Istanbul Technical University 15/11/2019

Department of Computer Engineering

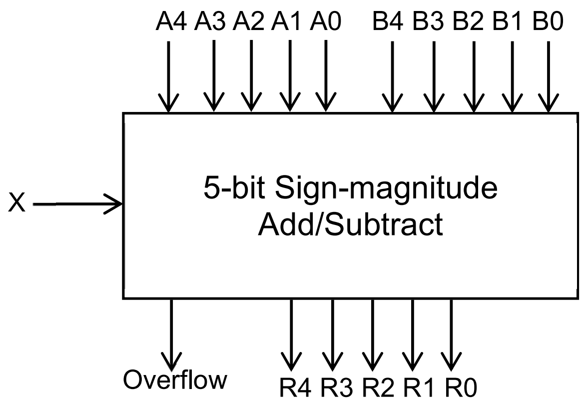
BLG 231E Digital Circuits

Take-Home Exam 4

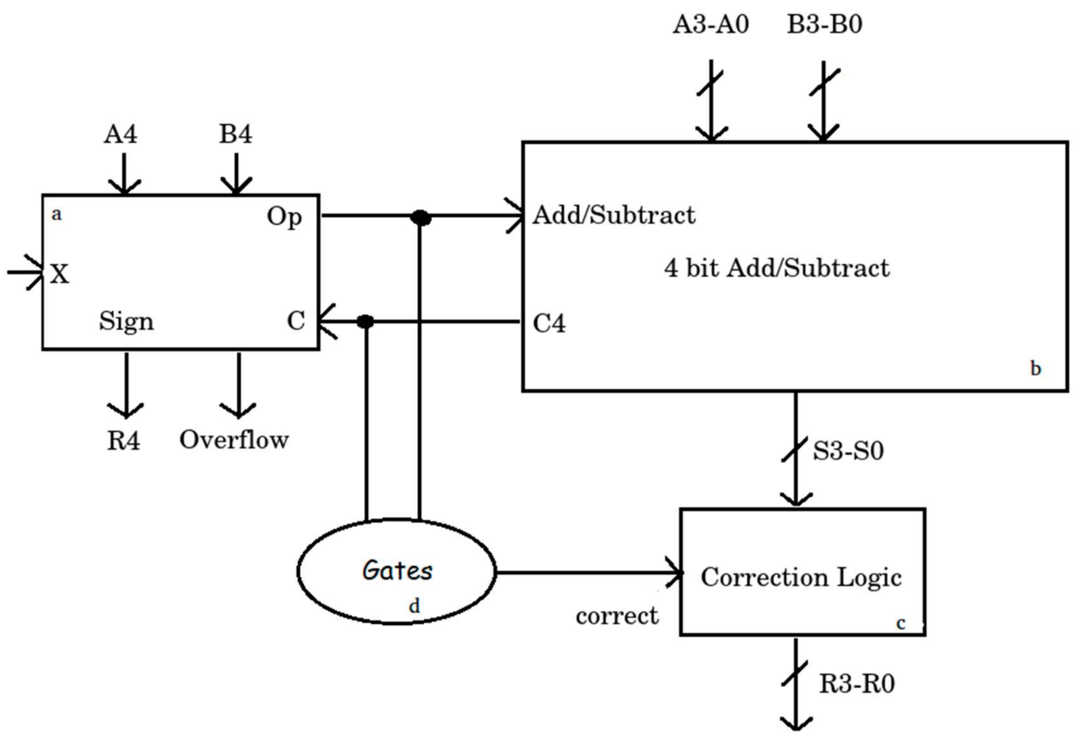
Due Date: Thursday, November 21, 2019 @ 16:00 PM

- Please be diligent and show all work.
- Show complement signs by inserting a dash over the character such as: $\bar{X}$
- **Consequences of plagiarism:** Any cheating will be subject to the university disciplinary proceedings.
- Late submissions will **NOT** be accepted.
  - **Submissions:** Please submit your solutions to the Digital Circuits Course Assignment Box.

