Node Analysis

Assumption: the elements are v-controlled

1. Write current (branch) equations at nodes (KCL)
2. Write element (branch) currents in terms of element (branch) voltages
3. Replace element (branch) voltages with node-to-datum voltages

① $i_1 + i_5 - I_{S1} = 0$

② $-\,i_1 + i_2 + i_3 + I_{S2} = 0$

③ $-\,i_2 + i_4 - i_5 = 0$

① $G_1 V_1 + \ldots = 0$

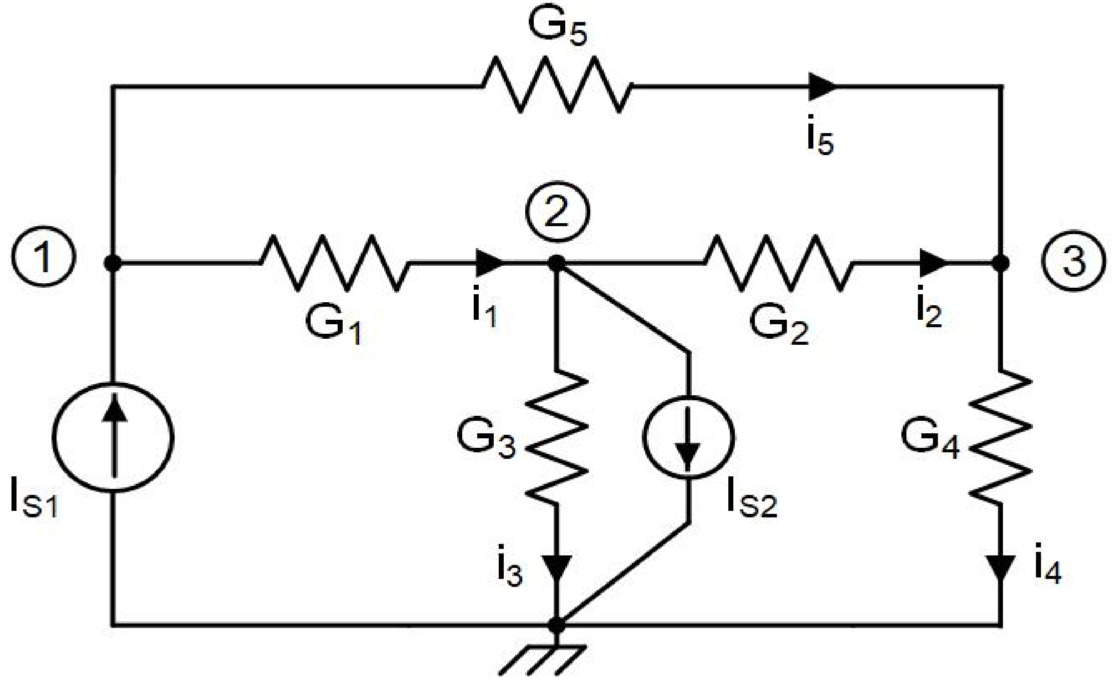
② $-\,G_1 V_1 + \ldots = 0$

③ $-\,G_2 V_2 + \ldots = 0$

① $G_1(e_1 - e_2) + \ldots = 0$

② $-\,G_1(e_1 - e_2) + \ldots = 0$

③ $-\,G_2(e_2 - e_3) + \ldots = 0$



$$\underbrace{\begin{bmatrix} G_1 + G_5 & -G_1 & -G_5 \\ -G_1 & G_1 + G_2 + G_3 & -G_2 \\ -G_5 & -G_2 & G_2 + G_4 + G_5 \end{bmatrix}}_{\boldsymbol{Y_n}} \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}}_{\boldsymbol{e}} = \underbrace{\begin{bmatrix} I_{S1} \\ -I_{S2} \\ 0 \end{bmatrix}}_{\boldsymbol{I_S}}$$

$\mathbf{Y_n}$: node admittance matrix

$\mathbf{I_S}$: equivalent source vector

Properties:

1. The *k*th diagonal element of $\mathbf{Y_n}$ is equal to the sum of all conductances attached to node *k*.
2. The *k*th off-diagonal element of $\mathbf{Y_n}$ is equal to the negative of the sum of the conductances between node *j* and node *k*.
3. The *k*th element of $\mathbf{I_S}$ is equal to the algebraic sum of currents of all independent current sources entering node *k*.
4. The matrix $\mathbf{Y_n}$ is symmetric ($\mathbf{Y_n} = \mathbf{Y_n}^T$).

