Independent Events

(symmetric)

E & F

$$\left\{\begin{array}{l} P(F \mid E) = P(F) \\ P(E \mid F) = P(E) \end{array}\right.$$

$$\frac{P(E \cap F)}{P(E)} = P(F)$$

$$\boxed{P(E \cap F) = P(E)P(F)}$$

independent.

Dependent

$$P(E|F) \neq P(E)$$

$$\boxed{\text{not ind.} = \text{dependent}}$$

3 events

A, B and C

Jointly mutually independent

① $P(A \cap B \cap C) = P(A)P(B)P(C)$

② pairwise ind

$$\begin{cases} P(A \cap B) = P(A)P(B) \\ P(B \cap C) = P(B)P(C) \\ P(A \cap C) = P(A)P(C) \end{cases}$$

Que If E, F and G are ind, E is ind of $F \cup G$ and F is ind of $E \cup G$

$$P(E \cap (F \cup G)) = P(E) \cdot P(F \cup G)$$

$$P(E \cap (F \cup G)) = P((E \cap F) \cup (E \cap G))$$

$$= P(E \cap F) + P(E \cap G) - P(E \cap F \cap G)$$

$$= P(E) \times P(F) + P(E) \times P(G) - P(F \cap G) \times P(E)$$

$$= P(E) \left[P(F) + P(G) - P(F \cap G) \right]$$

$$= P(E) \times P(F \cup G)$$

$$\cancel{P(F) \times P(\quad)) = P(E) \cdot P(}$$

$$P(F \cap (E \cup G)) = P(F) \times P(E \cup G)$$

$$= P(F \cap E) \cup P(F \cap G)$$

$$P(F) \times P(E) \cup P(F) \times P(G)$$

$$P(F) \left[P(E) \cup P(G) \right]$$

$$P(F) \left[P(E \cup G) \right]$$

Q

G_1 : 1st child is girl (elder)

G_2 : 2nd " " "

G : child seen with mother is a girl

$$P(G_1 G_2 | G) =$$

$\Rightarrow P(E)(1-P(F))$

$= P(E)P(F^c)$

$P(E \cap F^c) = P(E)P(F^c)$

Q2) If E and F are ind, then so are E and F^c

$$P(E) = P(EF) + P(EF^c)$$

$$P(E) = P(E)P(F) + P(EF^c)$$

$$P(E) - P(E)P(F) = P(EF^c)$$

$$P(E)P(F^c) = P(EF^c)$$