$S_1 \quad S_1 \subseteq S_2 \quad S_2$

$S_1 \supseteq S_2$

$$a \in A \cup (\underbrace{B \cap C}_{a}) = (\overbrace{A \cup B}^{a}) \cap (\overbrace{A \cup C}^{a}) = a$$

$$A \cup (A \cap B) \stackrel{?}{=} A$$

$$A \cap (A \cup B) = A$$

let $a \in (A \cup B) \cap (A \cup C)$

$a \in (A \cup B)$ and $a \in (A \cup C)$

$a \in A$ or $a \in B$ and $a \in A$ or $a \in C$

$a \in A$ or $a \in B$ and $a \in C$

$A \cup (B \cap C)$

$a \in A \cup (B \cap C)$

$a \in A$ + $a \in (B \cap C)$

$a \in (B \cap C)$

$a \in B$

$a \in C$

If $a \in A$ then $a \in (A \cup B)$

If $a \in A$ then $a \in (A \cup C)$

$a \in (A \cup B) \cap (A \cup C)$

For $i = 1, 2, \ldots$

$A_i = \{i, i+1, i+2, \ldots\}$

~~$A_n = \{$~~ $A_1 = \{1, 2 \ldots\}$ $A_2 = \{2, 3, \ldots\}$

$A_{10} = \{10, \ldots 1 \ldots\}$

Compute $\bigcup_{i=1}^{n} A_i = \{1, 2, 3, \ldots\}$

$\bigcap_{i=1}^{n} A_i = A_n$

$\boxed{\begin{array}{l} A_i = \{1, 2, \ldots i\} \\ \bigcup_{i=1}^{n} = \{1, 2, \ldots n\} = A_n \\ \bigcap_{i=1}^{n} = \{\cancel{A_i}\ A_i = \{1\} \end{array}}$

$A_1 \cap A_2 = \{2, 3 \ldots\}$

$A_1 \cap A_2 \cap A_3 = A_3$

Cartesian Product

$A = \{a_1 \ldots, a_n\}$ ordered n tuple

$B = \{b_1 \ldots \quad b_n\}$

$B \times A = \{(b, a) \mid b_1 \in B, a \in A\}$

(b, a)

$\{1, a, 6, f\}$

$\{a, f, 1, 6\}$

$A = B \Rightarrow a_1 = b_1 - a_n = b_n$

The cartesian product of $A \times B = \{(a, b) \mid (a \in A, b \in B\}$

$A = \{a, b, c\}$ how will elements of $(A \times B)$ look like

$B = \{\alpha, \beta, \gamma\}$ $\{(a, \alpha), (a, \beta), (a, \gamma), (b, \alpha) \ldots\}$

If $A \times B \overset{?}{\neq} B \times A$

$A = \{1, 2\}$

$A^2 = \{(1,1), (1,2), (2,1), (2,2)\}$

$(a, b) \in \underbrace{A \times B}$

$\{0, 1, 2, 3\}$

$(a,b) : a \leq b$

$(0,0), (0,1), (0,2)\ (0,3)$

$(1,2)\ (1,3)$

$p = 2x$

$p^2 = 4x^2 = 2q^2$

$p^2 = 2q^2 = 4 \cdot q$

Prove that $\sqrt{2}$ is irrational

. Statement : $\sqrt{2}$ is irrational

$p^2 = 2q^2$

Challenge the Statement

multiple of 4

$\frac{4}{6}$

$\sqrt{2} = p/q$

$2q^2 = p^2 \Rightarrow$ p is even

$\frac{60}{75}$

$2 = \frac{p^2}{q^2}$

p^2 even

$2q^2$

$p = 2 * X$

$12/15$ $4/5$ q is even then p is even

M M
Tu Tu
W W
Th Th
F F
S S
Su Su

Take a set of 15 days atleast 3 must fall on the same day

$2 \times 7 = 14$

(1) If p is even, p^2 is a multiple of 4.

