Sol:- (i) $y(xy+2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

$M = xy^2 + 2x^2y^3 \qquad N = x^2y - x^3y^2$

$M = yf_1(xy) \qquad N = xf_2(xy)$

$\because$ By applying Rule 3, i.e. if M is of the form $M = yf_1(xy)$ & N is of the form $N = xf_2(xy)$ then, $IF = \frac{1}{Mx - Ny}$.

$Mx = x^2y^2 + 2x^3y^3 \qquad Ny = x^2y^2 - x^3y^3$

$Mx - Ny = \cancel{x^2y^2} + 2x^3y^3 - \cancel{x^2y^2} + x^3y^3 = 3x^3y^3.$

$\therefore IF = \frac{1}{3x^3y^3}.$

for differential equation,

$$\frac{xy^2[1+2xy]dx}{3x^3y^3} + \frac{x^2y[1-xy]dy}{3x^3y^3} = \left(\frac{1}{3x^2y} + \frac{2xy}{3x^2y}\right)dx + \left(\frac{1}{3xy^2} - \frac{xy}{3xy^2}\right)dy.$$

$$= \left(\frac{1}{3x^2y} + \frac{2}{3x}\right)dx + \left(\frac{1}{3xy^2} - \frac{1}{3y}\right)dy = 0.$$

Its solution,

$$\int\frac{1}{3x^2y}dx + \int\frac{2}{3x}dx + \int\frac{-1}{3y}dy = c.$$

$$= \frac{-1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = c.$$

$$\Rightarrow \boxed{-\frac{1}{xy} + 2\log x - \log y = c} \text{ ans.}$$

(ii) $(2xy + e^y)dx + (x^2 + xe^y)dy = 0$

$\frac{\partial M}{\partial y} = 2x + e^y \qquad \frac{\partial N}{\partial x} = 2x + e^y$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

∴ this is an exact differential equation
so its solution is given by,

$$\int_{(y \text{ is const})} M\,dx + \int_{(\text{terms do not contain } x)} N\,dy = c.$$

$$\int (2xy + e^y)\,dx + \int 0 = c.$$

$$\Rightarrow \int 2xy\,dx + \int e^y\,dx = c$$

$$\Rightarrow \cancel{2}y\frac{x^2}{\cancel{2}} + e^y x \Rightarrow x^2y + xe^y = c$$

$$\Rightarrow \boxed{x^2y + xe^y = c}$$ ans

(iii) $(x^4y^4 + x^2y^2 + xy)\,y\,dx + (x^4y^4 - x^2y^2 + xy)\,x\,dy = 0.$

dividing equation by xy.

$$\frac{\cancel{x}y^{\cancel{2}}[x^3y^3 + xy + 1]\,dx}{\cancel{xy}} + \frac{x^{\cancel{2}}\cancel{y}[x^3y^3 - xy + 1]\,dy}{\cancel{xy}}$$

$$= \cancel{x^3y^4 + xy^2 + y}\; y(x^3y^3 + xy + 1)\,dx + x(x^3y^3 - xy + 1)\,dy.$$

By applying Rule 3, i.e if M is of the form $y f_1(xy)$ & N is of the form $x f_2(xy)$ then $IF = \frac{1}{Mx - Ny}$

$Mx = xy(x^3y^3 + xy + 1) \qquad Ny = xy(x^3y^3 - xy + 1)$

$$Mx - Ny = \cancel{x^4y^4} + x^2y^2 + \cancel{xy} - \cancel{x^4y^4} + x^2y^2 - \cancel{xy}$$

$$= 2x^2y^2.$$

$$\therefore IF = \frac{1}{2x^2y^2}$$

for differential equation

$$\frac{\cancel{y}(x^3y^3 + xy + 1)}{2x^2y^2}\,dx + \frac{\cancel{x}(x^3y^3 - xy + 1)}{2x^2y^2}\,dy$$

$$= \left(\frac{x^3y^3}{2xy} + \frac{xy}{2x^2y} + \frac{1}{2x^2y}\right)dx + \left(\frac{x^3y^3}{2xy^2} - \frac{xy}{2xy^2} + \frac{1}{2xy^2}\right)dy.$$

$$= \left(\frac{xy^2}{2} + \frac{1}{2x} + \frac{1}{2x^2y}\right)dx + \left(\frac{x^2y}{2} - \frac{1}{2y} + \frac{1}{2xy^2}\right)dy.$$

