

Example (2):

$$5 + 9 + 13 + \dots = n(2n+3)$$

$$\begin{aligned} n^{\text{th}} \text{ term} &= a + (n-1)d \\ &= 5 + (n-1)(4) \\ &= 5 + 4n - 4 \\ &= 4n + 1 \end{aligned}$$

Given series is

$$5 + 9 + 13 + \dots + (4n+1) = n(2n+3)$$

① $P(1)$

$$\begin{aligned} \text{LHS} \\ &= 5 \end{aligned}$$

RHS

$$\begin{aligned} &1(2(1)+3) \\ &= 5 \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

$\therefore P(1)$ is true.

② assume that $P(k)$ is true

$$5 + 9 + 13 + \dots + (4k+1) = k(2k+3)$$

③ Prove that $P(k+1)$ is true

$$\text{LHS} \quad 5 + 9 + 13 + \dots + (4k+1) + \{4(k+1)+1\}$$

$$= k(2k+3) + 4k + 4 + 1$$

$$= 2k^2 + 3k + 4k + 5$$

$$= 2k^2 + 7k + 5$$

$$= 2k^2 + 2k + 5k + 5$$

$$= 2k(k+1) + 5(k+1)$$

$$= (k+1)(2k+5)$$

$$= (k+1) \{2(k+1) + 3\}$$