An Adaptive Just Intonation Algorithm for Barbershop Quartets

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Abstract

Hier komt dus waarschijnlijk nog een abstract. Of is dat niet nodig volgens jullie?

1 Introduction

The piano is out of tune; every *musical interval* (i.e. distance between two tones) is slightly too small or too big in the standard tuning of the piano.[1] Music groups that have the possibility to, often attempt to tune their chords according to "just intonation" instead, adaptively retuning every note such that it works well in context. Barbershop quartets, for instance, explicitly try to achieve just intonation on every chord. However, when performers want to keep notes at the same pitch over different chords while tuning in just intonation, there is no unbiased answer to the question how exactly each note should be tuned.

The current study tries to answer that specific question for barbershop quartets: *what adaptive tuning algorithm would a mathematically ideal barbershop quartet follow, given a score?* (research question A). Afterwards, this study will implement this algorithm in order to answer research question B: *can we implement the tuning system from question A so that it models the ideal barbershop quartet?* In order to provide the necessary context for this question, the following paragraphs will expand upon the subjects of just intonation, adaptive just intonation algorithms and barbershop music.

1.1 Just Intonation

In many musical contexts, *just intonation* is seen as the ideal method for tuning. Rather than predefining each tone's pitch, just intonation defines specific frequency ratios for each musical interval. The most common intervals are the octave (defined as $2 : 1$), the fifth ($3 : 2$) and the major third ($5 : 4$). For example, if an A is tuned at 220 Hz, then an E a fifth above this A will be tuned at $220 \cdot \frac{3}{2} = 330$ Hz. The minor seventh, which is also prevalent in many types of music, is defined as $7 : 4$. These ratios are drawn from the smallest intervals of the *harmonic series*, which refers to the set of overtones that can be heard above nearly every sung, played or synthesised tone. When musical intervals follow these simple ratios, the overtones of different tones "lock together" and create a specific auditory experience that many musicians strive for.
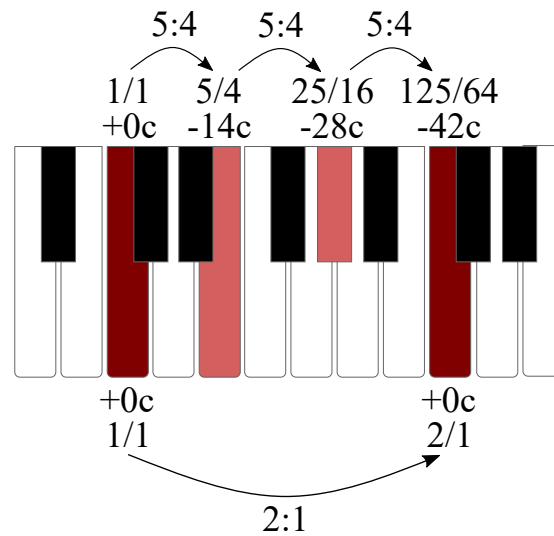
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Figure 1: 12 fixed tones are not compatible with just intonation. For each coloured tone, its frequency ratio with the low C is written. The difference with equal temperament in cents is also written. The bottom path shows that a high C should be tuned at exactly twice the frequency of the low C, when tuning the octave justly. The top path instead uses justly tuned major thirds with the standard 5 : 4 ratio. The two paths come out at a different tuning for the high C, proving that major thirds and octaves are incompatible.

Sadly, it is mathematically impossible to devise a fixed tuning for a set of twelve tones such that each pair of tones always exactly follows one of the simple ratios from just intonation. A consequence of this fact is that it is also impossible to provide a "perfect" tuning for the piano. To prove this impossibility, take a look at Figure 1. Suppose we want the major third to be equal to 4 semitones, as is the case in western music theory. Then the ratio between a low C and E on a piano should be 5 : 4, as should the ratio between E and G♯ and the ratio between G♯ and a high C. Therefore, the ratio between the low C and G♯ should be $\frac{5}{4} \cdot \frac{5}{4} = \frac{25}{16}$ and the ratio between the low C and the high C should be $\frac{25}{16} \cdot \frac{5}{4} = \frac{125}{64}$. However, the two Cs are one octave apart and should therefore have a ratio of $\frac{2}{1}$. These two ratios are not the same, which shows that it is impossible to devise a tuning system such that both major thirds and octaves are always in tune according to just intonation.

