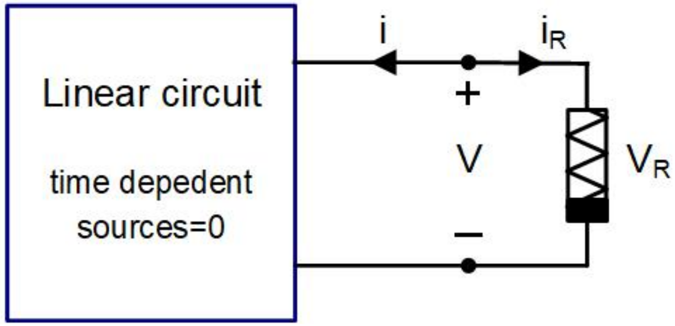
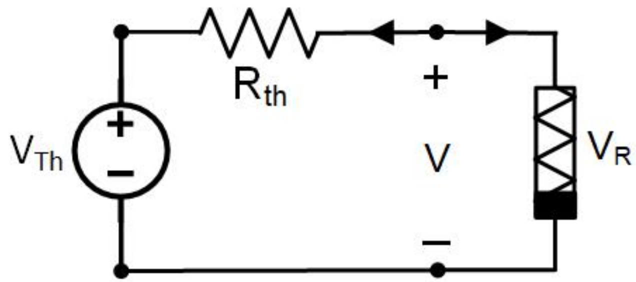
Nonlinear Circuits

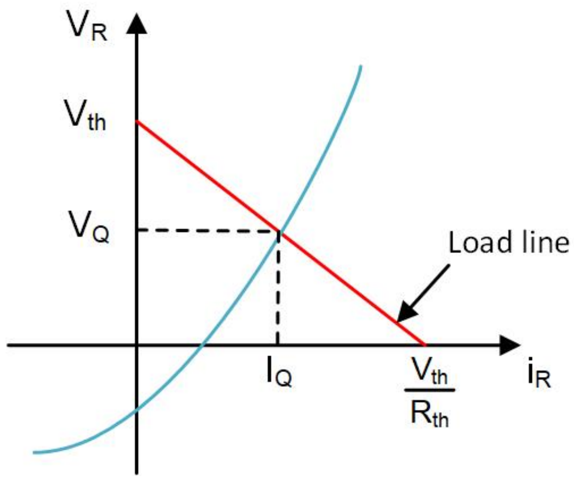
Operating point The solution (I_Q, V_Q) found when time dependent sources are suppressed in the circuit.



