Random Variable

Random experiment

Sample space = Set of possible outcomes of Random experiment.

Motivation :- We are not interested in the whole experiment, but value of a numerical quantity which comes out of random experiment.

Random Variable :- A function which takes input from sample space and map it to real numbers.

Or Real Valued function defined on sample space.

Random experiment.

E.g.> Rolling a dice twice.

Q1 What is the probability that sum is 5.

Q2 What is the probability that the smaller of the outcomes is 3.

$$S = \{(1,1), (1,2), \ldots, (1,6), (2,1), \ldots (2,6)$$

$$\ldots (6,1), \ldots (6,6)\}$$

X — Sum of outcomes of two rolls.

$$X : S \to \mathbb{R}$$

$$X \in \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$Y =$ Minimum of Outcome of a roll.

$Y \in \{1, 2, 3, 4, 5, 6\}$

X	Events	Probability
2	$\{(1,1)\}$	$\frac{1}{36}$
3	$\{(1,2),(2,1)\}$	$\frac{2}{36}$
4	$\{(1,3),(2,2),(3,1)\}$	$\frac{3}{36}$
5		$\frac{4}{36}$
$\vdots$		
12		$\frac{1}{36}$

X	P	X	P
2	$\frac{1}{36}$	9	$\frac{4}{36}$
3	$\frac{2}{36}$	10	$\frac{3}{36}$
4	$\frac{3}{36}$	11	$\frac{2}{36}$
5	$\frac{4}{36}$	12	$\frac{1}{36}$
6	$\frac{5}{36}$		
7	$\frac{6}{36}$		
8	$\frac{5}{36}$		

Y	Events	Probability
1	$\{(1,1), \ldots (1,6), (2,1), (3,1), (4,1), (5,1), (6,1)\}$	$\frac{11}{36}$
2	$\{(2,2), \ldots (2,6), (3,2), (4,2), (5,2), (6,2)\}$	$\frac{9}{36}$
3		$\frac{7}{36}$
4		$\frac{5}{36}$
5		$\frac{3}{36}$
6		$\frac{1}{36}$

Random experiment.

E.g.> Tossing a Coin three times.

$S = \{HHH, HHT, HTH, HTT, THT, THH, TTH, TTT\}$.

Q1 Of three tosses, how many tosses will be heads?

Q2. Of three tosses, which toss results in a head first?

X — No. of heads that appears.

	X	Y
HHH	3	1
HHT	2	1
HTH	2	1
HTT	1	1
THH	2	2
THT	1	2
TTH	1	3
TTT	0	Nil

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

$P(Y=1) = \frac{4}{8}$

$P(Y=2) = \frac{2}{8}$

$P(Y=3) = \frac{1}{8}$

$X \in \{0, 1, 2, 3\}$

Y — Toss in which head appears first.

$Y \in \{1, 2, 3\}$.

<u>Discrete random Variable</u> :- A random Variable is called discrete random Variable if it takes only countably finite or infinite values.

Apartment Complex data

- There are 12 apartments in a apartment Complex.

- Each floor has 3 apartments:
  One bedroom, two bedroom, three bedroom.

Apartment NO.	Floor NO.	No. of bedrooms.	Size of apartment	Distance of apartment from lift.
1	1	1	900.23	— 500.23
2	1	2	1175.11	325.3
3	1	3	1785.09	.
4	2	1	900.12	.
6	2	2	.	.
7	2	3	.	.
8	3	1	.	.
9	3	2	.	
10	3	3		
11	4	1		
12	4	2	1786.30	455.37
	4	3		

[900, 1800]

Random experiment - Choosing random apartment out of 12 apartments.

X — Floor no. — $\{1, 2, 3, 4\}$

Y - No. of bedrooms - $\{1, 2, 3\}$

Z - Size of apartment - $[900, 1800]$

W - distance of apartment from lift $[300, 501]$

Discrete Random Variables.

1. Maximum from a set of finite numbers.
2. Outcome of rolling a die.
3. No. of Students in Class.
4. No. of Spelling mistakes in a paragraph.