Its solution,

$$\int\left(\frac{xy^2}{2} + \frac{1}{2x} + \frac{1}{2x^2y}\right)dx + \int -\frac{1}{2y}\,dy = c.$$

$$= \frac{y^2}{2}\int x\,dx + \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2y}\int \frac{1}{x^2}\,dx - \frac{1}{2}\int \frac{1}{y}\,dy = c$$

$$= \frac{y^2}{2}\cdot\frac{x^2}{2} + \frac{1}{2}\log x - \frac{1}{2yx} - \frac{1}{2}\log y = c$$

$$= \frac{x^2y^2}{4} + \frac{1}{2}\log x - \frac{1}{2xy} - \frac{1}{2}\log y = c.$$

$$= \boxed{\frac{x^2y^2}{2} + \log x - \log y - \frac{1}{xy} = c} \text{ ans.}$$

(iv) $\left(\frac{y}{x}\sec y - \tan y\right)dx + (\sec y \log x - x)\,dy = 0.$

$M = \frac{y}{x}\cdot\sec y - \tan y \qquad N = \sec y \log x - x.$

$$\frac{\partial M}{\partial y} = \frac{1}{x}(y\cdot \sec y \tan y + \sec y) - \sec^2 y.$$

$$\frac{\partial N}{\partial x} = \frac{\sec y}{x} - 1$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = \cancel{\frac{\sec y}{x}} - 1 - \frac{y}{x}\sec y \tan y - \cancel{\frac{\sec y}{x}} + \sec^2 y.$$

$$= \sec^2 y - 1 - \frac{y}{x}\sec y \tan y.$$

$$= \tan^2 y - \frac{y}{x}\sec y \tan y \Rightarrow \tan y\left(\tan y - \frac{y}{x}\sec y\right)$$

$\frac{\partial N}{\partial x}$ → By applying Rule 2, i.e. if $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \phi(y)$ then

$IF = e^{\int \phi y \, dy}$.

$$\therefore \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{-\tan y(-\tan y + \cancel{\frac{y}{x}} \sec y)}{(\frac{y}{x}\sec y - \tan y)} = -\tan y.$$

$$\therefore IF = e^{\int -\tan y \, dy} = e^{-\log \sec y} = \frac{1}{\sec y} = \cos y$$

for exact differential equation

$$\cos y\left(\frac{y}{x}\sec y - \tan y\right)dx + \cos y(\sec y \log x - x)\,dy$$

$$= \left(\frac{y}{x} - \tan y.\cos y\right)dx + (\log x - \cos y.x)\,dy.$$

Its solution,

$$\int \frac{y}{x}dx - \int \tan y \cos y \, dx + \cancel{\log x}\int 0 = c$$

$$= y\log x - \tan y.\cos y \, x. = c$$

$$\Rightarrow \boxed{y\log x - \tan y \cos y \, x = c} \text{ ans.}$$

Sol2:- (i) put $\frac{d^2y}{dx^2} + a^2y = \sec ax$.

put $\frac{d}{dx} = D$

$$(D^2 + a^2)y = 0.$$

for Auxillary equation put $D = m$, then equating to zero.

