

Universidade Federal do Rio Grande do Norte
Instituto Metr pole Digital
IMD0601 - Bioestat stica

Dados normais

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Baixe a aula (e os arquivos)

- Para aqueles que não clonaram o repositório:

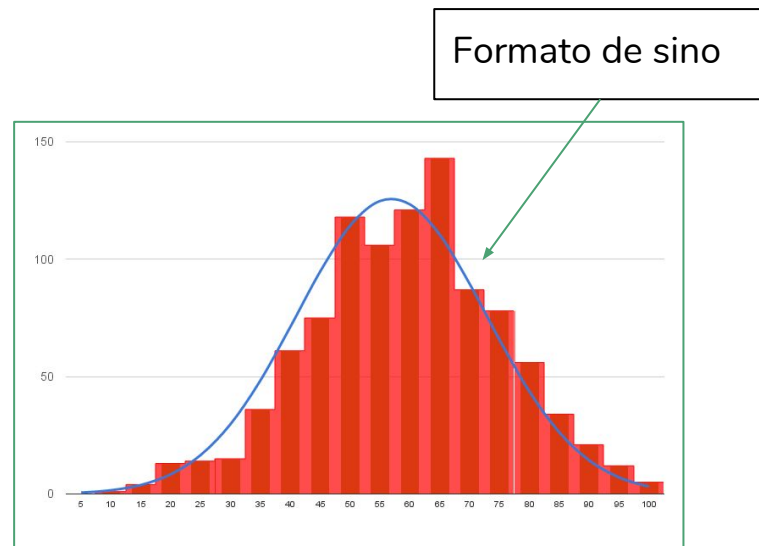
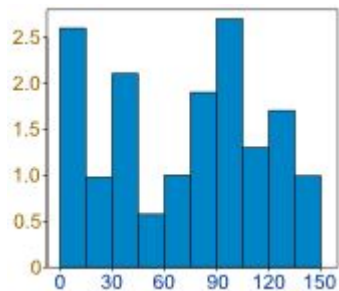
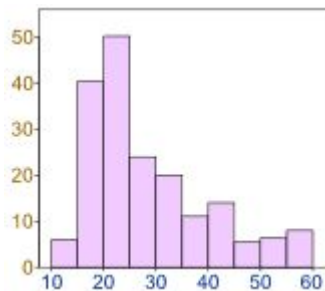
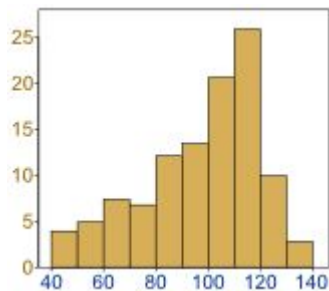
```
> git clone https://github.com/tetsufmbio/IMD0601.git
```

- Para aqueles que já tem o repositório local:

```
> cd /path/to/IMD0601
```

```
> git pull
```

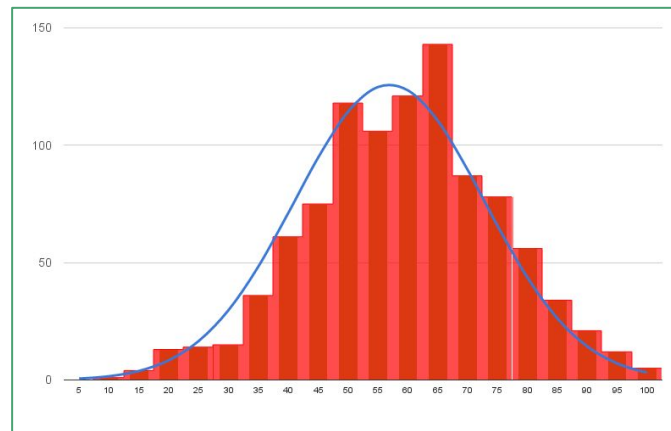
Distribuição dos dados



Dados com Distribuição Normal

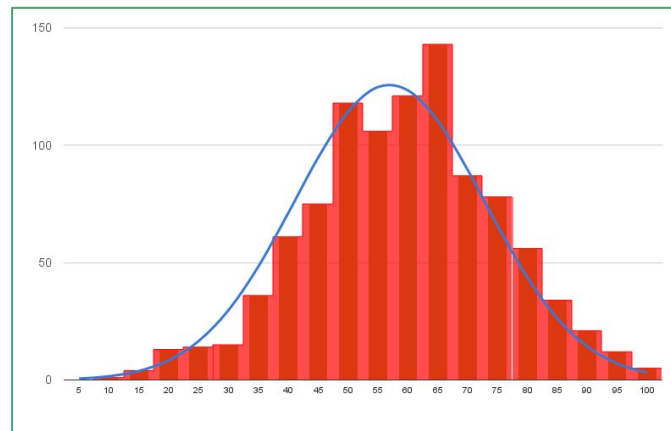
Exemplos de dados normais

- Altura das pessoas;
- Notas de prova;
- Duração da gravidez;
- Salário;
- Tamanho de peças produzidas em uma fábrica;
- etc...