Continuous random Variable

1. Area of apartment.
2. Height and Weight of students.
3. Temperature in a room.

Continuous Random Variable :-

A random Variable which takes Values in an interval.

Discrete RV.

Probability mass function :- Let "X" be a random variables which takes values $x_1, x_2, \ldots, x_N$, then Probability mass function (pmf) is defined as

$$\boxed{p(x_i) = P(X = x_i)}$$

Properties of pmf.

① $p(x_i) \geq 0$

② $\sum_{i=1}^{N} p(x_i) = 1$.

E.g. X takes values

X	0	1	2
$p(x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

⇓

Is it pmf?

Yes.

E.g.)

X	x_1 1	x_2 2	x_3 3	x_4 4	x_5 5
$p(x_i)$	0.4	0.1	0.2	0.1	0.3

— Not a pmf

E.g.)

X	1	2	3	4	5
$p(x_i)$	0.2	0.3	0.4	−0.1	0.2

< 0

Not a pmf.

Example. X $\quad$ $\begin{array}{ccccc} x_0 & x_1 & x_2 & x_3 & \cdots \\ 0 & 1 & 2 & 3 & \cdots \end{array}$ $\quad$ $\lambda > 0$

$p(x_i) = P(X = x_i)$ $\quad \dfrac{c\lambda^0}{0!} \quad \dfrac{c\lambda^1}{1!} \quad \dfrac{c\lambda^2}{2!} \quad \dfrac{c\lambda^3}{3!} \cdots$

Can you find c such that $p(x_i)$ is pmf.

① $p(x_i) \geq 0 \implies c \geq 0$.

$$C\frac{\lambda^0}{0!} + C\frac{\lambda^1}{1!} + C\frac{\lambda^2}{2!} + \dots = 1$$

$$\Rightarrow C\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots\right) = 1$$

$$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2!} + \dots$$

$$\Rightarrow C e^{\lambda} = 1$$

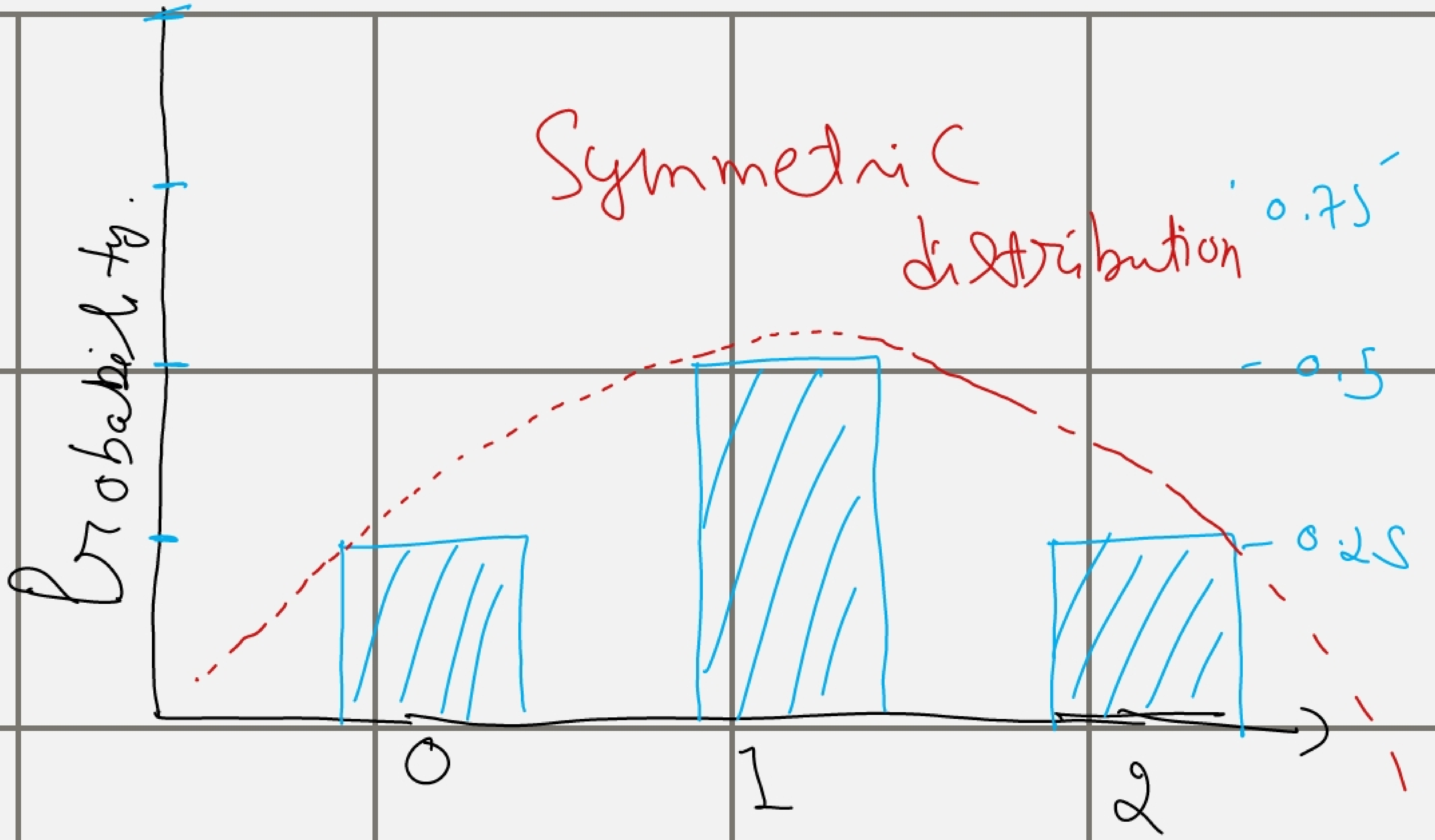
$$\Rightarrow \boxed{C = e^{-\lambda}}$$