Most modern music accepts the fact that just intonation cannot be achieved, using the tuning system of *equal temperament* as a substitute. In twelve-tone equal temperament, the octave is divided into twelve equal divisions. Since the octave is defined by the frequency ratio 2 : 1, this custom puts the smallest interval (the semitone) at exactly $\sqrt[12]{2} : 1$. For example, if an A is tuned at 220 Hz, then the lowest tone above this A, a B♭, will be tuned at $220 \cdot \frac{\sqrt[12]{2}}{1} \approx 233.08$ Hz.[1] Twelve-tone equal temperament works relatively well for some intervals, such as the fifth, which is only slightly too small compared to just intonation. On the contrary, intervals like the major third and the minor seventh are significantly smaller in just intonation than in equal temperament. Figure 2 shows the standard tuning in equal temperament for a part of the piano keyboard.

When talking about the difference between just intonation and 12-tone equal tem-

A4 = 440.00	G♯4 = 415.30
G4 = 392.00	A♯4 = 369.99
F4 = 349.23	
E4 = 329.63	D♯4 = 311.13
D4 = 293.67	C♯4 = 277.18
C4 = 261.63	
B3 = 246.94	A♯3 = 233.08
A3 = 220.00	G♯3 = 207.65
G3 = 196.00	F♯3 = 185.00
F3 = 174.61	
E3 = 164.81	D♯3 = 155.56
D3 = 146.83	C♯3 = 138.59
C3 = 130.81	

Figure 2: A selection of standard frequency values for piano in equal temperament, shown in Hertz. Equal temperament is the most common tuning method and ensures that each semitone is always the same relative size, namely $\sqrt[12]{2} : 1$. However, just intonation is not compatible with equal temperament; for example, this A3 and E4 have a frequency ratio that is slightly smaller than that of a just fifth, which would be $3 : 2$.

perament, the *cent* interval is often used as a unit. 1 cent is defined as $\frac{1}{100}$ of a semitone in equal temperament, meaning it has a frequency ratio of $\sqrt[1200]{2} : 1$. A justly tuned major third, for example, is 14 cents lower than a major third in equal temperament (Figure 1).[1]

1.2 Adaptive Just Intonation Systems

Although just intonation is impossible to achieve when all twelve tones have a fixed tuning, such as on an acoustic piano, it can be achieved when using instruments that can *adaptively* tune specific notes. In other words, if a note can be tuned differently based on which other notes are playing, the strict mathematical ratios of just intonation can be followed. This idea is, for example, applied to ensembles with human voices, trombones or string instruments such as the violin, since all of those instruments can continuously change their pitch by ear.[1]

Modern computational techniques also allow digital instruments to be tuned adaptively. Much research has been done into automatic *adaptive tuning systems*: programs that take sets of musical notes as input, and return their respective tunings according to just intonation as output. Sethares (1994) approached the problem very generally by minimising a loss function based on a sound's dissonance curve.[2] Løberg (2002) implemented a dynamic tuning system by the Norwegian composer Eivind Groven that dynamically chooses between 36 available tones to get just intervals.[3] Commercial software such as Hermode Tuning[4] and Pivotuner[5] can be integrated into music software to retune all played notes based on the sounding chord.