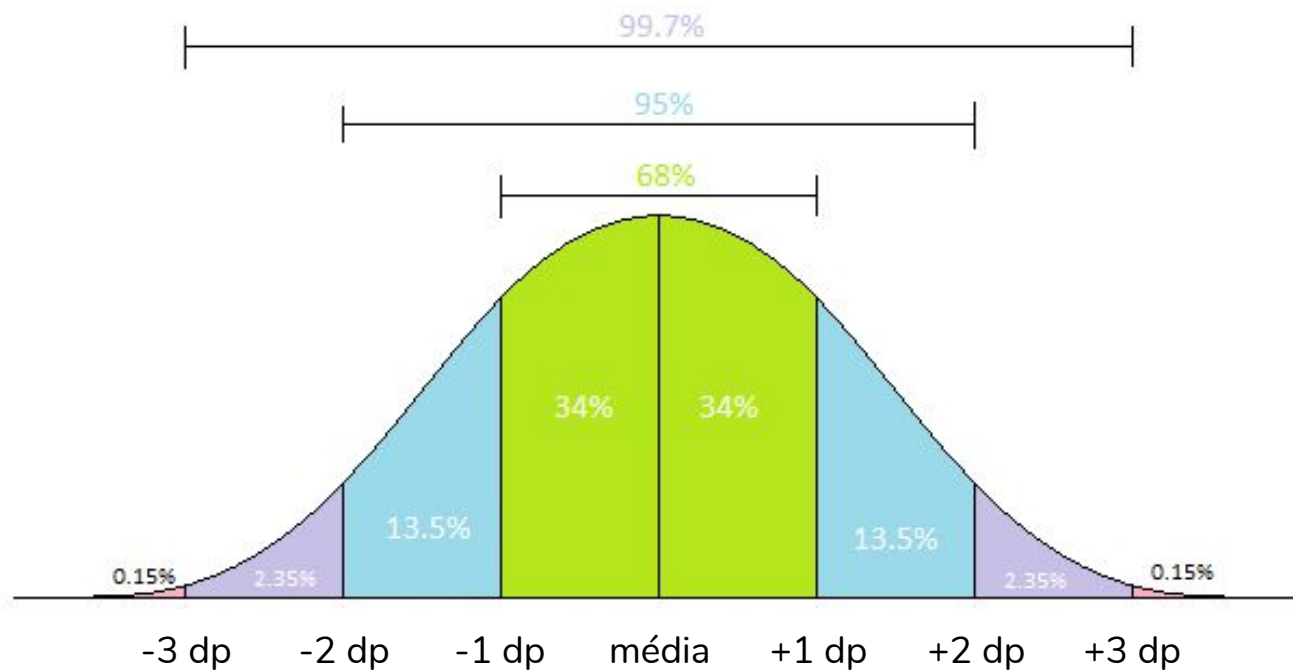


Propriedades de uma distribuição normal

- Formato de sino;
- Simétrico;
- Média = Mediana = Moda;
- Regra do desvio padrão.



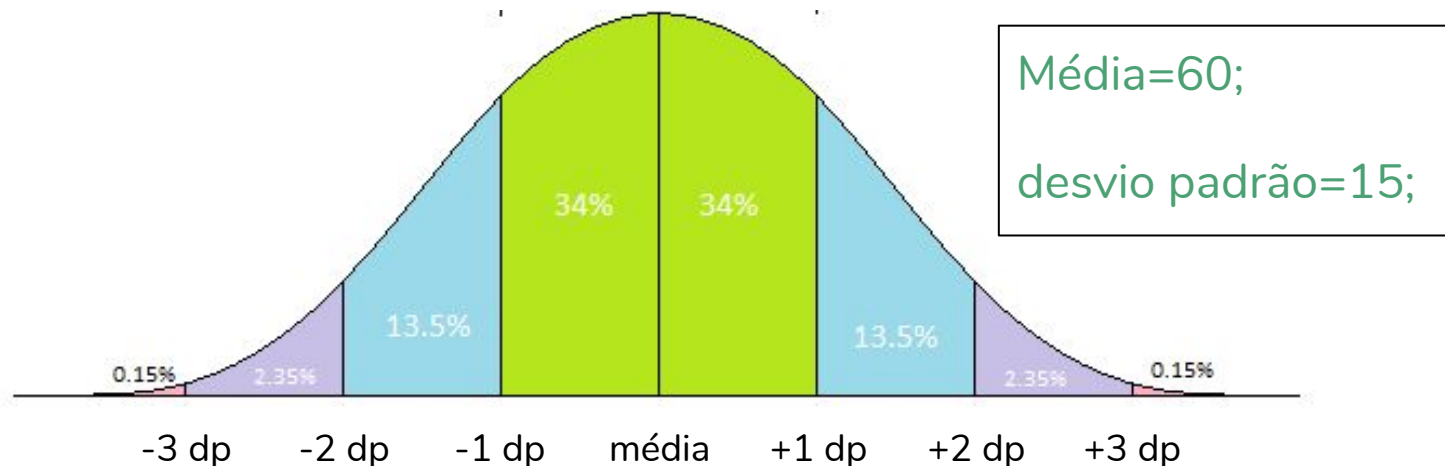
Regra do desvio padrão



Regra do desvio padrão

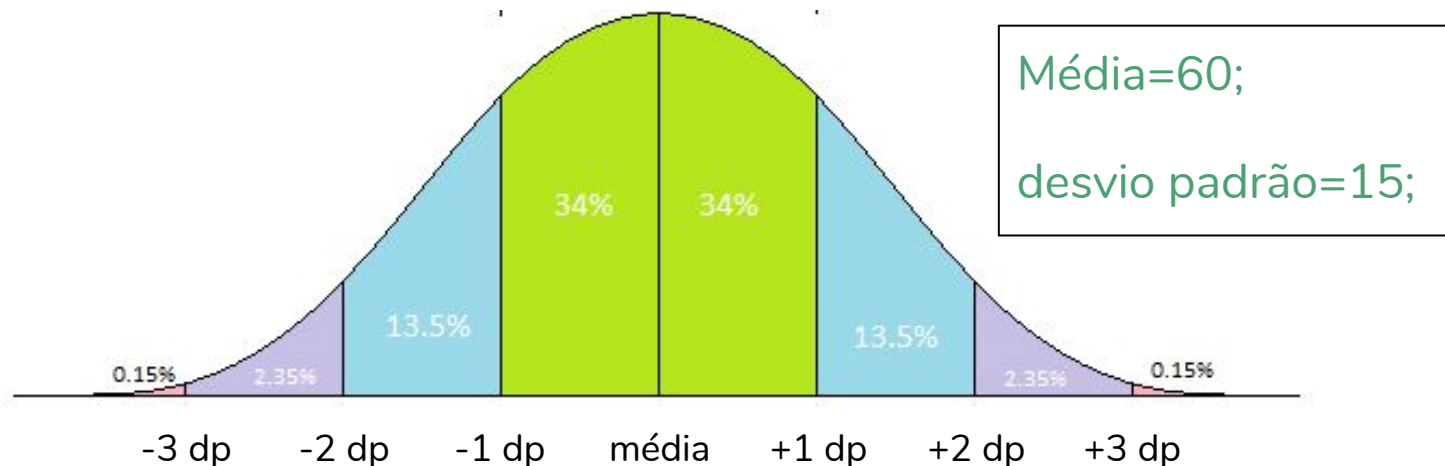
Tirei nota 75. Quantos alunos tiraram uma nota menor que a minha? 84%

