

functions

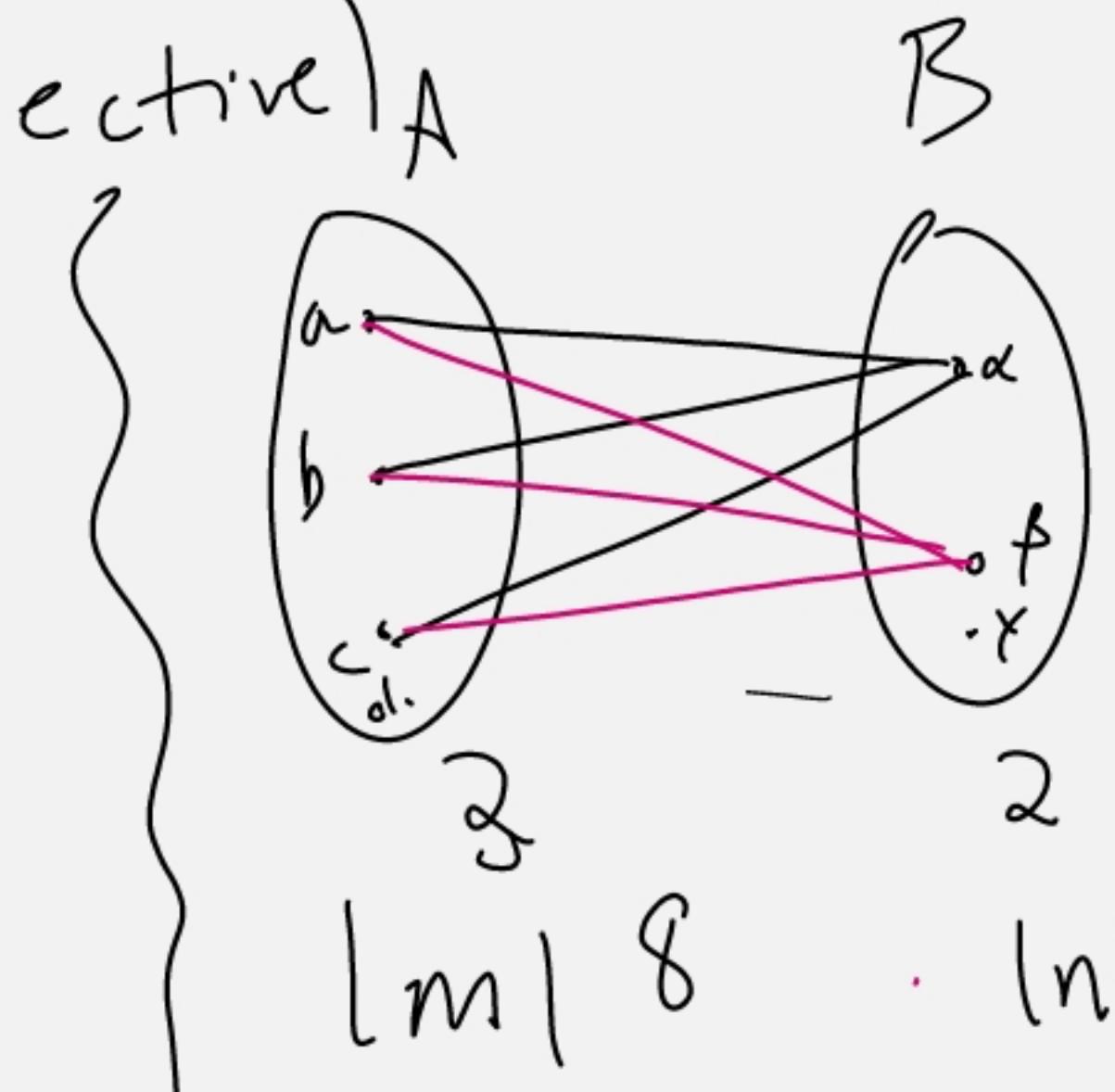
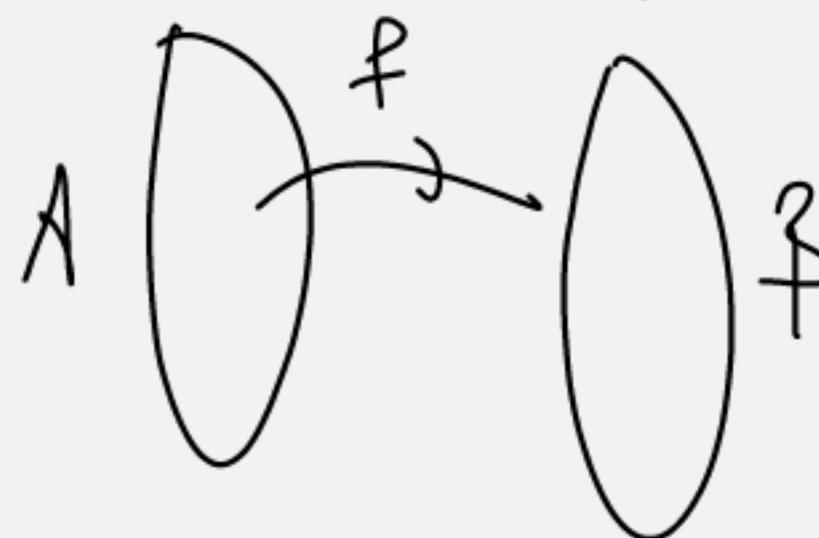
one-one function (Injective)

onto function (Surjective)

1-1 & onto function (bijective)

$$y = x^2$$

$f: \mathbb{R} \rightarrow \mathbb{R}$  as function



for every  $b \exists a: b = f(a)$

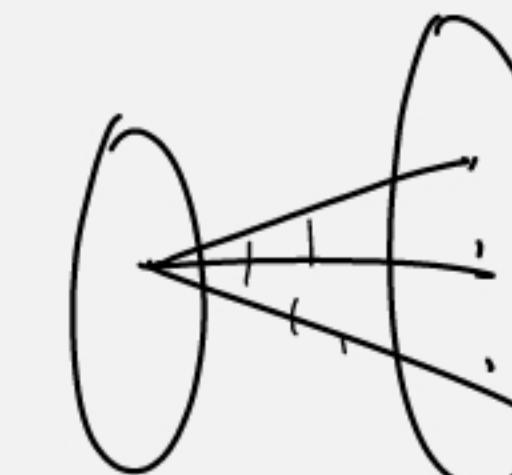
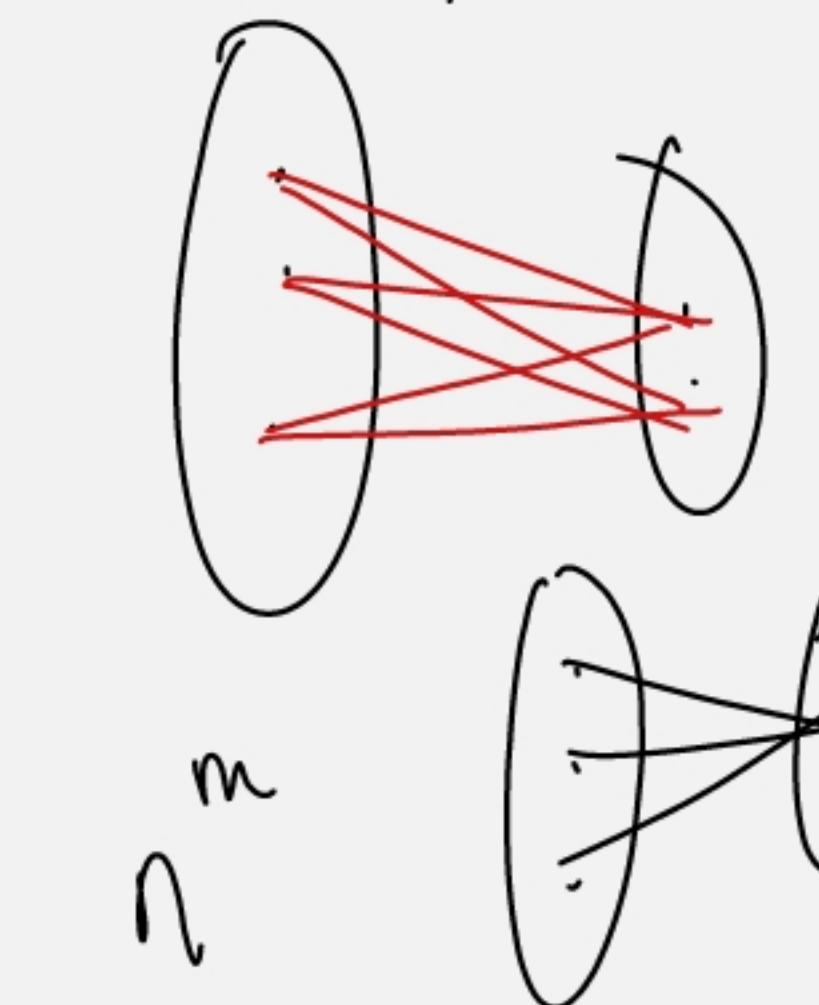
$A \xrightarrow{f} B$   
 domain      codomain  
 $f(a) = b$      $f(a_1) = f(a_2)$   
 $\Rightarrow a_1 = a_2$

$$|f| \leq |\text{co-domain}|$$

$$|f| \geq |\text{co-domain}|$$

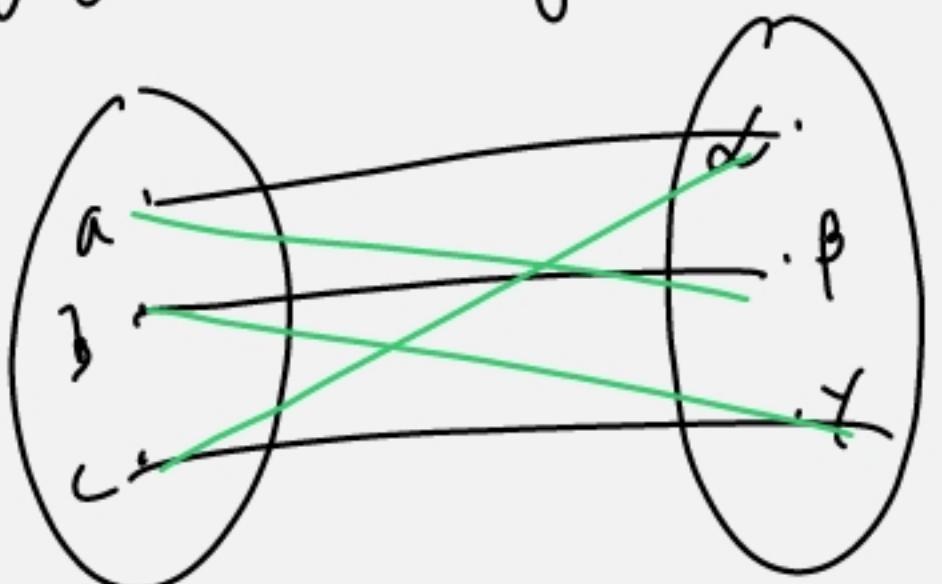
$$\begin{matrix} 2 & 3 \\ 3 & 4 \end{matrix} \quad |m| \quad |n| \quad |d| = |\text{co-domain}|$$

$$\begin{array}{c} a & b & c \\ \alpha & \alpha & \alpha \\ \alpha & \alpha & \beta \\ \alpha & \beta & \gamma \end{array}$$