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[1]The addition of the cent interval leaves us with three different ways to describe a tone's exact tuning: in Hertz, with a fraction or in cents. Hertz makes the most sense in phyics: an A3 can be tuned at 220 Hz, whereas an E4 would be tuned at 329.63 Hz on a piano. In just intonation, we often use fractions: when tuning an E4 to be a perfect fifth above this A3, the E4's tuning would need to be $\frac{3}{2}$ times the frequency of the A3 (equivalently, 330.00 Hz). The cent interval puts this abstract difference in perspective: 330.00 Hz is 2 cents higher than 329.63 Hz, meaning $\frac{2}{100}$ of a semitone.

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However, even adaptive just intonation does not have a single algorithm that completely solves the mathematical problem of tuning. Choices need to be made with regards to optimisation of melodic intervals, held notes and tonal centre drift. Optimisation of melodic intervals means that the intervals within the melody follow equal temperament as much as possible. Optimisation of held notes means notes that keep sounding in different chords do not need to be dramatically retuned for each new chord. Tonal centre drift is a gradual lowering or heightening of the average tuning of notes, which is generally seen as undesirable.[6, 7] If held notes are allowed to retune freely, just intonation without pitch drift is possible if the melody always follows equal temperament and the other voices adapt to tune justly with the melody.[6] In contrast, the constraint of held notes staying at the same pitch between different chords causes problems.

To illustrate the incompatibility of the held notes constraint with the other constraints, consider a C major triad (C-E-G) moving to an E major triad (E-G♯-B), with the E being held between the two chords. If the C is tuned according to equal temperament (e.g. 261.6 Hz), then the E will have to be a bit lower than equal temperament in order to create the standard $3 : 2$ ratio of the major third (as $261.6 \cdot \frac{3}{2} = 392.4$ Hz, rather than 392.6 Hz). If this lower E is held into the next chord, then the G♯ and B will also be tuned lower to tune justly with the E. Therefore, the overall tonal centre has moved slightly downwards ("flat"). This downwards shift shows that held notes and tonal centre drift cannot both be optimised at the same time.

1.3 Barbershop Music

Barbershop is a distinct genre of four-part singing that is mostly prevalent in North America. The music is primarily sung in quartets, though larger choirs are also often formed, of which quartets form smaller subsets.[8] Barbershop quartets are easily recognisable and are sometimes used to exemplify old-fashioned American culture. However, Barbershop is also very specifically defined and has an active community that aims to preserve the genre according to its general definition. Some of the largest central events where Barbershop enthusiasts gather are competitions, where quartets are judged according to this specific definition.[7] In this paper, the Barbershop Harmony Society's definition of barbershop music and its Contest and Judging Handbook will often be cited as the main source of information about priorities within the genre.

In this official definition, barbershop music is described as having four voices: tenor, lead, baritone and bass. The lead generally sings the melody, but exceptions can occur within a song. The melody is accompanied by mostly homorhythmic (in the same rhythm as the lead) harmonies in the other three voices. Chords generally do not get too complicated: most songs revolve around major and minor chords without many harmonic extensions. Additionally, the dominant seventh chord is often described as the most important chord in barbershop music.

The Contest and Judging Handbook (page 7-2, paragraph II.A.) prescribes intonation in barbershop music as follows:

> Barbershop singers adjust pitches to achieve perfectly tuned chords, and yet sing a melodic line that remains true to the tonal center. Barbershop singers strive for more precise tuning than is possible with the fixed 12-tones- per-octave of the equally tempered scale of fixed-pitched instruments, such as the piano. Essentially, we use just intonation for harmonic tuning while remaining true to the established tonal center.[7]

The above guideline describes just intonation as one of the key elements of barbershop music. Barbershop singers try to minimise "beats" in the sound of their harmonies, the auditory artefacts that appear when chords are not tuned justly. When quartets follow interval ratios as described in 1.1, the overtones of the different parts match and create a buzzing, unchanging auditory experience that is described as "lock and ring".