Quantos tiraram uma nota acima que a minha? 16%



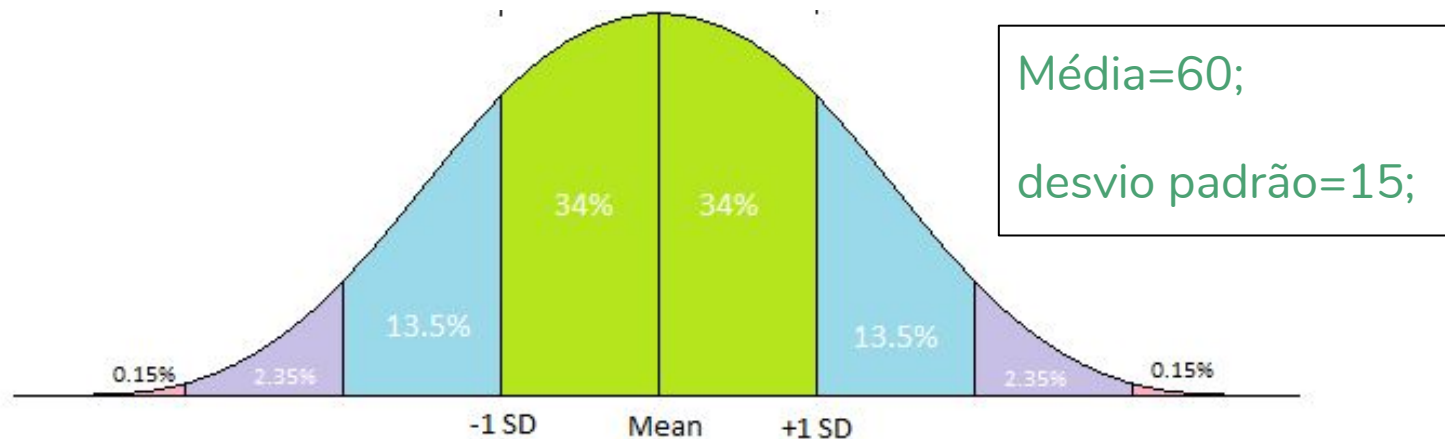
Regra do desvio padrão

Quero estar entre os 2.5% dos alunos que tiraram a maior nota. Que nota eu devo tirar? Resposta: 90



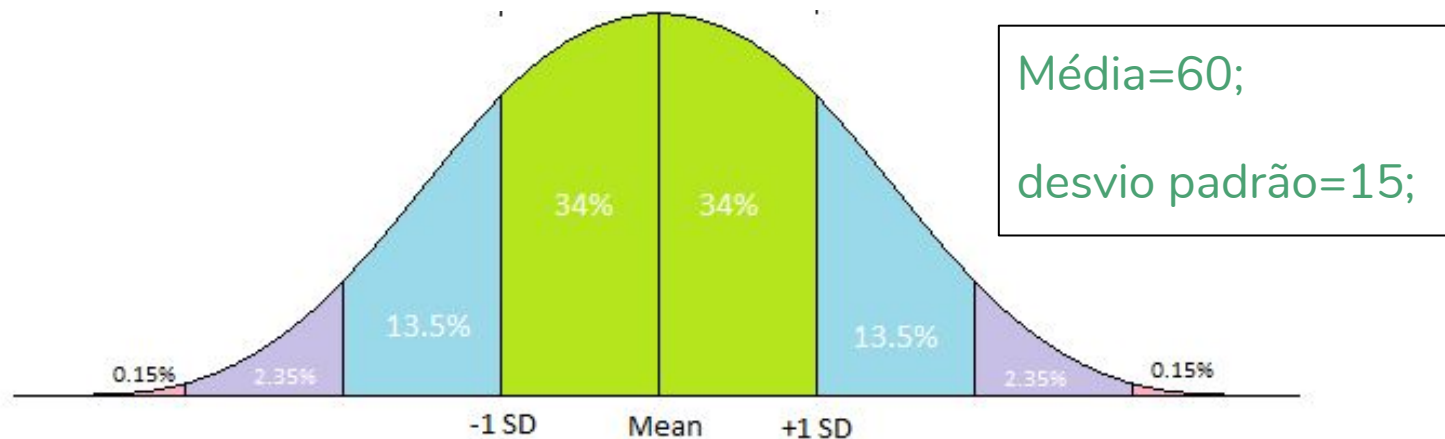
Regra do desvio padrão

Se eu tirei 80, quantos alunos ficaram abaixo da minha nota?