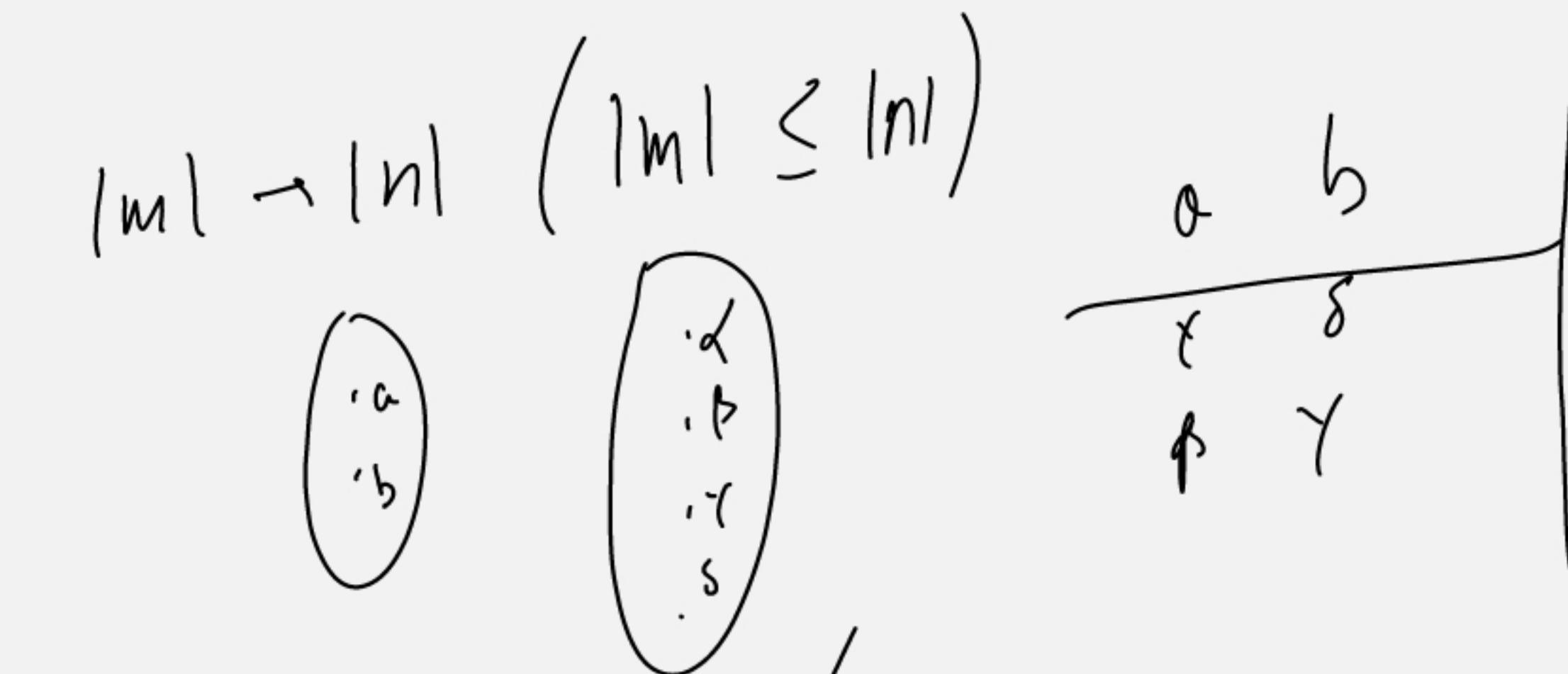
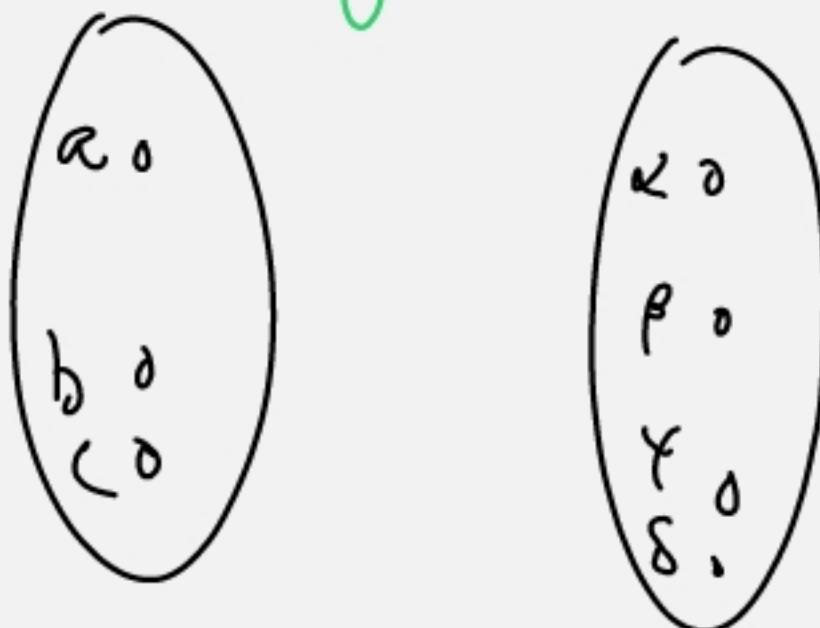


$|m| \rightarrow |n|$   
 $n^m$  functions.

① how many 1-1



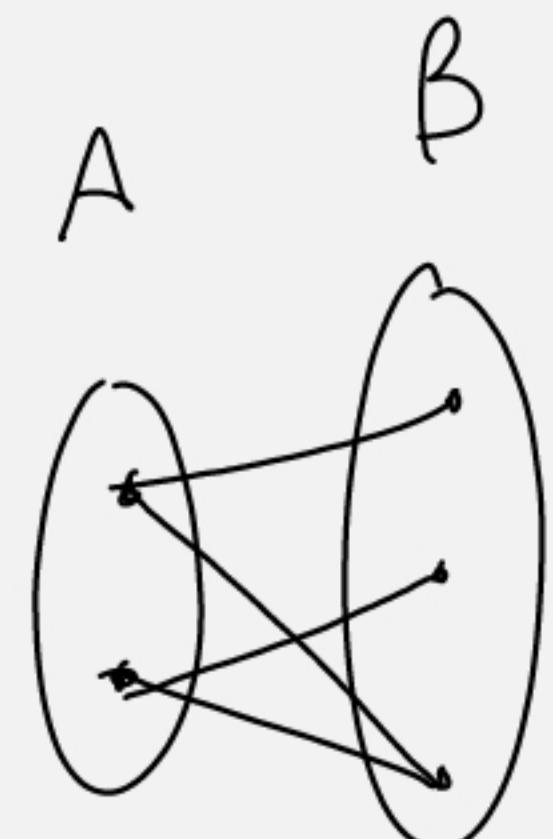
$|m|$  total 1-1 function  $|n|$



2 out of 4  
 $n P_m$

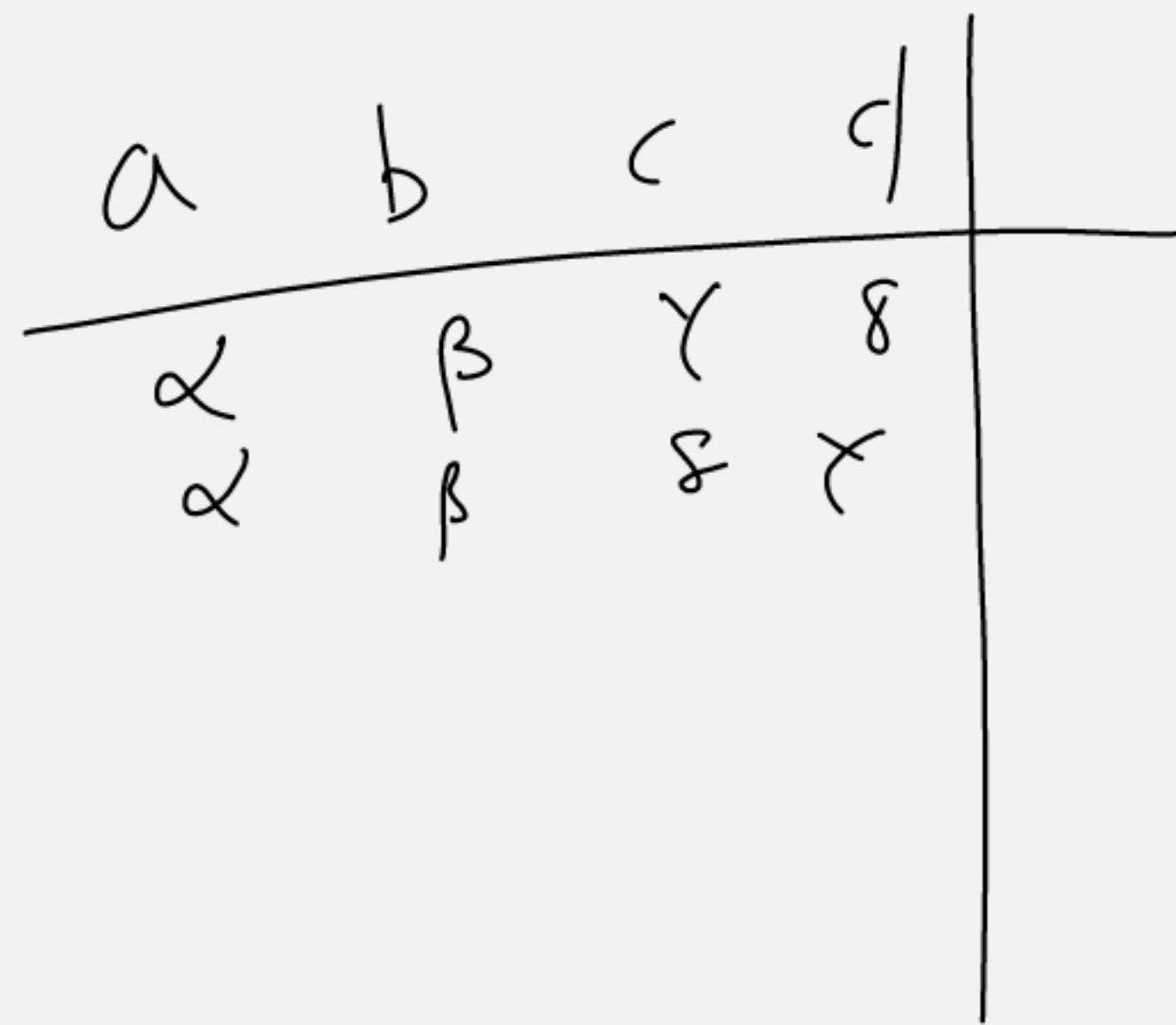
a	b	c
a	x	y
b	y	x
c	s	z
s	z	x
z	x	y
x	y	z
y	z	x