Modified node analysis

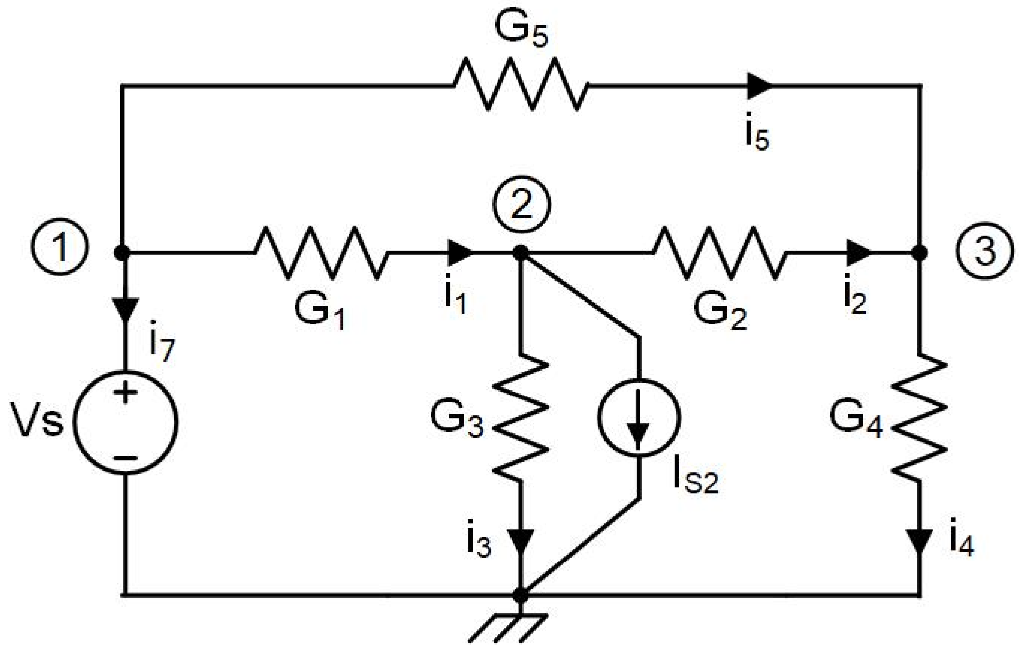
If there are also elements which are not v-controlled.

1. Write node equations using node voltages as variables.
2. Whenever an element is encountered that is not v-controlled, introduce in the node equation the corresponding branch current as a new variable.
3. Add the branch equation of that element as a new equation. The voltages of the elements which are not v-controlled, are written in terms of node voltages.

① $i_1 + i_5 + i_7 = 0$

② $-\,i_1 + i_2 + i_3 + I_{S2} = 0$

③ $-\,i_2 + i_4 - i_5 = 0$



① $G_1(e_1 - e_2) + G_5(e_1 - e_3) + i_7 = 0$

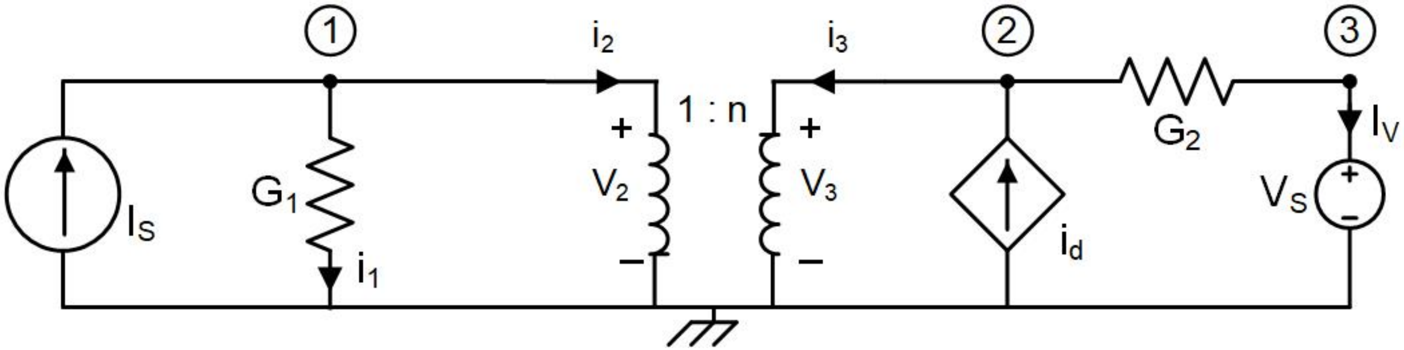
② $-\,G_1(e_1 - e_2) + G_2(e_2 - e_3) + G_3 e_2 = -\,I_{S2}$

③ $-\,G_2(e_2 - e_3) - G_5(e_1 - e_3) + G_4 e_3 = 0$

Extra equation: $e_1 = V_S$

$$\begin{bmatrix} G_1 + G_5 & -G_1 & -G_5 & 1 \\ -G_1 & G_1 + G_2 + G_3 & -G_2 & 0 \\ -G_5 & -G_2 & G_1 + G_4 + G_5 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ i_7 \end{bmatrix} = \begin{bmatrix} 0 \\ -I_{S2} \\ 0 \\ V_S \end{bmatrix}$$

Example 4.1



$$i_d = \alpha i_1$$

$$\begin{bmatrix} G_1 & 0 & 0 & 1 & 0 & 0 \\ -\alpha G_1 & G_2 & -G_2 & 0 & 1 & 0 \\ 0 & -G_2 & G_2 & 0 & 0 & 1 \\ -n & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & n & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ i_2 \\ i_3 \\ I_V \end{bmatrix} = \begin{bmatrix} I_S \\ 0 \\ 0 \\ 0 \\ 0 \\ V_S \end{bmatrix}$$