$m^2 + a^2 = 0 \Rightarrow m^2 = -a^2 = m = \pm ai$.

roots are imaginary

$\therefore CF = e^{0x}[c_1 \cos ax + c_2 \sin ax]$

⇒ CF =

here,

PI =

where

and

V =

putting

PI =

$\therefore y =$

$=$

$\boxed{Y = C_1}$

(ii) $\frac{d^2y}{dx^2}$

put $\frac{d}{dx}$

then

$\Rightarrow CF = C_1 \cos ax + \cancel{C_2 \cos} \; C_2 \sin ax.$

here, $y_1 = \cos ax$, $y_2 = \sin ax$

$PI = uy_1 + vy_2$

where $u = \int \frac{-y_2 \sec ax}{y_1 y_2' - y_1' y_2} dx.$

and $v = \int \frac{y_1 \sec ax}{y_1 y_2' - y_1' y_2} dx.$

$u = -\int \frac{\sin ax \sec ax}{a.} dx.$

$y_1 y_2' - y_1' y_2$
$= a.\cos ax \cos ax + \sin ax \cdot \sin ax \cdot a$
$= a(\cos^2 ax + \sin^2 ax)$
$= a$

$u = -\frac{1}{a} \int \tan ax \, dx = -\frac{\log(\cos ax)}{a^2}.(-1)$

$= \frac{1}{a^2} \log \cos ax$

$v = \int \frac{\cos ax \cdot \sec ax}{a} dx = \frac{1}{a} \int dx = \frac{x}{a}.$

putting the values of u and v,

$PI = \frac{1}{a^2} \cos ax . \log \cos ax + \frac{x}{a} . \sin ax.$

$\therefore y = CF + PI$

$= c_1 \cos ax + c_2 \sin ax + \frac{1}{a^2} \cos ax \log \cos ax + \frac{x}{a} \sin ax$ ans.

$\boxed{y = c_1 \cos ax + c_2 \sin ax + \frac{1}{a^2} \cos ax \log \cos ax + \frac{x}{a} \sin ax}$ ans.

to

(ii) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = \sinh x.$

put $\frac{d}{dx} = D$

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$(D^2 - 7D + 12)y = 0$

for Auxillary equation put $D = m$, then equating to zero.

$\Rightarrow m^2 - 7m + 12 = 0.$

$= m^2 - 3m - 4m + 12 = 0$

$= m(m-3) - 4(m-3) = 0.$

$= m - 3 = 0$ & $m - 4 = 0.$

$\Rightarrow \boxed{m = 3}$ $\boxed{m = 4}$.

$\because$ roots are real and distinct.

$\therefore$ $CF = C_1 e^{3x} + C_2 e^{4x}$.

here, $y_1 = e^{3x}$ $y_2 = e^{4x}$.

also, $\sinh xa = \dfrac{e^{ax} - e^{-ax}}{2}$

~~sh~~ $\sinh x = \dfrac{e^{x} - e^{-x}}{2}$ (X)

$PI = uy_1 + vy_2$

where $u = -\displaystyle\int \frac{y_2 X\, dx}{y_1 y_2' - y_2 y_1'}$ and $v = \displaystyle\int \frac{y_1 X}{y_1 y_2' - y_1' y_2} dx.$

$y_1 y_2' - y_2 y_1' = 4e^{3x} \cdot e^{4x} - 3e^{4x} e^{3x} = e^{7x}$

$u = -\displaystyle\int \frac{e^{4x}}{e^{7x}} \left(\frac{e^{x} - e^{-x}}{2}\right) dx = -\frac{1}{2}\int \left(e^{-3x} \cdot e^{x} - e^{-3x} \cdot e^{-x}\right) dx$

$u = -\dfrac{1}{2}\left[\displaystyle\int e^{-2x} dx - \int e^{-4x} dx\right] = -\dfrac{1}{2}\left[\dfrac{e^{-2x}}{-2} + \dfrac{e^{-4x}}{4}\right].$

