Number System

» Number :- It is group of symbols or digits.

maximum digit in any number system whose radix / base is R = R-1

Minimum digit is always '0'.

» Radix / base :- Total number of digits in any number system is called radix or base of that number system.

For Eg : In base 10 there are 10 digits from 0-9.
In base 7 there are 7 digits from 0-6
In hexadecimal system, there are digits from 0-15 (F) because base is 16.

» Most significant digit :-

Positional weight → For any number system whose radix is R in that number system positional weight = R^i

Eg :- $(\overset{3\,2}{1}\overset{1}{2}\overset{0}{4}.\overset{-1}{1}\overset{-2}{2})_{10}$ ← i

↑ R

In above positional weight of digit 1 is 10^2. It is highest and so it is called MSD.

MSD → The digit which has highest positional weight.

LSD → The digit which has lowest positional weight is called least significat digit.

In above example 2 is LSD because it has lowest positional weight i.e 10^{-2}. In binary number system, digit is called bit.

Q Determine LSD, MSD and positional weight of digit 5 if it is represented in base 7.

$(235.14)_7$

2 ↓ MSD

5 ↓ 7^0

4 ↓ LSD

26/07/17

>> Representation of Integers.

$$N = \sum_{i=0}^{n-1} a_i r^i$$

Eg: $\overset{3}{\underset{a_3}{1}}\ \overset{2}{\underset{a_2}{0}}\ \overset{1}{\underset{a_1}{1}}\ \overset{0}{\underset{a_0}{1}}$

$1 + 1\times 2^1 + 0\times 2^2 + 1\times 2^3$

$$= a_0 + a_1 r^1 + a_2 r^2 + a_3 r^3 + \dots$$

a_i = value of digit at position i.
r^i = value of positional weight at position i.
n = total no. of integer digits.

» Conversion of any Number System whose radix is R into Decimal Number System

i.e $(\ )_r \rightarrow (\ )_{10}$

Apply the formula $N = \sum_{i=0}^{n-1} a_i r^i$

Q. convert the following

(1) $(123)_4 \rightarrow (\ )_{10} \rightarrow 1 \times 4^2 + 2 \times 4^1 + 3 \times 4^0$

$\rightarrow 16 + 8 + 3 = (27)_{10}$

(2) $(AB)_{16} \rightarrow (\ )_{10} \rightarrow 10 \times 16^1 + 11 \times 16^0$

$\rightarrow (171)_{10}$

(3) $(271)_8 \rightarrow (\ )_{10} \rightarrow 2 \times 8^2 + 7 \times 8^1 + 1 \times 8^0$

$\rightarrow 128 + 56 + 1 = (185)_{10}$

») For fractional part

Apply the formula $N_{10} = \sum_{i=-1}^{-M} a_i r^i$

$(N)_{10\,frac} = a_{-1} r^{-1} + a_{-2} r^{-2} + a_{-3} r^{-3} + \ldots\ldots$

convert $(.23)_9 = (\ )_{10}$

$= 2 \times 9^{-1} + 3 \times 9^{-2}$

$= (.32)_{10}$

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Q convert $(121.32)_4 \longrightarrow (\ )_{10}$

$1 \times 4^2 + 2 \times 4^1 + 1 \times 4^0 . \ 3 \times 4^{-1} + 2 \times 4^{-2}$

$(16 + 8 + 1) . \left(\frac{3}{4} + \frac{1}{8}\right)$

$\rightarrow (25.875)_{10}$

$$\boxed{(N)_{10} = \Sigma(\text{value of digit at position } i) \times (\text{p.w. at position } i)}$$

> conversion from decimal number system into any number system whose radix is r.

Let $N = a_0 + a_1 r^1 + a_2 r^2$

divide N by r

N/r	quotient(q)	remainder
q/r	$a_1 + a_2 r$	a_0
q/r	a_2	a_1
q/r	0	a_2

For integer part divide decimal no. by r in which number system we want to convert. and separate quotient and remainder part. Again divide quotient by r until quotient becomes '0' and take the answer from bottom to top. (For integer part only).