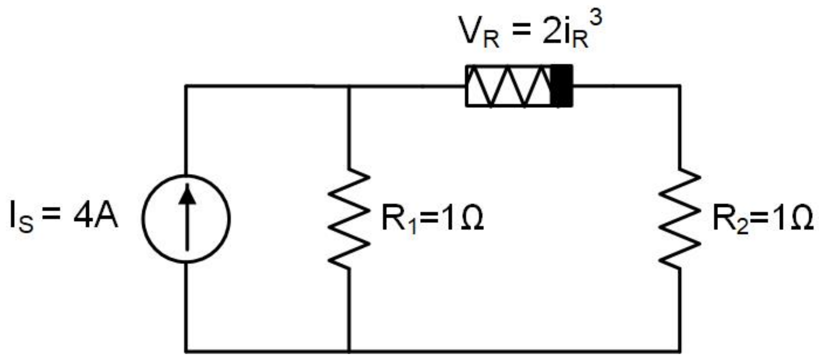


$V_R = f(i_R)$ $\qquad V = V_R$ $\qquad i + i_R = 0$

$V_{th} - R_{th}i - V_R = 0 \rightarrow V_R = -R_{th}i_R + V_{th}$



Example 8.1

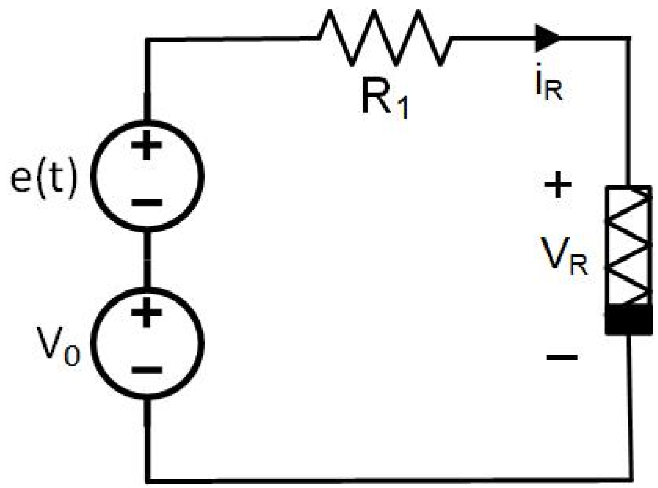


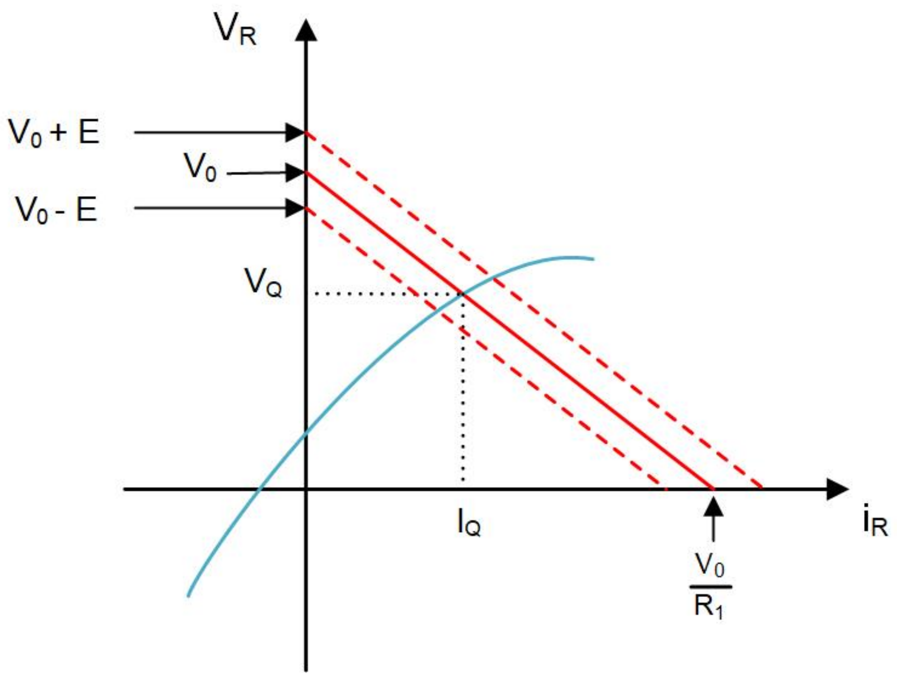
Small signal analysis

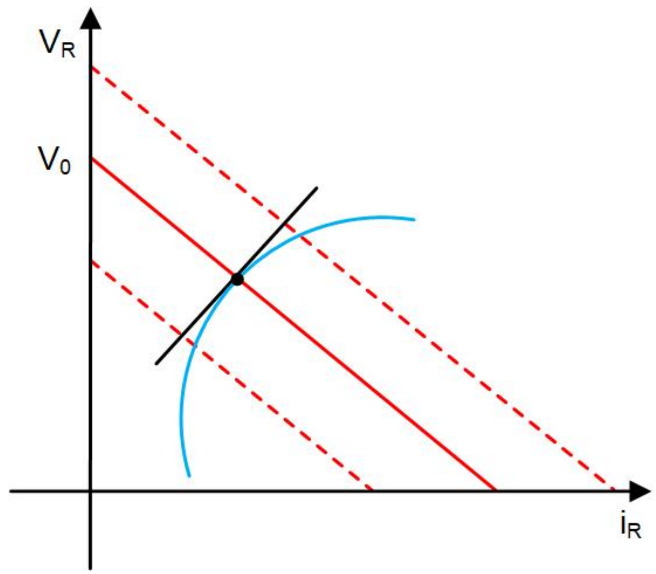
$V_S = V_0 + e(t)$

$e(t) = E\cos(\omega t)$

$|E| << |V_0|$ for all the time







1. Time dependent sources are suppressed. Operating point (I_Q, V_Q) is found.
2. Nonlinear elements are linearized around their operating points. Small signal equivalents are found.