$u = \dfrac{e^{-2x}}{4} - \dfrac{e^{-4x}}{8}$

$v = \displaystyle\int \frac{e^{3x}}{e^{7x}} \left(\frac{e^{x} - e^{-x}}{2}\right) dx = \frac{1}{2}\left\{\int e^{-4x} \cdot e^{x} - e^{-4x} \cdot e^{-x}\, dx\right\}$

$$= \frac{1}{2}\left\{\int e^{-3x}\,dx - \int e^{-5x}\,dx\right\}$$

$$= \frac{1}{2}\left[\frac{e^{-3x}}{-3} + \frac{e^{-5x}}{5}\right] = \frac{e^{-3x}}{-6} + \frac{e^{-5x}}{10}.$$

$$PI = \left(\frac{e^{-2x}}{4} - \frac{e^{-4x}}{8}\right).e^{3x} + \left(\frac{e^{-5x}}{10} - \frac{e^{-3x}}{6}\right).e^{4x}$$

$$= \frac{e^{x}}{4} - \frac{e^{-x}}{8} + \frac{e^{-x}}{10} - \frac{e^{x}}{6} = \frac{30e^{x} - 15e^{-x} + 12e^{-x} - 20e^{x}}{120}$$

$$= \frac{10e^{x} - 3e^{-x}}{120}$$

$\therefore\ y = CF + PI$

$$\boxed{y = c_1 e^{3x} + c_2 e^{4x} + \frac{1}{12}e^{x} - \frac{e^{-x}}{40}}$$ ans.

(iii) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} = e^{2x}\sin x.$

put $\dfrac{d}{dx} = D$

$(D^2 - 2D)y = 0$

for Auxillary equation, put $D = m$, then equating to 0.

$m^2 - 2m = 0.$

$m(m-2) = 0 \Rightarrow \boxed{m = 0, 2}$

$\because$ roots are real and distinct.

$\therefore\ CF = c_1 e^{0x} + c_2 e^{2x} = c_1 + c_2 e^{2x}.$

here $y_1 = e^{0x} = 1.$ $\quad y_2 = e^{2x}.$

$PI = uy_1 + vy_2$

where, $u = -\displaystyle\int \frac{y_2 \times X}{y_1y_2' - y_1'y_2}\,dx$ and $v = \displaystyle\int \frac{y_1 \times X}{y_1y_2' - y_1'y_2}\,dx.$

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(iii) $y_1 y_2' - y_1' y_2 = 2e^{2x}$

$$u = -\int \frac{e^{2x} \cdot e^{2x} \sin x \, dx}{2e^{2x}} = -\frac{1}{2}\int \underset{II}{e^{2x}} \cdot \underset{I}{\sin x} \, dx$$

$$I = -\frac{1}{2}\int e^{2x} \cdot \sin x \, dx = -\frac{1}{2}\left[\sin x \int e^{2x}\,dx - \int \frac{d}{dx}\sin x \int e^{2x}\,dx\right]$$

$$= -\frac{1}{2}\left[\frac{\sin x \cdot e^{2x}}{2} - \frac{1}{2}\int \cos x \cdot e^{2x}\,dx\right]$$

$$= -\left[\frac{\sin x \cdot e^{2x}}{4} - \frac{1}{4}\int \cos x \cdot e^{2x}\,dx\right]$$

$$= -\left[\frac{\sin x \cdot e^{2x}}{4} - \frac{1}{4} I_1 \text{ (say)}\right]$$

$$I_1 = \int e^{2x} \cos x \, dx = \frac{\cos x \cdot e^{2x}}{2} + \frac{1}{2}\int \sin x \, e^{2x}\,dx$$

$$= \frac{1}{2}\cos x \cdot e^{2x} + \frac{1}{2} I_B$$

$$I = -\left[\frac{\sin x \, e^{2x}}{4} - \frac{1}{4}\left(\frac{1}{2}\cos x \cdot e^{2x} + \frac{1}{2} I\right)\right]$$

$$= -\left(\frac{\sin x \, e^{2x}}{4} - \frac{1}{8}\cos x \cdot e^{2x} - \frac{1}{8} I\right)$$