3 of 4



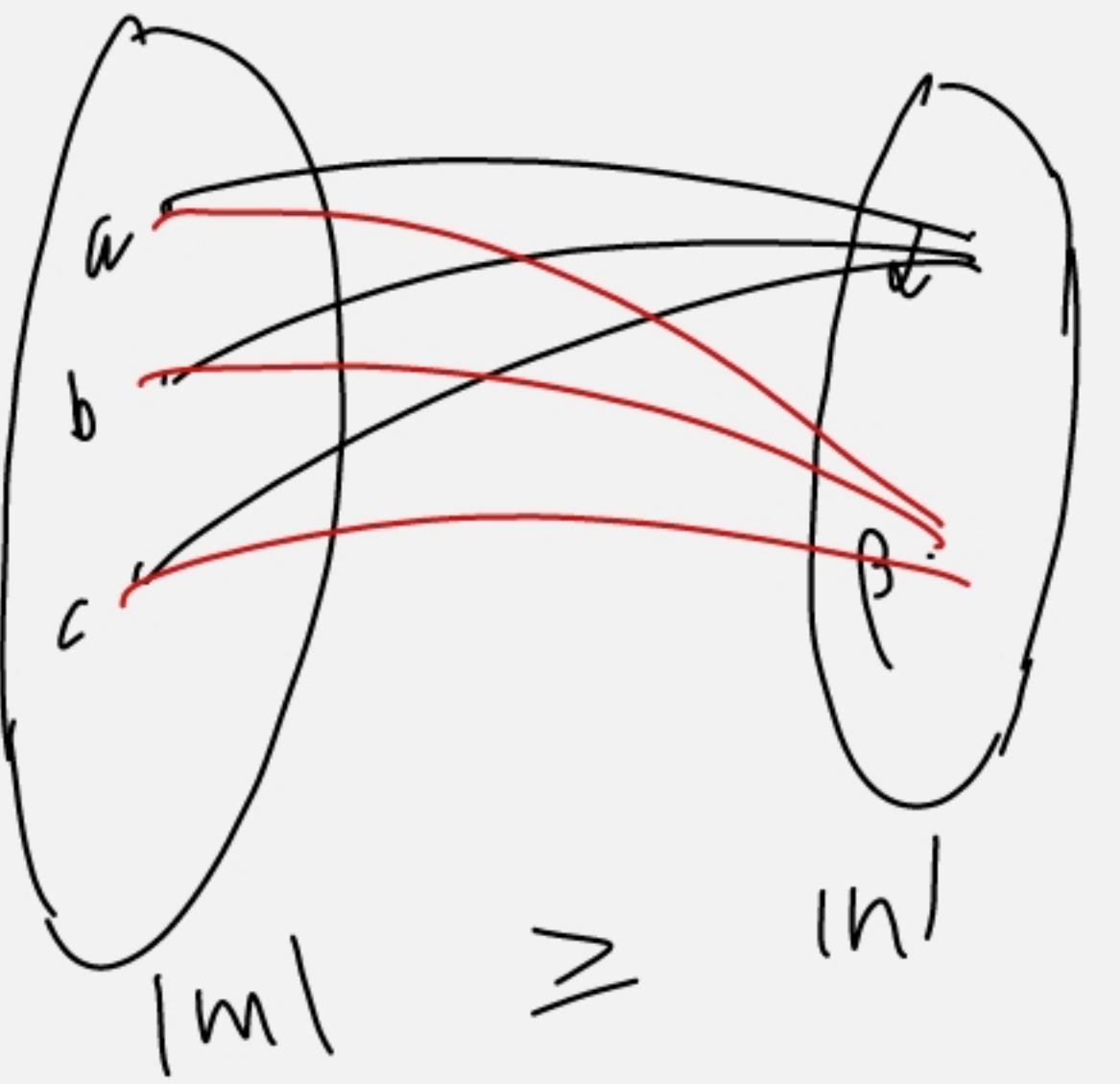
$n_0 \cdot q$  1-1 + onto function  
=  $n$