However, very few barbershop quartets consistently achieve the high standard of just intonation.[8] The above quote immediately describes one of the major dilemmas that singers face when attempting to follow just intonation: tonal centre drift (see 1.2). Besides attempting to maintain a constant tonal centre, quartets may also try to have the lead follow consistent melodic intervals similar to those on the piano, and refrain from moving held notes around too much.

1.4 Adaptive Just Intonation for Barbershop Music

This thesis project will answer two separate questions. Question A concerns the solution to barbershop quartets' problem: *what adaptive tuning algorithm would a mathematically ideal barbershop quartet follow, given a score?* In this first part of the thesis, I will construct an adaptive tuning system for barbershop music. It will optimise four criteria (outlined in 2) which are important to the genre. In order to validate the algorithm, I will write a Results section in which I show how the algorithm accounts for these four criteria and where concessions need to be made in order to minimise conflicts between them.

Research question B is: *can we implement the tuning system from question A so that it models the ideal barbershop quartet?* This second part of the project should result in a C♯ program that, given a sequence of chords, can play them back justly tuned and following the genre's limitations, possibly serving as a suggestion for performers.

1.5 Academic Context and Outline

This thesis project is part of the bachelor Kunstmatige Intelligentie at Utrecht University. The Utrecht University AI programmes focus on Human-Centered Artificial Intelligence (HCAI), rather than trying to keep up with the latest innovations in machine learning algorithms. HCAI targets to understand, reproduce and possible enhance human intelligence.[9] Modelling something specifically human such as barbershop music is inherently AI research, especially when trying to build an algorithm that is inspired by human performance, but performs better. This study involves a linguistic view of musical grammar, written as a computer science project, and analysed afterwards from a philosophical perspective. All three of those sciences are important areas of HCAI.

Despite the extensive research into adaptive tuning systems, an algorithm that takes the limitations of barbershop quartets into account, such as the roles of the four different voices, has not yet been made. Adaptive tunings generally do not model human choirs. All previous work in adaptive just intonation takes a general approach, maximising dynamicity in trying to work for every performer, but they are not quite applicable to barbershop. Therefore, this thesis project adds a new dimension to the field of adaptive just intonation. At the same time, methods from Hermode Tuning and Pivotuner are important inspirations for the solution to research question A.

The following section will describe the exact methodology and further describe the algorithm's constraints. Sections 3 and 4 will answer research questions A and B, respectively. Section 5 will attempt to evaluate the algorithm by analysing results such

as global pitch drift and particularly hard-to-tune passages. Section 6 will then place this research in context of other literature about choirs, tuning and philosophy.

2   Methodology

This paper will propose an algorithm that adaptively tunes barbershop scores to just intonation. Figure 3 shows the pipeline that the algorithm should follow. The tuning system will optimise the following criteria:

1. maintain just intonation on every chord [*just intonation constraint*]

2. account for the different roles of the four parts: specifically, the lead's intervals should mostly stick to 12-tone equal temperament [*lead constraint*]

3. minimise (but not necessarily eliminate) retuning of held notes [*tie constraint*]

4. minimise (but not necessarily eliminate) tonal centre drift [*drift constraint*]

The input of the algorithm consists of `.txt` files that describe a barbershop score. The `.txt` files should be written using a newly-defined language specific to this project that contains information about note pitches, harmonies, tied notes and durations. This language is described in Appendix A. Importantly, the input splits the score into *Chords*, which in this case are periods of time where none of the notes change. A new Chord begins any time one of the four voices stops singing, starts singing or moves to a different pitch. Figure 4 visualises such a split score.

Because of this newly-defined language, a parser must be programmed to convert the input `.txt` files into the program's internal data structure. Conversion from original sheet music to the input language is, for now, a human job.

The output of the algorithm consists of a MIDI file[10] and some general statistics such as how much the song had to drift in general and which chords caused the most dramatic retunings. The MIDI files mostly contain the original notes from the score, now with extra pitch bend messages. Section 4 will describe the MIDI conversion in further detail.

