functions

$\overset{a}{A} \xrightarrow{f} \overset{b}{B}$ codomain

domain

$f(a) = b$

$f(a_1) = f(a_2) \Rightarrow a_1 = a_2$

$\overset{m}{|d|} \leq \overset{n}{|\text{co-domain}|}$

one - one function (Injective)

$\boxed{\text{on-to function (Surjective)}}$

for every b $\exists a$: $b = f(a)$

$|d| \geq |\text{co-domain}|$

$|d| = |\text{co-domain}|$

1-1 & Onto function (bijective)

2^3

3^4

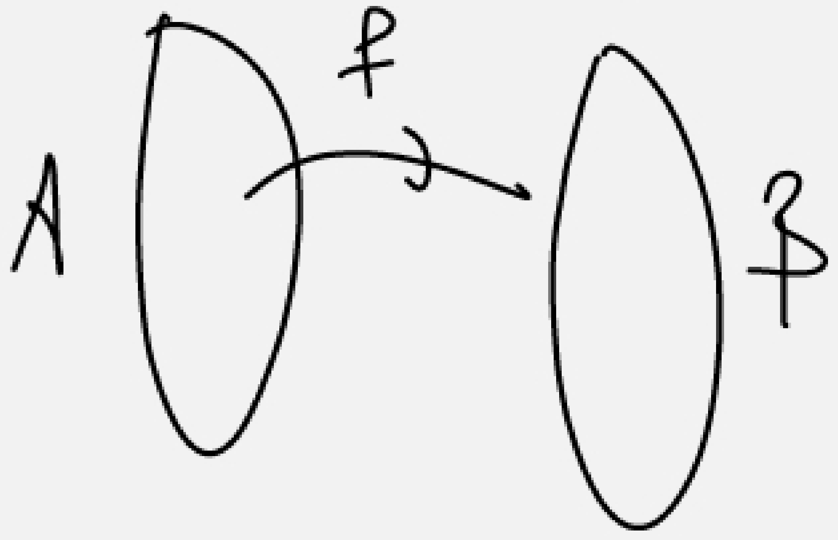
$|n|$

$|m|$

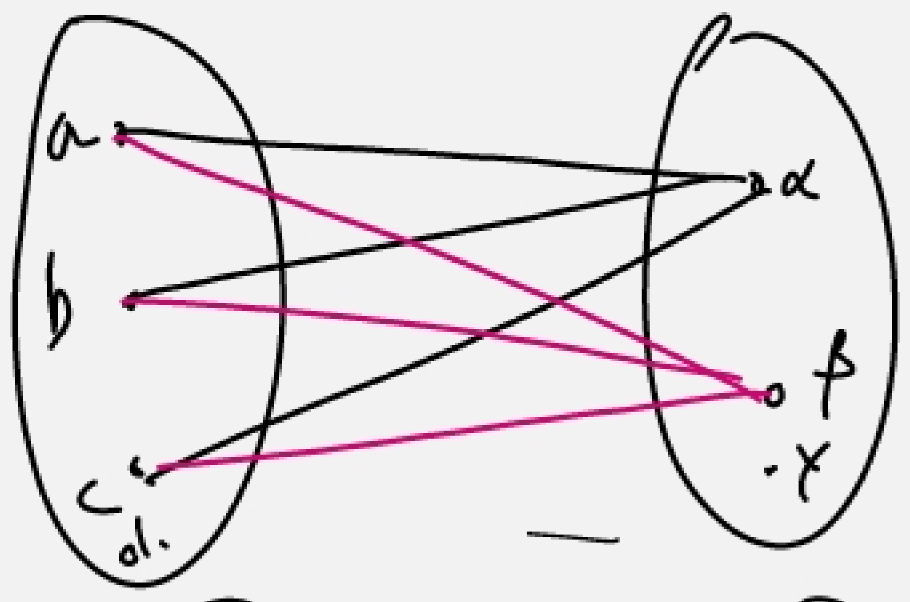