$$V_R(t) - V_{R_Q} = f'(I_{R_Q})\left(i_R(t) - I_{R_Q}\right)$$

$$\left.\begin{aligned}\widetilde{i_R}(t) &= i_R(t) - I_{R_Q}\\ \widetilde{V_R}(t) &= V_R(t) - V_{R_Q}\end{aligned}\right\}\text{Small signal components}$$

$$\widetilde{V_R}(t) = f'(I_{R_Q})\widetilde{i_R}(t)$$

$f'(I_{R_Q})$: small signal equivalent of the nonlinear element at the operating point Q $\quad (f'(I_{R_Q}) = R_Q)$

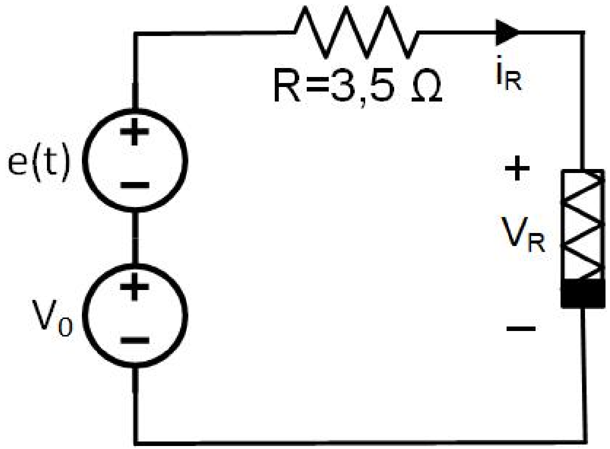
3. Time independent sources are suppressed. Small signal components ($\widetilde{V_R}(t)$, $\widetilde{i_R}(t)$) are found.

4. $V_R(t) \cong V_Q + \widetilde{V_R}(t) \qquad i_R(t) \cong I_Q + \widetilde{i_R}(t)$

Example 8.2

$e(t) = 0,5 \sin(1000t)$

$V_R = i_R^3 - 6i_R^2 + 9i_R$



Small signal model of nonlinear multi-terminal elements

n+1 terminal element → **y, x**

$$\begin{aligned} y_1 &= f_1(x_1, x_2, \dots, x_n) \\ y_2 &= f_2(x_1, x_2, \dots, x_n) \\ &\vdots \\ y_n &= f_n(x_1, x_2, \dots, x_n) \end{aligned}$$

Small signal model around (x_Q, y_Q)

$$y_1 - y_{1Q} = \frac{\partial f_1}{\partial x_1}\left(x_1 - x_{1Q}\right) + \frac{\partial f_1}{\partial x_2}\left(x_2 - x_{2Q}\right) + \cdots + \frac{\partial f_1}{\partial x_n}\left(x_n - x_{nQ}\right)$$

$$y_2 - y_{2Q} = \frac{\partial f_2}{\partial x_1}\left(x_1 - x_{1Q}\right) + \frac{\partial f_2}{\partial x_2}\left(x_2 - x_{2Q}\right) + \cdots + \frac{\partial f_2}{\partial x_n}\left(x_n - x_{nQ}\right)$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$y_n - y_{nQ} = \frac{\partial f_n}{\partial x_1}\left(x_1 - x_{1Q}\right) + \frac{\partial f_n}{\partial x_2}\left(x_2 - x_{2Q}\right) + \cdots + \frac{\partial f_n}{\partial x_n}\left(x_n - x_{nQ}\right)$$

$$\frac{\partial f}{\partial x} = J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$$J_{ij} = \frac{\partial f_i}{\partial x_j}$$

