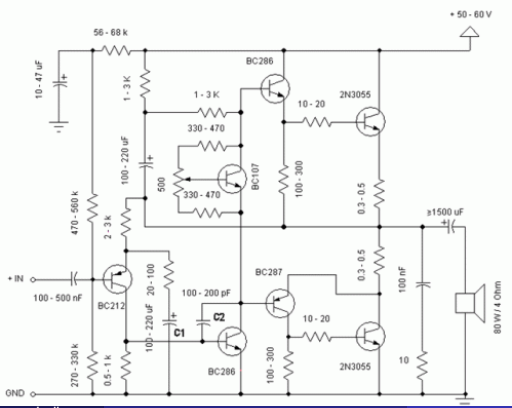
Introduction: Physical Circuit

Physical Circuit

Any interconnection of (physical) electric device.

Electric devices are resistors, diodes, transistors , ...



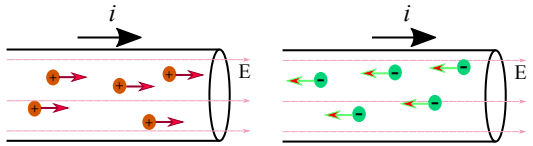
The Fundamental Variables

The variables are used to describe the behavior of a circuit.

- Charge (MKS units of charge is the Coulomb) contained on 6.24 10^{18} electrons.
- Current is simply a measure of the net amount of positive charge that passes a plane in space per second in a reference direction (analogous to measuring river).

Symbol : i
Unit: Ampere (A)= 1 Coulomb / second

$$i = \frac{dq}{dt}$$



The Fundamental Variables

- Voltage is the ratio of the energy required to move charge between two points.

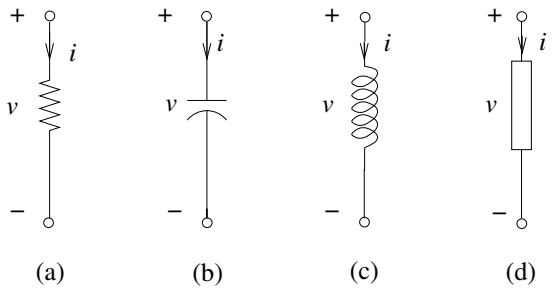
Symbol : v ; Unit: Volt (V)= 1 Joule / Coulomb

$$v = \frac{dw}{dq}$$

In a uniform electrical field pulling electrons to the left at 9.8 Newton/Coulomb. How much energy would it take to move this charge 2 meters to the right?
$9.8 Newton/Coulomb \times 2 = 19.6 Joule/Coulomb = 19.6 Volts$
There is a gravitational field pulling some matter downward at 9.8 Newton/kg. How much energy/kg would it take to move this matter meters upwards ? $9.8 Newton/kg \times 2 = 19.6 Joule/kg$

Electric Circuit and Circuit Elements

Two-terminal element



Typical examples of two-terminal element are resistor,inductor, capacitor, diode, voltage and current sources.

The instantaneous branch voltage (v) across a two-terminal and the instantaneous branch current (i) flows through a two-terminal.

Physical Circuit

The goal of circuit theory is to predict the electrical behavior of physical circuits.

Circuit theory focuses on the electrical behavior of circuits (... thermal, mechanical, chemical effects...).

An Electrical Circuit might be

Distributed circuit

A distributed circuit is one in which all dependent variables are functions of time and one or more spatial variables.

or

Lumped circuit

A lumped circuit is one in which the dependent variables of interest are a function of time alone.

Physical Circuit

Lumped circuit

Let l be the largest dimension of the circuit, λ the shortest wavelength of interest. If

$$\lambda >> l$$

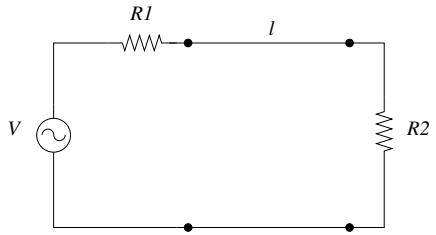
then the circuit may be considered to be lumped.

While Lumped circuit is analyzed by solving a set of ordinary differential equations (ODEs), Distributed circuit is analyzed by solving partial differential equations.

Typical examples of distributed circuits are circuits made of waveguides and transmission lines.

In this course we shall consider only lumped circuit.

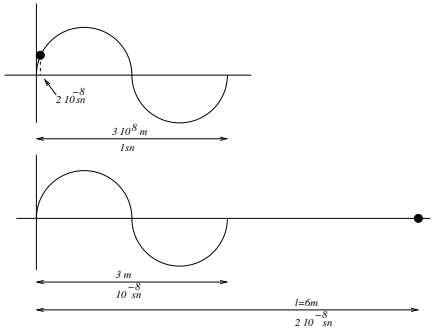
Physical Circuit



How long does it take to reach end of line ?