$$p(x_i) = P(X = x_i) = c \cdot \frac{\lambda^i}{i!}$$

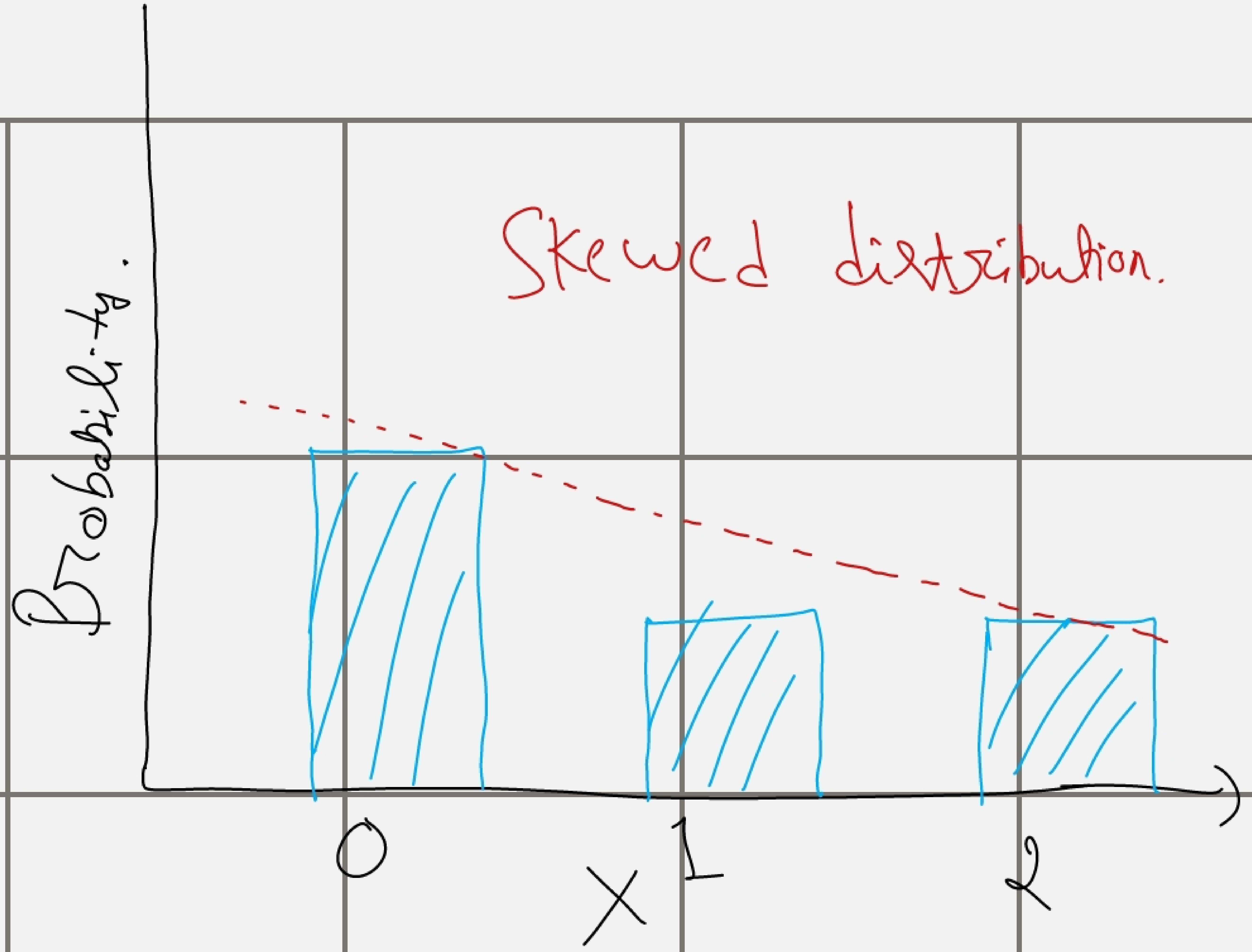
$$\boxed{p(x_i) = e^{-\lambda} \frac{\lambda^i}{i!}}$$