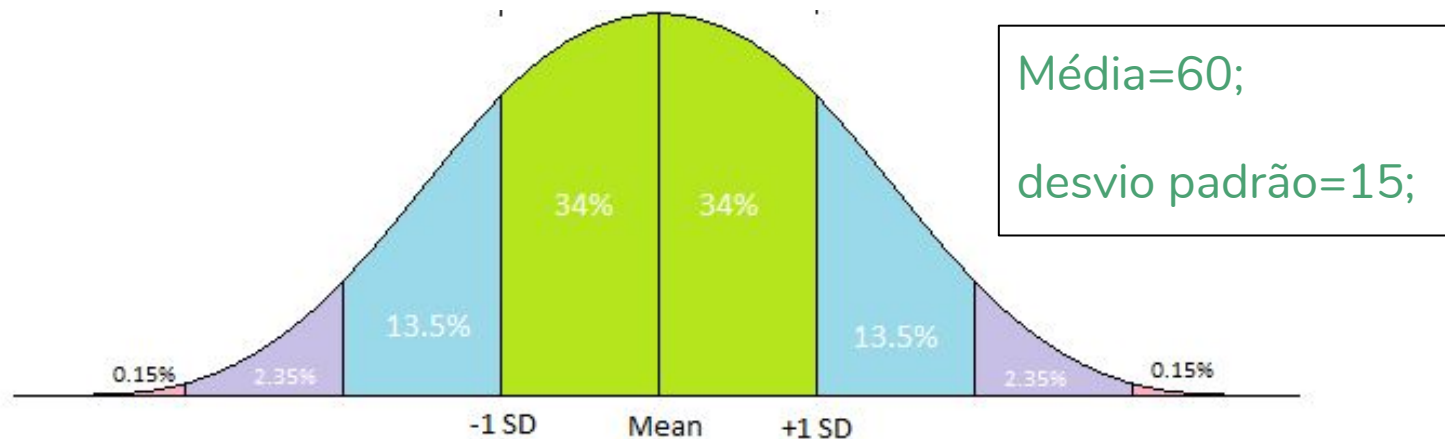


Função da distribuição normal

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Se eu tirei 80, quantos desvios padrão estou da média?

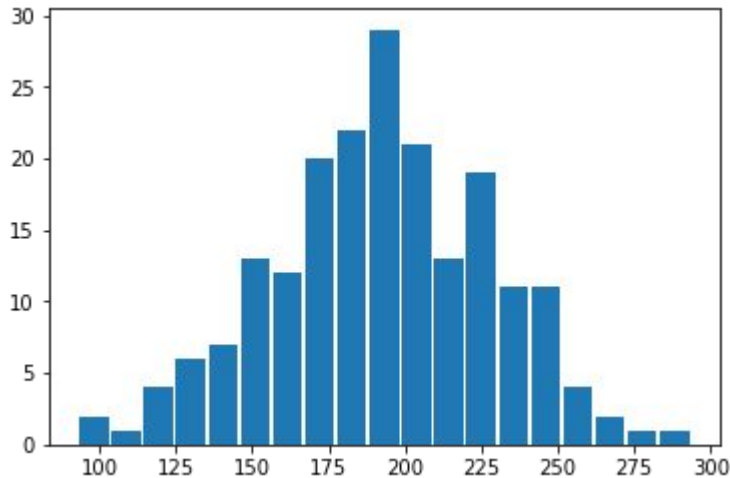


Z-score

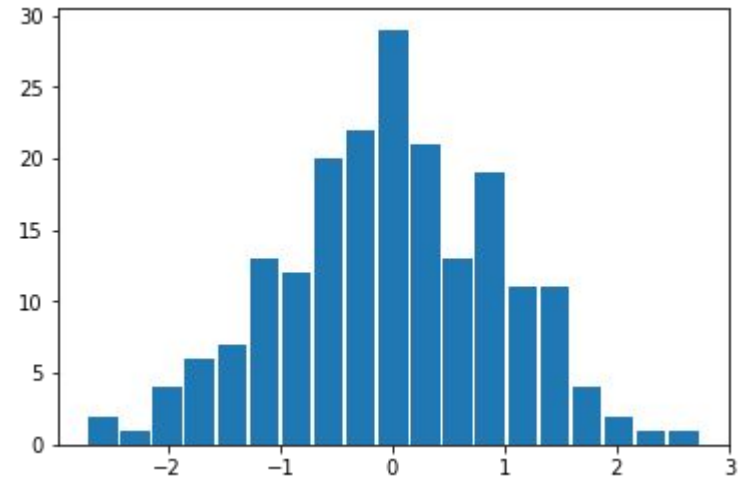
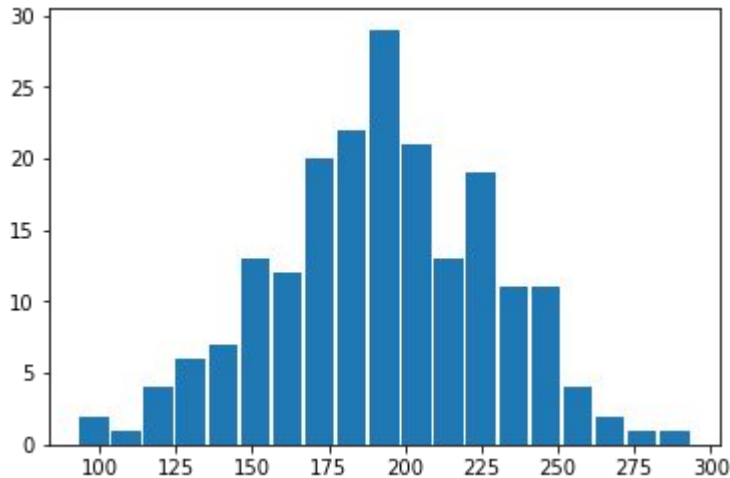
Z-score → unidades de desvio padrão de um determinado valor em um conjunto de dados de média μ e desvio padrão σ .

$$z = \frac{x - \mu}{\sigma}$$

O que acontece se eu aplicar o z-score em todo dado...?



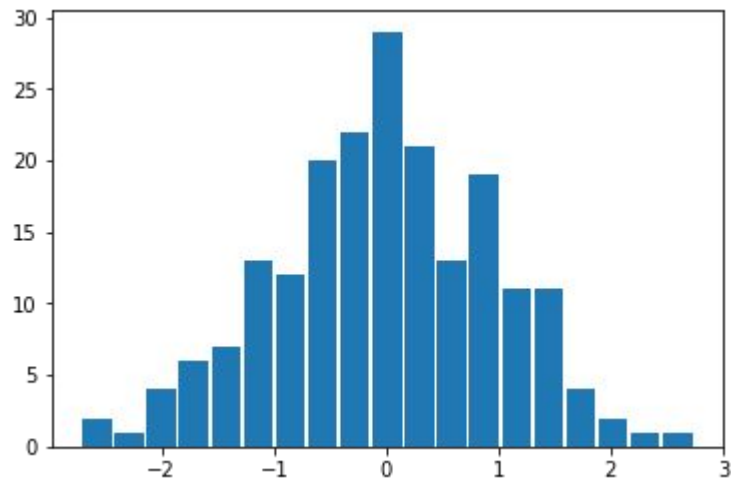
O que acontece se eu aplicar o z-score em todo dado...?