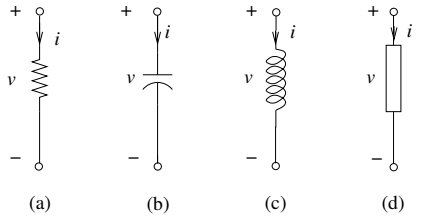
Physical Circuit

Electromagnetic waves travel at the velocity of light $c = 3\,10^8$ meters per second to travel l the time elapsed is $20ns$.



$f = 1Hz \rightarrow \frac{3\,10^8}{1} >> 6$ then the circuit may be considered to be lumped.
$f = 100MHz \rightarrow \frac{3\,10^8}{100\ 10^6} < 6$ then the circuit may be considered to be disturbuted.

Electric Circuit and Circuit Elements



The term $v(t)$ (or v) represent the instantaneous branch voltage, $i(t)$ the instantaneous branch current of the element. The voltage reference plus and mines symbol and the current reference arrow symbol. These symbols do not necessarily represent the actual direction of positive voltage drop or positive current flow.

The associated reference direction

The direction of positive current flow coincides with the direction of positive voltage drop.

The associated reference direction

The associated reference direction (passives sign convention)

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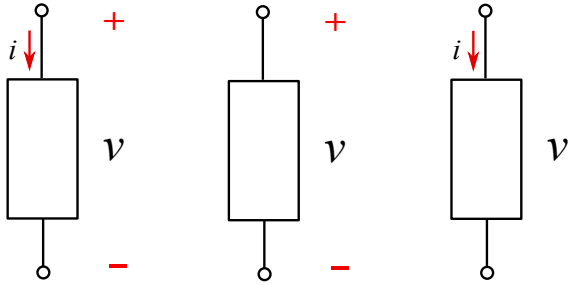

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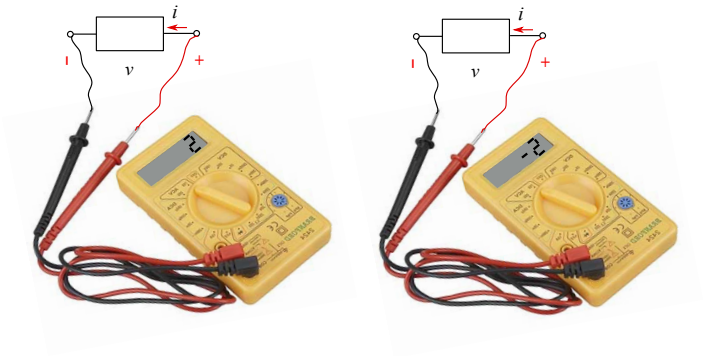
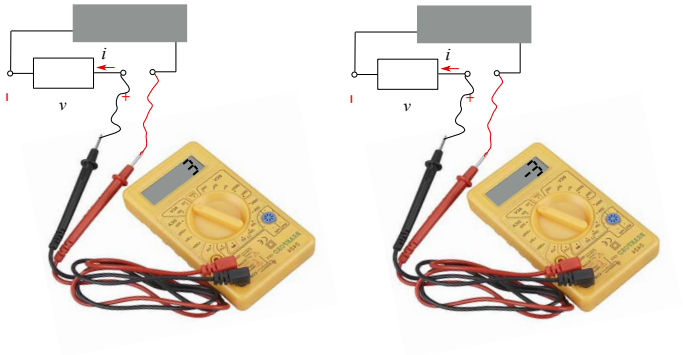


A voltmeter, also known as a voltage meter, is an instrument used for measuring the potential difference. A voltmeter is placed in parallel with a circuit element to measure the voltage drop across it.



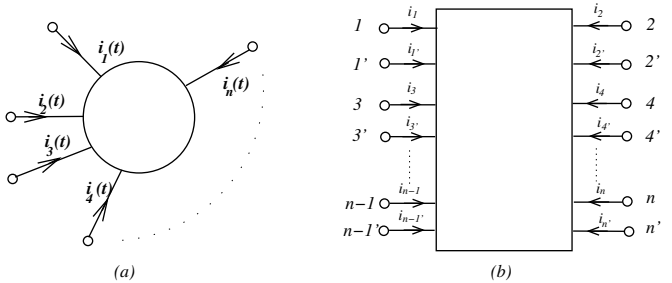
An ammeter is a device that is used to measure the amount of electric current that runs through a circuit. It is measured in the unit of the Ampere, simply called "Amp," (A). The most common way to measure current in a circuit is to break the circuit open and insert an "ammeter" in series (in-line) with the circuit.
Read: Nilsson Riedel, Section 3.2, page 68

