

Abstract

We present an example of how to solve a physics problem. Involving a block sliding on an inclined plane, a pulley with rotational inertia, and a second block serving as a counterweight, the problem is moderately complex. No calculus is required. The example shows the approach in which no numerical value for any parameter is assumed in the solution. The solution is obtained for the general case. The specific, limiting cases are explored. Finally, a summary of the parameter space is presented in a collection of graphs. This approach allows one to grasp all of the modes of behavior that various individual systems of the same type can display and how the different behaviors relate to the parameters defining the type.

1 Introduction

Many a physics problem aimed at the high-school student is stated so that every element of the problem has a specified numerical value. There are good reasons to state a problem with numerical specificity and to ask for a numerical solution. Numerical specificity allows for the easy multiplication of different example problems of the same general type. This is convenient for the teacher. Further, the student should be able not only to demonstrate conceptual mastery of the subject but also to calculate an accurate answer in a specific case. Numerical practice is good for the student. Perhaps, for practical reasons, the majority of problems should be stated with numerical specificity.

In any event, the ultimate purpose of giving a physics problem to a student is to allow him to demonstrate conceptual mastery of the subject. However useful numerical calculation may be as a skill, it can be learned and demonstrated elsewhere, entirely apart from a physics curriculum. The conceptual mastery necessary for the demonstration of ability in physics, though, is inherently abstracted away from the numerical particulars. The teacher of physics ought therefore to emphasize a conceptual method of solution, even for a problem that specifies numerical particulars. That is, the teacher might well require that the student solve every problem in a perfectly general way and then, perhaps as a final step, make numerical substitutions and calculations. At least on occasion, the teacher should ask the student to reflect on the general solution in order to answer various conceptual questions, such as whether the dimensions make sense, whether there might be, for a given quantity, a critical value beyond which the behavior of the system changes qualitatively, etc.

The problem considered below has no numerical specificity, but the behavior of the physical system is, at the end of the paper, analyzed for a wide range of numerical specifics.

2 Problem

The flat bottom of a block B1 of mass m lies on a plane inclined at an angle θ to the horizontal. See Figure 1. The coefficient of kinetic friction between the plane and the block is μ . A massless, taut, inelastic

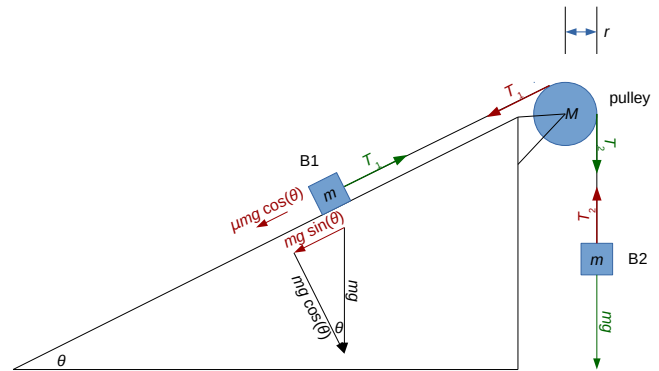


Figure 1: Diagram of system. Some of the system parameters (θ , m , and M) are indicated. The coefficient μ of friction corresponds to the surface on which block B1 sits. Although the radius r of the pulley's disc is indicated, r is eliminated from all of the relevant expressions and is therefore not a key parameter of the system's behavior.

cable runs from B1 toward the top of the plane, around a pulley, and straight down to another block B2, which also has mass m . The pulley's disc has infinite static friction against the cable but requires no energy to separate from the cable as it moves. The pulley's bearing is frictionless. The disc of the pulley has uniform density, total mass M , and radius r . Find the acceleration a of the blocks; express a in terms of μ , $\frac{M}{m}$, θ , and the acceleration g of gravity.

3 Solution

Each of B1, the pulley, and B2 must undergo the same acceleration because they move as a rigid system: B1 and B2 maintain a constant distance of separation along the length of the cable, and the cable does not slip along the surface of the pulley. The net force on each of B1, the pulley, and B2, when divided by the relevant mass, must yield the same acceleration.

3.1 Forces at B1

Suppose that the tension in the cable attached to B1 is T_1 . There are three forces, parallel to the plane, acting on B1:

1. toward the pulley, a tensile force of magnitude T_1 ,
2. away from the pulley, the frictional force $\mu mg \cos(\theta)$ due to the plane-perpendicular component of B1's weight, and
3. away from the pulley, the plane-parallel component $mg \sin(\theta)$ of B1's weight.