a	b	c
α	α	α
α	α	β
α	β	α

$y = x^2$

$f: \mathbb{R} \to \mathbb{R}$ a function

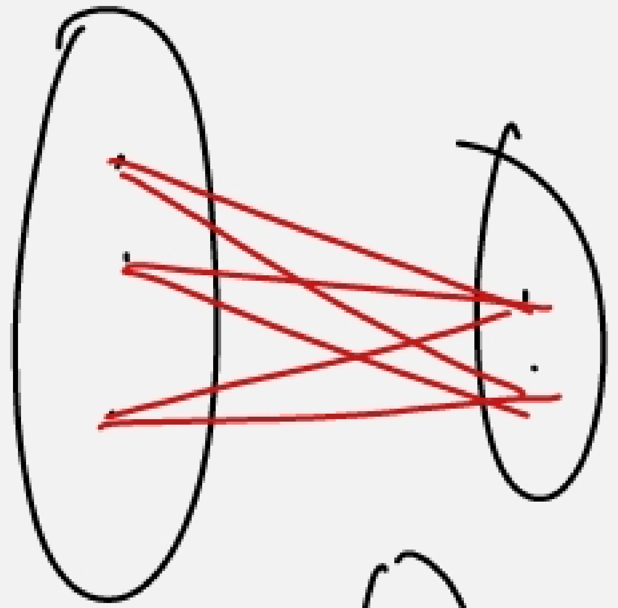


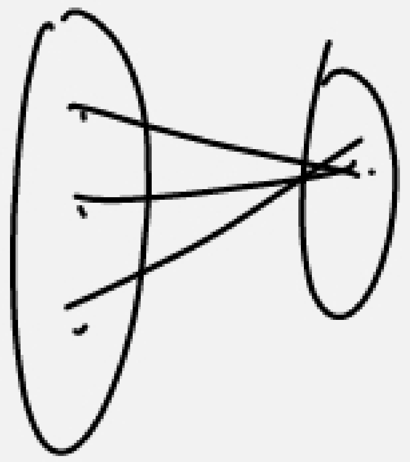
A B

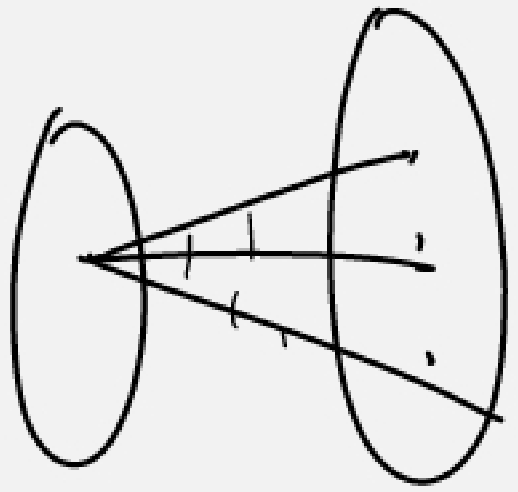


3 2

$|m|$ 8 $|n|$ n^m



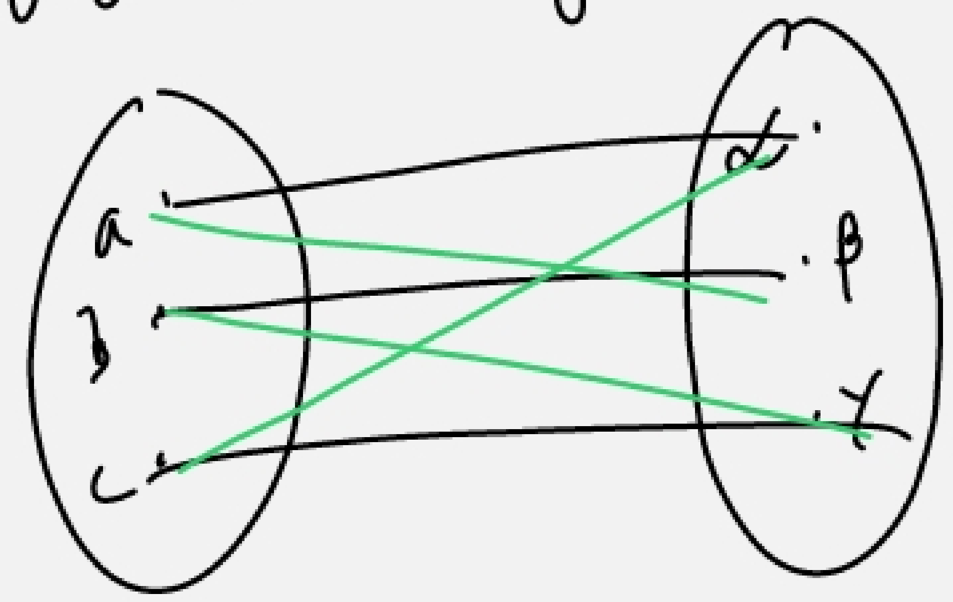




$|m| \to |n|$

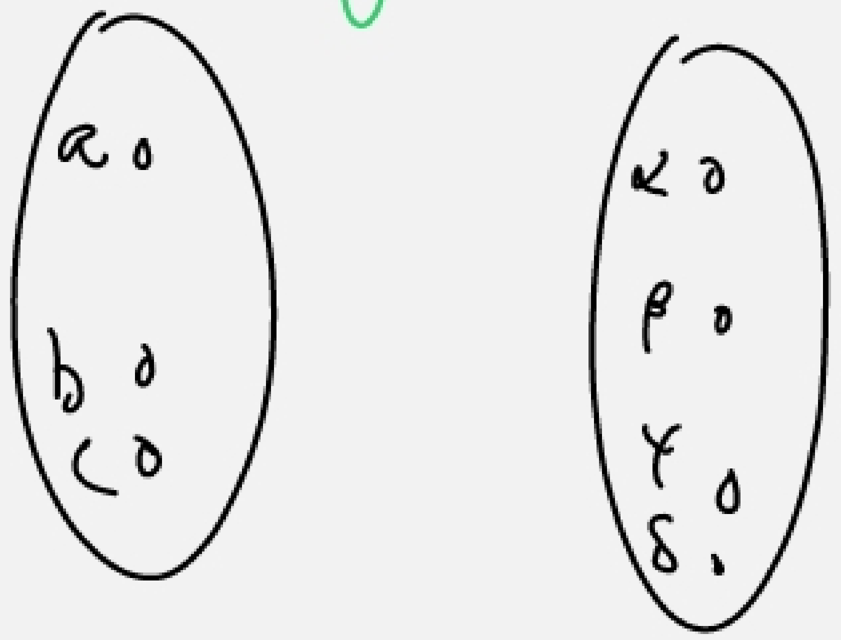
n^m function.

① how many 1-1

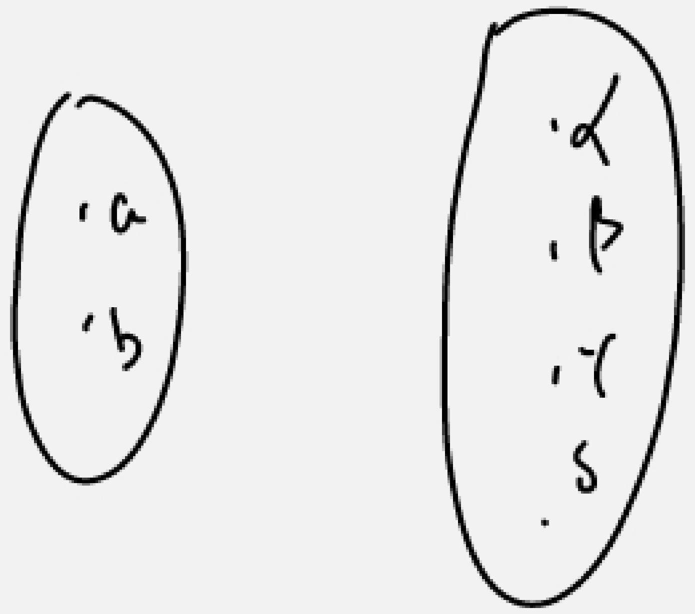


$|m|$ 3 $\qquad$ $|n|$ 3

total 1-1 function 3!