The associated reference direction

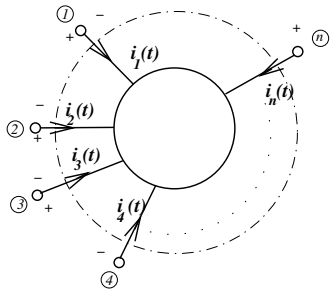
The instantaneous voltage and current have the same sign and power ($P = v\ i$) is being instantaneously delivered to the element.
When the instantaneous voltage and current have opposite sign, the element is the instantaneously delivering power to the remainder of the circuit.
A two-terminal circuit element is represented in the term of its associated reference directions just by an oriented branch. the direction of the arrow indicating both the voltage drop and the current flow reference polarities.

Electric Circuit and Circuit Elements

n-terminal element



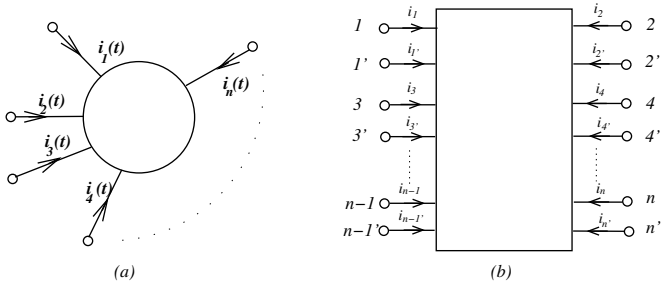
we assign arbitrarily a reference direction to each current variable by an arrow, and a reference polarity to each voltage variable by a pair of plus (+) and minus (-) sign.



If $i_1 = 3A$ it means that a current of 3A flows into the n-terminal element by node 1. If $i_1 = -0.3A$ it means that a current of 3A flows out of the n-terminal element by node 1.
If $V_1 = 3V$ it means that the electrical potential of terminal 1 is 3V larger than the electrical potential of terminal 2. If $V_1 = -3V$ it means that the electrical potential of terminal 1 is 3V smaller than the electrical potential of terminal 2.

Electric Circuit and Circuit Elements

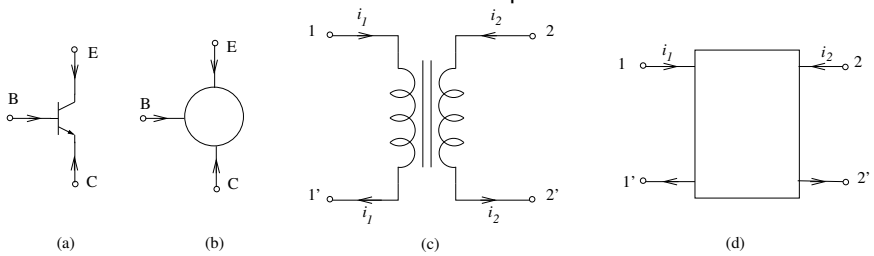
n-port element



port currents $i_1 = -i_1^{'}$,... and port voltages $V_{1,1'}$,...

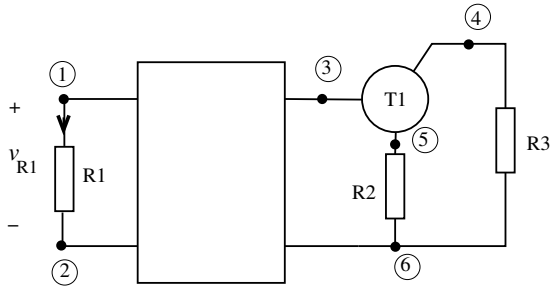
Electric Circuit and Circuit Elements

3-terminal element and 2-port element



Electric Circuit and Circuit Elements

We use conducting wires to tie the terminals together as shown in Figure...



A node is any junction in a circuit where terminals are joined together or any isolated terminal of a circuit element, which is not connected.

Modelling Circuit Element

A mathematical model can be develop for each circuit element. The mathematical model is obtained after performing certain tests on the element.