$m$



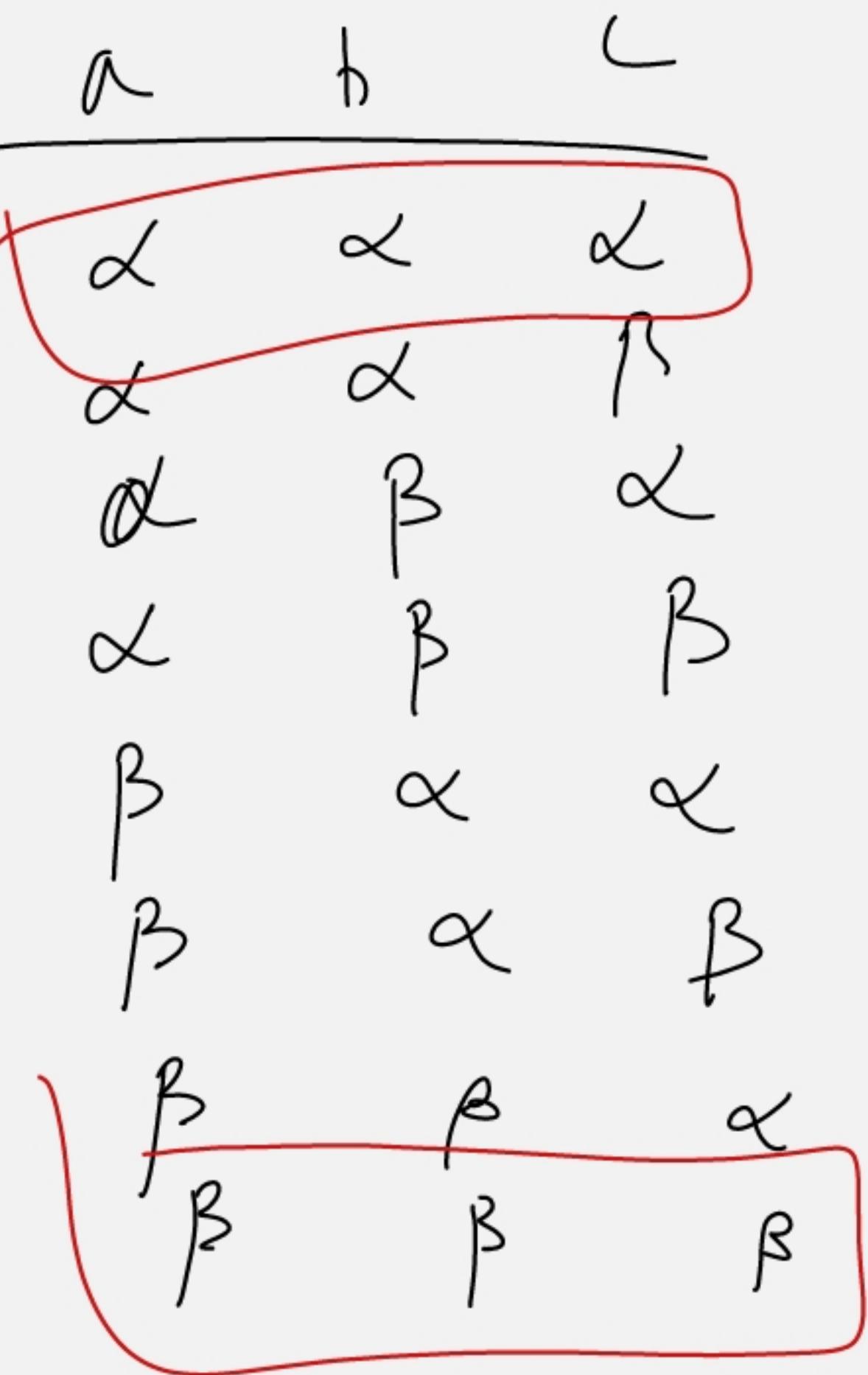
$f!$

$n!$  or  $m!$



$$8 - 2$$

$$= 6$$



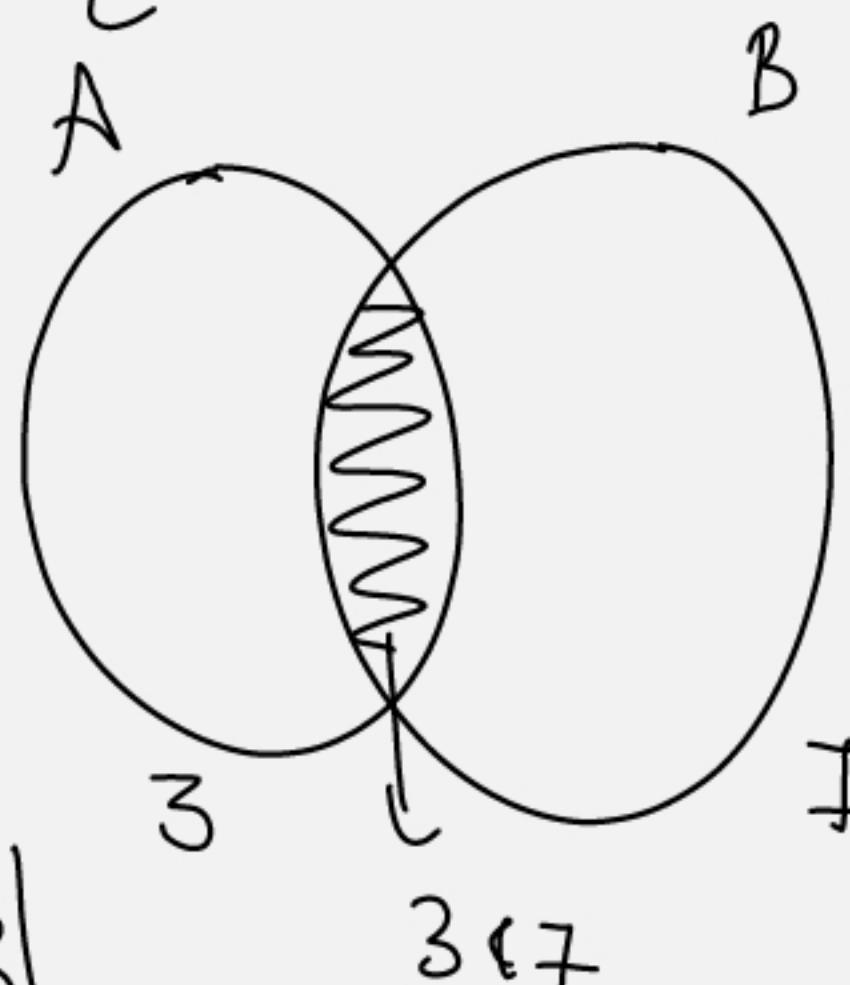
How many positive integers not exceeding 1000  
are divisible by 3 or 7?

$$|A| \left\lfloor \frac{1000}{3} \right\rfloor = 333$$

$$|B| \left\lfloor \frac{1000}{7} \right\rfloor = 142$$

428

$$|A \cup B| = |A| + |B| - |A \cap B|$$



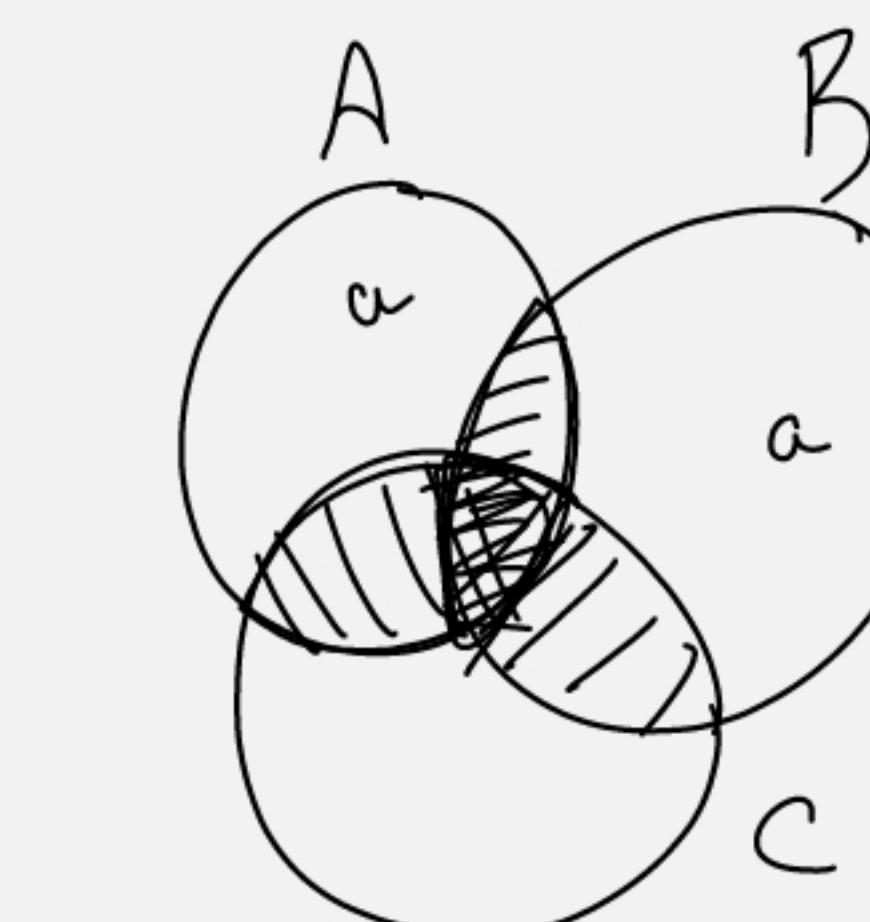
$|A + B|$  Principle of Inclusion-Exclusion

$$\left| A \cap B \right| \left\lfloor \frac{1000}{3 \cdot 7} \right\rfloor = 47$$

$$3 \text{ or } 7$$

$$[3 \text{ or } 7 \text{ or } 11]$$

$$476 + 4 \\ = 480$$



$$\begin{aligned} & |A \cup B \cup C| \\ &= |A| + |B| + |C| \\ &\quad - |A \cap B| - |B \cap C| - |A \cap C| \end{aligned}$$

$$+ |A \cap B \cap C|$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n \quad nC_0 = 1$$

$$\sum_{k=0}^n (-1)^k \binom{n}{k} = 0 \quad (1+(-1))^n$$

$$\begin{aligned} & |A_1 \cup A_2 \dots \cup A_n| \\ &= \sum_{1 \leq i \leq n} |A_i| - \sum_{1 \leq i < j \leq n} |A_i \cap A_j| \\ &\quad + \sum_{1 \leq i < j < k \leq n} |A_i \cap A_j \cap A_k| - \\ &\quad + (-1)^{n+1} |A_1 \cap A_2 \dots \cap A_n| \end{aligned}$$

$$nC_0 = 1$$

'a' is a member of  $r$  sets  $1 \leq r \leq n$

This 'a' is counted  $r$  times in  $\sum |A_i|$   $rC_1$   $rC_m$  times for a summation involving ' $m$ ' sets of  $A_i$

Counted  $rC_2$  times in  $|A_i \cap A_j|$

$$rC_1 - rC_2 + rC_3 - \dots - (-1)^{r+1} rC_r = 1$$

$$\left\{ \frac{nC_0 - nC_1 + nC_2 - \dots}{nC_0 - [\text{Count}]} = 0 \right\} = 0$$

$$x_i \geq 0$$

$$x_1 + x_2 + x_3 = 11$$

$$78 = \binom{13}{2} = \binom{13}{11}$$

$$x_1 \leq 3, x_2 \leq 4, x_3 \leq 6$$

$$\underline{x_1 \geq 4, x_2 \geq 5, x_3 \geq 7}$$

$$x_1 \geq 4, x_2 \geq 0, x_3 \geq 0$$

$$[\underbrace{x_1 + y_1}_{x_1}] + x_2 + x_3 = 11 ; y_1 + x_2 + x_3 = 7 \quad \begin{pmatrix} 9 \\ 2 \end{pmatrix}$$

$$78 \boxed{\dots}$$

$$9 \quad | \quad 1$$

so as a sum of  
4 (positive) integers?

$$x_1 + x_2 + x_3 + x_4 = 50$$

$$x_i \geq 0 \quad \binom{53}{3} \quad x_1 + x_2 = 1$$

$$x_i \geq 0 \quad \binom{49}{3} \quad x_2 - x_1 = 6$$

$$y_1 + 1 + y_2 + 1 + y_3 + 1 + y_4 + 1 = 50$$

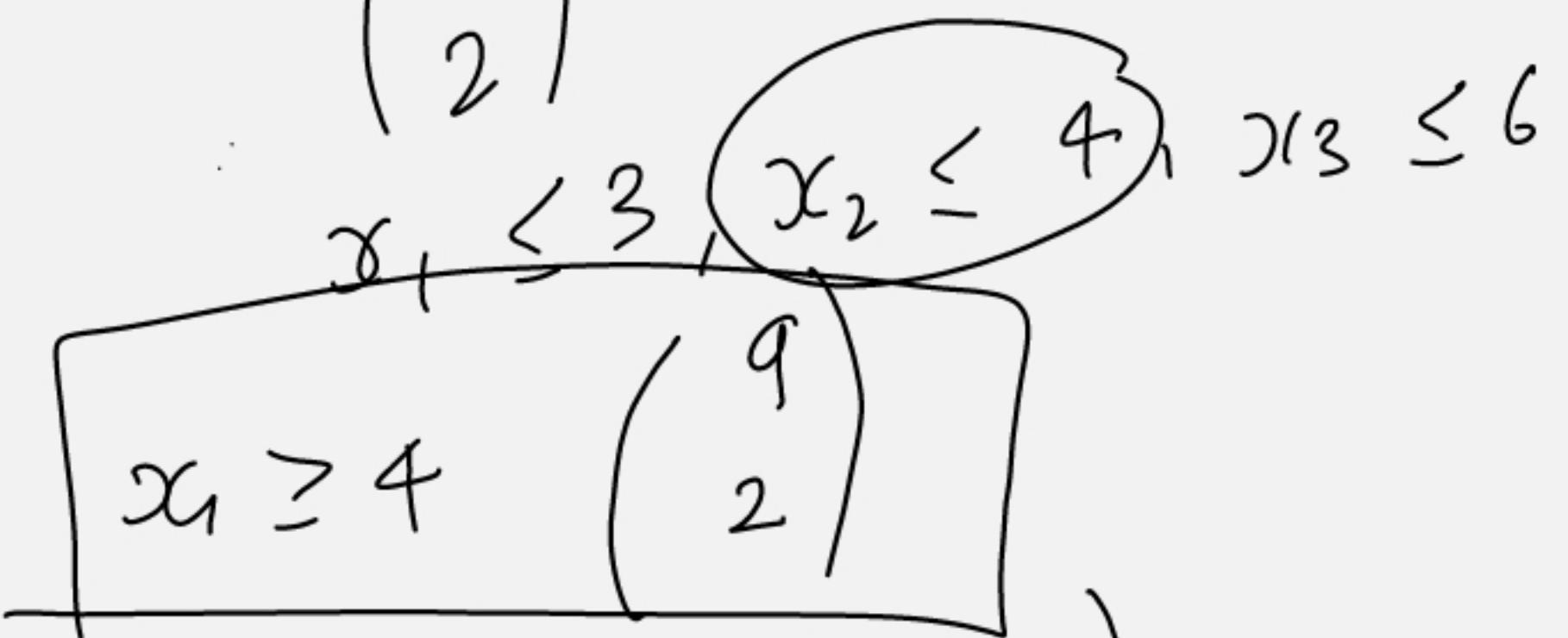
$$y_1 + y_2 + y_3 + y_4 = 46 \quad x_1 + x_2 \geq 1$$

$$y_i \geq 0$$

nr of onto functions.