In this exam, you need to design a combinational circuit that performs ADD and SUBTRACT operations on 5-bit binary signed integers that are given **in sign-magnitude format** (different than 2's complement representation). In sign-magnitude format, the first bit represents the sign (0: positive, 1: negative) and the remaining bits represent the absolute value (magnitude) of the number. For example; 5-bit $+5_{10}$ = 0 0101 and -5_{10} = 1 0101. The block diagram of the circuit is shown below. A and B are 5-bit signed numbers **in sign-magnitude format** (A4 and B4 for the signed bits and other 4 bits for the magnitudes). Result R is also in sign-magnitude format.
If there is an overflow (the result cannot be represented in 5-bit sign-magnitude format) than the Overflow output should be 1. The input X is used to control the operation of the circuit (X=0: Add, X=1: Subtract).



You must design your circuit so that it consists of 4 subcircuits (a, b, c, and d) as explained below.



X is the operation that we wanted to do. If X = 0, it is addition; if X=1, it is subtraction.

Op is the real operator that the circuit b (2's complement adder/subtracter) will do. If Op=0, it is addition; if Op = 1, it is subtraction.

Circuit a: It determines the operator (Op) (Add or Subtract) input for the Circuit b and generates the sign of the result R based on its input values. It also generates the Overflow output.
Circuit b: It is a 4-bit parallel 2's complement Adder-Subtracter (as shown in the lecture).
Circuit c: This circuits converts the result (S3-S0) from Circuit b in 2's complement form to sign-magnitude format to obtain R3-R0 (if needed).
Circuit d: This circuits determines whether it is necessary or not to convert (S3-S0) from 2's complement form to sign-magnitude form (Correct=0 not necessary; Correct=1: necessary).

Examples:

- A = 00101 (+5) B = 10111 (-7) X = 0 (Addition)

Circuit a: Op = 1 (subtraction, because the required operation is addition and one of the numbers is negative)
Circuit b: A-B; 5-7= 0101-0111= 1110; S=1110 C4=0.
Circuit a: Because after subtraction C=0 (no carry means borrow), B>A so Sign is R4=1, Overflow=0.
Circuit d: Correct=1 correction is necessary.
Circuit c: Convert S=1110 to R3-R0= 0010
Result = R4-R0=10010 (-2)

- A = 00101 (+5)   B = 10111 (-7)   X = 1 (Subtraction)

Circuit a: Op = 0 (addition, because the required operation is subtraction and one of the numbers is negative)
Circuit b: A+B; 5+7= 0101+0111= 1100; S=1100 C4=0.
Circuit a: Sign is R4=0, Overflow=0.
Circuit d: Correct=0 correction is not necessary.
Circuit c: Do not convert. R3-R0 = 1100
Result = R4-R0=01110 (+12)

- A = 01000 (+8)   B = 01001 (+9)   X = 1 (Subtraction)

Circuit a: Op = 0 (Subtraction, because the required operation is subtraction and both numbers are positive)
Circuit b: A-B; 8-9= 1000-1001= 1111; S=1111 C4=0.
Circuit a: Because after subtraction C=0 (no carry means borrow), B>A so Sign is R4=1, Overflow=0.
Circuit d: Correct=1 correction is necessary.
Circuit c: Convert S=1111 to R3-R0= 0001
Result = R4-R0=10001 (-1)

- A = 01000 (+8)   B = 01001 (+9)   X = 0 (Addition)

Circuit a: Op = 0 (addition, because the required operation is addition and both numbers are positive)
Circuit b: A+B; 8+9= 1000+1001= 0001; S=0001 C4=1.
Circuit a: Sign is R4=Φ (don't care), Overflow=1. There is a carry after addition; this value cannot be represented in a 5-bit sign-magnitude format.
Circuit d: Correct=Φ (don't care).
Circuit c: R3-R0= Φ (don't care).

a) Draw the truth table for the Circuit a.
Write the simplest expression for the outputs Op, Sign, and Overflow.
Draw the circuit using any type of logic gates.
Fully label all input and outputs.

b) Design and draw the Circuit c using only half adders and minimum number of logic gates. Fully label all input and outputs.

c) Design and draw the Circuit d using any type of logic gates.
Fully label all input and outputs.