$|m| \to |n| \quad (|m| \leq |n|)$



a	b
γ	δ
β	γ

2 out of 4

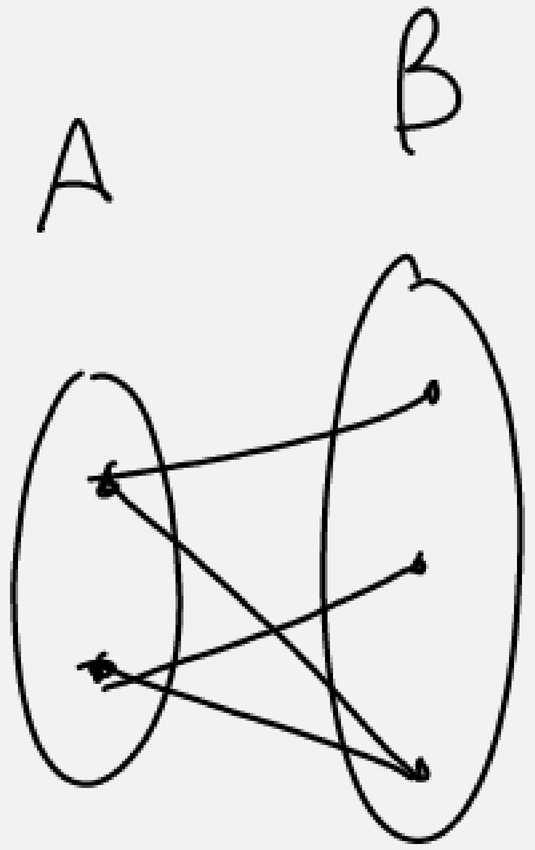
nP_m

2 of 3

a	b
α	β
α	γ
β	γ
β	α
γ	β
γ	α

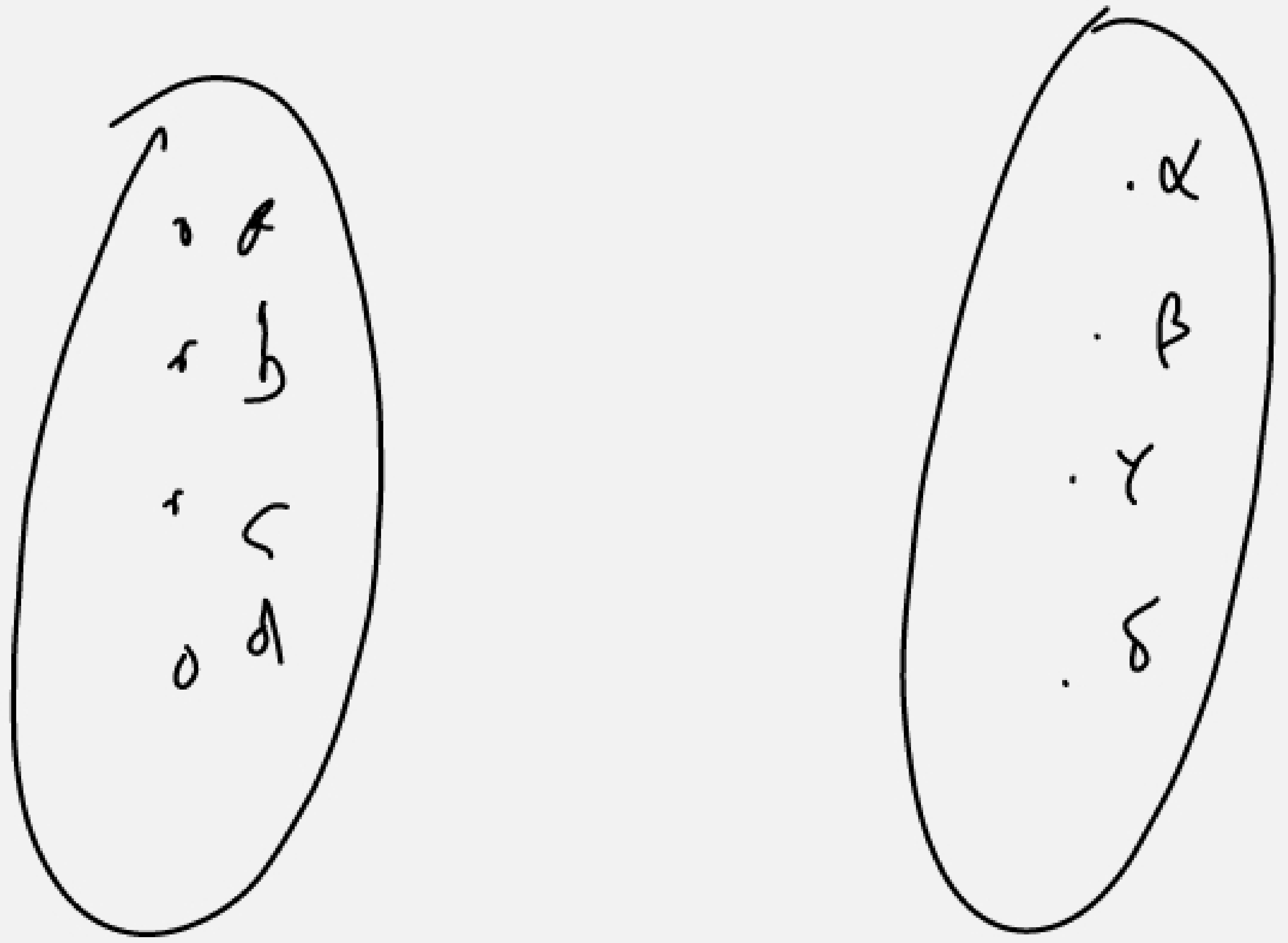
a	b	c
α	β	γ

3 of 4



no. of 1-1 + onto functions

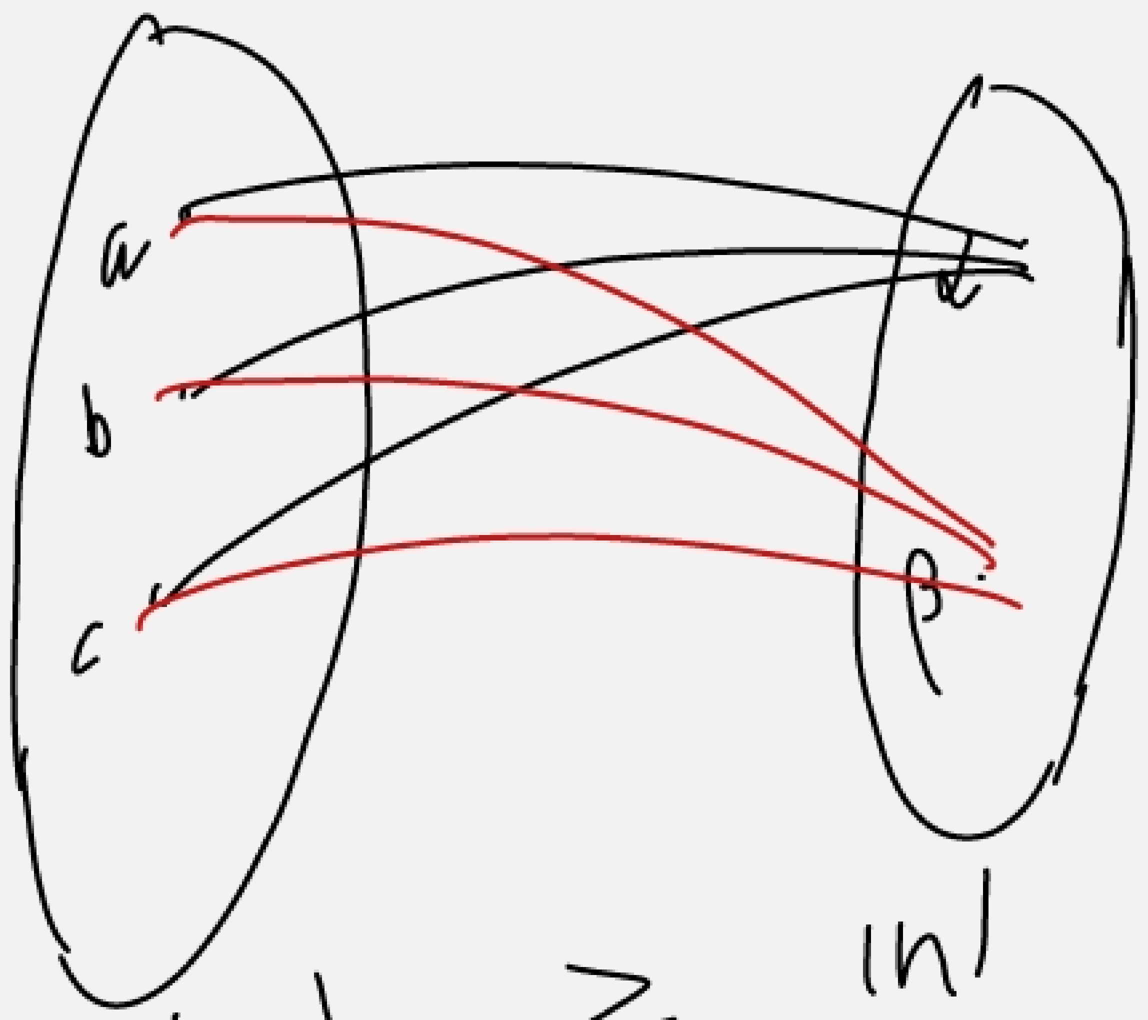
$m = n$



a	b	c	d
α	β	γ	δ
α	β	δ	γ

$4!$

$n!$ OR $m!$



$|m| \geq |n|$

$8 - 2$

$= 6$

a	b	c
α	α	α
α	α	β
α	β	α
α	β	β
β	α	α
β	α	β
β	β	α
β	β	β

How many positive integers not exceeding 1000 are divisible by 3 or 7? [3 or 7 or 11]

