Lec-2 Conditional prob

8R, 4W

Urn has 8 Red & 4 White balls. We draw 2 balls without replacement.

a) if we assume equally likely, at each draw, prob of both balls to be red (if equally likely)

b) Suppose balls have different weight r (red) & w (white) & prob. of selecting a ball is its weight divided by sum of weights of all balls. Find prob. of both balls being red?

$P(R_1 R_2)$

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(AB) = P(A|B) \cdot P(B)$$

Sol: $P(R_1 R_2) = ?$

$$P(R_1 R_2) = P(R_2|R_1)\, P(R_1)$$

$\frac{7}{11}$ $\frac{8}{12}$

$$P(A|B) = \frac{P(AB)}{P(B)} \quad \text{--- } ①$$

$$P(B|A) = \frac{P(AB)}{P(A)} \quad \text{--- } ②$$

$$P(A|B)P(B) = P(B|A) \cdot P(A)$$

$$P(R_2|R_1)P(R_1) = P(R_1|R_2) \cdot P(R_2)$$

$$\text{Ans: } \left(\frac{8r}{8r+4w}\right)\left(\frac{7r}{7r+4w}\right)$$

$$\sum_{i=1}^{8} B_i = R$$

Red $= 8r$

White $= 4w$

Total $= 8r+4w$

$$P(R_1) = \frac{8r}{8r+4w}$$

$$\boxed{P(R_2|R_1) = \frac{7r}{7r+4w}}$$

$8r+4w$

$7r+4w$ $\qquad r$

Ans: $P(R_1R_2) = P(R_2|R_1)\cdot P(R_1)$

$$= \frac{P(R_1R_2)}{\cancel{P(R_1)}}\cdot \cancel{P(R_1)}$$

$$\frac{8r}{8r+4w} = \sum_{i=1}^{8} \frac{r}{8r+4w} \Longleftarrow \frac{r}{8r+4w} \quad \frac{r}{8r+4w}$$

1 2 8

Multiplication rule

$$P(E_3 E_2 E_1) = P(E_1)\,P(E_2 \mid E_1) \cdot P(E_3 \mid E_1 E_2)$$

$$= \cancel{P(E_1)}\,\frac{\cancel{P(E_1 E_2)}}{\cancel{P(E_1)}} \cdot \frac{P(E_3 E_1 E_2)}{\cancel{P(E_1 E_2)}}$$

$$P(E_2 \mid E_1)$$

$$= \frac{P(E_1 E_2)}{P(E_1)}$$

$$= \frac{\cancel{P(E_1)}\,P(E_2)}{\cancel{P(E_1)}}$$

$$P(E_1 E_2 E_3 \cdots E_n) = P(E_1)\,P(E_2|E_1)\,P(E_3|E_1 E_2) \cdots P(E_n|E_1 E_2 \cdots E_{n-1})$$

2 events

$$P(E_1 E_2) = P(E_1)\,P(E_2|E_1) \Rightarrow P(E_2|E_1) = \frac{P(E_1 E_2)}{P(E_1)}$$

D : person has disease.

E : test is +ve

Given: $P(D) = 0.005$

$P(E|D^c) = 0.01$

$P(E|D) = 0.95$

To find: $P(D|E) = \dfrac{P(DE)}{P(E)} = \dfrac{P(E|D)\cdot P(D)}{P(E)}$

FP for 1% of healthy

$\Downarrow$

D^c ← Given healthy, still test says disease

$\Downarrow$

E

$P(E|D) = \dfrac{P(DE)}{P(D)}$

$DE + D^cE$

$$P(E) = P(E|D)\,P(D) + P(E|D^c)\,P(D^c)$$

$$= P(ED) + P(ED^c) =: P(E)$$

E

D occur

D not occur