Qual é a média e o desvio padrão dos dados transformados em z-score?

$$\mu = 0$$

$$\sigma = 1$$



Quando os dados são transformados em z-score...

Eles se transformam em um conjunto de dados de:

- $\mu = 0$
- $\sigma = 1$

Transformar os dados em Z-score é uma forma de padronizar (normalizar) os dados;

Gera uma distribuição normal padrão.

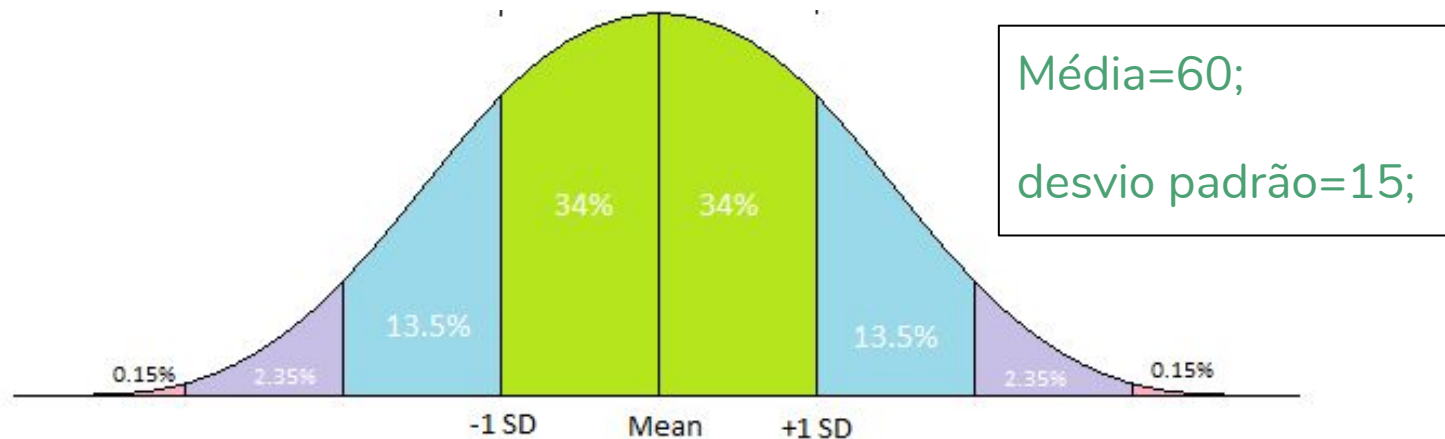
A normal distribution curve is shown. The area under the curve to the left of a point z on the horizontal axis is shaded gray. The point z is marked on the horizontal axis with a dot and labeled below it.

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2} \quad -\infty \leq z \leq \infty$$