3   Algorithm

This section will describe the tuning algorithm proposed by this paper, thereby answering research question A: *what adaptive tuning algorithm would a mathematically ideal barbershop quartet follow, given a score?* The algorithm returns a set of bend values that prescribe how much each note in the song should deviate from 12-tone equal temperament. The algorithm consists of a vertical step and a horizontal step. The vertical step satisfies the just intonation constraint from Section 2, whereas the horizontal step satisfies the other three constraints. The vertical step is a relatively standard procedure for just intonation, whereas the horizontal step is a completely new proposal. Figure 4 shows an outline of the full algorithm.

The algorithm distinguishes between two levels of tuning: *individual bend* and *master bend*. The individual bend describes how much the notes within a chord differ from each other and is determined in the vertical step of the algorithm. A note's individual bend is only relative to the tuning of the other three notes in the chord, where a chord only spans a period of time where not a single note changes. The master bend, on
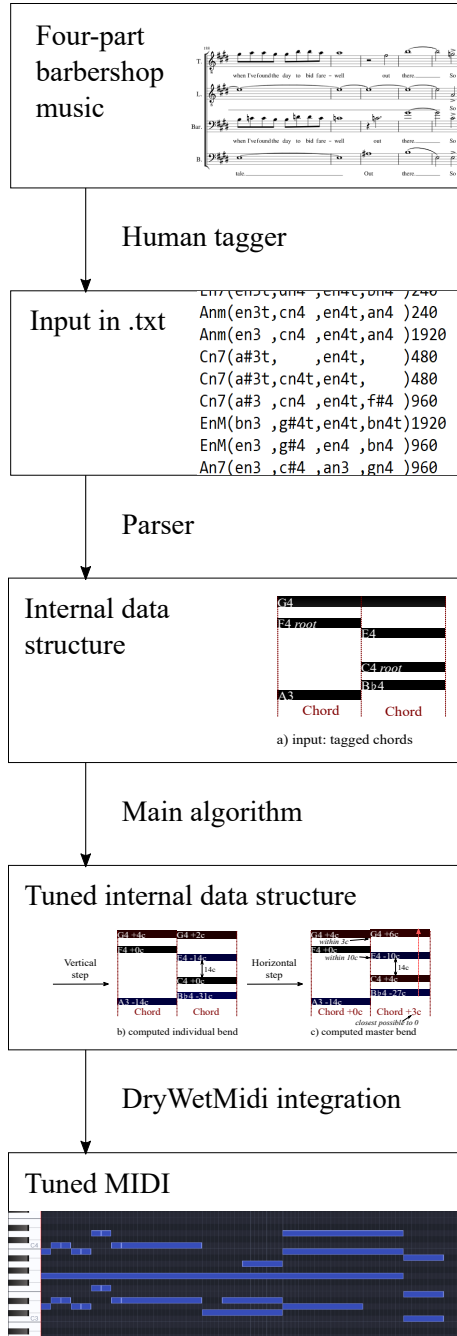
Figure 3: Pipeline for an adaptive just intonation algorithm for barbershop music. The program works on any four-part score, but requires a human tagger to translate it to a language that is specific to this algorithm. A parser can then interpret this language and build the internal data structure that the main algorithm uses to determine how each note should be tuned. At last, the library DryWetMidi is used to convert the tuned internal data structure to a MIDI file with pitch bend messages.
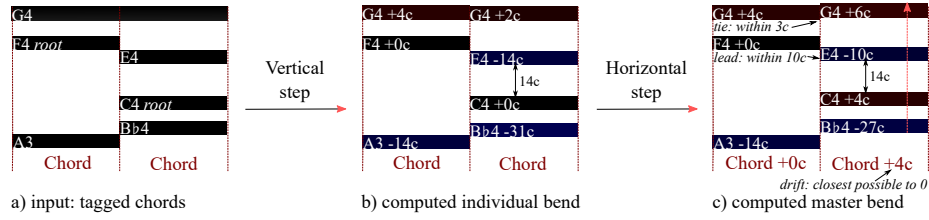
Figure 4: A basic view of the algorithm. tieRadius = 3c, leadRadius = 10c

the other hand, is used to move an entire chord's tuning up or down and is determined in the horizontal step. A note's pitch is ultimately calculated by adding its individual bend to the chord's master bend (*posterior bend*) and using the result as a degree of deviation from just intonation. Importantly, a note's individual bend never changes after the vertical step of the algorithm is done, making sure the notes of one chord always stay justly tuned with one another. Therefore, constraint 1 is always satisfied.