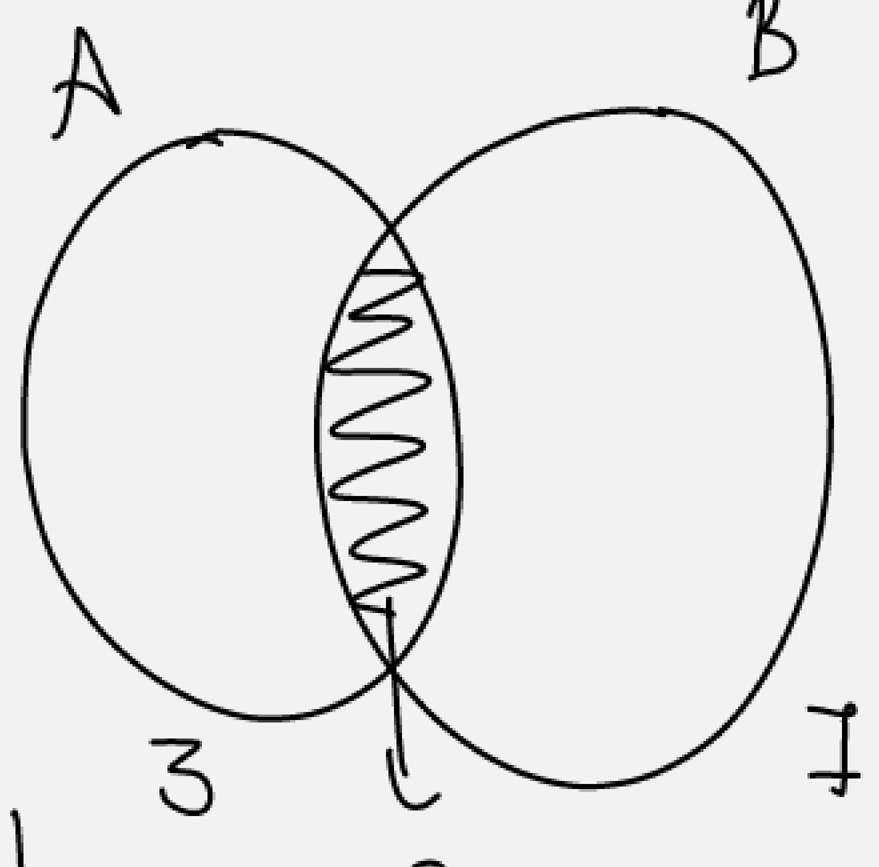
$476 + 4 = 480$

$|A \cap B| \left\lfloor \frac{1000}{3 \cdot 7} \right\rfloor = 47$

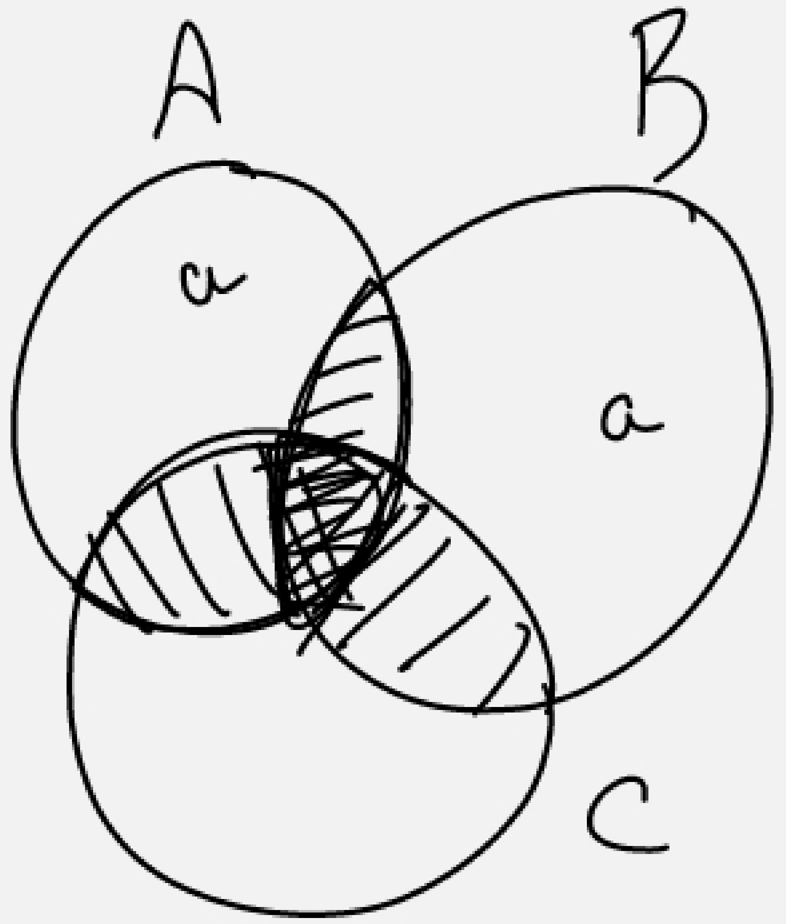
$|A| \left\lfloor \frac{1000}{3} \right\rfloor = 333$

$|B| \left\lfloor \frac{1000}{7} \right\rfloor = 142$

428



$|A \cup B| = |A| + |B| - |A \cap B|$



$|A \cup B \cup C|$

$= |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| \boxed{+ |A \cap B \cap C|}$

Principle of Inclusion - Exclusion

$|A + B + C + D|$

$(1+1)^n$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n \qquad {}^nC_0 = 1$$

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0 \qquad (1+(-1))^n$$

$|A_1 \cup A_2 \ldots \cup A_n|$ $\qquad {}^nC_0 = 1$

$$= \sum_{1 \le i \le n}^{n} |A_i| - \sum_{1 < i < j < n}^{n} |A_i \cap A_j|$$

$$+ \sum_{1 \le i < j < k \le n}^{n} |A_i \cap A_j \cap A_k| - $$

$$+ (-1)^{n+1} |A_1 \cap A_2 \ldots \cap A_n|$$

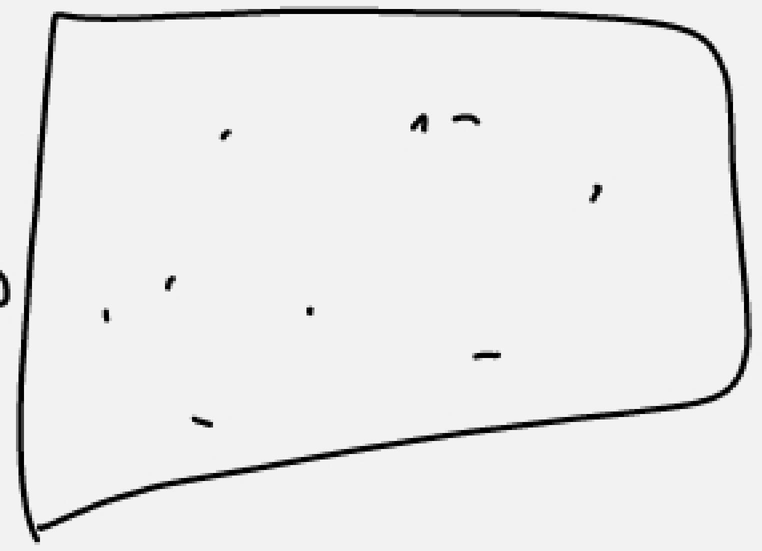
'a' is a member of r sets $1 \le r \le n$

This 'a' is counted r times in $\sum |A_i|$ $\quad {}^rC_1 \quad {}^rC_m$ times for a summation involving 'm' sets of A_i

counted ${}^rC_2$ times in $|A_i \cap A_j|$

$${}^rC_1 - {}^rC_2 + {}^rC_3 + \ldots (-1)^{r+1}\, {}^rC_r \stackrel{?}{=} 1$$

$$\left\{ \begin{array}{l} {}^nC_0 - {}^nC_1 + {}^nC_2 + \ldots = 0 \\ \hline {}^nC_0 - [\quad \text{count} \quad] = 0 \end{array} \right.$$

78 

$x_i \geq 0$

$$x_1 + x_2 + x_3 = 11$$

$$78 = \binom{13}{2} = \binom{13}{11}$$

$$x_1 \leq 3, \quad x_2 \leq 4, \quad x_3 \leq 6$$

$$\underset{P_1}{x_1 \geq 4}, \quad \underset{P_2}{x_2 \geq 5}, \quad \underset{P_3}{x_3 \geq 7}$$

$$x_1 \geq 4, \; x_2 \geq 0, \; x_3 \geq 0$$

$$[\underbrace{4 + y_1}_{x_1}] + x_2 + x_3 = 11 \; ; \; y_1 + x_2 + x_3 = 7$$

$$\binom{n+r-1}{n} = \binom{n+r-1}{r-1}$$

9 1 1

$$\binom{9}{2}$$

50 as a sum of 4 (positive) integers?

$$x_1 + x_2 + x_3 + x_4 = 50$$

$x_i \geq 0 \quad \binom{53}{3}$

$x_i > 0 \quad \binom{49}{3}$

$X_1 + X_2 = 1$

$X_2 - X_1 = 6$

$2X_1 - X_2 = 19$

$$y_1 + 1 + y_2 + 1 + y_3 + 1 + y_4 + 1 = 50$$

$$y_1 + y_2 + y_3 + y_4 = 46 \qquad x_1 + x_2 = 1$$

$y_i \geq 0$

no of onto functions

$x_1 + x_2 + x_3 = 11$

$\binom{13}{2} = 78$

$x_1 \le 3,\ x_2 \le 4,\ x_3 \le 6$

$x_1 \ge 4 \quad \binom{9}{2}$

$x_2 \ge 5 \quad \binom{8}{2}$

$x_1 + y_2 + 5 + x_3 = 11 \quad x_1 + y_2 + x_3 = 6$

$x_3 \ge 7 \quad \binom{6}{2} \quad x_1 + x_2 + (y_3 + 7) = 11 \quad x_1 + x_2 + y_3 = 4$