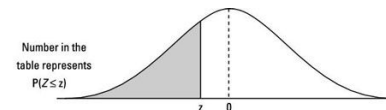
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0010	0.0010	0.0009	0.0008	0.0008	0.0008	0.0008	0.0008	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0029	0.0027	0.0026
-2.6	0.0047	0.0046	0.0044	0.0044	0.0043	0.0042	0.0041	0.0041	0.0039	0.0038
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0227	0.0221	0.0217	0.0212	0.0207	0.0202	0.0198	0.0193	0.0189	0.0185
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0656	0.0643	0.0630	0.0616	0.0602	0.0588	0.0573	0.0559	0.0549
-1.4	0.0808	0.0795	0.0778	0.0764	0.0749	0.0735	0.0719	0.0704	0.0684	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1445	0.1423	0.1401	0.1379
-0.9	0.1841	0.1797	0.1751	0.1706	0.1661	0.1617	0.1573	0.1529	0.1485	0.1439
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3403	0.3358	0.3313	0.3268	0.3224	0.3179	0.3134	0.3089	0.3043
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4600	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.5400	0.5359	0.5319	0.5278	0.5237	0.5196	0.5155	0.5114	0.5073	0.5032
0.2	0.5808	0.5758	0.5717	0.5675	0.5633	0.5591	0.5549	0.5507	0.5465	0.5423
0.3	0.6227	0.6168	0.6108	0.6047	0.5986	0.5925	0.5864	0.5803	0.5741	0.5679
0.4	0.6657	0.6588	0.6518	0.6447	0.6376	0.6305	0.6234	0.6163	0.6091	0.6019
0.5	0.7097	0.7019	0.6940	0.6861	0.6781	0.6701	0.6621	0.6541	0.6460	0.6379
0.6	0.7547	0.7459	0.7370	0.7280	0.7189	0.7098	0.7007	0.6916	0.6824	0.6732
0.7	0.7997	0.7899	0.7800	0.7700	0.7599	0.7497	0.7395	0.7293	0.7190	0.7087
0.8	0.8447	0.8339	0.8230	0.8120	0.8009	0.7897	0.7784	0.7671	0.7557	0.7443
0.9	0.8897	0.8779	0.8660	0.8540	0.8419	0.8297	0.8174	0.8050	0.7926	0.7801
1.0	0.9347	0.9219	0.9090	0.8960	0.8829	0.8697	0.8564	0.8430	0.8295	0.8160
1.1	0.9797	0.9659	0.9519	0.9378	0.9236	0.9093	0.8949	0.8804	0.8658	0.8512
1.2	1.0247	1.0099	0.9949	0.9798	0.9646	0.9493	0.9339	0.9184	0.9028	0.8871
1.3	1.0697	1.0539	1.0379	1.0218	1.0056	0.9893	0.9729	0.9564	0.9398	0.9231
1.4	1.1147	1.0979	1.0810	1.0640	1.0469	1.0296	1.0122	0.9947	0.9771	0.9594
1.5	1.1597	1.1419	1.1239	1.1058	1.0876	1.0692	1.0507	1.0321	1.0134	0.9946
1.6	1.2047	1.1859	1.1669	1.1478	1.1285	1.1091	1.0896	1.0699	1.0501	1.0302
1.7	1.2497	1.2299	1.2099	1.1898	1.1695	1.1490	1.1284	1.1077	1.0868	1.0658
1.8	1.2947	1.2739	1.2529	1.2318	1.2105	1.1890	1.1673	1.1454	1.1234	1.1013
1.9	1.3397	1.3179	1.2959	1.2738	1.2515	1.2290	1.2063	1.1834	1.1604	1.1373
2.0	1.3847	1.3619	1.3389	1.3158	1.2925	1.2690	1.2453	1.2214	1.1974	1.1732
2.1	1.4297	1.4059	1.3819	1.3578	1.3335	1.3090	1.2843	1.2594	1.2344	1.2092
2.2	1.4747	1.4499	1.4249	1.3998	1.3745	1.3490	1.3233	1.2974	1.2714	1.2452
2.3	1.5197	1.4939	1.4679	1.4418	1.4155	1.3890	1.3623	1.3354	1.3084	1.2812
2.4	1.5647	1.5379	1.5109	1.4838	1.4565	1.4290	1.4013	1.3734	1.3454	1.3172
2.5	1.6097	1.5819	1.5539	1.5258	1.4975	1.4690	1.4403	1.4114	1.3824	1.3532
2.6	1.6547	1.6259	1.5969	1.5678	1.5385	1.5090	1.4793	1.4494	1.4194	1.3892
2.7	1.6997	1.6699	1.6399	1.6098	1.5795	1.5490	1.5183	1.4874	1.4564	1.4252
2.8	1.7447	1.7139	1.6829	1.6518	1.6205	1.5890	1.5573	1.5254	1.4934	1.4612
2.9	1.7897	1.7579	1.7259	1.6938	1.6615	1.6290	1.5963	1.5634	1.5304	1.4972
3.0	1.8347	1.7999	1.7649	1.7298	1.6945	1.6590	1.6233	1.5874	1.5514	1.5152
3.1	1.8797	1.8439	1.8079	1.7718	1.7355	1.6990	1.6623	1.6254	1.5884	1.5512
3.2	1.9247	1.8879	1.8509	1.8138	1.7765	1.7390	1.7013	1.6634	1.6254	1.5872
3.3	1.9697	1.9319	1.8939	1.8558	1.8175	1.7790	1.7403	1.7014	1.6624	1.6232
3.4	2.0147	1.9759	1.9369	1.8978	1.8585	1.8190	1.7793	1.7394	1.6994	1.6592
3.5	2.0597	2.0199	1.9799	1.9398	1.8995	1.8590	1.8183	1.7774	1.7364	1.6952
3.6	2.1047	2.0639	2.0229	1.9818	1.9405	1.8990	1.8573	1.8154	1.7734	1.7312
3.7	2.1497	2.1079	2.0659	2.0238	1.9815	1.9390	1.8963	1.8534	1.8104	1.7672
3.8	2.1947	2.1519	2.1089	2.0658	2.0225	1.9790	1.9353	1.8914	1.8474	1.8032
3.9	2.2397	2.1959	2.1519	2.1078	2.0635	2.0190	1.9743	1.9294	1.8844	1.8392
4.0	2.2847	2.2399	2.1949	2.1498	2.1045	2.0590	2.0133	1.9674	1.9214	1.8752
4.1	2.3297	2.2839	2.2379	2.1918	2.1455	2.0990	2.0523	2.0054	1.9584	1.9112
4.2	2.3747	2.3279	2.2809	2.2338	2.1865	2.1390	2.0913	2.0434	1.9954	1.9472
4.3	2.4197	2.3719	2.3239	2.2758	2.2275	2.1790	2.1303	2.0814	2.0324	1.9832
4.4	2.4647	2.4159	2.3669	2.3178	2.2685	2.2190	2.1693	2.1194	2.0694	2.0192
4.5	2.5097	2.4599	2.4099	2.3598	2.3095	2.2590	2.2083	2.1574	2.1064	2.0552
4.6	2.5547	2.5039	2.4529	2.4018	2.3505	2.2990	2.2473	2.1954	2.1434	2.0912
4.7	2.5997	2.5479	2.4959	2.4438	2.3915	2.3390	2.2863	2.2334	2.1804	2.1272
4.8	2.6447	2.5919	2.5389	2.4858	2.4325	2.3790	2.3253	2.2714	2.2174	2.1632
4.9	2.6897	2.6359	2.5819	2.5278	2.4735	2.4190	2.3643	2.3094	2.2544	2.1992
5.0	2.7347	2.6799	2.6249	2.5698	2.5145	2.4590	2.4033	2.3474	2.2914	2.2352
5.1	2.7797	2.7239	2.6679	2.6118	2.5555	2.4990	2.4423	2.3854	2.3284	2.2712
5.2	2.8247	2.7679	2.7109	2.6538	2.5965	2.5390	2.4813	2.4234	2.3654	2.3072
5.3	2.8697	2.8119	2.7539	2.6958	2.6375	2.5790	2.5203	2.4614	2.4024	2.3432
5.4	2.9147	2.8559	2.7969	2.7378	2.6785	2.6190	2.5593	2.4994	2.4394	2.3792
5.5	2.9597	2.8999	2.8399	2.7798	2.7195	2.6590	2.5983	2.5374	2.4764	2.4152
5.6	3.0047	2.9439	2.8829	2.8218	2.7605	2.6990	2.6373	2.5754	2.5134	2.4512
5.7	3.0497	2.9879	2.9259	2.8638	2.8015	2.7390	2.6763	2.6134	2.5504	2.4872
5.8	3.0947	3.0319	2.9689	2.9058	2.8425	2.7790	2.7153	2.6514	2.5874	2.5232
5.9	3.1397	3.0759	3.0119	2.9478	2.8835	2.8190	2.7543	2.6894	2.6244	2.5592
6.0	3.1847	3.1199	3.0549	2.9898	2.9245	2.8590	2.7933	2.7274	2.6614	2.5952
6.1	3.2297	3.1639	3.0979	3.0318	2.9655	2.8990	2.8323	2.7654	2.6984	2.6312
6.2	3.2747	3.2079	3.1409	3.0738	3.0065	2.9390	2.8713	2.8034	2.7354	2.6672
6.3	3.3197	3.2519	3.1839	3.1158	3.0475	2.9790	2.9103	2.8414	2.7724	2.7032
6.4	3.3647	3.2959	3.2269	3.1578	3.0885	3.0190	2.9493	2.8794	2.8094	2.7392
6.5	3.4097	3.3399	3.2699	3.1998	3.1295	3.0590	2.9883	2.9174	2.8464	2.7752
6.6	3.4547	3.3839	3.3129	3.2418	3.1705	3.0990	3.027			