Q Convert ~~271~~ $(185)_{10} \longrightarrow (\ )_8$

8	185	
8	23	1
8	2	7
	0	2

$(271)_8$

$(72)_{10} \rightarrow (\ )_2$

2	72	
2	36	0
2	18	0
2	9	0
2	4	1
2	2	0
2	1	0
	0	1

$(1001000)_2$

$(43)_{10} \longrightarrow (\ )_{16}$

16	43	
16	2	11
	0	2

$(2B)_{16}$

>> for fractional part

To convert from base 10 to any radix number system, multiply by Rx in which we want to convert and separate integer and fractional part. Again repeat above process untill fractional part is zero or any above step repeats. & take ans in top to bottom

Q Convert $(.175)_{10}$ into $(\ )_4$

.175 × 4	=	.700	0
.700 × 4	=	2.800	2
.800 × 4	=	3.200	3
.200 × 4	=	.800	0
.800 × 4	=	3.200	3

$(.02303)_4$

Q Convert $(412.35)_{10}$ into $(\ )_{16}$

16	412	
16	25	12
16	1	9
	0	1

.35 × 16	=	5.6	5
.6 × 16	=	9.6	9
.6 × 16	=	9.6	9

$(19C.599)_{16}$

» To convert $( )_{b_1} \longrightarrow ( )_{b_2}$

Step → $( )_{b_1} \longrightarrow ( )_{10} \longrightarrow ( )_{b_2}$

Q $(124)_5 \longrightarrow ( )_4$

$1 \times 5^2 + 2 \times 5^1 + 4 \times 5^1 = 25 + 10 + 4$
$= (39)_{10}$

4	39	
4	9	3
4	2	1
	0	2

$(213)_4$

» If base is in 2^i, then

Eg:- $( )_2 \xrightarrow[\text{group}]{\text{make 3 bit}} ( )_8$ $\{ 8 = 2^3 \}$

$( )_8 \xrightarrow[\text{digit by 3 bit}]{\text{Replace every}} ( )_2$
421 code

Q $(1011011)_2 \longrightarrow ( )_8$

$\underbrace{\overset{421}{001}}, \underbrace{\overset{421}{011}}, \underbrace{\overset{421}{011}} \longrightarrow (133)_8$

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Q convert $(10101.10101)_2 \longrightarrow (\ )_8$

$$\begin{array}{l} 421 \\ \underbrace{010}\,\underbrace{101}\,.\,\underbrace{101}\,\underbrace{010} \rightarrow (25.52)_8 \end{array}$$

>> $(\ )_2 \xrightarrow[\text{group}]{\text{make 4 bit}} (\ )_{16}$

$\longleftarrow$ Replace every digit with 8421 code.

Q convert $(AB.9)_{16} \longrightarrow (\ )_2$

8421 8421 8421

A B . 9

$(1010\ 1011.\ 1001)_2$

convert $(55)_8 \longrightarrow (\ )_{16}$

421 421

5 5

$(101\ 101)_2 \rightarrow \underbrace{0010}_{8421}\,\underbrace{1101}_{8421}$

$(2D)_{16}$

Q. $\sqrt{41} = (5)_{10}$ in which base system it is possible

$$41 = 25$$

$$1 \times b^0 + 4 \times b^1 = 25$$

$$4b + 1 = 25$$

$$4b = 24$$

$$\boxed{b = 6}$$

Q. If $\frac{55}{5} = 11$ in which base system it is possible

$$\frac{55}{5} = 11$$

$$\cancel{11 = 11}$$

$$\cancel{1 \times b_1^0 + 1 \times b_1^1 = 1 \times b_2^0 + 1 \times b_2^1}$$

$$\cancel{b_1^1 = b_2^1}$$

$$\frac{5 \times b^0 + 5 \times b^1}{5 \times b^0} = 1 \times b^0 + 1 \times b^1$$

$$\frac{5 + 5b}{5} = b + 1$$

$$5 + 5b = 5(b+1)$$

$$b + 1 = b + 1$$

$$\Rightarrow \boxed{b \geq 6}$$

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Q $\frac{302}{20} = 12.1$ find N.S.

$$\frac{3\times b^2 + 0\times b^1 + 2\times b^0}{2\times b^1 + 0\times 2^0} = 1\times b^1 + 2\times b^0 . 1\times b^{-1}$$

$$\frac{3b^2+2}{2b} = b + 2 + \frac{1}{b}$$

$$\frac{3b^2+2}{2\cancel{b}} = \frac{b^2+2b+1}{\cancel{b}}$$

$$3b^2 + 2 = 2b^2 + 4b + 2$$

$$b^2 - 4b = 0$$

$$b(b-4) = 0$$

$$b = 0 \times \quad \boxed{b=4}$$

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Q In any no. system $xy = 20$ and $yx = 31$ and if y is successive digit of x. what will be value of x, y and base b. in which no. sys. it is represented.