(2) p^2 is even, multiple of 4

$4 \cdot \text{Even} = \cancel{2} q^2$

(3) $2 \cdot \text{Even} = q^2 \Rightarrow$ q is also even

If p is even $\Rightarrow$ q even

$\sqrt{2}$ is irrational

$\sqrt{2}$ is rational $\rightarrow$

$\sqrt{2} = p/\underline{q}$

$2 = \frac{p^2}{q^2}$

$p^2 = 2\underline{q}^2 \Rightarrow p$ is even

Square of any even number is a multiple of 4. $2q^2 = 4 \cdot X$ (CONTRADICTION)

$q^2 = 2 \cdot X \Rightarrow q$ is even

If $3n+2$ is odd then n is odd.

n is NOT odd. $\Rightarrow n$ is even

$n = 2K$

$3 \cdot 2K + 2 = 2(3K+1)$

$\Rightarrow 3n+2$ is even

$\underset{(\text{odd})}{3n+2} \Rightarrow \underline{n \text{ is odd}}$

even

N is even

MATHEMATICAL LOGIC

DM is NOT an interesting course

p Statement

Proposition

negation $\neg p$

$\bar{p}$

OR operation

P, Q ORM

S/W or H/W

Two simple statements

P	Q	S
0	0	0
0	1	1
1	0	1
1	1	1

P	Q	M	S
0	0	0	0

Q: $6+4 = 3$ (F-0)

$\neg$Q: $6+4 \neq 3$ (T-1)

$\binom{3}{0}+\binom{3}{1}+\binom{3}{2}+\binom{3}{3}$

Power set 2^3

and.

SW + HW

p	Q	S
0	0	0
0	1	0
1	0	0
1	1	1

(p AND q) OR NOT p

OR $p \vee q$

AND $p \wedge q$

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \vee \neg p$
0	0	0	1	1
0	1	0	1	1
1	0	0	0	0
1	1	1	0	1

Truth table

p	q	$\neg p$	$\neg q$	$\neg q \to \neg p$
0	0			1
0	1			1
1	0			0
1	1			1

Inverse

$\neg p \to \neg q$

CONTRAPOSITIVE

When is a CONDITIONAL Statement $P \to q$ false? | $\neg q \to \neg p$ false

When p $Q = F$ & $P = T$

If $3n+2$ is odd then n is odd

Q

If NOT Q then NOT P

$H \to E$

$p \wedge (q \vee r)$

$= (p \wedge q) \vee (q \vee r)$

Prove $P \to Q$ for $\neg Q \to \neg P$

Logical Implication

$P \to Q$

$E \to S$

E	S	$E \to S$
0	0	1
0	1	1
1	0	0
1	1	1

$H \to E$

not eating $\to$ not hungry

H	E	$H \to E$	$\neg E$	$\neg H$	$\neg E \to \neg H$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

$\neg q \to \neg p$

If $3n+2$ is odd $\to$ n is odd.