The vertical step (Figure 4b) determines each note's individual bend using just intonation intervals. The bend value for the chord root is always set to 0 (again, since these are relative values, that does not actually mean the root will be tuned to the regular 12-tone equal temperament value). The bend values for the other playing notes are decided using a set of interval values based on fractions relative to the chord root. This set of fractions can be given by the user, but a standard set is provided. The fractions are taken from simple ratios in the harmonic series. To illustrate: the dominant seventh chord is usually described with a $4:5:6:7$ ratio between its chord tones. Therefore, in an A dominant seventh chord (A-C♯-E-G), the G's frequency will be tuned to be $\frac{7}{4}$ times the A's frequency. On the other hand, a half-diminished chord is usually described with a $5:6:7:9$ ratio, so in an A half-dimished chord (A-C-E♭-G), the G will be equal to $\frac{9}{5}$ times the A.

The horizontal step (Figure 4c) moves all chord notes around evenly to satisfy constraints 2-4 (lead, tie and drift; see Section 2). Specifically, it outputs a master bend value for the current chord by comparing it to the previous chord. For the lead and tie constraints, a collection of lead notes and tied notes from the previous chord is made in order to calculate how much the current master bend can move around while satisfying those constraints. Each of those notes has a desired space that the new note should move into. To illustrate: in the example, *tieRadius* is set to 3 and the previous G4 is tied over, so the current G4 should be within 3 cents of the old G4. Meanwhile, *leadRadius* is set to 10 and the previous F4 has a posterior bend of 0 cents, so the current E4 should deviate less than 10 cents from equal temperament. After the collection of lead and tied notes has been completely run through, the algorithm chooses a value for the master bend that is as close to 0 as possible. This last step helps the drift constraint, because a master bend value of 0 means that there is no tonal centre drift.

Ultimately, the algorithm's results depend on a set of parameters given by the user. The first parameter is a set of lists of fractions that should be used for the vertical step. This first parameter is important to note, because opinions vary as to which exact tunings are the best for each interval. The other three parameters are referred to as *priority*, *tieRadius* and *leadRadius*. *priority* is either "lead" or "tie" and determines whether the lead or tie constraint should be satisfied first. *tieRadius*, given in a unit similar to cents, determines how much a tied note is allowed to be retuned compared to its predecessor. *leadRadius* is similar: it determines by how many cents an interval in

```
 1: procedure SETMASTERBEND(MB_{i-1}, Notes_{i-1}, Notes_i, tieRange, leadRange)
 2:     ties ← [all notes in Notes_{i-1} that have the tie property, ordered lead-bass-tenor-
        baritone]
 3:     ranges ← [for all tied notes, the range MB_i could move into in order to satisfy
        its parameter]
 4:     ranges.Add(the range in which MB_i could move according to leadDiff)
 5:     if 0 is within ⋂ ranges then
 6:         return MB_i ← 0
 7:     else
 8:         currRange ← ranges[0]
 9:         for all range r in ranges[1 :] do
10:             currRange ← r ⋂ currRange
11:             if no intersection then
12:                 return MB_i ← the closest number to r within currRange
13:         return MB_i ← the closest number to 0 within currRange
```

Figure 5: Pseudocode for the horizontal step. *Hier komt nog meer uitleg, hoort bij het implementatie-hoofdstuk*

the lead voice is allowed to deviate from an interval in equal temperament, relatively.