$$y = x+1$$

$$x\times b^1 + y\times b^0 = 20$$

$$xb + y = 20 \rightarrow ① \rightarrow xb + x+1 = 20$$

$$y\times b^1 + x\times b^0 = 31$$

$$yb + x = 31 \rightarrow ② \rightarrow (x+1)b + x = 31$$

$$eq^n ① = (x+1)b + 1 = 20$$

$$eq^n ② = (x+1)b + x = 31$$

$$① - ②$$

$$-b+1 = -11$$

$$-b = -12$$

$$\boxed{b=12}$$

$$xy = 20$$
$$x \times 12^1 + y \times 12^0 = 20$$
$$12x + y = 20 \rightarrow (3)$$

$$yx = 31$$
$$y \times 12^1 + x \times 12^0 = 31$$
$$12y + x = 31 \rightarrow (4)$$

on solving (3) & (4)

$$x = 19/13$$
$$y = 22/13$$

» Arithematical Operations in different base.

<u>Addition</u> :-

$$\begin{array}{cc} y_1 & x_1 \\ y_2 & x_2 \end{array}$$

if $x_1 + x_2 > b-1 \Rightarrow x_1 + x_2 - b$ & 1 forward carry

Eg:

$$\begin{array}{r} \overset{1}{(24)_8} \\ + (55)_8 \\ \hline 8 \quad 9 \\ -8 \quad -8 \\ \hline 1 \, 0 \, 1 \end{array} = (101)_8$$

Eg :

$$
\begin{array}{rccc}
 & (3 & A & B)_{16} \\
+ & (9 & 6 & 5)_{16} \\
\hline
 & 13 & 17 & 16 \\
 & & -16 & -16 \\
\hline
 & & 1 & 0
\end{array}
$$

$= (D10)_{16}$

Subtraction :-

$$
\begin{array}{cc}
y_1 & x_1 \\
y_2 & x_2
\end{array}
$$

if $x_1 < x_2$ borrow base from y_1

Q Perform the following opn.

(1)

$$
\begin{array}{r}
(3\ \ \cancel{4}^{3}\ \ \cancel{1}^{5})_5 \\
-(1\ \ 2\ \ 3)_5 \\
\hline
(2\ \ 1\ \ 3)_5
\end{array}
$$

(2)

$$
\begin{array}{r}
(\cancel{3}^{2}\ \ \cancel{6}^{5+8}\ \ \cancel{5}^{+8})_8 \\
-(1\ \ 7\ \ 6)_8 \\
\hline
(1\ \ 6\ \ 7)_8
\end{array}
$$

Q If $35 + 53 = 110$ then how much $137 + 731 = ?$

$$(3 \times b^1 + 5 \times b^0) + (5 \times b^1 + 3 \times b^0) = 1 \times b^2 + 1 \times b^1 + 0 \times b^0$$

$$3b + 5 + 5b + 3 = b^2 + b + 0$$

$$8b + 8 = b^2 + b + 0$$

$$b^2 - 7b - 8 = 0$$

$$b^2 - 8b + b - 8 = 0$$

$$b(b-8) + (b-8) = 0$$

$$b = -1, 8$$

$$\boxed{b = 8}$$

$$
\begin{array}{rccc}
 & & 1 & \\
 & (1 & 3 & 7)_8 \\
+ & (7 & 3 & 1)_8 \\
\hline
 & 8 & 7 & 8 \\
 & -8 & & -8 \\
\hline
1 & 0 & & 0
\end{array}
$$

$(1070)_8$

Multiplication :-

Eg:

$$
\begin{array}{rcc}
 & +3 & \\
 & (2 & 5)_6 \\
\times & (1 & 4)_6 \\
\hline
 & 18 & 20 \\
 & -6 & -6 \;\; 1 \\
\hline
 & 5 & 14 \\
 & & -6 \;\; 1 \\
\hline
 & & 8 \\
 & & -6 \;\; 1 \\
\hline
 & & 2
\end{array}
$$

$$
\begin{array}{rccc}
 & & (2 & 5)_6 \\
 & & (1 & 4)_6 \\
\hline
 & {}^{1}1 & 5 & 2 \\
+ & 2 & 5 & \\
\hline
 & 10 & & 2 \\
 & -6 & & \\
\hline
4 & 4 & & 2
\end{array}
$$

$= (442)_6$

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$(25)_6 \rightarrow (\ )_{10} \rightarrow 2\times 6^1 + 5\times 6^0$