Voltando a questão

Se eu tirei 80, quantos alunos ficaram abaixo da minha nota?



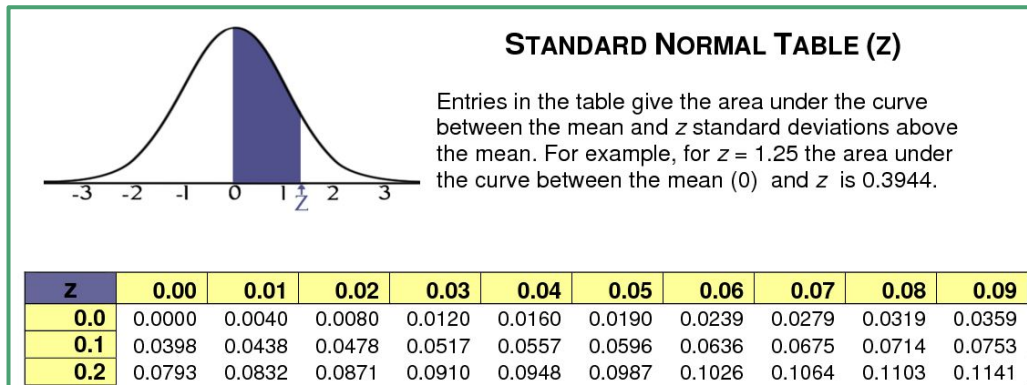
Diferentes tabelas de z-score



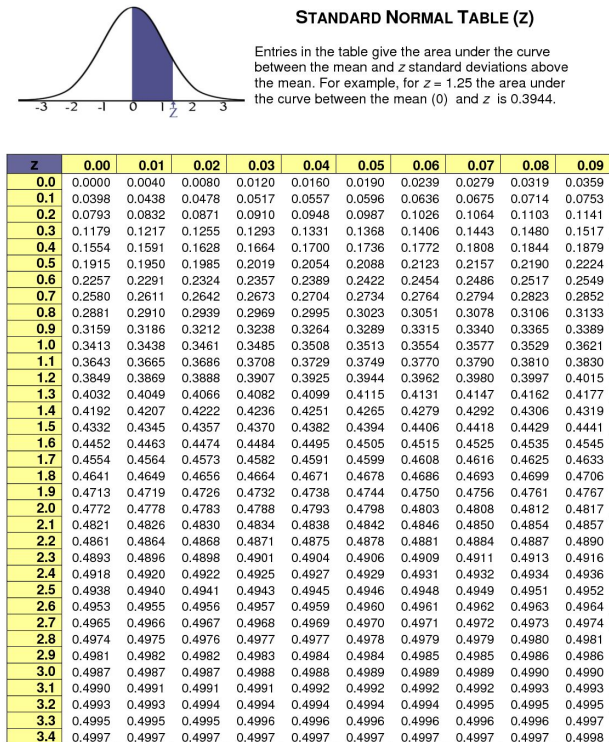
-0.5	.1841	.1814	.1786	.1762	.1738	.1711	.1685	.1660	.1633	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Diferentes tabelas de z-score



■ ■ ■

[illegible]

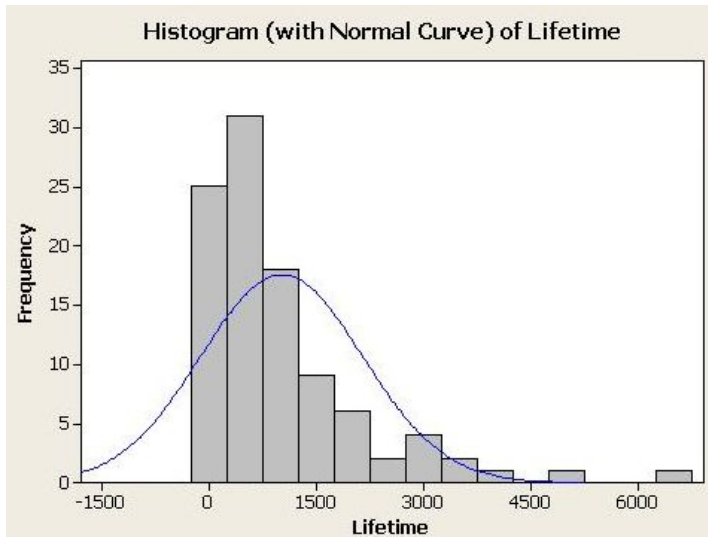
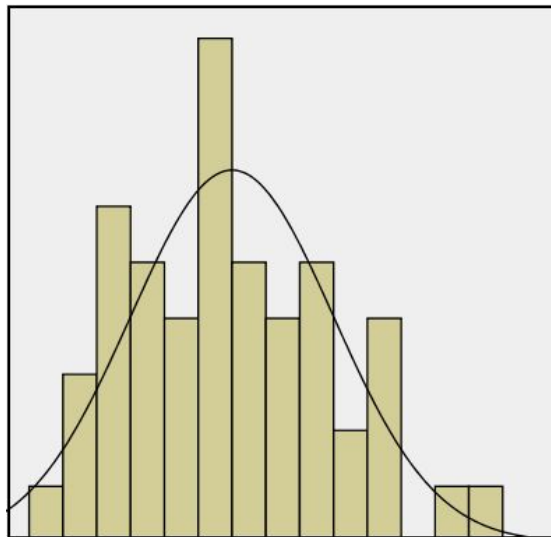
Os valores tabelados do z-score só podem ser utilizados para dados que possuem distribuição normal.