4 C♯ Implementation

Now that Section 3 has described the just intonation algorithm for barbershop music, we can look at how the algorithm was implemented into the programming language C♯. This section therefore answers research question B: *can we implement the tuning system from question A so that it models the ideal barbershop quartet?*

Dit ga ik nog schrijven, maar is niet het interessantste deel om te lezen

5 Results

This section will show the proposed algorithm's effect on an example score, namely the tag of *Ring-a-Ding-Ding*, arranged by Anthony Bartholomew. Several good recordings of this song can be found on YouTube. Figure 6 shows the sheet music for this tag. Since both the bass and tenor voice hold notes for a long time while the lead and baritone sing phrases inbetween, *Ring-a-Ding-Ding* is a good example to show how the proposed algorithm makes choices in ambiguous scenarios.

Figure 7 shows the effects that the three parameters (tieRadius, leadRadius and priority, see)

6 Discussion

6.1 Results evaluation

Iets met dat dit best oké is, logisch dat er zo'n sprong in de blauwe lijnen zit want dat is waar het algoritme het oké gaat vinden dat tied notes volledig herstemmen. Iets van discussie over dat drift steeds lager wordt als je minder streng bent, maar dat betekent
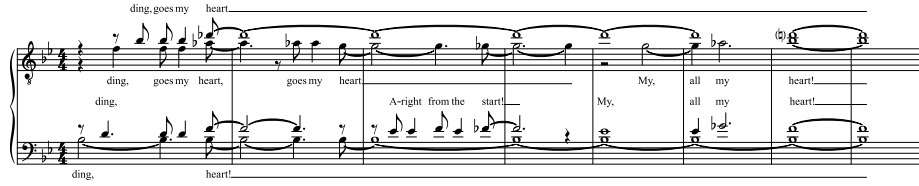
Figure 6: Sheet music for the tag of Ring-a-Ding-Ding, a barbershop arrangement by Anthony Bartholomew.
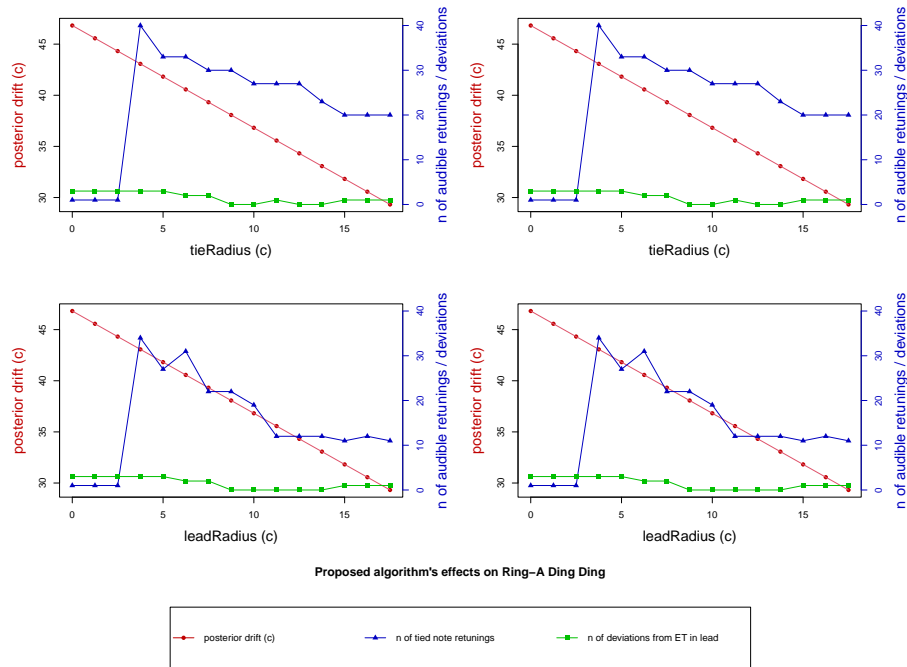