$= 12+5 = (17)_{10}$

$(14)_6 \rightarrow (\ )_{10} \rightarrow 1\times 6^1 + 4\times 6^0 = (10)_{10}$

$$\begin{array}{r} 17 \\ \times\ 10 \\ \hline = (170)_{10} \end{array}$$

$(170)_{10} \rightarrow (\ )_6$

$$\begin{array}{r|c|c} 6 & 170 & \\ \hline 6 & 28 & 2 \\ \hline 6 & 4 & 4 \\ \hline & 0 & 4 \end{array} \uparrow \qquad (442)_6$$

① $(15)_6 \times (23)_4 = (\quad)_4$

$1\times 6^1 + 5\times 6^0 = 6+5 = (11)_{10}$

$$\begin{array}{r|c|c} 4 & 11 & 3 \\ \hline 4 & 2 & 2 \\ \hline & 0 & \end{array} \uparrow \qquad (23)_4$$

$$\begin{array}{r} (\overset{2}{2}\ 3)_4 \\ \times\ (2\,3)_4 \\ \hline 8\quad 9 \\ -4\ -4 \quad ① \\ \hline 4\quad 5 \quad ① \\ -4\ -4 \\ \hline 2\quad 0\quad 1 \end{array} \qquad \begin{array}{r} 2\,3 \\ 2\,3 \\ \hline 2\,0\,1 \\ 5\,6 \\ -4\ -4 \\ \hline 1\ 1\ 2 \end{array}$$

$$\begin{array}{r} 2\,0\,1 \\ 1\,1\,2 \\ \hline 1\,3\,2\,1 \end{array} \Rightarrow (1321)_4$$

>> $(r-1)$'s complement

$r \rightarrow$ 2, 4, 6, 8, 10, 16 .....

r is always even

$(r-1)$'s comp $\rightarrow$ subtract every digit from $(r-1)$

Q Determine 1's complement of 1001

$$\begin{array}{r} 1111 \\ -\ 1001 \\ \hline 0110 \end{array}$$

Q Determine 9's complement of 257

$$\begin{array}{r} 999 \\ -\ 257 \\ \hline 742 \end{array}$$

Q Determine 7's complement of 156

$$\begin{array}{r} 777 \\ -\ 156 \\ \hline 621 \end{array}$$

Q Determine F's complement of A27

$$\begin{array}{ccc} F & F & F \\ A & 2 & 7 \\ \hline 5 & D & 8 \end{array}$$

>> r's complement

r → always even
r's comp = (r−1)'s comp + 1

Q Determine 2's complement of 1011

$(r-1)$'s 1 1 1 1
1 0 1 1
0 1 0 0
+ 1
r's 0 1 0 1

Q Determine 8's complement of 7000
7 7 7 7
− 7 0 0 0
$(r-1)$'s 0 7 7 7
+ 1
r's → 8 8 8
−8 −8 −8
1 0 0 0

r's → $(1000)_8$

Q Determine 16's complement of 25F0
F F F F
− 2 5 F 0
D A 0 F

D A 1 0 F
+ 1
D A 1 0

» (r-1)'s complement of N = $r^m - r^{-n} - N$
where m = total no. of integer digit
n = total no of fractional digit

9's complement of 34 = $10^2 - 10^0 - 34$
= 100 − 1 − 34
= 99 − 34

» subtraction by using (r−1)'s complement method.
(1 or 9's method) A − B

Step 1: Determine (r−1)'s complement (1's or 9's) of B and add this to A.

Step 2: If A > B, in this case always carry is generated and add this carry to LSD of result obtained from step 1

If A < B, in this case carry is not generated and final answer will be again (r−1)'s complement of result obtained from step 1. with negative sign.

Q Perform the following operation by using 1's complement method.

(i)
```
  1011  (A)
- 1010  (B)
  ?
```

1's comp. of B = 0101

```
     1011
     0101
     2222
    -2-2-2-2
MSD (1)0000
       +1
     0001
```

Answer: 0001

(v)
$$\begin{array}{r} 0110 \\ -1110 \\ \hline -1000 \end{array}$$

1's comp of B = 0001

$$\begin{array}{r} 0110 \\ +0001 \\ \hline 0111 \end{array}$$

comp of (0111) = −1000 (Answer)

Q Perform the following operation by using 9's complement method

(i)
$$\begin{array}{r} 35 \\ -85 \\ \hline -50 \end{array}$$

9's comp of B =
$$\begin{array}{r} 99 \\ 85 \\ \hline 14 \end{array}$$

$$\begin{array}{r} 14 \\ 35 \\ \hline 49 \\ -9 \\ \hline -50 \end{array}$$

Ans = −50

(ii)
$$\begin{array}{r} -235 \\ -814 \\ \hline 221 \end{array}$$

9's comp of B =
$$\begin{array}{r} 999 \\ 014 \\ \hline 985 \end{array}$$

$$\begin{array}{r} 185 \\ 235 \\ \hline 220 \end{array}$$

~~2220~~

~~42~~

~~222~~

~~999~~

~~320~~

329

320

−10

~~9's comp of 320~~

$$\begin{array}{r} 985 \\ +235 \\ \hline 1220 \\ +1 \end{array}$$

>> subtraction by using r's complement method A − B
(2's & 10's method)

Step 1: Determine r's complement of B and add this to A.