$$I + \frac{1}{8} I = -\frac{\sin x \, e^{2x}}{4} + \frac{1}{8}\cos x \, e^{2x}.$$

$$9I = -\frac{\cancel{8}^{\,2}}{\cancel{4}}\sin x \, e^{2x} + \frac{\cancel{8}}{\cancel{8}}\cos x \, e^{2x}$$

$$I = -\frac{2}{9}\sin x \, e^{2x} + \frac{1}{9}\cos x \, e^{2x}.$$

$$\Rightarrow u = -\frac{2}{9}\sin x \, e^{2x} + \frac{1}{9}\cos x \, e^{2x}.$$

$$v = \int \frac{1}{2\cancel{e^{2x}}} \cancel{e^{2x}} \sin x \, dx = \frac{1}{2}\int \sin x \, dx = \frac{1}{2}(-\cos x)$$

$$v = -\frac{\cos x}{2}.$$

$$\therefore PI = \left(-\frac{2}{9}\sin x \, e^{2x} + \frac{1}{9}\cos x \, e^{2x}\right) \cdot 1 - \frac{\cos x}{2} e^{2x}$$

$$\boxed{y = c_1 + c_2 e^{2x} - \frac{2}{9}\sin x\, e^{2x} + \frac{1}{9}\cos x\, e^{2x} - \frac{1}{2}\cos e^{2x}}$$ ans

Sol 3:- (i) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$

This is cauchy Euler's homogeneous LDE.

So, let $x = e^z$ $\quad z = \log x$ $\quad \frac{dz}{dx} = \frac{1}{x}$ and $\frac{d}{dx} = D$.

and $x^2\frac{d^2y}{dx^2} = D(D-1)y$ $\quad x\frac{dy}{dx} = \frac{dy}{dz} = Dy$

$$D(D-1)y + Dy + y = \sin(\log x^2)$$

$$(D^2 \cancel{-D} + \cancel{D} + 1)y = \sin(\log x^2)$$

$$D^2 + 1 = 0$$

for Auxillary equation put $D = m$

$m^2 + 1 = 0$ $\quad m^2 = -1$ $\quad \Rightarrow m = \pm i$

$\because$ roots are complex

$\therefore$ CF $= e^{0x}[c_1 \cos z + c_2 \sin z] = c_1 \cos z + c_2 \sin z$.

PI of $\frac{1}{D^2+1}(\sin(\log x^2))$.

In $\sin(\log x^2)$ put $x = z$. $\cancel{\sin \log z^2)}$ $\Rightarrow \sin 2z$.

$\therefore \frac{1}{(D^2+1)}(\sin 2z) = \frac{1}{-4+1}(\sin 2z) = \frac{-1}{3}\sin 2z$.

$$PI = \frac{-1}{3}\sin 2(\log x)$$

$\therefore y = CF + PI = c_1 \cos z + c_2 \sin z + \left(-\frac{1}{3}\sin 2(\log x)\right)$

$$\boxed{y = c_1 \cos(\log x) + c_2 \sin(\log x) - \frac{1}{3}\sin 2(\log x)}$$ ans.

(ii) $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$.

This is cauchy Euler's homogeneous LDE

So let $x = e^z \Rightarrow z = \log x$. $\frac{dz}{dx} = \frac{1}{x}$ & $\frac{d}{dx} = D$

& $x^2 \frac{d^2y}{dx^2}$ $\cancel{\pm}$ ~~x~~ $D(D-1)y$ $\quad x\frac{dy}{dx} = Dy$.

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = 12\log x.$$

$\cancel{B}$ $12 \log e^z = 12z$.

$D(D-1)y + Dy = 0$.

$(D^2 - \cancel{D} + \cancel{D})y = 0$

for auxillary equation put $D = m$

$m^2 = 0. \Rightarrow \boxed{m = 0, 0}$

$\because$ roots are real & equal

$\therefore$ $CF = (C_1 + C_2 z)e^{0z} = C_1 + C_2 z$.