$x_1 \ge 4,\ x_2 \ge 5$ —

$(y_1 + 4) + (y_2 + 5) + x_3 = 11$

$y_1 + y_2 + x_3 = 2$

$x_2 \ge 5,\ x_3 \ge 7 \quad (0)$

$x_1 \ge 4,\ x_3 \ge 7 \quad \binom{2}{2}$

78

$-\binom{9}{2} - \binom{8}{2} - \binom{6}{2}$

$+\binom{4}{2} + 0 + \binom{2}{2}$

-0

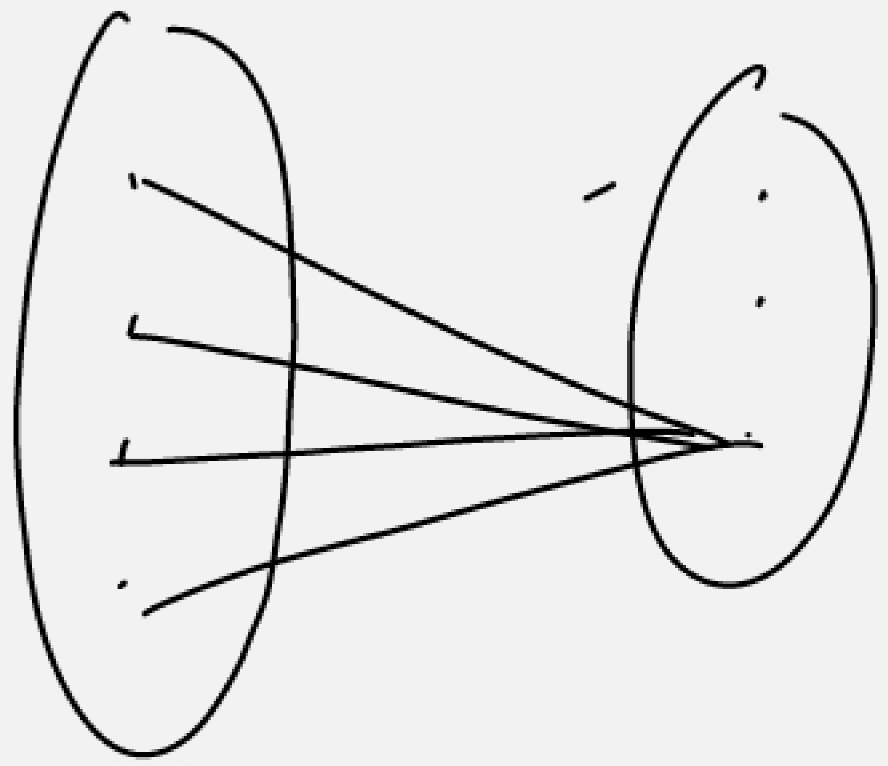
$= 6$

$x_1 \ge 4,\ x_2 \ge 5,\ x_3 \ge 7 \quad (0)$

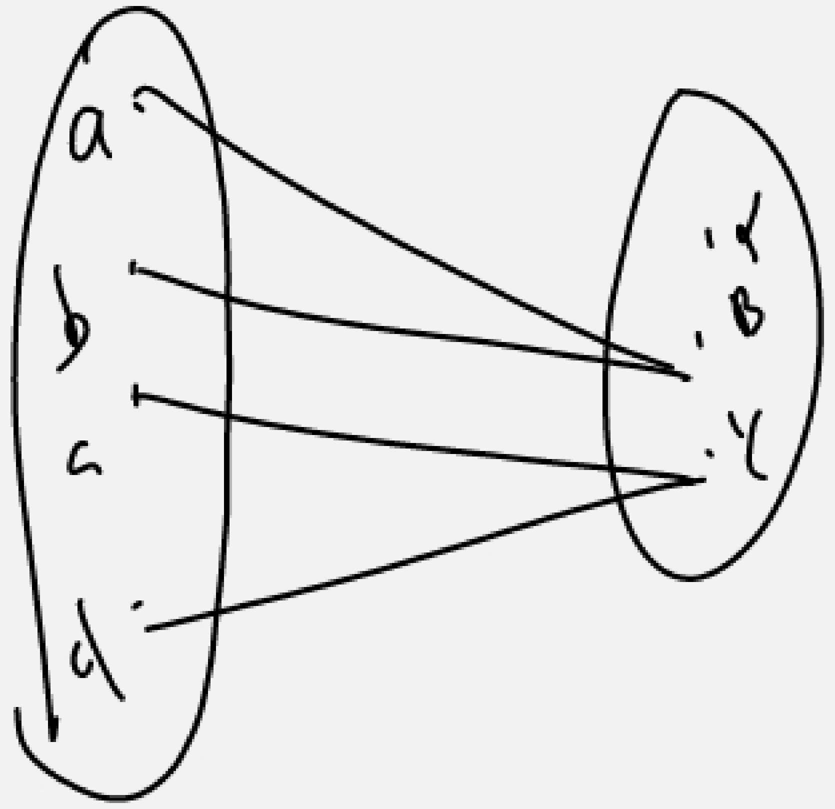
$$20$$

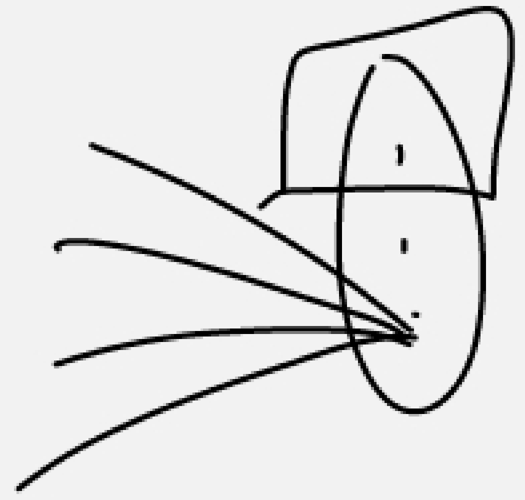
$$x_1 + x_2 + x_3 + x_4 = \cancel{2}0$$

$$0 \le x_i \le \cancel{5}\ 10$$



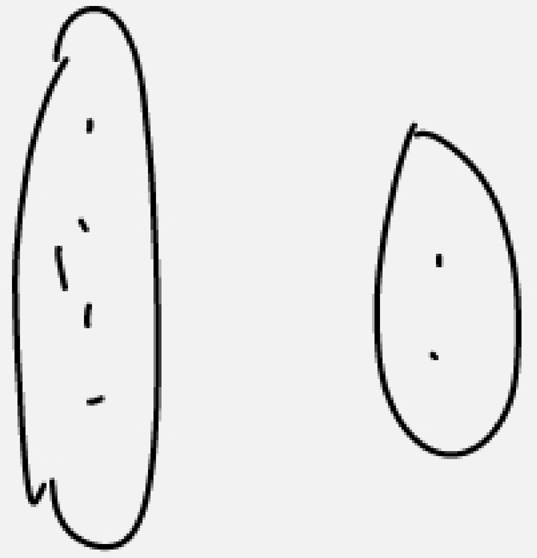
no. of onto function:





$$3^4$$

$\underline{P(\alpha)}$

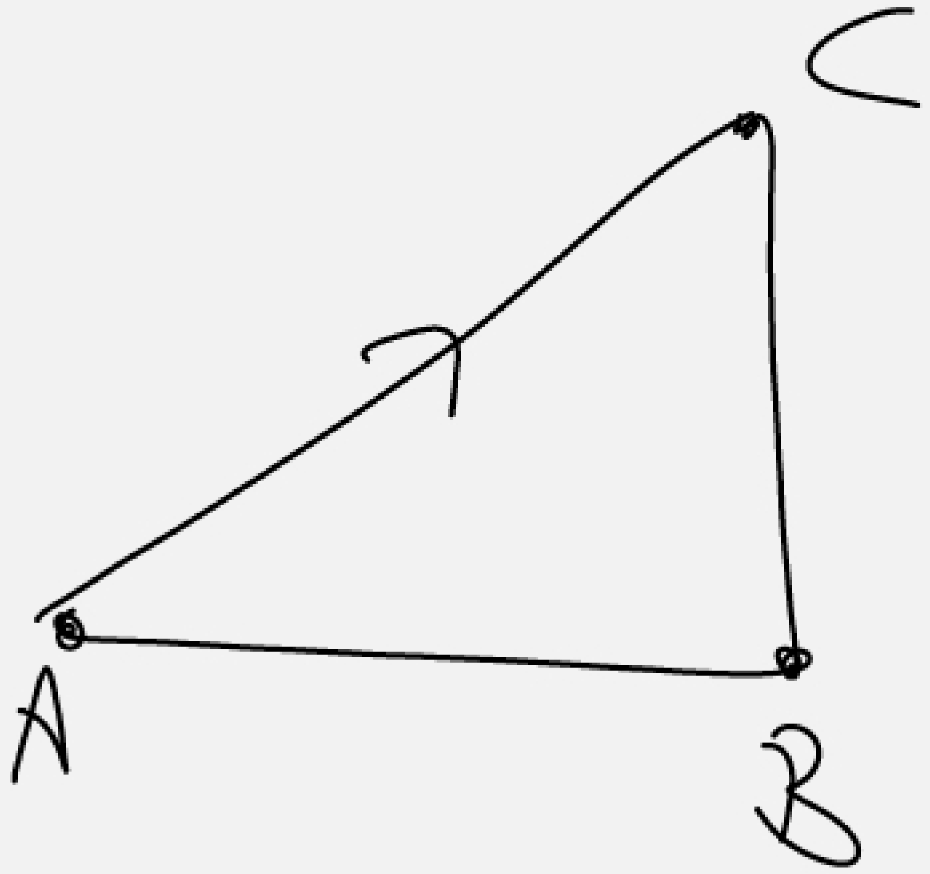


$$- 3 \cdot 2^4$$

$$+ 3 \cdot 1^4$$

$$=$$

Student: what is application of Pythagorous theorem



How many positive integers from 1 - 2000 are not divisible by 5, 7, 11.