Step 2: If A>B, in this case carry is always generated and discard / remove carry from result obtained in above step.
If A<B, in this case carry is not generated and final answer will be again r's complement of result obtained from step1 with negative sign.

Q Perform the following opⁿ using 2's comp.

```
  1011        2's comp of B = 1's comp + 1
- 1010                      = 0101 + 1
   ?                        = 0110
```

```
      0110
    + 1011
   x(1)0001    = (0001)
Answer = 0001
```

>> Advantage of 2's complement over 1's complement

Computer always uses 2's complement method instead of 1's complement method because in 2's complement method we discard carry but in 1's complement method we add again carry to LSD so only one adder circuit is required in 2's complement method. Hence, less hardware, high speed.

Q Perform the following operation using 10's comp.

$$\begin{array}{r} 298 \\ -\ \ 9 \\ \hline 289 \end{array}$$

10's comp of 9 = 991

$$\begin{array}{r} 999 \\ -009 \\ \hline 990 \\ +\ \ \ 1 \\ \hline 991 \end{array}$$

$$\begin{array}{r} 991 \\ +\ 298 \\ \hline 1289 \end{array}$$

discard 1

Answer = 289

» Signed Number

There are three methods to represent any signed number

(1) sign Magnitude

MSB bit represents signed bit & remaining bits represent magnitude of that number. If signed bit (MSB) is '1', it indicates no. is negative and if it's value is '0', it indicates no. is positive.

Eg: +5 ⇒ 0 101 } → Binary representation
−5 ⇒ 1 101
↳ sign

128 64 32 16 8 1
1 1 1

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8 bit +57 ⇒ 00111001 } Total should be 8 bit
-57 ⇒ 10111001 }

(2) 1's complement method
In this positive number is represented same as in case of sign magnitude method and for negative number determine 1's complement of N.

Eg: +5 ⇒ 0101
-5 ⇒ 1010

Range of Number in 1's complement method is
$+(2^{n-1}-1)$ to $-(2^{n-1}-1)$

(3) 2's complement method
For positive number representation is same as in case of above two method and for negative number determine 2's complement of N.

Eg: +5 ⇒ 0101
-5 ⇒ 1010
+ 1
1011
-5 ⇒ 1011

Range :- $+(2^{n-1}-1)$ to $-(2^{n-1})$

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Q How many bits are required to represent −64 by using 1's complement method and 2's complement method?

+64 = 01 00 0000

−64 = 10111111 (1's complement)

+1

−64 = 11000000 (2's complement)

8 bits required.

(1's) range $= +(2^{n-1}-1)$ to $-(2^{n-1}-1)$

$n = 7 \Rightarrow -63$ ✗

$n = 8 \Rightarrow -127$ ✓

(2's) range $= +(2^{n-1}-1)$ to $-(2^{n-1})$

$n = 7 = -64$ ✓

For 1's = 8 bits

2's = 7 bits

Q Assuming all numbers in 2's complement form which number is divisible by 11111 011

a) 11100100 b) 11100111

c) both a & b d) None of these.

$$11111011 \xrightarrow{\text{2's comp}} \begin{array}{r} 00000100 \\ +1 \\ \hline 00000101 \end{array}$$

$= -5$

$$11100100 \longrightarrow \begin{array}{r} 00011011 \\ +1 \\ \hline 00011100 \end{array}$$

16 8 4 2 1

$= -28$

$$11100111 \longrightarrow \begin{array}{r} 00011000 \\ +1 \\ \hline 00011001 \end{array}$$

$= -25$

» <u>Arithematical</u> Operation in signed number

i) $+5-6$

ii) $-5-6$

iii) $6-5$

} by using 2's comp. Method.