$$x_1 + x_2 + x_3 = 11$$

$$\binom{13}{2} = 78$$



$$x_2 \geq 5$$

$$\binom{8}{2}$$

$$x_1 + y_2 + 5 + x_3 = 11 \quad x_1 + y_2 + x_3 = 6$$

$$x_3 \geq 7 \quad \binom{6}{2} \quad x_1 + x_2 + (y_3 + 7) = 11 \quad x_1 + x_2 + y_3 = 4$$

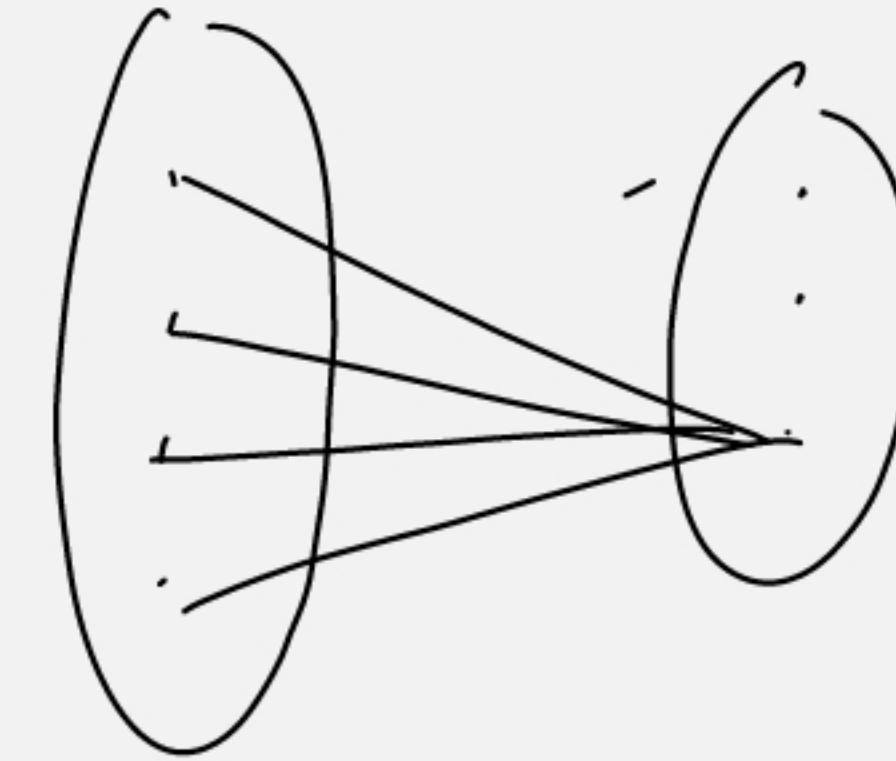
$$\left| \begin{array}{l} x_1 \geq 4, x_2 \geq 5 \\ (y_1 + 4) + (y_2 + 5) + x_3 = 11 \\ y_1 + y_2 + x_3 = 2 \\ x_2 \geq 5, x_3 \geq 7 \end{array} \right. \quad \left( \begin{array}{l} 0 \\ 0 \\ 0 \end{array} \right)$$

$\frac{78 - \binom{9}{2} - \binom{8}{2} - \binom{6}{2}}{\binom{4}{2} + 0 + \binom{2}{2}} = 0$

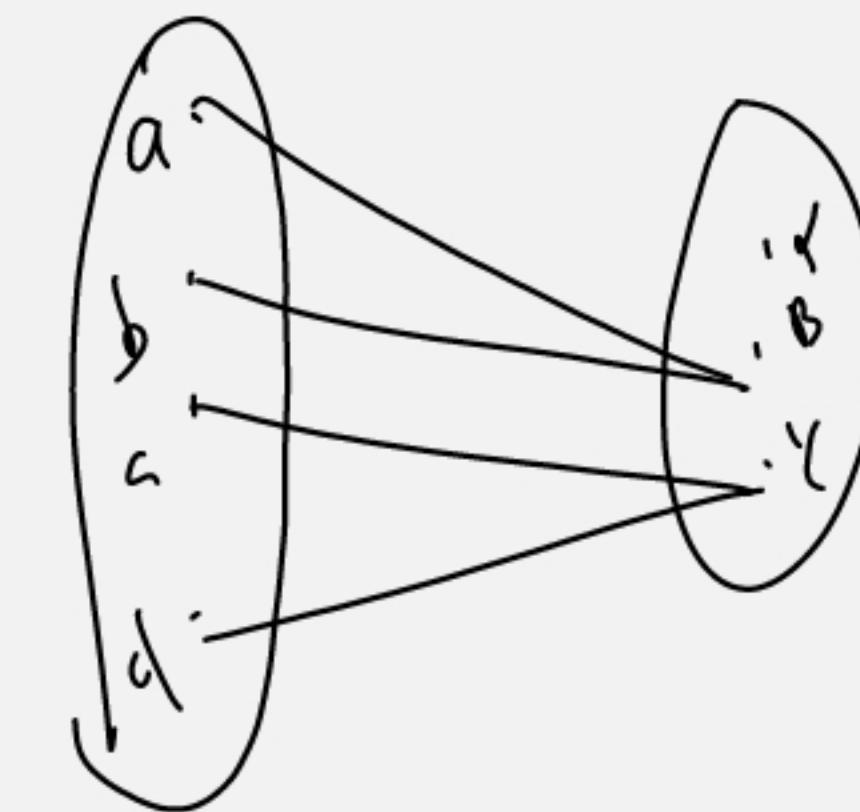
$$\left. \begin{array}{l} x_1 \geq 4, x_2 \geq 5, x_3 \geq 7 \end{array} \right. \quad \left( \begin{array}{l} 0 \\ 0 \\ 0 \end{array} \right)$$

$$x_1 + x_2 + x_3 + x_4 = \cancel{26}$$

$$0 \leq x_i \leq \cancel{10}$$



No. of onto functions

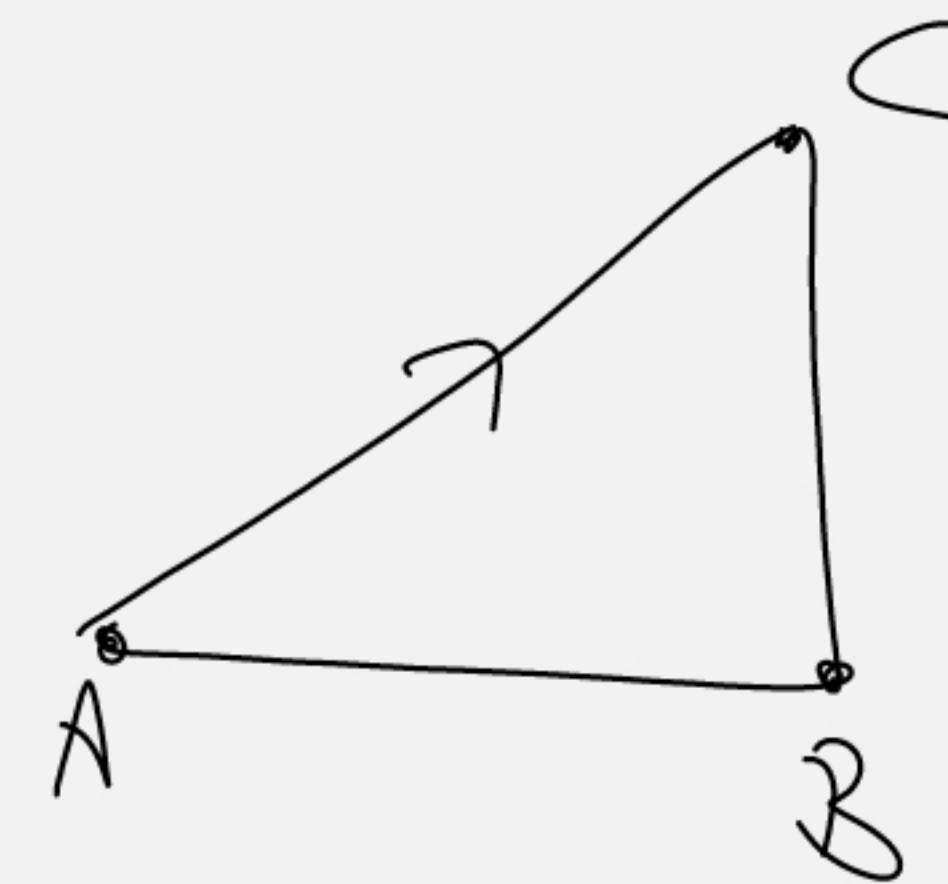


$$\begin{aligned} & 3^4 \\ & - 3 \cdot 2^4 \\ & + 3 \cdot 1^4 \end{aligned}$$

=



Student: What is application of Pythagoras theorem



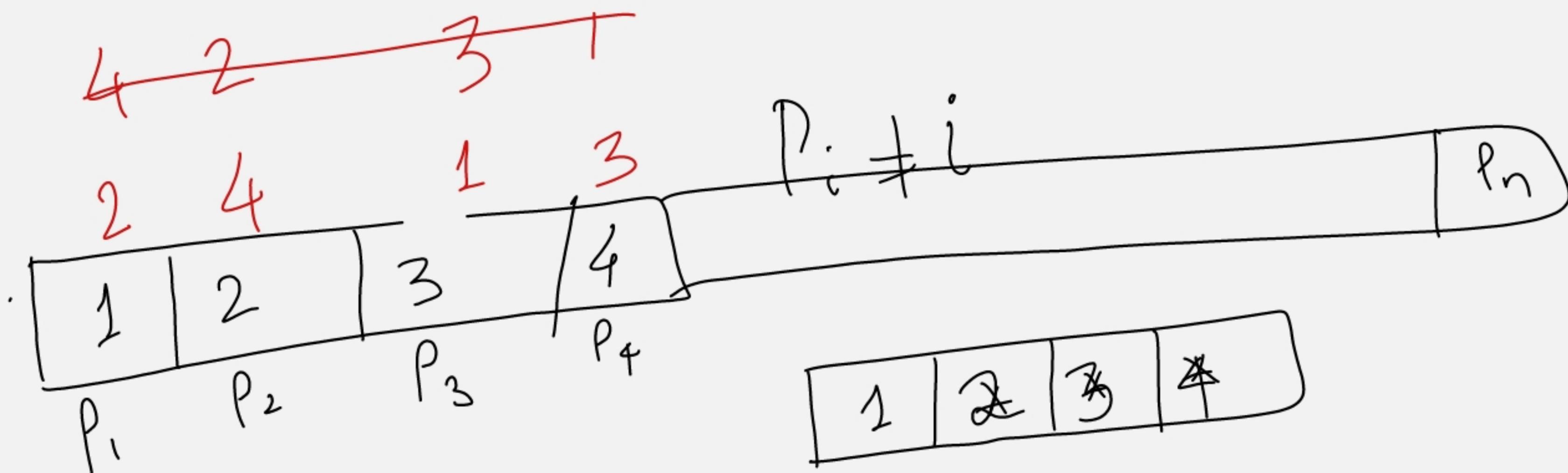
How many positive integers from 1-2000 are not divisible by 5, 7, 11.