$\therefore$ PI of $\frac{1}{D^2} z \Rightarrow PI = (D^{-2})^{-1} z$

$$\boxed{y = C_1 + C_2 \log x + 2(\log x)^3} \text{ ans.}$$

(iii) $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin\log(1+x)$

This is Legendre's Homogeneous Differential equat^n

$(1+x) = e^z \Rightarrow z = \log(1+x)$ $\quad \frac{dz}{dx} = \frac{1}{(1+x)}$ $\quad \frac{d}{dz} = D$.

or $(1+x)^2 \frac{d^2y}{dx^2} = D(D-1)y$. &

$(1+x)\frac{dy}{dx} = Dy$

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$D(D-1)y + Dy + y = 2\sin z.$

$(D^2 - \cancel{D} + \cancel{D} + 1)\, y = 2\sin z.$

$(D^2+1)\, y = 2\sin z$

for Auxillary equation put $D = m$

$m^2 + 1 = 0$

$m^2 = -1 \Rightarrow m = \pm i.$

$\because$ roots are complex.

$\therefore CF = e^{0x}[c_1 \cos z + c_2 \sin z]$

$CF = c_1 \cos z + c_2 \sin z.$

$\therefore$ PI of $\dfrac{1}{D^2+1} \cdot 2\sin z$

$= 2 \dfrac{1}{D^2 - i^2} \sin z = 2 \dfrac{1}{(D-i)(D+i)} \sin z.$

$= \dfrac{1}{(D-i)(D+i)} = \dfrac{A}{(D\pm i)} + \dfrac{B}{(D+i)}$

$= \dfrac{1}{(D-i)(D+i)} = \dfrac{A(D+i) + B(D \mp i)}{(D+i)(D-i)}$

$\Rightarrow 1 = AD + Ai + BD \mp Bi$

$\Rightarrow 1 = AD + BD + i(B - A)$

$A + B = 0 \qquad -Ai + Bi = 1. \Rightarrow \boxed{A = -B}$

$\boxed{A = +1/2i}, \boxed{B = 1/2i}$

$\dfrac{1}{2i}\left[\dfrac{1}{D\mp i} \mp \dfrac{1}{D+i}\right] \sin z = \dfrac{1}{2i}\left[\dfrac{1}{D\mp i}\sin z - \dfrac{1}{D+i}\sin z\right]$

$= \dfrac{1}{2i}\left[e^{+iz}\int e^{-iz}\sin z\, dz - e^{-iz}\int e^{iz}\sin z\, dz\right]$

$= \dfrac{1}{2i}\left[e^{iz}\int(\cos z \mp i\sin z)\sin z\, dz - e^{-iz}\int(\cos z + i\sin z)\sin z\, dx\right]$

$$= \frac{1}{2i}\left[e^{iz}\int \cos z \sin z - i\sin^2 z \, dz - e^{-iz}\int \cos z \sin z + i\sin^2 z \, dz \right.$$

$$= \frac{1}{2i}\left[e^{iz}\left(-\frac{1}{4}\cos 2z\right) - \frac{i}{2}\left[z - \frac{1}{2}\sin 2z\right] - \left\{ e^{-iz}\left(-\frac{1}{4}\cos 2z\right) + \frac{i}{2}\left[z - \frac{1}{2}\right.\right.\right.$$

$$= \frac{1}{2i}\left[-\frac{e^{iz}}{4}\cos 2z - \frac{z i e^{iz}}{2} + \frac{ie^{iz}}{4}\sin 2z + \frac{e^{-iz}}{4}\cos 2z - \frac{iz}{2}e^{-iz} + \frac{ie^{-iz}}{4}\sin 2z \right]$$