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(i) +5 + (-6)

+5 → 0101
+6 → 0110
-6 → 1001
+1
~~00~~10
0101
1111

(ii) -5 -6

+5 → 0101
-5 → 1010
+ 1
1011

-6 → 1010

= 1011
+ 1010
x 1 0101
+1
0101

⇒ ~~0110~~

1001
+1
1010

(iii) 6-5

+6 → 0110
-5 → 1010
x 1 0000

= 0000

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Q Perform the following operation using 2's complement & make 8 bit result.

i) +32 − 49 → −17

+32 = 00100000.
+49 = 00110001

−49 = 11001110
+1
11001111

Verify
11101111
00010000.
+1
00010001

00100000
+ 11001111
11101111 you cannot see ye wahi Hai

00010001 = −17

» BCD Number (8421 code) (Decimal code)
There are 10 digits from 0 to 9
Binary coded Decimal − BCD.
Replace every digit by equivalent 8421 code

Q Represent 275.4 into BCD

2	7	5	.	4
8421	8421	8421		8421

(001001110101.0100)

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» Addition of two BCD number
Add two digits of BCD number if sum is greater than 9, add 6 otherwise add 0.

Q Perform the following operation of BCD number

37 → 100101
25 → 011001

37	0011	0111
+ 25	0010	0101
12		1100
+6		0110
18	0110	0010
-16		
6 2		

» ~~Access~~ XS-3 Code

Add ~~every~~ three in every digit and convert with 8421 code.

Q Convert 1) 125.9 into XS-3 2) 97.5 into XS-3

125 . 9
\+ 333 . 3
8421 458 . 12

01000101 1000 . 0000

97.5
33.3
1210 . 8

(~~000100110000.1000~~)

10001010.1000

» BCD Subtraction

It is performed by using either 9's or 10's complement method.

Q Subtract a) 235 − 187 b) 87 − 6

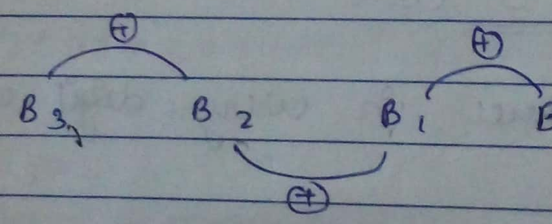
$$\begin{array}{r} 999 \\ -187 \\ \hline 812 \end{array} \rightarrow 9\text{'s comp.}$$

$$\begin{array}{r} 812 \\ +235 \\ \hline 1047 \\ +1 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 99 \\ -06 \\ \hline 93 \end{array} \leftarrow 10\text{'s comp.}$$

$$\begin{array}{r} 93 \\ +1 \\ \hline 94 \\ +87 \\ \hline \times 181 \end{array}$$

» Conversion from Binary to Gray code



$G_3 = B_3$

$G_2 = B_3 \oplus B_2$

$G_1 = B_2 \oplus B_1$

$G_0 = B_1 \oplus B_0$

Q Convert 10111 into Grey code

11100 ← Ans in grey code

Q. Convert $(32)_8 \rightarrow (\ )_G$

$(32)_8 \rightarrow (011010)_2$

Grey code $\rightarrow$ 010111

>> Conversion from Grey to Binary code

B_3 $\quad$ B_2 $\quad$ B_1 $\quad$ B_0

$\oplus$ $\quad$ $\oplus$ $\quad$ $\oplus$

G_3 $\quad$ G_2 $\quad$ G_1 $\quad$ G_0

$$G_3 = B_3$$
$$G_2 = G_3 \oplus B_2$$
$$G_1 = G_2 \oplus B_1$$
$$G_0 = G_1 \oplus B_0$$

Q. Convert $(1010)_G \rightarrow (\ )_2$

1010

Ans $\rightarrow$ 1100

No	8421	XS-3	5421	2421
0	0000	0011	0000	0000
1	0001	0100	0001	0001
2	0010	0101	0010	0010
3	0011	0110	0011	0011
4	0100	0111	0100	0100
5	0101	1000	1000	1011
6	0110	1001	1001	1100
7	0111	1010	1010	1101
8	1000	1011	1011	1110
9	1001	1100	1100	1111

» sequential code
If we add 1 to nth code and we get next code of n+1. Ex: 8421, XS-3

» Unit Distance code (cyclic code)
From going previous number to next number if only 1 variable changes that code is called unit distance code. Eg: G code

3 → 0010 → 0011
4 → 0100 → 0110
6 → 0110 → 0101
7 → 0111 → 0100

It is used in K map.