$$|S| = 2000$$

$$\left\{ \begin{array}{l} N(P_1) : \text{nrs. divisible by } 5 \\ N(P_2) : \text{nrs. divisible by } 7 \\ N(P_3) : \text{nrs. divisible by } 11 \\ N(P_1, P_2) = \frac{2000}{35} = 57; \quad N(P_2, P_3) = \frac{2000}{77} = 25 \\ N(P_1, P_3) = \frac{2000}{119} = 181 \\ N(P_1, P_2, P_3) = \frac{2000}{385} = 5 \end{array} \right. \quad \bar{N} = N - [N(P_1) + N(P_2) + N(P_3)] + [N(P_1, P_2) + N(P_1, P_3) + N(P_2, P_3)] - N(P_1, P_2, P_3)$$

~~$\frac{2000}{2 \cdot 2 \cdot 2 \cdot 3}$~~

$$\left. \begin{array}{l} \frac{2000}{4} = 500 \\ \frac{2000}{6} = 333 \end{array} \right\} \text{LCM}(4, 6)$$



$$4!$$

$$\binom{4}{0} 4! - \binom{4}{1} 3! + \binom{4}{2} 2! - \binom{4}{3} 1! + \binom{4}{4} 0!$$

$$N(P_1) \quad P_1 \hookrightarrow L \quad 3^1$$

$$N(p_2)$$

$$N(p_3)$$

$$31 \quad N(P_4) = 3$$

$$|m| \rightarrow |n|$$

$$|S| = 2^b$$

$$|S| = 2^6!$$

ONE, QUIZ, PART

$N(P_1) \rightarrow$  no. of words which contain ONE

$N(P_2)$

$N(P_3)$

$N(P_1 P_2)$

$N(P_2 P_3)$

ONEQUIZ

ONE QUIZ

QUIZ PART

QUIZ

PART

$$19 + 1 + 1 = 21$$

$N(P_1 P_2)$

$$|26!! - (24! + 23! + 23!) + (21! + 20! + 2!) - (18!)|$$

AUNERBC - .

24!

$$\frac{\text{ONE} \cdot \text{ABCDEFG} \dots}{23}$$

$$\frac{\text{QUIZ} \{ A \dots Y \}}{22}$$

$$N(P_1 P_2 P_3) = 18!$$

23!  
23!

$$\frac{\text{ONE PART QUIZ}}{18}$$

$$20! \\ = 21!$$

floor  $\lfloor x \rfloor$

Celing  $\lceil x \rceil$

$$\lfloor 1.2 \rfloor = 1$$

$$\lceil 2.1 \rceil = 3$$

$$\lceil 1.2 \rceil = 2$$

$$\lfloor -\frac{1}{2} \rfloor = -1$$

$$\lceil -\frac{1}{2} \rceil = 0$$

floor function  
 $x \in \mathbb{R}, f(x) \rightarrow \mathbb{Z}$

it assigns the largest integer  $\leq x$

Celing function

assign the smallest intgr  $\geq x$

$$\lfloor x+n \rfloor = \lfloor x \rfloor + n$$

$$\lfloor x \rfloor = m$$

$$m \leq x < m+1$$

$$m+n \leq \lfloor x+n \rfloor < m+1+n$$

$$\lfloor x+n \rfloor = m+n = \lfloor x \rfloor + n$$

$$\lceil x+y \rceil \geq \lceil x \rceil + \lceil y \rceil$$

$$\lfloor x \rfloor = n \text{ iff } n \leq x < n+1$$

$$\lfloor x \rfloor = n \text{ iff } x-1 < n \leq x$$

$$\lceil x \rceil = n \text{ iff } n-1 < x \leq n.$$

iff  $x \leq n < x+1$

$$x-1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x+1$$

$$\begin{aligned} \lfloor x+n \rfloor &= \lfloor x \rfloor + n \\ \lceil x+n \rceil &= \lceil x \rceil + n \end{aligned}$$

$$\begin{aligned} \lfloor x \rfloor &= n \\ \lfloor x + \frac{1}{2} \rfloor &= n+1 \end{aligned}$$

$$\lceil x+y \rceil \stackrel{?}{=} \lceil x \rceil + \lceil y \rceil \quad x = \frac{1}{2}, y = \frac{1}{2}$$

$\lceil \cdot \rceil \neq \lceil \cdot \rceil$

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$$\begin{aligned} \lfloor x \rfloor &= \lfloor n \rfloor \\ \lfloor 2x \rfloor &= \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor \\ 2x &= 2n + 2\varepsilon \quad \& \quad \lfloor 2n \rfloor = 2^n. \quad \left\{ \begin{array}{l} \lfloor x \rfloor = n \\ 0 \leq \varepsilon < \frac{1}{2} \end{array} \right. \\ \lfloor x + \frac{1}{2} \rfloor &= \lfloor n + \varepsilon + \frac{1}{2} \rfloor = n \quad \frac{1}{2} \leq \varepsilon < 1 \end{aligned}$$

$$\begin{aligned} \lfloor 2x \rfloor &= \lfloor 2n + 2\varepsilon \rfloor = \lfloor 2n+1 + 2\varepsilon - 1 \rfloor = 2^{n+1} \\ \lfloor x + \frac{1}{2} \rfloor &= \lfloor n + \frac{1}{2} + \varepsilon \rfloor = \lfloor n+1 + \varepsilon - \frac{1}{2} \rfloor = \lfloor n+1 \rfloor \end{aligned}$$

1, 2, 4, 8, 16. -

$$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16}$$

Arithmetic progression

$$2 \quad 4 \quad 6 \quad 8 \quad \dots$$

$$a_0 = 1$$

$$a_1 = 2$$

$$\boxed{\frac{dx}{dt} = -x}$$



$$x(0) = 1$$

$$\sum_{n=1}^{10}$$

$$x(1) = e^{-1} \cdot 1$$

$$1 + 2 + \dots$$

$$x(t) = \left( e^{-t} \right) \left( x(0) \right) = e^{-t} \cdot 1$$

Geometric Progression

$$a_{10}, a_2, a_3, a_n = r a_{n-1}$$

$$a, ar, ar^2, \dots$$

Sequences  $r=4$

$$a_2 = r a_1$$

$$a_3 = r a_2 = r^2 a_1$$

$$a_n = r a_{n-1}$$

$$a_1 = 1$$

$$r = 4$$

$$a_n =$$

$$n! = n(n-1)!$$

$$a+b, a+2b, a+3b, \dots$$

$$n=0 \quad n=1 \quad n=2 \quad n=3$$

$$\frac{10(10+1)}{2} = 55$$

$$a_n = a_{n-1} - a_{n-2}$$

$$a_2 = 2 - 1 = 1$$

$$a_3 = a_2 - a_1 = -1$$

recurrence relations

$$a_n = r \cdot a_{n-1}$$

$$a_0 = 1 \quad A_4 = \underbrace{1 \cdot 1}_{r=4} A_3 \\ r = 4 \quad = (1 \cdot 1) A_1$$