The acceleration, as derived from forces acting on B1, is

$$\begin{aligned} a &= \frac{T_1 - \mu mg \cos(\theta) - mg \sin(\theta)}{m} \\ a &= \frac{T_1}{m} - [\mu \cos(\theta) + \sin(\theta)]g. \end{aligned} \quad (1)$$

3.2 Forces at the Pulley

Suppose that the tension in the cable attached to B2 is T_2 . There are two forces, along the direction of the cable, acting on the pulley:

1. toward B1, a tensile force of magnitude T_1 and
2. toward B2, a tensile force of magnitude T_2 .

There are two corresponding torques:

1. a torque $\tau_1 = rT_1$ tending to spin the pulley so that B1 accelerates down the plane and
2. a torque $\tau_2 = rT_2$ tending to spin the pulley so that B1 accelerates up the plane.

The angular acceleration, derived from torques acting on the pulley, is

$$\alpha = \frac{\tau_2 - \tau_1}{I} = \frac{2[T_2 - T_1]}{Mr}, \quad (2)$$

where $I = \frac{1}{2}Mr^2$ is the rotational inertia of the pulley's disc. The acceleration, as derived from forces acting on the pulley, is

$$a = r\alpha = \frac{2}{M}[T_2 - T_1]. \quad (3)$$

3.3 Forces at B2

There are two forces acting on B2:

1. toward the pulley, a tensile force of magnitude T_2 and
2. away from the pulley, B2's weight mg .

The acceleration, as derived from forces acting on B2, is

$$a = \frac{mg - T_2}{m} = g - \frac{T_2}{m}. \quad (4)$$

3.4 Tension Acting on B2

We can, by combining Equation 3 and Equation 4, solve for the tension T_2 acting on B2.

$$\begin{aligned} \frac{2}{M}[T_2 - T_1] &= g - \frac{T_2}{m} \\ 2[T_2 - T_1] &= \frac{M}{m}[mg - T_2] \\ [2 + \frac{M}{m}]T_2 &= [\frac{M}{m}]mg + 2T_1 \\ T_2 &= \left[\frac{\frac{M}{m}}{2 + \frac{M}{m}} \right] mg + \left[\frac{2}{2 + \frac{M}{m}} \right] T_1 \end{aligned} \quad (5)$$

Here we have expressed the result in terms of the ratio $\frac{M}{m}$, which, along with μ and θ , is one of the three fundamental parameters that determine the solution.

3.5 Tension Acting on B1

Now, combining Equation 1, Equation 4, and Equation 5, we can solve for the tension T_1 acting on B1.

$$\begin{aligned} g - \frac{T_2}{m} &= \frac{T_1}{m} - [\mu \cos(\theta) + \sin(\theta)]g \\ \frac{T_1 + T_2}{mg} &= 1 + \mu \cos(\theta) + \sin(\theta) \\ \frac{T_1}{mg} \left[\frac{4 + \frac{M}{m}}{2 + \frac{M}{m}} \right] &= \frac{2}{2 + \frac{M}{m}} + \mu \cos(\theta) + \sin(\theta) \\ T_1 &= \left[\frac{2 + [\mu \cos(\theta) + \sin(\theta)] [2 + \frac{M}{m}]}{4 + \frac{M}{m}} \right] mg \end{aligned} \quad (6)$$

3.6 Acceleration of Blocks

Finally, combining Equation 6 and Equation 1, we can solve for the acceleration.

$$\begin{aligned} \frac{a}{g} &= \frac{2 + [\mu \cos(\theta) + \sin(\theta)] [2 + \frac{M}{m}]}{4 + \frac{M}{m}} - [\mu \cos(\theta) + \sin(\theta)] \\ \frac{a}{g} &= \left[\frac{2}{4 + \frac{M}{m}} \right] [1 - \mu \cos(\theta) - \sin(\theta)] \end{aligned} \quad (7)$$

Again, we see that the acceleration depends on three parameters: $\frac{M}{m}$, μ , and θ . The form of the solution shows that the acceleration can range from a minimum of zero to a maximum of $\frac{g}{2}$.

3.6.1 Minimum Acceleration

The minimum acceleration ($a = 0$) obtains when either of the following is true:

- $\frac{M}{m}$ approaches infinity, or
- $\mu \cos(\theta) + \sin(\theta) = 1$.

Considering the second case, we see that, for a given θ , when μ is larger than the critical value $\mu_{\text{crit}} = \sec(\theta) - \tan(\theta)$, the kinetic friction is large enough to decelerate the system. In such a case, the deceleration would stop when the system comes to rest, for at that moment the kinetic frictional force would cease to exist. Figure 2 shows the how μ_{crit} varies with θ . For small θ , μ can approach unity, and the system will still accelerate. For values of θ approaching 90 degrees, however, only a small μ allows the system to accelerate.