» self complementing code

n's code = 1's comp. of (9's comp. of n)

Eg: for 2421 code
4 → 0100
1's comp of (9's comp of 4)
1's comp of (5 code)
1's comp of 1011
= 0100

Eg: 2421 code, XS-3 code

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>> weighted code and Non- weighted code
weighted codes are those code which follow positional weight rule and non - weighted code are those which do not follow non weighted code.

weighted code → 8421, 2421, 5421
Non- weighted code → XS-3, Grey code

>> Alpha Numeric code
It contains alphabet, letters as well as number.
Eg:

ASCII → American standard code for information Interchange. It is 7 bit code.

EBCDIC → Extendend Binary coded decimal information change. It is 8 bit code.

Hollerith → it is used in punch machine

9 4 5
2421
1011
1111
1011
0100
5

» Error Detecting and correcting code.
when message signal (data) is transmitted at receiver from transmitter noise (error) may introduce in the system, due to this bits are changed either from 0 to 1 or 1 to 0.

one (1) parity bit is added along with message signal and at receiver we check whether there is even parity or odd parity.

EVEN PARITY → if total no. of 1's are even at receiver end.

ODD PARITY → if total no. of 1's are odd at receiver end.

ASCII

0 → 30H (0110000)   A → 41H (65D)(1000001)

1 → 31H   B → 42H (1000010)

2 → 32H

9 → 39H

Q ASCII digit D is transmitted from transmitter along with parity bit. what will be code at receiver end for A) even parity
B) odd parity

D → 44H → 1000100

i) 01000100 → even parity
ii) 11000100 → odd parity

Q An 8 bit code 11000111 is received at receiver end check whether this code is correct or not for even parity?

Total no of 1s are odd so for even parity code is not correct.

>> Imp Hamming code (single bit error detecting and correcting code)

$$n = m + p$$

$$m \leq 2^p - p - 1$$

n : total no. of bits received at receiver end

m : total no of bits of message signal

p : total no. of parity bits

Parity bits are added along with message signal and transmitted to receiver.

Q Imp If total no of received bits are 7 determine value of m and p

Let $m = 4$ $\quad 4 \leq 2^p - p - 1$

if $p = 2$

$$4 \leq 2^2 - 2 - 1$$

$$4 \leq 1 \quad \times$$

if $p = 3$

$$4 \leq 2^3 - 3 - 1$$

$$4 \leq 4 \quad \checkmark$$

$n = 7$, $m = 4$, $p = 3$

Total message bits = 4
Total parity bits = 3

NOTE: Parity bits are placed at 2^i position
i.e

7	6	5	4	3	2	1
M_7	M_6	M_5	P_4	M_3	P_2	P_1

For 15 bit hamming code

Q if message bits are 4 determine sequence of all the bits

Shortcut
maximum value = 15 and to represent 15 we require 4 bits, so total parity bits are 4
$\therefore$ $p = 4$, Hence, $m = 11$

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
M_{15}	M_{14}	M_{13}	M_{12}	M_{11}	M_{10}	M_9	P_8	M_7	M_6	M_5	P_4	M_3	P_2	P_1

To check parity bit corresponds to which digit

$P_1 \rightarrow$ 1, 3, 5, 7 (LSB 1)
$P_2 \rightarrow$ 2, 3, 6, 7 (Middle 1)
$P_4 \rightarrow$ 4, 5, 6, 7 (MSB 1)

same for 15 bit

$1 \rightarrow 001$
$2 \rightarrow 010$
$3 \rightarrow 011$
$4 \rightarrow 100$
$5 \rightarrow 101$
$6 \rightarrow 110$
$7 \rightarrow 111$
8

Q. A 4 bit message signal 1011 is transmitted at receiver end. What will be hamming code at receiver end for A) even parity
B) odd parity

$4 \leq 2^p - p - 1$

if $p = 2$ ✗

$4 \leq 4$ if $p = 3$ ✓

$\Rightarrow m = 4$

$p = 3$

$n = 7$

M_7	M_6	M_5	P_4	M_3	P_2	P_1
1	0	1	0	1	0	1

For even parity

$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (1, 1, 1, 1)$

$P_2 \rightarrow 2, 3, 6, 7 \rightarrow (0, 1, 0, 1)$

$P_4 \rightarrow 4, 5, 6, 7 \rightarrow (0, 1, 0, 1)$

For odd parity

$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (0, 1, 1, 1)$

$P_2 \rightarrow 2, 3, 6, 7 \rightarrow (1, 1, 0, 1)$

$P_4 \rightarrow 4, 5, 6, 7 \rightarrow (1, 1, 0, 1)$

M_7	M_6	M_5	P_4	M_3	P_2	P_1
1	0	1	1	1	1	0

Ans → for even parity = 1010101

for odd parity = 1011110

Q. 4 bit message signal is 0101 for odd parity. What will be 7 bit hamming code at reciver end.

M_7	M_6	M_5	P_4	M_3	P_2	P_1
0	1	0	0	1	1	0

$P_1 \rightarrow 1,3,5,7 \rightarrow (0,1,0,0)$

$P_2 \rightarrow 2,3,6,7 \rightarrow (1,1,1,0)$

$P_4 \rightarrow 4,5,6,7 \rightarrow (0,0,1,0)$

Ans → 0100110

10/08/17

Q. Message signal 11001100110 is transmitted from transmitter. what will be 15 bit hamming code at receiver end for even parity.