Graph of probability mass function

X	0	1	2
$P(X=x_i)$	0.25	0.5	0.25



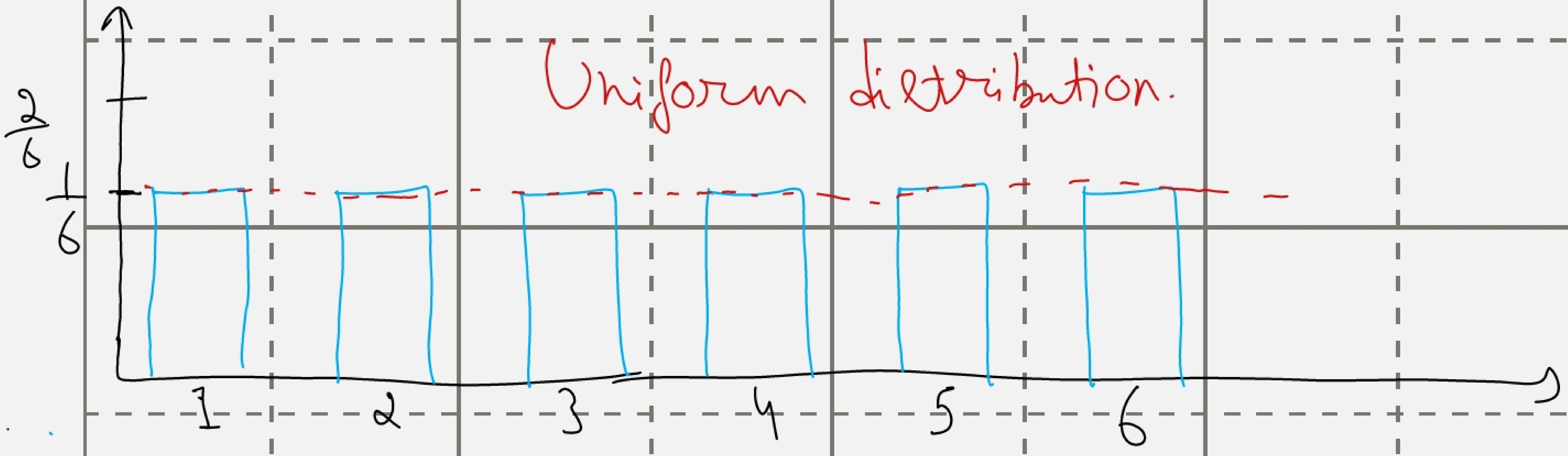
X	0	1	2
$P(X=x_i)$	0.5	0.25	0.25



X	0	1	2
$P(X=x_i)$	0.35	0.55	0.10

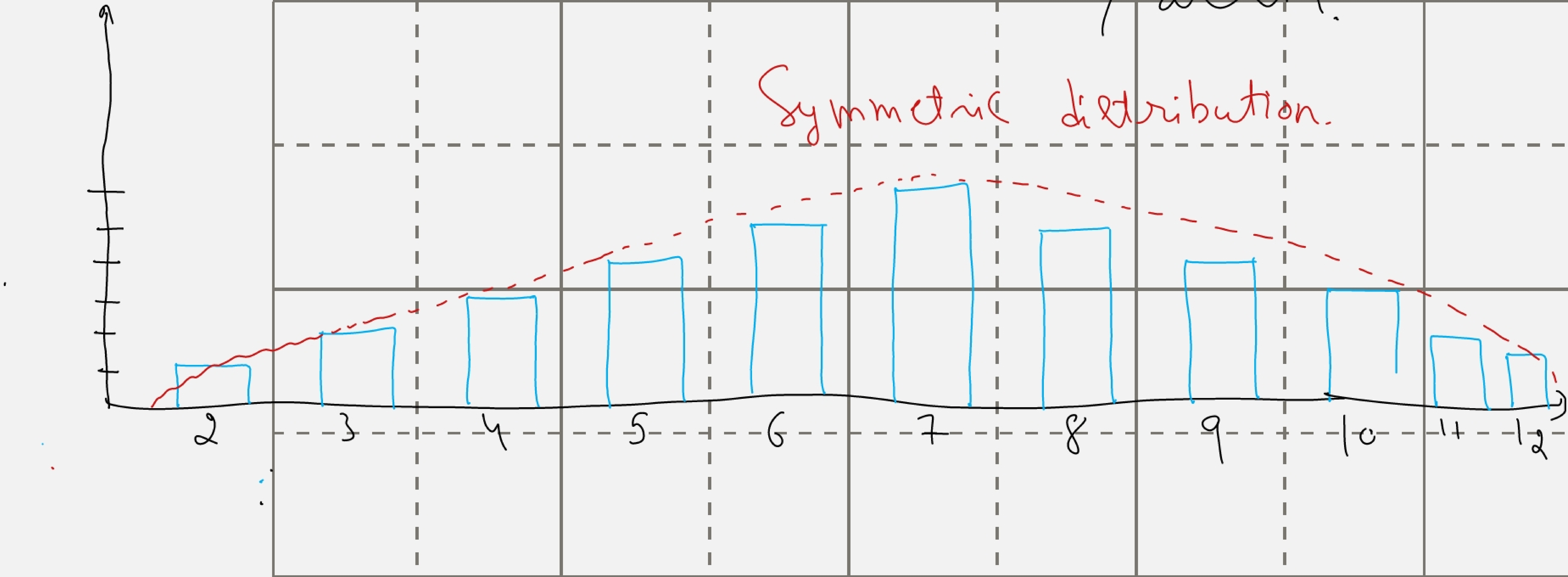
E.g. Rolling a die

X	1	2	3	4	5	6
$P(X=x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



x	2	3	4	5	6	7	8	9	10	11	12
$P(X=x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Try it for Y also?



X

$P(X=x_i)$

$\|$

$p(x_i)$

$P(X \le a)$, $P(X \ge a)$

$P(X \ge 5)$

$$\boxed{F(a) = P(X \le a)}$$