$$\underline{y} = \underbrace{J\Big|_Q}_{\text{Jacobian}} \underline{x}$$

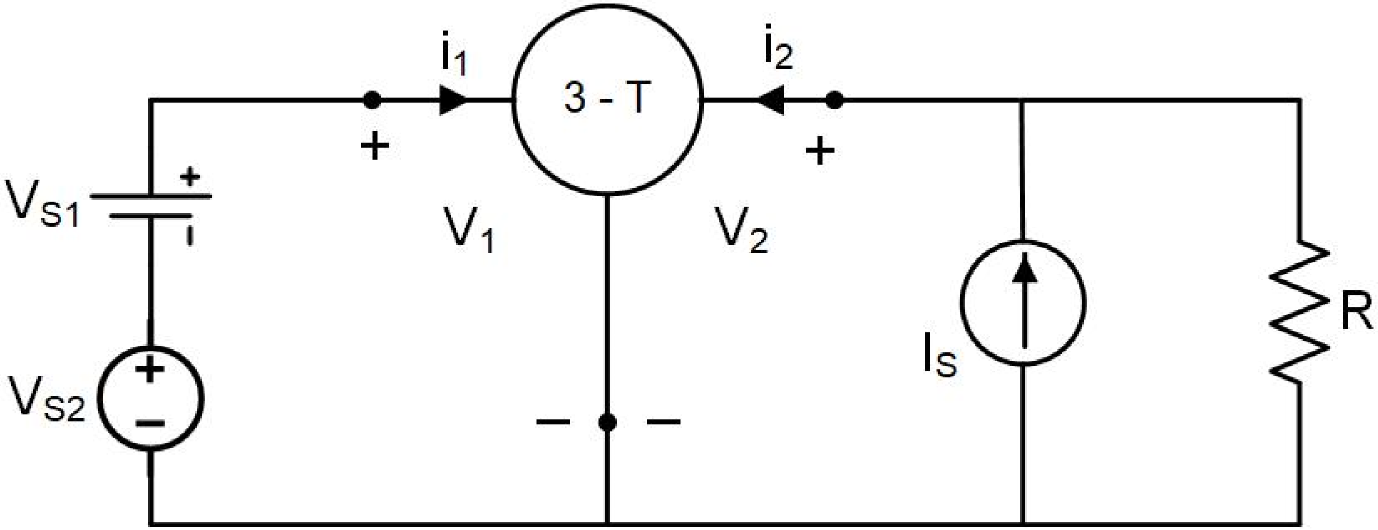
Example 8.3

$R = 1\Omega$

$V_{S1} = 2V$

$V_{S2} = 0{,}2\sin(10t)$

$I_S = 20A$



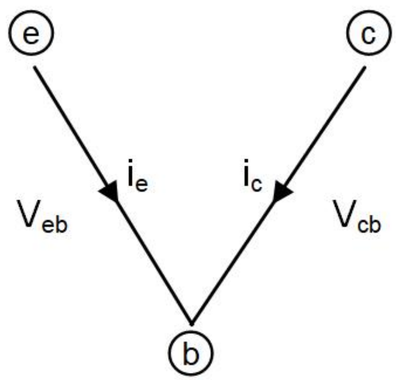
Equations of the 3 – T element: $V_1 = 2i_1$

$i_2 = v_2 + 8v_2 i_1$

Small signal components of the Ebers Moll Model of transistors

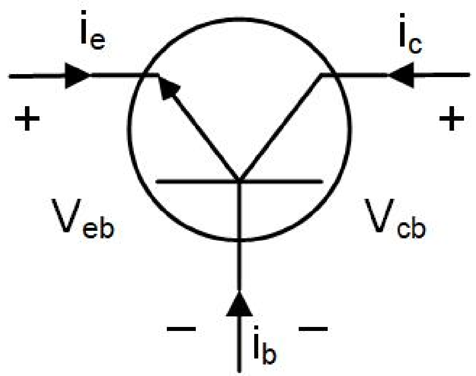
$$i_e = -I_{ES}\left(e^{-\frac{V_{eb}}{V_T}} - 1\right) + \alpha_R I_{CS}\left(e^{-\frac{V_{cb}}{V_T}} - 1\right)$$