Como saber se seus dados seguem uma distribuição normal?

Como saber se seus dados seguem uma distribuição normal?

Análise visual

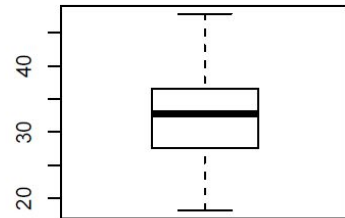
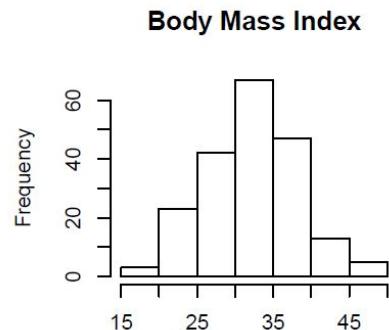
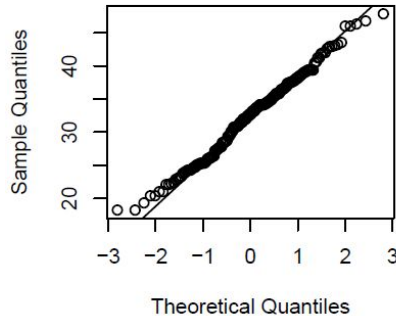
- **Histogramas e a curva normal**



Como saber se seus dados seguem uma distribuição normal?

Análise visual

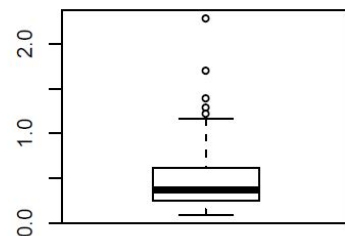
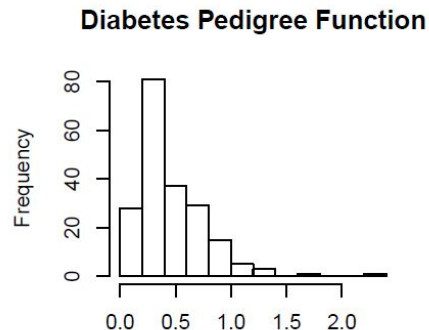
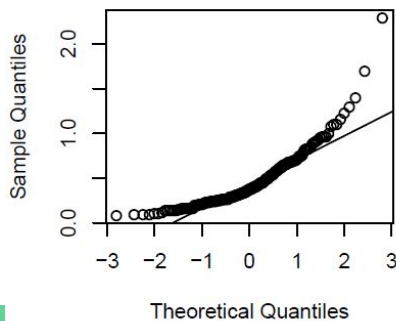
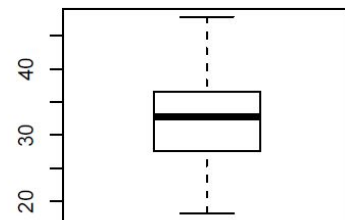
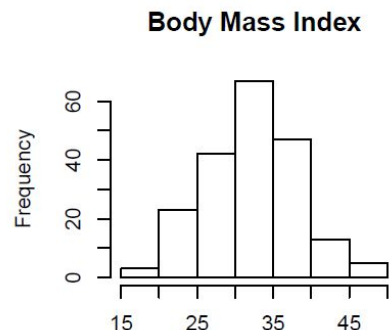
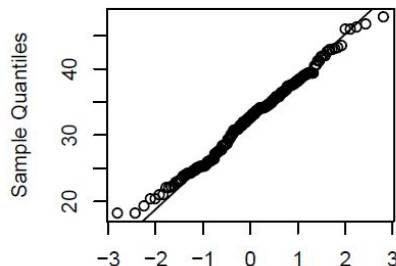
- Histogramas e a curva normal
- **QQ-plot**



Como saber se seus dados seguem uma distribuição normal?

Análise visual

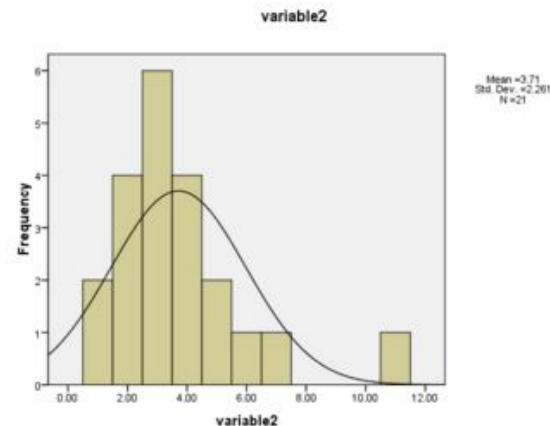
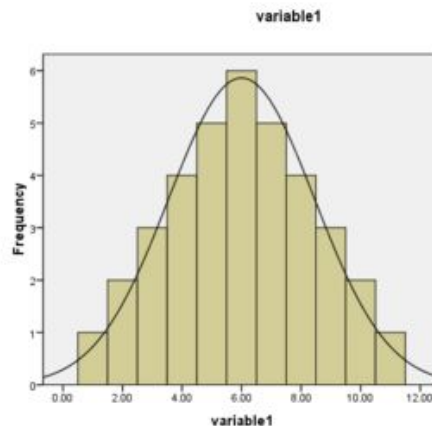
- Histogramas e a curva normal
- **QQ-plot**



Como saber se seus dados seguem uma distribuição normal?

Testes estatísticos

- Kolmogorov-Smirnov (K-S)
- Shapiro-Wilk (S-W)



Tests of Normality

	Kolmogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
variable1	.083	36	.200 [*]	.981	36	.782
variable2	.223	21	.008	.805	21	.001

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.