3.6.2 Maximum Acceleration

The maximum ($a = \frac{g}{2}$) obtains when all of the following are true at the same time:

- $\frac{M}{m}$ approaches zero;
- $\mu = 0$; and
- $\theta = 0$.

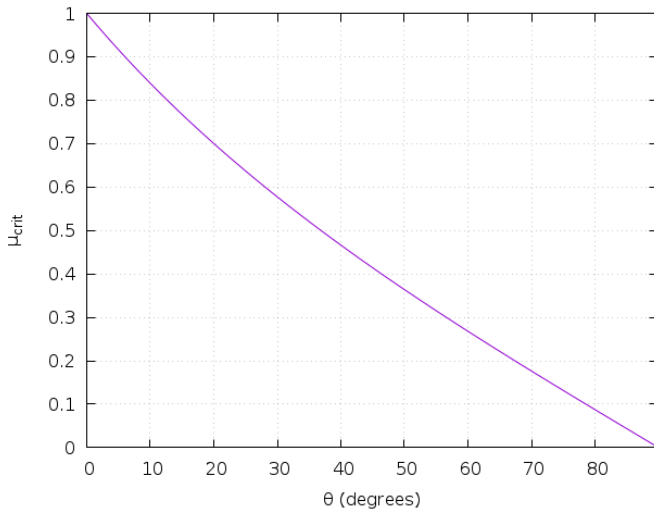
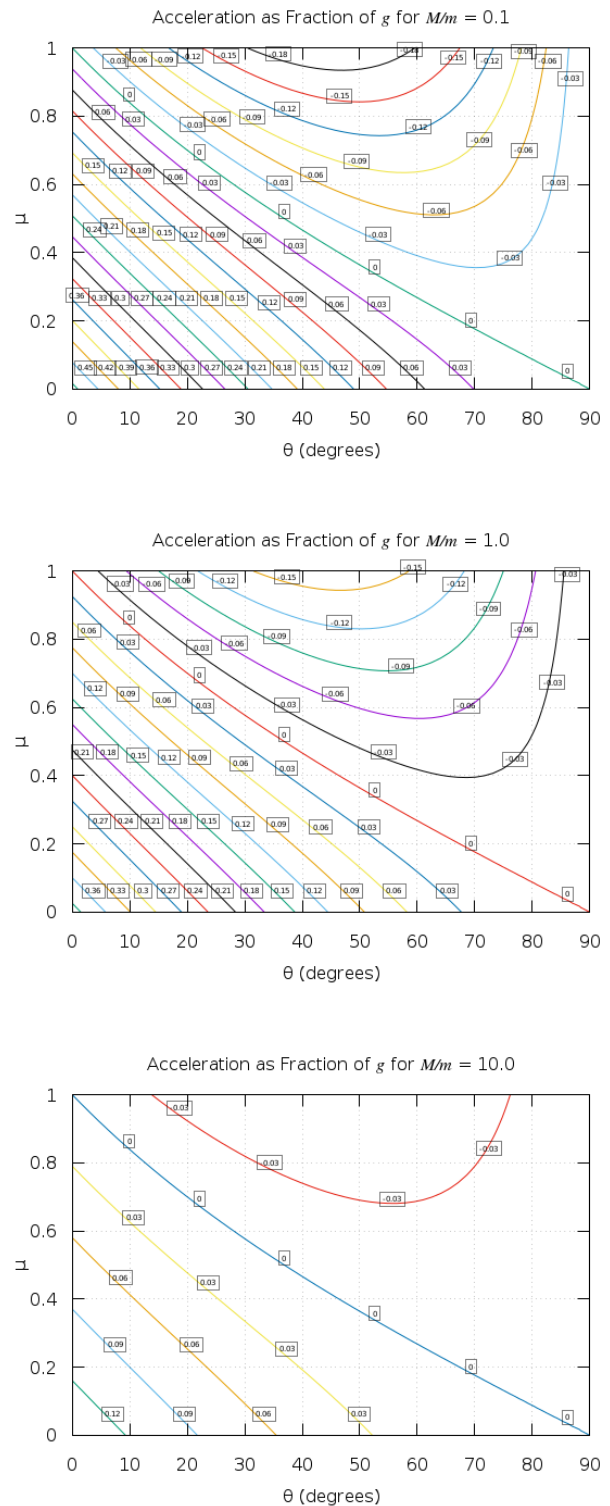


Figure 2: Plotted against the angle of inclination of the plane, the limiting value μ_{crit} of the coefficient of kinetic friction. For any coefficient above this limit, a moving system decelerates, and a stationary system remains stationary.

3.6.3 Summary

Figure 3 summarizes the result. Notice that the curve for zero acceleration in each of the three plots is the same as the curve in Figure 2. Each curve for negative acceleration applies only to a system that is initially moving and indicates frictional deceleration.



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