

# Vector Differentiation

## Vector function

A vector function is like a normal function of any variable.

like :

$$\vec{r} = \vec{r}(t)$$

$\vec{r}$  is a function of scalar  $t$ .

And when  $\vec{r}$  move along a curved path then position vector of  $\vec{r}$  is given by :

$$\vec{r}(t) = f_1(t)\hat{i} + f_2(t)\hat{j} + f_3(t)\hat{k}$$

where;

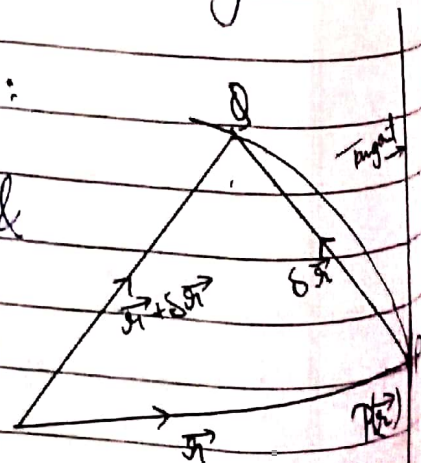
$f_1(t)$  is a component along  $x$ -axis

$f_2(t)$  is a component along  $y$ -axis

$f_3(t)$  is a component along  $z$ -axis

## Vector Differentiation :

Let  $O$  be the origin &  $P$  is the initial position of a moving particle at time  $t$ .



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and  $Q$  is the position of the particle after " $\delta t$ " time when it travel a distance of  $\delta \vec{r}$

Thus,

we can write position vectors of as :

$$\vec{OP} = \vec{r}$$

$$\vec{OQ} = \vec{r} + \delta \vec{r}$$

$$\begin{aligned}\vec{PQ} &= \vec{OQ} - \vec{OP} \\ &= (\vec{r} + \delta \vec{r}) - \vec{r} \\ &= \delta \vec{r}\end{aligned}$$

$\frac{\delta \vec{r}}{\delta t}$  is a vector, as  $\delta t \rightarrow 0$ ,  $P \rightarrow Q$

and the chord  $PQ$  becomes a tangent at  $P$ .

We define,  $\frac{d\vec{r}}{dt} = \lim_{t \rightarrow 0} \frac{\delta \vec{r}}{\delta t}$

$\frac{d\vec{r}}{dt}$  is a directional vector of the tangent at  $P$ , which is differential (derivative) of  $r$  (w.r.t) with respect to ' $t$ '. It gives velocity at  $P$ .

also  $\frac{d^2 \mathbf{r}}{dt^2}$  is second order derivative of  $\mathbf{r}$ . It gives acceleration of particle at P.