$|S| = 2000$

$N(P_1)$: nos. divisible by 5 $\quad \frac{2000}{5\ 4} = 400$

$N(P_2) \quad 7 \quad \left\lfloor \frac{2000}{6\ 7} \right\rfloor = 285$

$LCM\ of\ (4,6)$

$N(P_3) \quad \left\lfloor \frac{2000}{11\ 9} \right\rfloor = 181$

$N(P_1, P_2) = \frac{2000}{35} = 57; \quad N(P_2 P_3) = 2000/77_{6,9} = 25$

$N(P_1 P_3) = 2000/55_{4,9} = 36$

$N(P_1 P_2 P_3) = \frac{2000}{385} = 5$

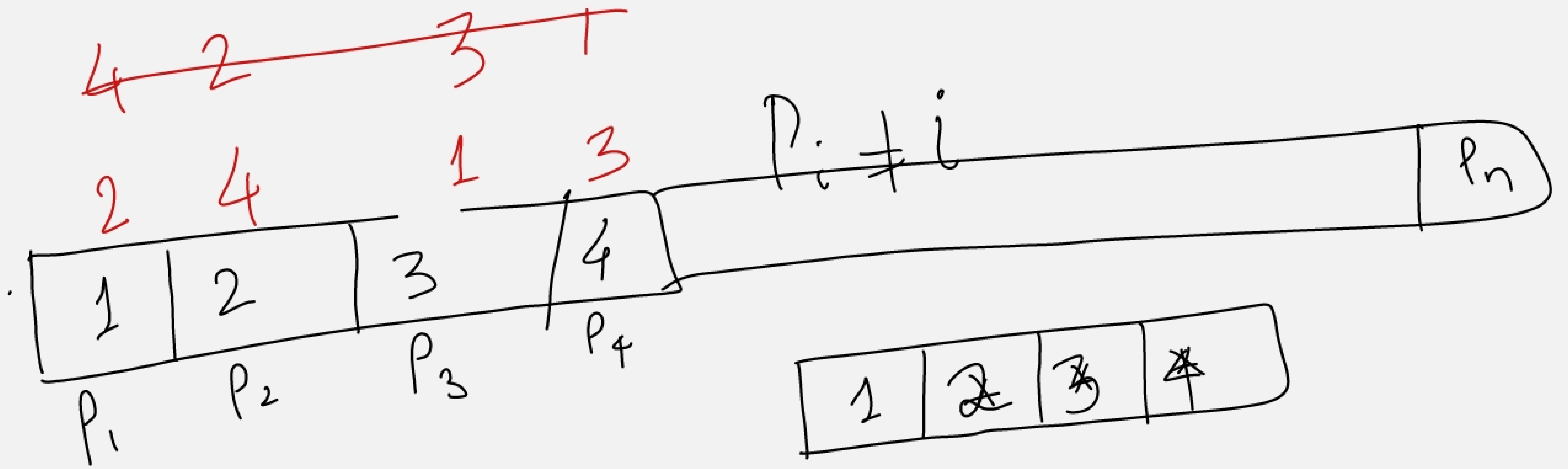
$\bar{N} = N - [N(P_1) + N(P_2) + N(P_3)]$
$+ [N(P_i, P_j)]$
$- N(P_1 P_2 P_3)$

$\frac{2000}{2 \cdot 2 \cdot 2 \cdot 3}$

4, 6, 9.

$\frac{2000}{4} \quad \frac{2000}{6}$

$\frac{2000}{LCM(4,6)}$



$|m| \to |n|$

n^m

$4!$

$|S| = 24$

$$\binom{4}{0}4! - \binom{4}{1}3! + \binom{4}{2}2! - \binom{4}{3}1! + \binom{4}{4}0!$$

$N(P_1)$ $P_1 \Leftrightarrow 1$ $3!$

$N(P_2)$ $3!$

$N(P_3)$ $3!$ $N(P_4) = 3$

$S = 26$

$\boxed{|S| = 26!}$

ONE, QUIZ, PART

$|26|! - (24! + 23! + 23!)$

$+ (21! + 20! + 21!)$

$- (18!)$

AUNEBC . . .

$24!$

$\underbrace{\text{ONE}}_{1} \cdot \underbrace{\text{ABCDFG} \ldots \quad \text{Z}}_{23}$

$N(P_1) \to$ no. of words which contain ONE

QUIZ

$\underbrace{\text{QUIZ}}_{1} [\underbrace{\text{A} \ldots \quad \text{Y}}_{22}]$

PART

$N(P_2)$

$N(P_3)$

$23!$

$N(P_1 P_2 P_3) = 18!$

$23!$

OQNUEIZ

$N(P_1 P_2)$

$\underbrace{\text{ONE}}_{1} \; \underbrace{\text{QUIZ}}_{1} \; \underbrace{\_\_\_\_}_{19}$

$19 + 1 + 1 = 21!$

$\underbrace{\text{ONE}}_{1} \; \underbrace{\text{PART}}_{1} \; \underbrace{\text{QUIZ}}_{1} \; \underbrace{\_\_\_\_}_{15}$

$N(P_2 P_3)$

$\underbrace{\text{QUIZ}}_{1} \; \underbrace{\text{PART}}_{1} \; \underbrace{\_\_\_\_}_{18}$ $N(P_1 P_2)$

$20!$

$= 21!$

floor $\lfloor x \rfloor$

Celing $\lceil x \rceil$

$\lfloor 1.2 \rfloor = 1$

$\lceil 2.1 \rceil = 3$ $\quad \lfloor 2.1 \rfloor = 2$

$\lceil 1.2 \rceil = 2$ $\quad \lfloor -\frac{1}{2} \rfloor = -1$

$\lceil -\frac{1}{2} \rceil = 0$

floor function:

$x \in \mathbb{R}$. $\quad f(x) \to \mathbb{Z}$.

it assigns the largest integer $\leq x$

Celing function:

assigns the Small integer $\geq x$

$\lceil x + y \rceil \stackrel{?}{=} \lceil x \rceil + \lceil y \rceil$

$\lfloor x+n \rfloor = \lfloor x \rfloor + n$ $\quad \lfloor x \rfloor = m$

$m \leq x < m+1$

$m+n \leq \lfloor x+n \rfloor < m+1+n$

$\lfloor x+n \rfloor = m+n = \lfloor x \rfloor + n$

$$\lfloor x \rfloor = n \quad \text{iff} \quad n \le x < n+1$$

$$\lfloor x \rfloor = n \quad \text{iff} \quad x-1 < n \le x$$

$$\lceil x \rceil = n \quad \text{iff} \quad n-1 < x \le n.$$

$$\text{iff} \quad x \le n < x+1$$

$$x-1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x+1$$

$$\lfloor x+n \rfloor = \lfloor x \rfloor + n$$

$$\lceil x+n \rceil = \lceil x \rceil + n$$

$$\lceil x+y \rceil \overset{?}{=} \lceil x \rceil + \lceil y \rceil \qquad x = \tfrac{1}{2},\ y = \tfrac{1}{2}$$

$$1 \ne 2$$

$$\lfloor x \rfloor = \lfloor n \rfloor \qquad x = n + \varepsilon \qquad \boxed{0 \le \varepsilon < \tfrac{1}{2}}$$

$$\boxed{\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \tfrac{1}{2} \rfloor}$$

$$2x = 2n + 2\varepsilon \quad \& \quad \lfloor 2x \rfloor = 2n. \quad \left\{ \lfloor x \rfloor = n \right.$$

$$\lfloor x + \tfrac{1}{2} \rfloor = \lfloor n + \varepsilon + \tfrac{1}{2} \rfloor = n$$

$$\tfrac{1}{2} \le \varepsilon < 1$$

$$\lfloor x \rfloor = n$$

$$\lfloor x + \tfrac{1}{2} \rfloor = n+1$$

$$\lfloor 2x \rfloor = \lfloor 2n + 2\varepsilon \rfloor = \lfloor 2n+1 + 2\varepsilon - 1 \rfloor = 2n+1$$

$$\lfloor x + \tfrac{1}{2} \rfloor = \lfloor n + \tfrac{1}{2} + \varepsilon \rfloor = \lfloor n+1 + \varepsilon - \tfrac{1}{2} \rfloor = \lfloor n+1 \rfloor$$