$$a_1 = 4 \cdot a_0$$

$$a_2 = 4 \cdot a_1 = 4^2 \cdot a_0$$

$$a_3 = 4 \cdot a_2 = 4^3 \cdot a_0 \quad \vdots \quad (n)$$

$$a_n = 4 \cdot a_{n-1} \dots = 4 \cdot a_0$$

Sequence

$$a_n = r^n \cdot a_0$$

$$x(t) = e^{rt} \cdot x(0)$$

Compound

$$M_1 = 100$$

$$M_2 = 110 = (1.1)100$$

$$M_3 = (1.1)110 = 121$$

$$M_4 = (1.1) \cdot 121 = 133.1$$

$$\frac{dx}{dt} = -ax \quad x(t)$$

Simple Interest

$$M_1 = 100 \$$$

$$M_2 = 110$$

$$M_3 = 120$$

$$A_2 = 1.1 A_1$$

$$A_3 = 1.1 \cdot A_2 = (1.1)^2 A_1$$

10%

$$A_n = (1.1) A_{n-1}$$

$$A_n = (1.1)^{n-1} A_1$$

$$a_n = a_{n-1} + a_{n-2}$$

6, 3, 9, 18, 30, 45, -

$$a_2 - a_1 = 3$$

$$a_3 - a_2 = 6$$

$$a_4 - a_3 = 9$$

$$a_5 - a_4 = 12$$

1, 2, 3, 5, 8, 13, 21, -

$$a_n = a_{n-1} + a_{n-2}$$

$$a_n = 3(n-1) + a_{n-1}$$

$$n \quad 2^n$$

$$P \left(1 + \frac{r}{100}\right)^n$$

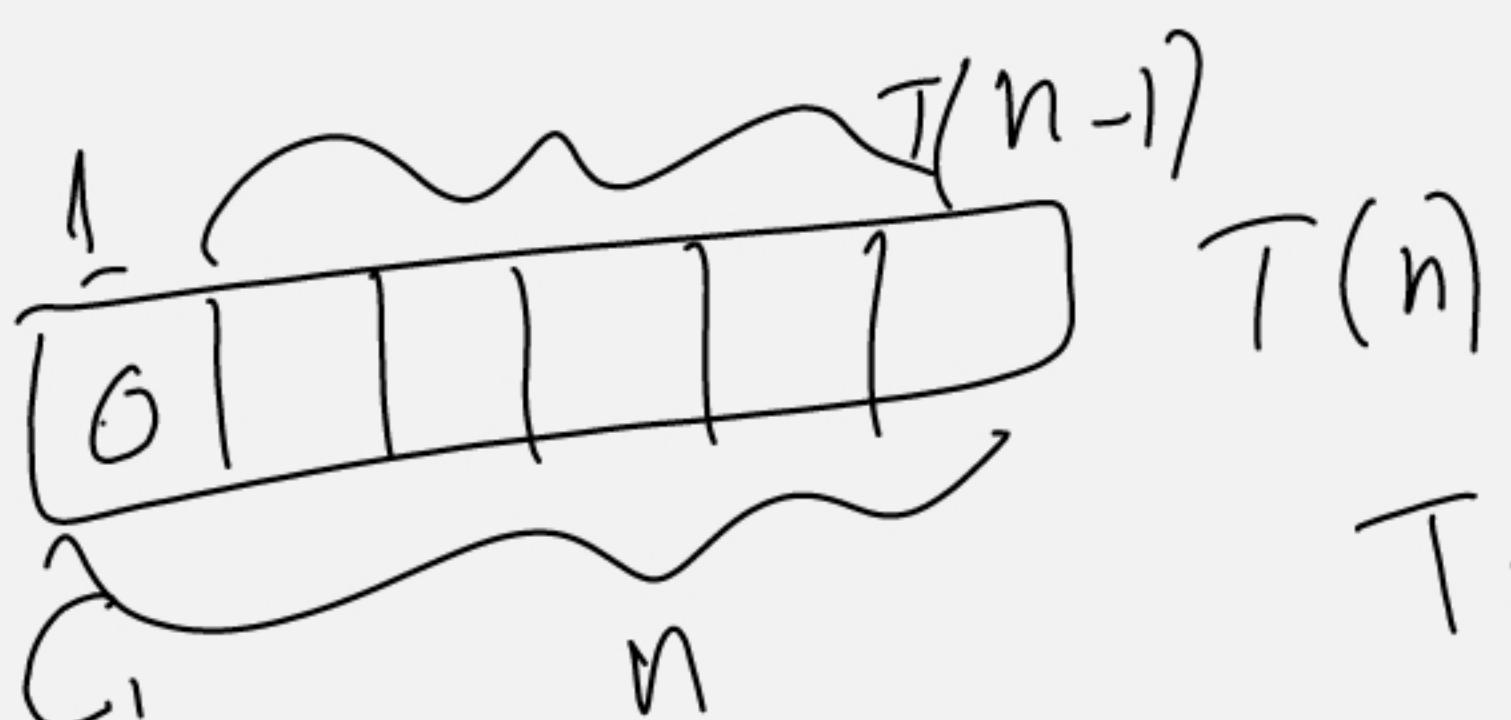
$$(a_1 + a_n a)$$

$$a_n = a_{n-1} + 3n$$

$$(a_0, a_1 - 1)$$

$$a_n = \frac{n}{2} \cdot n +$$

$$a_n = f(a_{n-1}, a_{n-2})$$

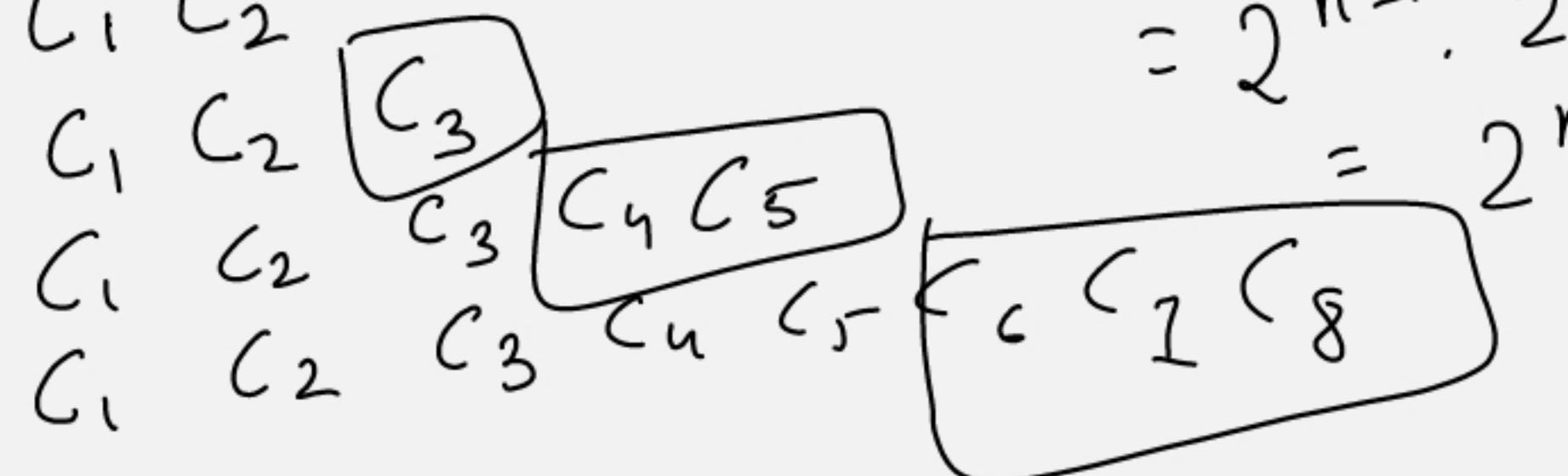


$$T(1) = 2$$

$$T(n) = 2^{n-1} \cdot T(1)$$

$$= 2^{n-1} \cdot 2$$

$$= 2^n$$



$$n - K = 1$$

$$C_n = C_{n-1} + C_{n-2}$$

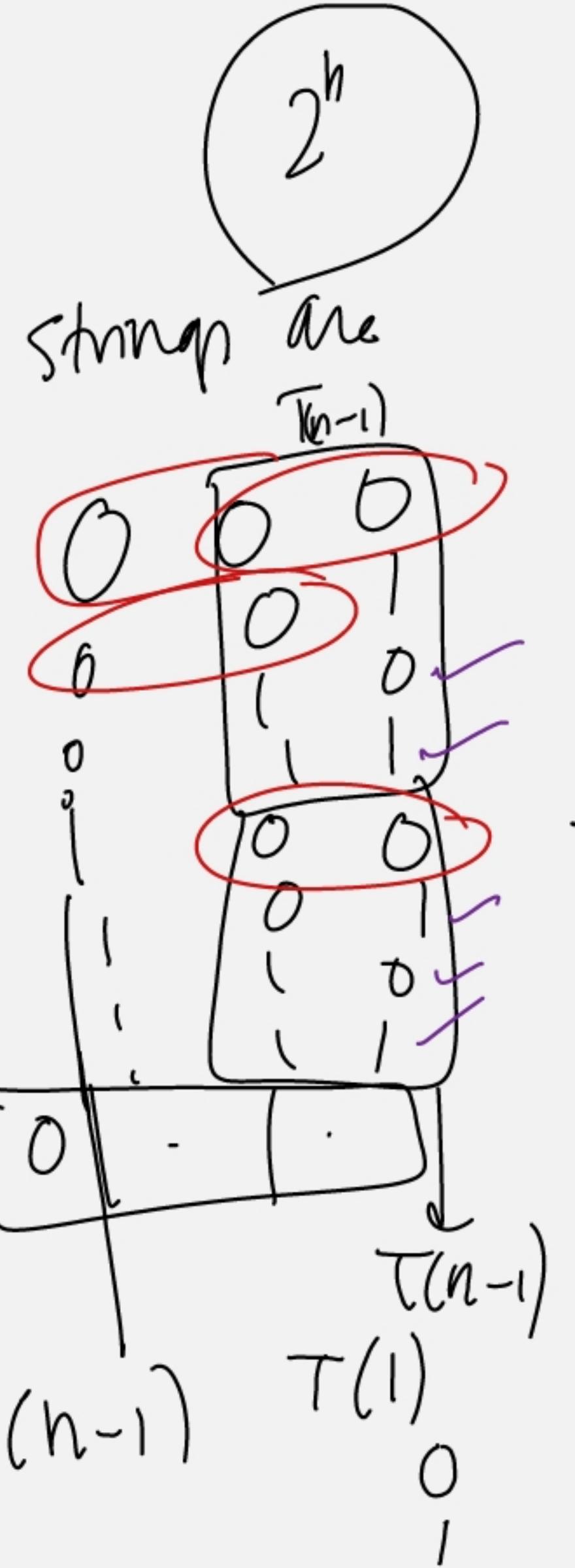
no. of pairs in Prev. month

New born,

$$T(n) \quad T(n-1)$$

How many binary digit strings are possible of length  $n$ .

$$\begin{aligned} & T(n) = T(n-1) + T(n-1) \\ & = 2 \cdot T(n-1) \\ & = 2 \cdot 2 [T(n-2)] = 2^2 T(n-2) \\ & = 2^{n-1} T(n-K) \end{aligned}$$