$$i_c = \alpha_F I_{ES}\left(e^{-\frac{V_{eb}}{V_T}} - 1\right) - I_{CS}\left(e^{-\frac{V_{cb}}{V_T}} - 1\right)$$



$$i_e = f_1(V_{eb}, V_{cb})$$

$$i_c = f_2(V_{eb}, V_{cb})$$



$$i_e - i_{eQ} = \left.\frac{\partial f_1}{\partial V_{eb}}\right|_Q \left(V_{eb} - V_{ebQ}\right) + \left.\frac{\partial f_1}{\partial V_{cb}}\right|_Q \left(V_{cb} - V_{cbQ}\right)$$

$$i_c - i_{cQ} = \left.\frac{\partial f_2}{\partial V_{eb}}\right|_Q \left(V_{eb} - V_{ebQ}\right) + \left.\frac{\partial f_2}{\partial V_{cb}}\right|_Q \left(V_{cb} - V_{cbQ}\right)$$

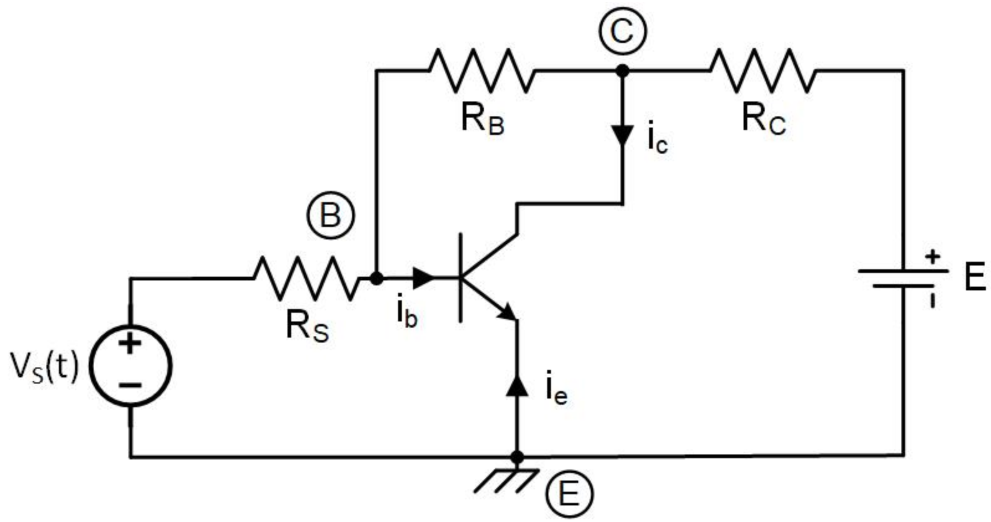
$$\begin{bmatrix} \widetilde{i_e} \\ \widetilde{i_c} \end{bmatrix} = \begin{bmatrix} \frac{I_{ES}}{V_T} e^{-\frac{V_{ebQ}}{V_T}} & -\frac{\alpha_R I_{CS}}{V_T} e^{-\frac{V_{cbQ}}{V_T}} \\ -\frac{\alpha_F I_{ES}}{V_T} e^{-\frac{V_{ebQ}}{V_T}} & \frac{I_{CS}}{V_T} e^{-\frac{V_{cbQ}}{V_T}} \end{bmatrix} \begin{bmatrix} \widetilde{V_{eb}} \\ \widetilde{V_{cb}} \end{bmatrix}$$

Example 8.4

$\alpha_F = 0{,}99$ $\alpha_R = 0{,}6$

$I_{CS} = 12\,pA$ $I_{ES} = 7\,pA$

$R_S = 600\Omega$ $E = 15V$



a) Find the values of R_B and R_C such that $V_{beQ} = 0{,}6V$ and $V_{cbQ} = 5V$.

b) Find the small signal model of the transistor around the operating point. Is the transistor locally active ?

c) Find $V_{CE}(t) \cong V_{CEQ} + \tilde{V}_{CE}(t)$