$p \to q$

A B

p	q	$p \to q$		
0	0	1	1	
0	1	1	1	
1	0	0	0	$\to$ NOT POSSIBLE
1	1	1	1	POSSIBLE

0 1

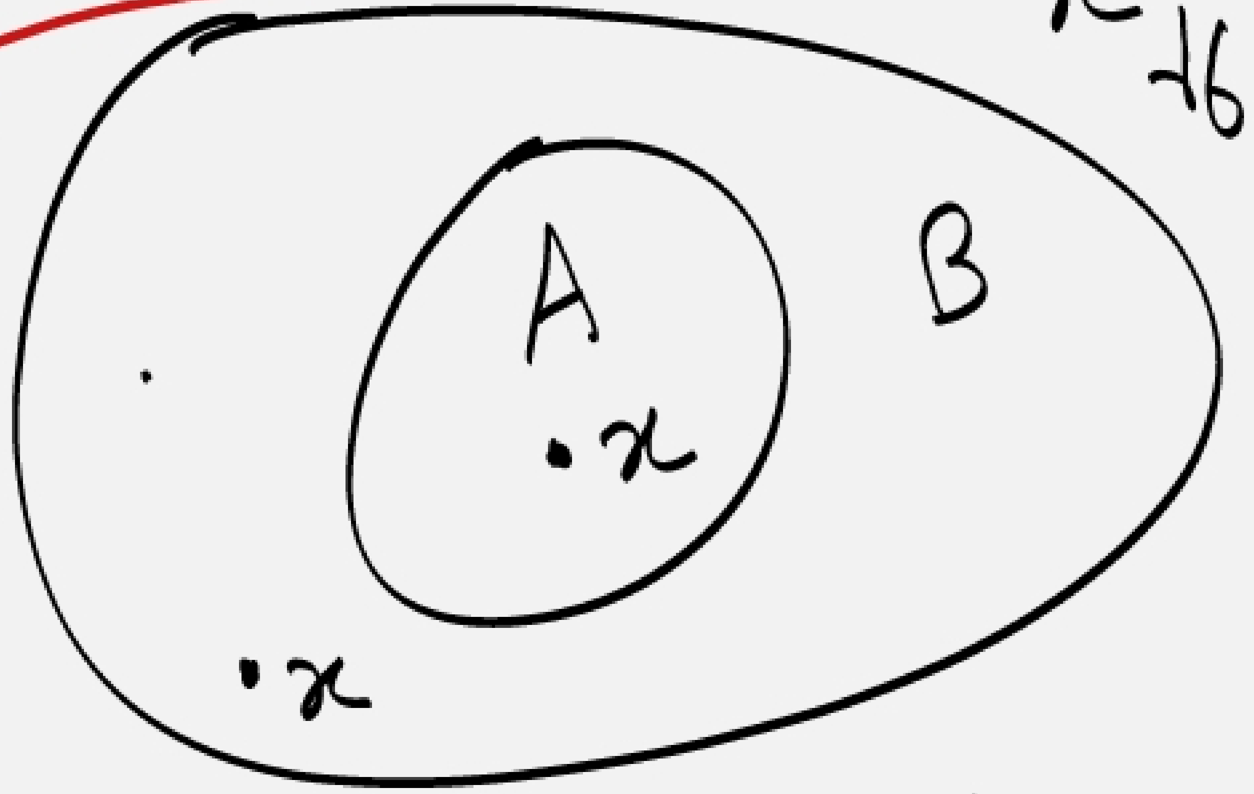
$x \notin A$, but $x \in B$

$x \in A$, $x \notin B$

If $x \in A$ then $x \in B$

$\to$ If $x \in A \to x \in B$

$\cdot x$ If $x \in B \xrightarrow{?} x \in A$



$A \subset B$

If $x \notin A$, $x \notin B$

0 0

$\neg q \to \neg p$ Contrapositive $p \to q$

$\neg q \to \neg p$

p	q	$\neg q$	$\neg p$	$\neg q \to \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

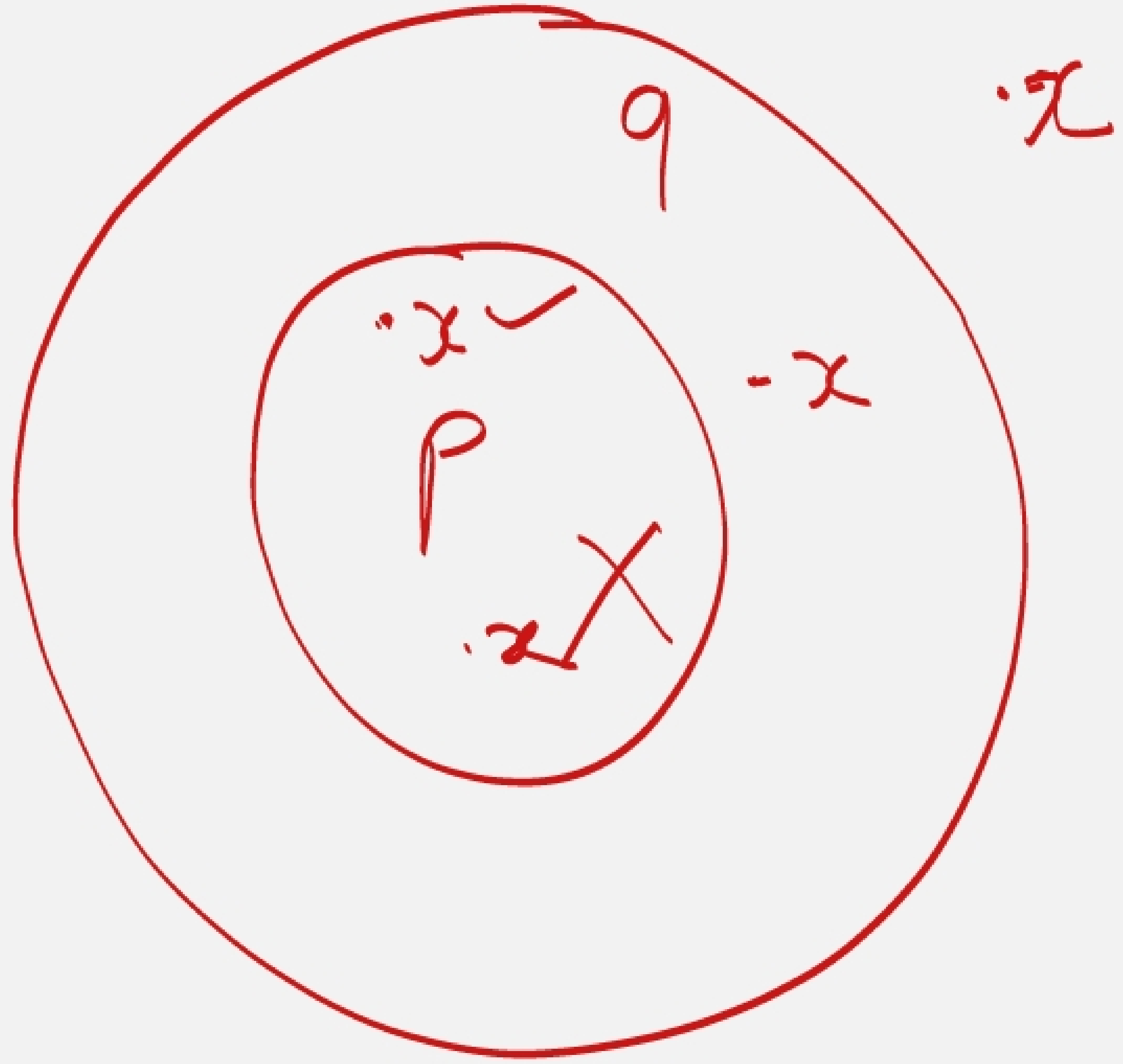
$\overset{p}{x \in A} \Rightarrow \overset{q}{x \in B}$