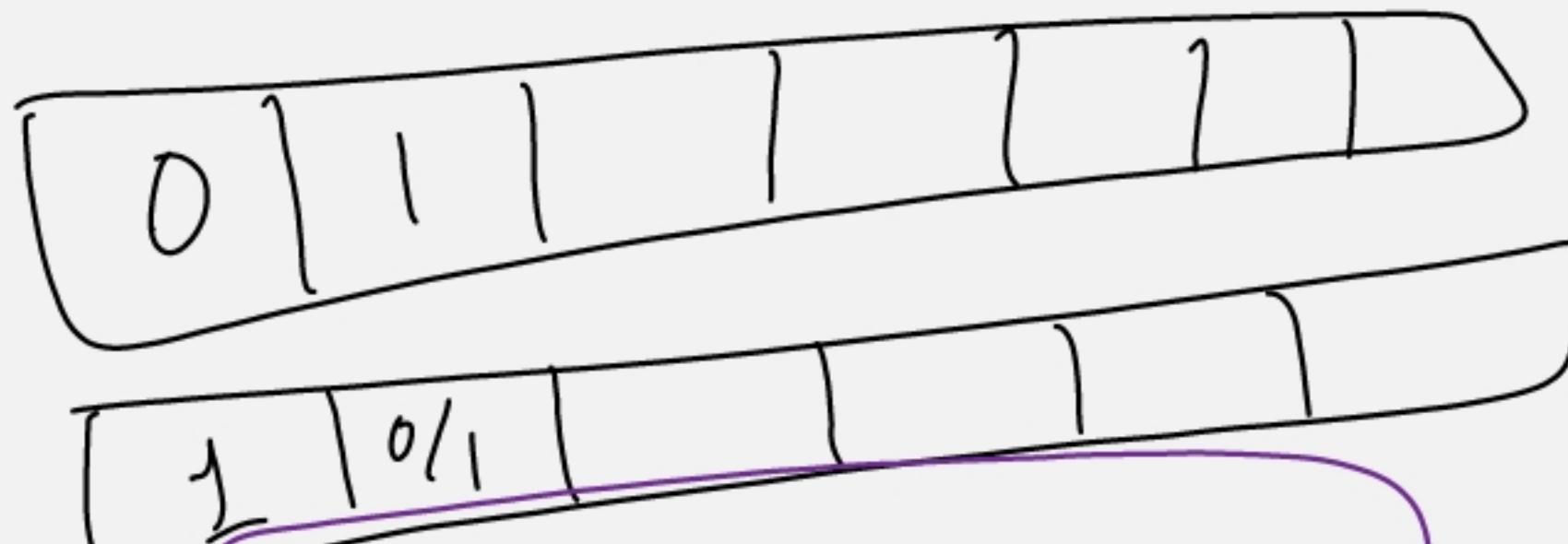


How many n-digit strings are possible without consecutive zeros & consecutive 1's

$$T(n)$$

$$T(n) = T(n-1) + T(n-2)$$

0 0  
0 1  
1 0  
1 1



$$T_n = T(n-1) + T(n-2)$$

$$T(n-1) + T(n-2)$$

$$T(3) = 5 \quad T(4) = 8$$

$$\begin{aligned} T(1) &= 2 \\ T(2) &= 3 \end{aligned}$$

$$T(n)$$

$$T(n-1)$$

$$T(1) = 0$$

$$1$$

$$T(2) = 3$$

0 0 0
0 0 1
0 1 0 ✓
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1

0 0 0 0
0 0 0 1
0 0 1 0
0 0 1 1
0 1 0 0
0 1 0 1
0 1 1 0
1 0 0 0
1 0 0 1
1 0 1 0
1 0 1 1
1 1 0 0
1 1 0 1
1 1 1 0
1 1 1 1

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\frac{dx}{dt} = -x$$

$$x(0) = 0$$

$$x'(0) = 1$$

$$\boxed{\frac{d^2x}{dt^2} - 2 \frac{dx}{dt} - 3x = 0}$$

Solve for  $x(t)$

$$2 \left[ x(t) = A e^{\lambda_1 t} + B e^{\lambda_2 t} \right]$$

$$\lambda^2 - 2\lambda - 3 = 0; \quad \lambda_1 = 3$$

$$\boxed{x(t) = \frac{1}{4} e^{3t} - \frac{1}{4} e^{-t}}$$

$$\left. \begin{aligned} x(t) &= \left( \frac{1}{4} e^{3t} - \frac{1}{4} e^{-t} \right) \\ -2 \left( \frac{dx}{dt} \right) &= -\left( \frac{3}{4} e^{3t} + \frac{1}{4} e^{-t} \right) \\ \frac{d^2x}{dt^2} &= -\frac{9}{4} e^{3t} - \frac{1}{4} e^{-t} \end{aligned} \right\}$$

$$\left. \begin{aligned} x(0) &= \frac{1}{4} - \frac{1}{4} = 0 \\ x'(0) &= \frac{3}{4} + \frac{1}{4} = 1 \end{aligned} \right\}$$

$$\left. \begin{aligned} x(0) &= e^0 \\ x(t) &= e^{-t} \end{aligned} \right\}$$

$$\left. \begin{aligned} x(t) &= Ae^{3t} + Be^{-t} \\ 0 &= A + B \\ 1 &= 3A - B \end{aligned} \right\}$$

$$-t$$

$$x \text{ at } t=0 = 1$$

$$A = \frac{1}{4}; \quad B = -\frac{1}{4}$$

Recurrence relation -

10% (Compound) Interest

$$a_n = (1.1) a_{n-1}$$

$$a_0 = 1000$$

$$a_1 = 1.1 a_0$$

$$a_2 = 1.1 a_1 = (1.1)^2 a_0$$

$$a_{14} = \frac{(1.1) a_{13}}{(1.1)(1.1) \cdot a_n}$$

$$a^n = (1.1)^n a_0 = (1.1)^n \cdot 1000$$

$$a_{14} = (1.1)^{14} \cdot 1000$$

$$c_n = c_{n-1} + c_{n-2}$$

$$a_n = (1.1)^n a_0$$

Solution to the  
recurrence relation -

$$a_n = c_1 a_{n-1} + c_2 a_{n-2}$$

Solve  $\left( x^2 - c_1 x - c_2 = 0 \right)$

$x_1, x_2$  (roots of quad eq)

$$a_n = \alpha_1 x_1^n + \alpha_2 x_2^n$$

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

$$a_n - c_1 a_{n-1} - c_2 a_{n-2} = 0$$

$$a_n = 3a_{n-1} - 2a_{n-2}, \begin{cases} a_0 = 1 \\ a_1 = 2 \end{cases}$$

$$x^2 - 3x + 2 = 0$$

$$x_1 = 2 \quad x_2 = 1.$$

$$a_n = \alpha_1 (2)^n + \alpha_2 (1)^n$$

$$a_0 = 1 = \alpha_1 + \alpha_2$$

$$a_1 = 2 = 2\alpha_1 + \alpha_2$$

$$\alpha_1 = 1, \quad \alpha_2 = 0$$

$$a_n = (2)^n$$

$$a_n = a_{n-1} + a_{n-2}$$

$$a_n = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\begin{aligned} a_0 &= 0 \\ a_1 &= 1 \end{aligned}$$

$$x^2 - x - 1 = 0$$

$$\alpha_1 = \frac{1+\sqrt{5}}{2}$$

$$\alpha_2 = \frac{1-\sqrt{5}}{2}$$

$$\alpha_1 + \alpha_2 = 0$$

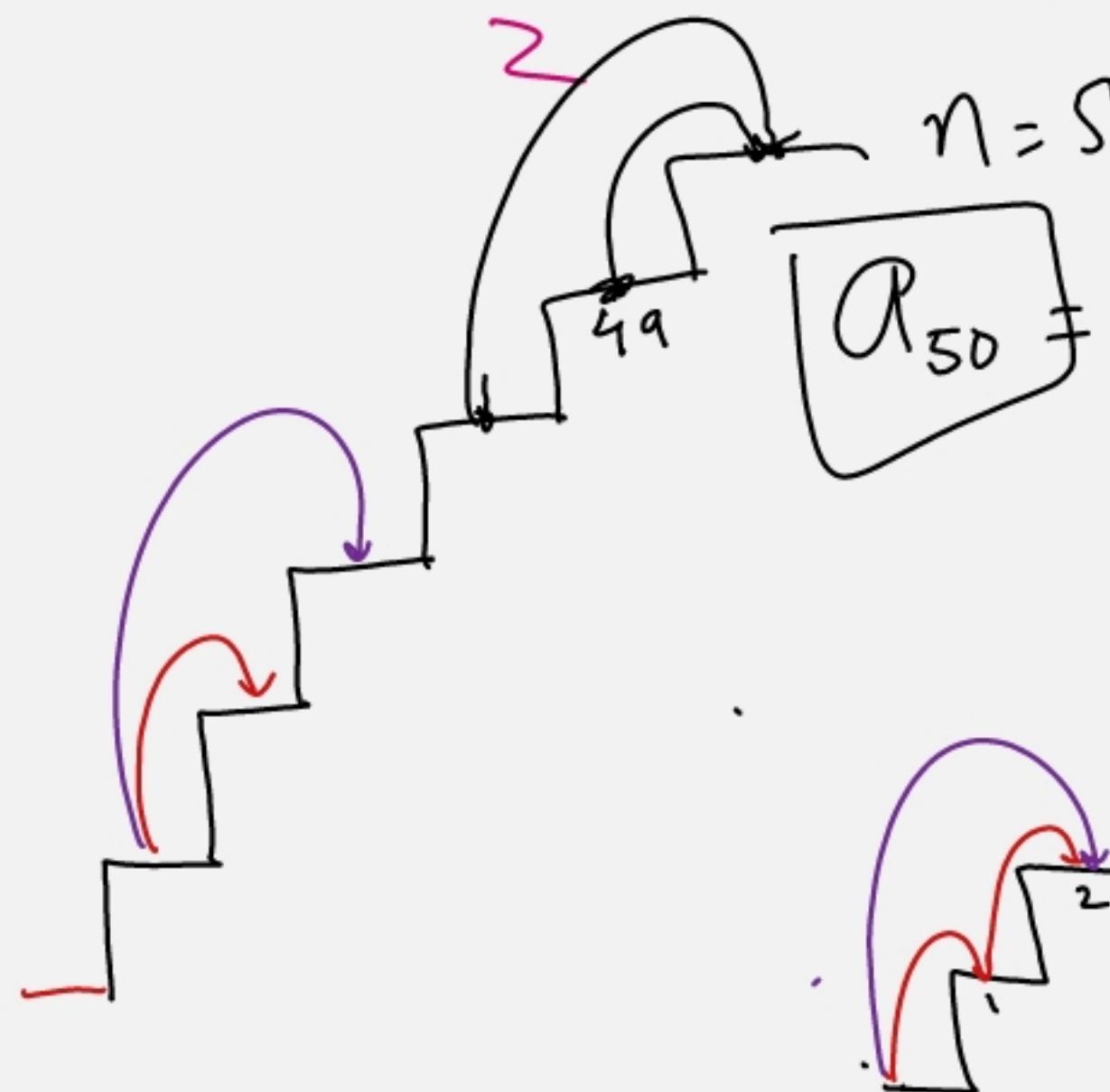
$$a_n = a_{n-1} + a_{n-2}$$

$$\alpha_1 = \frac{1}{\sqrt{5}}$$

$$\frac{\alpha_1 + \alpha_2}{2} + \frac{\sqrt{5}\alpha_1}{2} - \frac{\sqrt{5}\alpha_2}{2} = 1$$

$$\alpha_2 = -\frac{1}{\sqrt{5}}$$

$a_n$



$$a_{49} + a_{48}$$

$$a_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\boxed{\begin{aligned} a_1 &= \frac{1}{2} \\ a_2 &= 2 \\ a_3 &= 3 \end{aligned}}$$

