 4 & 6 \end{bmatrix} \xrightarrow[\text{Reduction}]{\text{Gaussian}} \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 5/3 & 0 \\ 0 & 0 & 1 & -3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

linearly dependent on the first 3

Row 'RANK' $\rightarrow$ max no. of linearly independent rows

Col 'Rank' $\longrightarrow$ max no. of l. i. columns

$$\begin{array}{c} \overbrace{}^{3} \\ \begin{bmatrix} 1 & \cdots & 0 \\ \cdots 1 & & 0 \\ \cdots & \cdots 1 & 0 \\ & & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 3 & 0 & 3 \\ 4 & 6 & 4 & 6 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -3 & -5 & 0 \\ 1 & 1 & -3 & 2 \\ 4 & -2 & -8 & 2 \end{bmatrix} \longleftarrow \quad \longrightarrow \begin{matrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & -1/3 & & \\ 4 & 2/3 & & \end{matrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ - & 1 & 0 & 0 \\ - & - & 1 & 0 \\ - & - & - & 0 \end{bmatrix}$$

Row Rank = Column Rank (= Rank)

Lower diagonal

Algorithm for Gaussian Redn

1) Rearrange rows so that $a_{11} \neq 0$

2) Divide Row 1 by a_{11} to produce a new Row 1 : $a'_{11} = \frac{a_{11}}{a_{11}} = 1$

3) Mult new Row 1 successively by

→ $-a_{21}$ and add to Row 2 to get new row 2

→ $-a_{31}$ —— " —— 3 —— " —— 3

4) Consider the submatrix obtd by leaving out Row 1 & Col 1

5) Repeat steps 1 – 3 on the submatrix

$$\begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 0 & a'_{22} & & \\ 0 & a'_{32} & & \\ 0 & & & \\ 0 & \cdots & & a_{mn} \end{bmatrix}$$