$$= \frac{1}{2i}\left[-\frac{\cos 2z}{4}\left(e^{iz} - e^{-iz}\right) + \frac{i}{4}\sin 2z\left(e^{iz} + e^{-iz}\right) - \frac{zi}{2}\left(e^{iz} + e^{-iz}\right)\right]$$

$$= \frac{1}{2i}\left(-\frac{\cos 2z}{\cancel{4}_2}\cancel{2}i\sin z + \frac{i}{\cancel{4}_2}\sin 2z \cdot \cancel{2}\cos z - \frac{zi}{\cancel{2}}\cancel{2}\cos z \right]$$

$$= \frac{1}{2i}\left(\frac{i\sin 2z \cdot \cos z}{2} - \frac{i\cos 2z \sin z}{2} - iz\cos z \right]$$

$$= \frac{1}{2i}\left[\frac{2i\sin z\cos^2 z}{2} - \frac{i(2\cos^2 z - 1)}{2}\sin z - iz\cos z \right]$$

$$= \frac{1}{2i}\left[\frac{\cancel{2}i\sin z\cos^2 z}{\cancel{2}} - \frac{i\cancel{2}\sin z\cos^2 z}{\cancel{2}} + \frac{i\sin z}{2} - iz\cos z \right]$$

$$= 2\left(\frac{\cancel{i}\sin z}{2\times 2\cancel{i}} - \frac{1}{2\cancel{i}}\cancel{i}z\cos z \right) = 2\left(\frac{1}{4}\sin z - \frac{1}{2}\cos z \cdot z\right)$$

$y = CF + PI.$

$PI = 2\left(\frac{1}{4}\sin z - \frac{1}{2}\cos z \cdot z\right)$

$CF = C_1 \cos z + C_2 \sin z$

$y = C_1\cos z + C_2\sin z + \frac{\cancel{2}}{\cancel{4}_2}\sin z - \frac{\cancel{2}}{2}z\cdot\cos z$

$y = C_1\cos z + C\sin z - z\cos z$

$$\boxed{y = C_1\cos\log(1+x) + C_2\sin\log(1+x) - \log(1+x)\cos\log(1+x)}$$

Sol 4:-

put

D^2

for

m^2

m

$\therefore$

$\therefore$

$\Rightarrow$

PI

PI

=

=

=

=

Sol4:- (i) $\frac{d^2y}{dx^2} + 4y = e^{2x}\cos x$.

put $\frac{d}{dx} = D$

$D^2y + 4y = e^{2x}\cos x$.

for auxillary equation, put $D = m$.

$m^2 + 4 = 0$.

$m^2 + 4 = 0 \Rightarrow m^2 = -4 \Rightarrow m = \pm 2i$

$\because$ roots are complex

$\therefore \quad CF = e^{0x}\left[C_1\cos 2x + C_2\sin 2x\right] = C_1\cos 2x + C_2\sin 2x$

$\Rightarrow \boxed{CF = C_1\cos 2x + C_2\sin 2x}$

PI of $\frac{1}{D^2+4}\left\{e^{2x}\cos x\right\}$

$$PI = \frac{1}{D^2+4}\cdot e^{2x}\cos x = e^{2x}\frac{1}{(D+2)^2+4}\cos x.$$

$$= e^{2x}\cdot\frac{1}{D^2+4+4D+4}\cdot\cos x = e^{2x}\cdot\frac{1}{D^2+4D+8}\cos x.$$

$$= e^{2x}\cdot\frac{1}{-1+4D+8}\cos x = e^{2x}\cdot\frac{1}{4D+7}\cos x.$$

$$= e^{2x}\cdot\frac{D-\frac{7}{4}}{D^2-\frac{49}{16}}\cos x. = e^{2x}\cdot\frac{D-\frac{7}{4}}{-1-\frac{49}{16}}\cos x$$

$$= e^{2x}\frac{D-\frac{7}{4}}{\frac{-65}{16}}\cos x = e^{2x}\cdot\frac{4D-7}{\cancel{4}\left(\frac{-65}{\cancel{16}_4}\right)}\cos x.$$