$\underset{\neg q}{x \notin B} \Rightarrow \underset{\neg p}{x \notin A}$

If $x \in A \to x \in B$

If $x \in B \to x \in A$ $A = B$

$A = B$



$\overset{p}{3n+2 \text{ is odd}} \to \overset{q}{n \text{ is odd}}$

$\neg q \Rightarrow \neg p$

n is even $\Rightarrow$ $3n+2$ is also even

$p \to q$ 

$q \to p$

p	q	$p \to q$	$q \to p$	AND $p \to q \wedge q \to p$	
0	0	1	1	1	
0	1	1	$\boxed{0}$	0	IMPOSSIBLE
1	0	$\boxed{0}$	1	0	IMPOSSIBLE
1	1	1	1	1	

~~Ne~~ Sufficient

$p \leftrightarrow q$

If n is even $\Rightarrow n$ is divisible by 2.

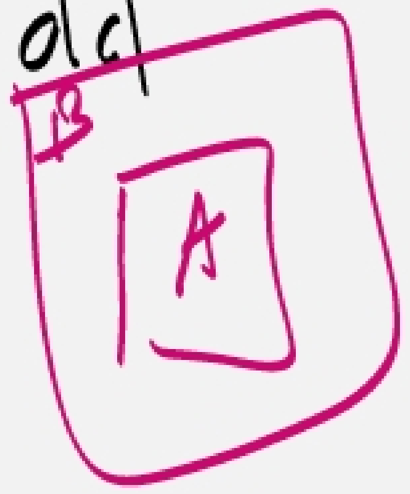
If n is div by 2 $\Rightarrow n$ is even

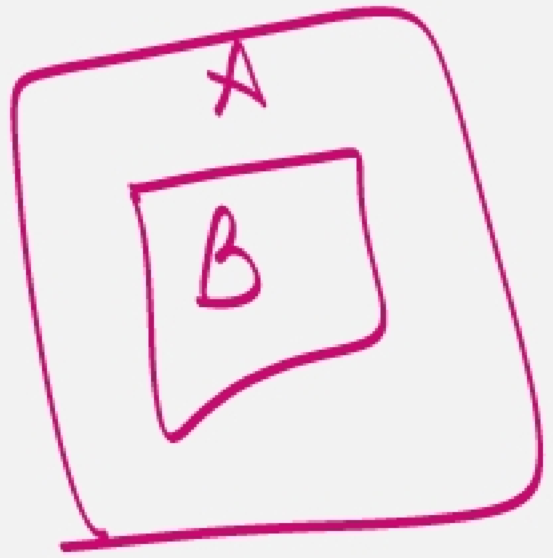
$q \to p$

$p \Longleftrightarrow q$

n is odd

$x \in A, x \notin B$





$p \to \neg p$

p	$\neg p$	$p \to \neg p$
0	1	1
1	0	0

$p \to q$

Converse $q \to p$

Contrapositive $\neg q \to \neg p$

Inverse: $\neg p \to \neg q$

Tautology	Logical Equivalance
Converse	

$p \to p \vee q$

1
1
1
1

TAUTOLOGY

ALWAYS TRUE

$\neg(p \to q) \wedge q$ ALWAYS FALSE

$\neg(p \to q) \wedge q$
0
0
0
0

CONTRADICTION

A is Logically eq. to B

A $p \to q$	B $\neg p \vee q$
1	1
1	1
0	0
1	1