Figure 7: Results from the proposed algorithm on the tag of Ring-a-Ding-Ding (sheet music in Figure 6). The graphs show the effects of parameter changes on three measures: overall tonal centre drift in cents (red), number of tied note retunings over 3 cents (blue) and number of deviations from equal temperament in the lead voice larger than 10 cents (green). The tieRadius and leadRadius parameters determine how loosely the algorithm allows the most important held notes to retune (tie constraint) and how loosely it allows the lead voice's intervals to deviate from equal temperament. The left two graphs have the *priority* set to "tie", whereas the right two graphs have it set to "lead". Both parameters appear to have a similar effect on how many notes need to retune, how many lead intervals are outside equal temperament and how much the song drifts overall. *Tjaaaa, dit kan mooier en beschrijvender. Ik wil in ieder geval nog iets doen met het feit dat blauw en groen bij elkaar horen, suggesties zijn zeer welkom*

6.2   In musical contexts

The use of a model such as the one proposed is debatable. For human performers, this algorithm would be impossible to realise; the exact calculations required would be too complicated to carry out while singing and the tiniest intonation changes cannot be replicated using the human voice. Also, the algorithm's goal is not to "capture theories of cognitive functioning in mathematical or computational form", which means it is not exactly a cognitive model.[11] However, the algorithm's results could serve as a reference for real singers to hear what a potentially ideal method of tuning could be. Additionally, high-level findings from the algorithm such as described in section 5 could give singers a frame of reference as to how they should generally tune their notes in a barbershop context.

Even the concept of just intonation itself is a subject of debate: are the intervals from the harmonic series really the best way to tune? Hagerman & Sundberg analysed recordings of barbershop quartets and found that the tenor, baritone and bass use the lead as a reference to adjust their pitches, but their intervals often deviate from just intonation.[12] Thanks to new technologies in music information retrieval, intonation and drift in choirs have been analysed in recent years.[13, 14, 15] Results from these analyses have even led some researchers to conclude that just intonation as a whole is not the ideal tuning method.[16] However, not enough research has been done to completely isolate these findings from twelve-tone bias.

At the same time, it is important to note that twelve-tone bias does play a role in this article's proposed tuning system. Barbershop music is composed with the standard twelve-tone tuning system in mind and this algorithm does tend to return to them as a result of the lead and drift constraints. A tuning method that does not adhere to the twelve-tone system and is therefore more culturally independent, was proposed by Sethares. Instead of using fractions from the harmonic series, he proposes to analyse a note's timbre using its *dissonance curve*.[2] However, since this article focuses specifically on barbershop, dissonance curves will always adhere to the natural harmonic series and using the twelve-tone system is commonplace. For these reasons, it was decided not to use Sethares' proposal in the current research.

6.3   Philosophical Framework

6.4   Future Work

The input for this algorithm was a newly-defined language, described in Appendix A. Translating a score into this language has turned out to be a painstaking process and could be optimised better. One more commom way to describe music in plain text is abc notation, which also allows chord annotation.[17] However, since automatic chord recognition has improved in the past couple of decades[18], an implementation of this algorithm could easily be built that simply takes MIDI as input. Essentially, the only thing that the tuning system has to add is pitch bend messages.

The scope of this paper did not allow for a comparison to real-world barbershop quartets. In recent years, systems such as AMPACT can be used to evaluate the tuning tendencies of real choirs.[13] Analysing the difference between mathematically ideal algorithms such as the one proposed and the tendencies of professional barbershop quartets could lead to an even better tuning system. Eventually, a tuning system might be developed that all barbershop musicians can refer to when singing their genre.

Nog niet alle referenties werken goed, vooral die van webpagina's, Jos help!!

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Appendices

A Input language

Ook dit ga ik nog schrijven en ook dit is niet zo interessant

A.1 Songs

A.2 Fractions