$$= e^{2x}\frac{(16D-28)\cos x}{-65} = \frac{e^{2x}\cdot 16\frac{d}{dx}\cos x - 28e^{2x}\cos x}{-65}$$

$$= \frac{e^{2x}\, 16(-\sin x) - 28e^{2x}\cos x}{-65} = \frac{-16e^{2x}\sin x - 28e^{2x}\cos x}{-65}$$

$$= \frac{16e^{2x}\sin x + 28e^{2x}\cos x}{65} = \frac{4}{65}e^{2x}(4\sin x + 7\cos x)$$

$PI = \frac{4}{65}e^{2x}(4\sin x + 7\cos x)$ ans

$y = CF + PI$

$\boxed{y = c_1\cos 2x + c_2\sin 2x + \frac{4}{65}e^{2x}(4\sin x + 7\cos x)}$ ans

(ii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2e^{2x}\cos 2x$

put $\frac{d}{dx} = D$

$D^2y - 4Dy + 4y = 8x^2e^{2x}\cos 2x$

for auxiliary equation put $D = m$

$m^2 - 4m + 4 = 0$

$m^2 - 2m - 2m + 4 = 0$

$m(m-2) - 2(m-2) = 0$

$\boxed{m = 2, 2}$

$\because$ roots are real and equal

$\therefore CF = (c_1 + c_2x)e^{2x}$

PI of $\frac{1}{D^2 - 4D + 4}\, 8x^2e^{2x}\cos 2x = 8\frac{1}{(D-2)^2}x^2e^{2x}\cdot\cos 2x$

$= e^{2x}8\frac{1}{(D-2+2)^2}x^2\cos 2x = 8e^{2x}\frac{1}{D^2}x^2\cdot\cos 2x$

$$8e^{2x}\frac{1}{D}\left[x^2\left(\frac{\sin 2x}{2}\right) + 2x\left(\frac{+\cos 2x}{\cancel{4}\,2}\right) \mp 2\left(\frac{\sin 2x}{8}\right)\right]$$

$$= 8e^{2x}\frac{1}{D}\left[\frac{x^2}{2}\sin 2x + \frac{x\cos 2x}{2} - \frac{\sin 2x}{4}\right]$$

$$= \frac{\cancel{8}^{4} e^{2x}}{\cancel{2}}\frac{1}{D}\left[x^2\sin 2x + x\cos 2x - \frac{\sin 2x}{2}\right]$$

$$= 4e^{2x}\left[-\frac{x^2}{2}\cos 2x + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + \frac{1}{4}\cos 2x\right]$$

$$= e^{2x}\left[-\frac{\cancel{4}^{2} x^2\cos 2x}{\cancel{2}} + \frac{\cancel{4}^{2} x\sin 2x}{\cancel{2}} + \frac{\cancel{4}\cos 2x}{\cancel{4}} + \frac{\cancel{4}^{2} x\sin 2x}{\cancel{2}} + \frac{\cancel{4}\cos 2x}{\cancel{4}} + \frac{\cancel{4}\cos 2x}{\cancel{4}}\right]$$

$$= e^{2x}\left[-2x^2\cos 2x + 2x\sin 2x + \cos 2x + 2x\sin 2x + 2\cos 2x\right]$$

$$= e^{2x}\left[-2x^2\cos 2x + 4x\sin 2x + 3\cos 2x\right]$$

$$= +e^{2x}\left[4x\sin 2x - (2x^2-3)\cos 2x\right]$$

$$\boxed{PI = e^{2x}\left[4x\sin 2x - (2x^2-3)\cos 2x\right]}$$

$y = CF + PI$

$$\boxed{y = [C_1 + C_2 x]e^{2x} + e^{2x}\left[4x\sin 2x - (2x^2-3)\cos 2x\right]}$$ ans.

(iii) $\frac{d^4y}{dx^4} - y = x\sin x.$