$1, 2, 4, 8, 16 \ldots$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16}$

Geometric progression

$a_1 \quad a_2 \quad a_3 \quad a_n = r a_{n-1}$

$a, \; ar, \; ar^2 \ldots$

Sequences

$a_1 = 1$

$r = 4$

$a_2 = r a_1$

$a_3 = r a_2 = r^2 a_1$

$\boxed{a_n = r a_{n-1}}$

$a_n =$

$\boxed{n! = n(n-1)!}$

Arithmetic progression

$2 \quad 4 \quad 6 \quad 8 \; \ldots$

$a_0 = 1$

$a_1 = 2$

$\underbrace{a + b, \; a + 2b}_{b}, \; \underbrace{a + 3b}_{b}, \; \underbrace{\ldots}_{b}$

$n=0 \quad n=1 \quad n=2 \quad n=\ldots$

$a_n = a_{n-1} - a_{n-2}$

$a_2 = 2 - 1 = 1$

$a_3 = a_2 - a_1 = -1$

$x(0) = 1$

$\sum_{n=1}^{10}$

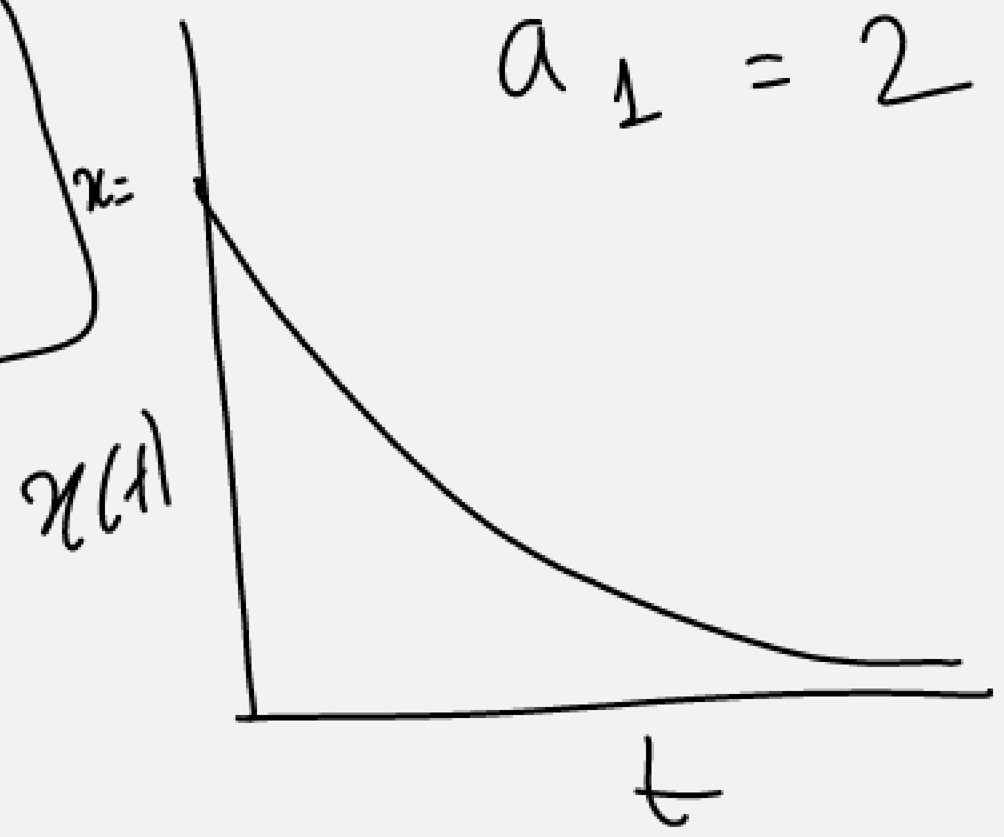
$1 + 2 + \ldots + 10$

$\frac{10(10+1)}{2} = 55$

$\boxed{\frac{dx}{dt} = -x}$

$x(1) = e^{-1} \cdot 1$

$x(t) = e^{-t} x(0) = e^{-t} \cdot 1$



Sequence

recurrence relations

$$\boxed{a_n = r^n \cdot a_0}$$

$$\frac{dx}{dt} = -aX \qquad X(0)$$

$$a_n = r\, a_{n-1}$$

$$A_4 = 1.1\, A_3 = (1.1)^3 A_1$$

$$X(t) = e^{-at} \cdot X(0)$$

$$a_0 = \frac{1}{4}$$

$$r = 4.$$

Compound

$M_1 = 100$

$M_2 = 110 = (1.1)100$

$M_3 \quad (1.1)110 = 121$

$M_4 = (1.1) \cdot 121$

$= 133.1$

$$a_1 = 4 \cdot a_0$$

$$a_2 = 4 \cdot a_1 = 4^2 \cdot a_0$$

$$a_3 = 4 \cdot a_2 = 4^3 \cdot a_0$$

$$\vdots$$

$$a_n = 4 \cdot a_{n-1} \cdots = 4^{(n)} \cdot a_0$$

Simple Interest

$M_1 \quad 100\$ \quad 10\%$

$M_2 = 110$

$M_3 = 120$

$A_n = (1.1) A_{n-1}$

$A_n = (1.1)^{n-1} A_1$

$A_2 = 1.1 A_1$

$A_3 = 1.1 \cdot A_2 = (1.1)^2 A_1$

$a_n = a_{n-1} + a_{n-2}$

$0, 3, 9, 18, 30, 45, \ldots$

$a_2 - a_1 = 3$

$a_3 - a_2 = 6$

$a_4 - a_3 = 9$

$a_5 - a_4 = 12$

$a_n = 3(n-1) + a_{n-1}$ $(a_1 + a_2 \, a)$

n 2^n

$a_n = a_{n-1} + 3n$

$(a_0, a_1 \ldots)$

$P\left(1 + \frac{r}{100}\right)^n$

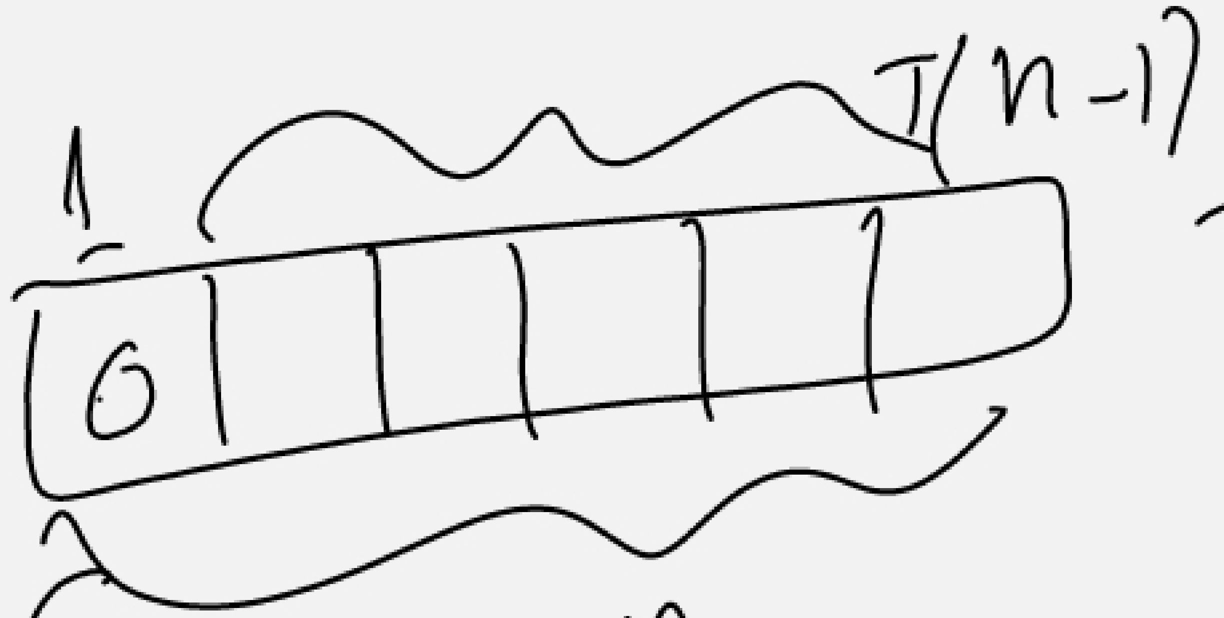
$a_n = \frac{n}{2} \cdot n +$

$a_n = f(a_{n-1}, a_{n-2})$

a_n

$1, 2, 3, 5, 8, 13, 21, \ldots$

$\boxed{a_n = a_{n-1} + a_{n-2}}$



$T(n)$

$T(1) = 2$

$T(n) = 2^{n-1}\,T(1)$

$= 2^{n-1} \cdot 2$

$= 2^n$

C_1

C_1

$C_1 \; C_2$

$C_1 \; C_2 \; \boxed{C_3}$

$C_1 \; C_2 \; C_3 \; \boxed{C_4 \; C_5}$

$C_1 \; C_2 \; C_3 \; C_4 \; C_5 \; \boxed{C_6 \; C_7 \; C_8}$

$n - K = 1$

$K = n - 1$

$$C_n = \underbrace{C_{n-1}}_{\text{no of pairs in prev. month}} + \underbrace{C_{n-2}}_{\text{new born}}$$