$n = 15$

$m = 11$

$p = 4$

M_{15}	M_{14}	M_{13}	M_{12}	M_{11}	M_{10}	M_9	P_8	M_7	M_6	M_5	P_4	M_3	P_2	P_1
1	1	0	0	1	1	0	0	0	1	1	0	0	1	1

$P_1 \rightarrow 1,3,5,7,9,11,13,15$

$P_2 \rightarrow 2,3,6,7,10,11,14,15$

$P_4 \rightarrow 4,5,6,7,12,13,14,15$

$P_8 \rightarrow 8,9,10,11,12,13,14,15$

110011000110011

Message signal

for even parity
for 5-10 marks do for both.

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Q. A 7 bit hamming code received at receiver is 1011011. determine whether this code is correct or not, if not correct; correct it for even parity

$n = 7$

$p = 3$

$m = 4$

1	0	1	1	0	1	1
M_7	M_6	M_5	P_4	M_3	P_2	P_1

$P_1 \rightarrow 1, 3, 5, 7$ (odd) $\rightarrow (1, 0, 1, 1)$ not E.P $\Rightarrow$ 1

$P_2 \rightarrow 2, 3, 6, 7 \rightarrow (1, 0, 0, 1) \rightarrow$ EP $\Rightarrow$ 0

$P_3 \rightarrow 4, 5, 6, 7 \rightarrow (1, 1, 0, 1) \rightarrow$ Not E.P $\Rightarrow$ 1

$(101) \Rightarrow 5$

Hence there is error at 5th bit and change that bit

Correct code $\rightarrow$ 1001011

Q. A 7 bit hamming code received at receiver 1010110 determine this code is correct or not if not correct it for odd parity

1	0	1	0	1	1	0
M_7	M_6	M_5	P_4	M_3	P_2	P_1

$P_1 \rightarrow 1, 3, 5, 7 \rightarrow (0, 1, 1, 1) \rightarrow 0$

$P_2 \rightarrow (2, 3, 6, 7) \rightarrow (1, 1, 0, 1) \rightarrow 0$

$P_3 \rightarrow (4, 5, 6, 7) \rightarrow (0, 1, 0, 1) \rightarrow 1$

$(100) \rightarrow 4$

Error at 4 bit, correct code $\rightarrow$ 1011110

Q. 15 bit hamming code at receiver end is 010011000110011. determine whether this code is correct or not for even parity. If not correct it

0	1	0	0	1	1	0	0	0	1	1	0	0	1	1
M_{15}	M_{14}	M_{13}	M_{12}	M_{11}	M_{10}	M_9	P_8	M_7	M_6	M_5	P_4	M_3	P_2	P_1

$P_1 \rightarrow 1,3,5,7,9,11,13,15 \rightarrow (1,0,1,0,0,1,0,0) \rightarrow 1$

$P_2 \rightarrow 2,3,6,7,10,11,14,15 \rightarrow (1,0,1,0,1,1,1,0) \rightarrow 1$

$P_4 \rightarrow 4,5,6,7,12,13,14,15 \rightarrow (0,1,1,0,0,0,1,0) \rightarrow 1$

$P_8 \rightarrow 8,9,10,11,12,13,14,15 \rightarrow (0,0,1,1,0,0,1,0) \rightarrow 1$

$(1111) \rightarrow 15$

Error at 15th bit

correct code → 110011000110011

Q. A 7 bit hamming code is received as 1011110. determine whether this code is correct for odd parity

1	0	1	1	1	1	0
M_7	M_6	M_5	P_4	M_3	P_2	P_1

$P_1 \rightarrow 1,3,5,7 \rightarrow (0,1,1,1) \rightarrow 0$

$P_2 \rightarrow 2,3,6,7 \rightarrow (1,1,0,1) \rightarrow 0$

$P_4 \rightarrow 4,5,6,7 \rightarrow (1,1,0,1) \rightarrow 0$

$(000) \rightarrow 0$

No error in this code