Table: Electrical measurements for 2-terminal circuit element

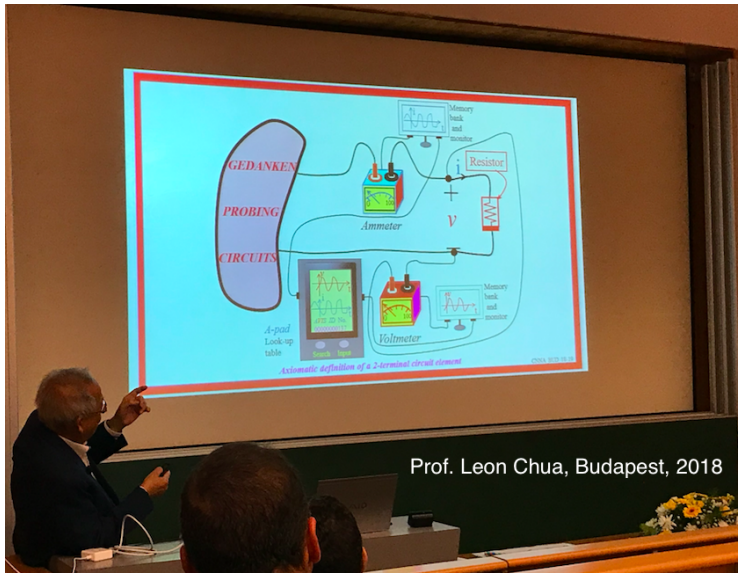
Measurements	Terminal variables i	v
1	-	-
2	-	-
3	-	-

The relation between the terminal variables is called terminal equation

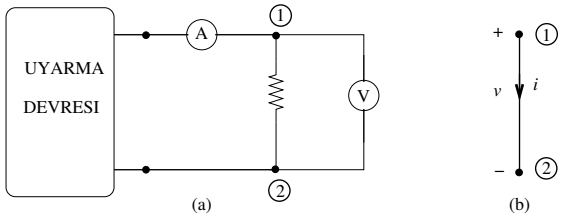
$$f(v, i) = 0$$

or

$$f(v, i, \frac{dv}{dt}, \frac{di}{dt}) = 0$$



Modelling Circuit Element



Terminal graph with two nodes and one branch (the arrow on the branch indicating the reference direction of the current).

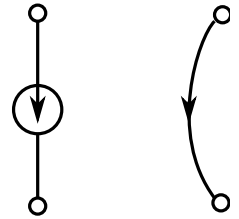
Mathematical Model

The terminal graph and the terminal equation are the mathematical model of the circuit element.

Modelling Circuit Element

Mathematical Model of Current Source :

Terminal graph:



and the terminal equation:

$$i = i_k$$

Modelling Circuit Element

Mathematical Model of Voltage Source :

Terminal graph:

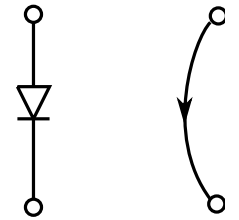
and the terminal equation:

$$v = v_k$$

Modelling Circuit Element

Mathematical Model of Diode:

Terminal graph of the diode:



and the terminal equation:

$$i = I_0 e^{(v/v_T - 1)}$$

Modelling Circuit Element

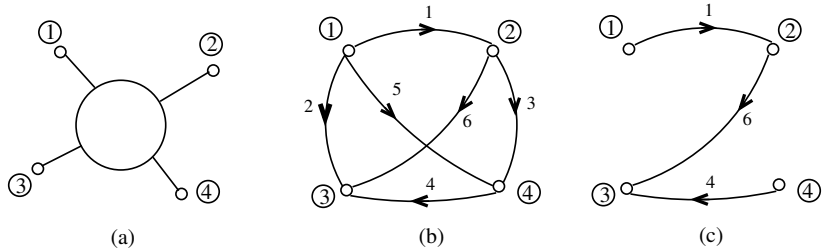
Power delivered at time t to the two-terminal circuit element:

$$P = vi$$

If the voltage $v(t)$ is expressed in volt and the current in amperes then the power is expressed in Watt.

Modelling Circuit Element

Measurement Graphes:



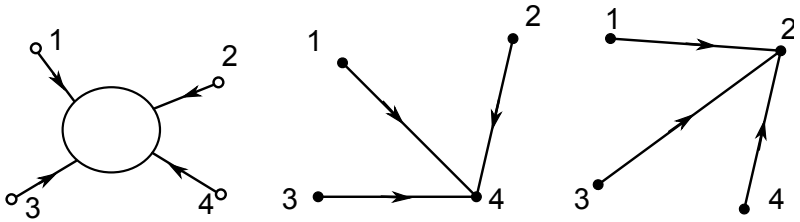
(a) (b) (c)

Measurement Graph $\rightarrow$ Measurement $\rightarrow$ Terminal Equation

Measurement graph = terminal graph

Modelling Circuit Element

Typical measurement and terminal graph:



One of the arbitrarily node is chosen as the datum node (as a reference for measuring electric potentials). The node-to-datum voltages (branches voltages) for the others will be independent voltages. Also branches currents will be independent currents.