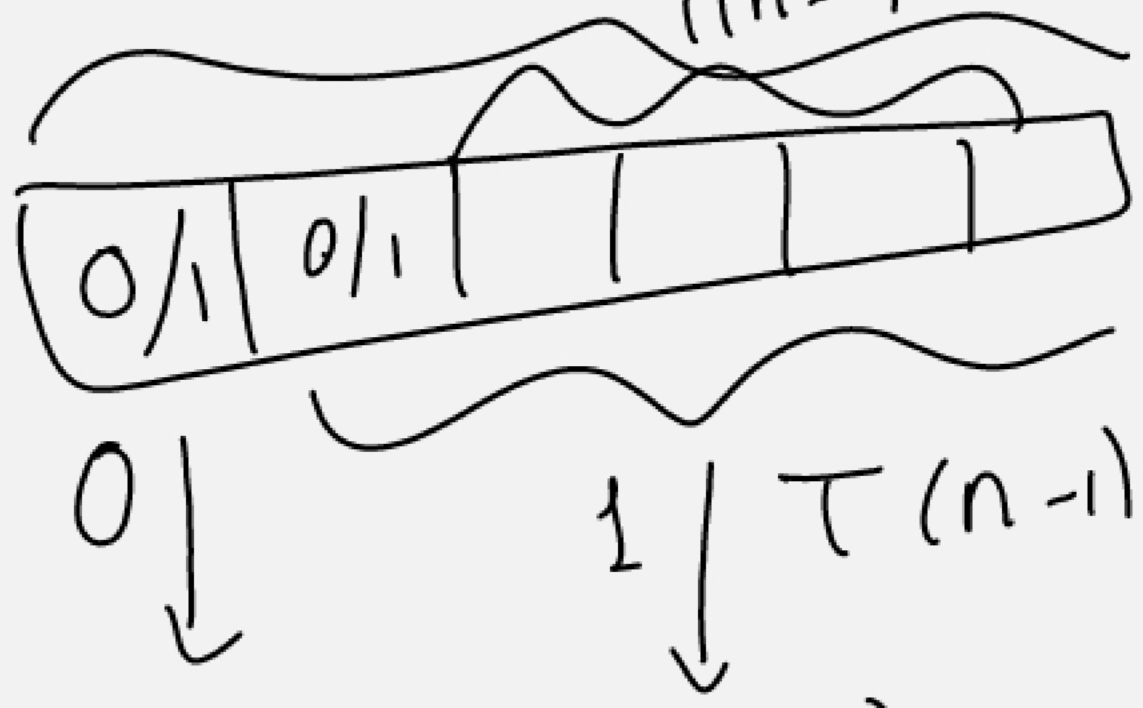
no of pairs in prev. month

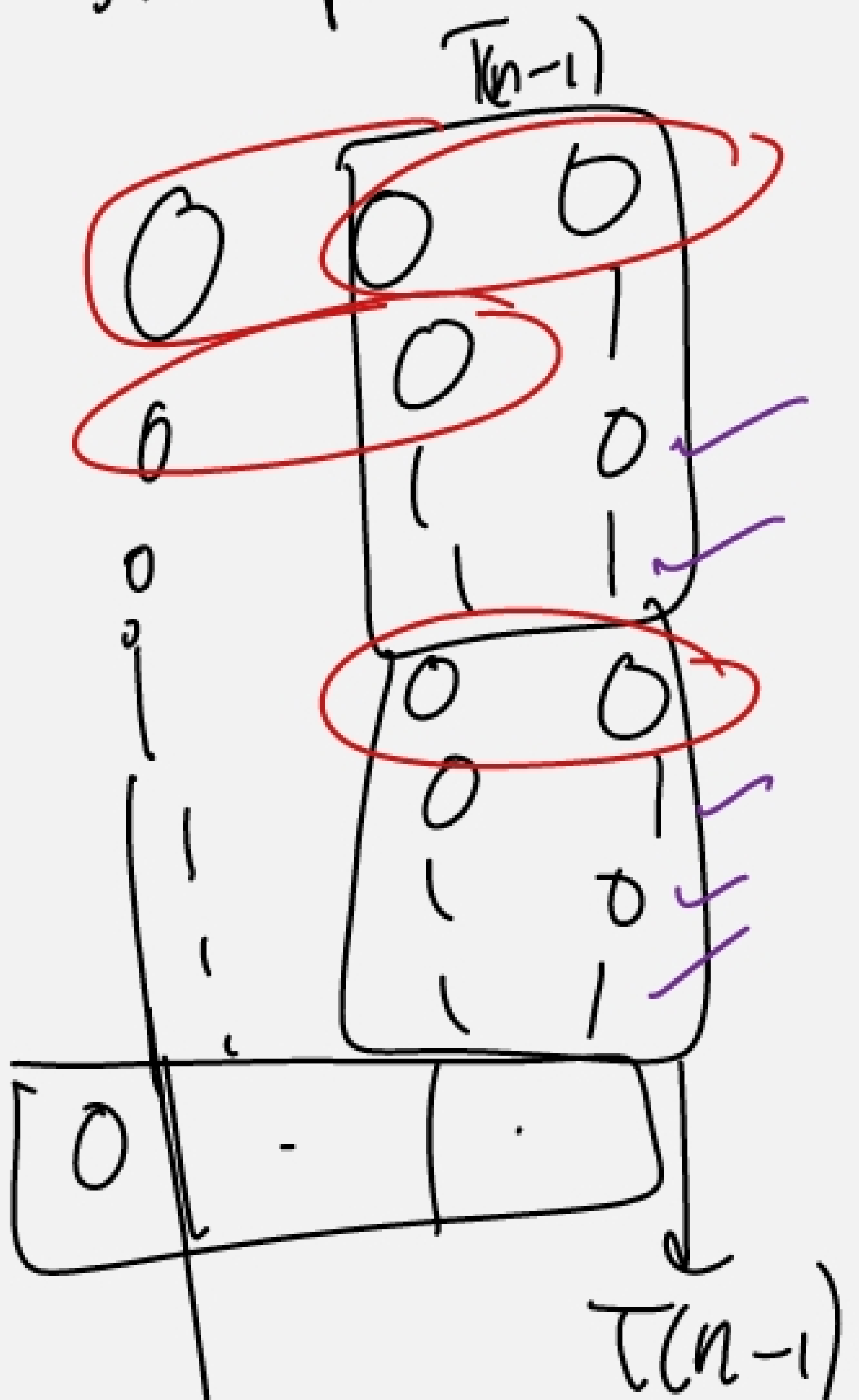
new born,

$T(n) \quad T(n-1)$

2^n

How many binary digit strings are possible of length n.





$T(n) = T(n-1) + T(n-1)$

$= 2\,T(n-1)$

$= 2 \cdot 2\,[T(n-2)] = 2^2\,T(n-2)$

$= 2^K\,T(n-K)$

$= 2^{n-1}\,T(1)$

$T(n-1) \quad T(1)$

$T(1)$

0

1

T(n) T(n-1)

T(1) 0
1

T(2) = 3

How many n-digit strings are possible without consecutive zeros

& consecutive 1's

T(n)

$T(n-1) = T(n-2) + T(n-?)$

0	0
0	1
1	0
1	1

0	1					

1	0/1					

$T_n = \boxed{T(n-1) + T(n-2)}$

$T(n-1) + T(n-?)$

$T(3) = 5 \quad T(4) = 8$

$T(1) = 2$
$T(2) = 3$

1	0	0	0	
1	0	0	1	
1	0	1	0	1 ✓
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	1
0	1	1	0	
0	1	1	1	

$$\frac{dy}{dt} = -x$$

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$$