Simplify the following expression using Boolean algebra

(1) $Z = AB + A(B+C) + B(B+C)$

$= AB + AB + AC + BB + BC$

$= AB + AC + B + BC$ | $AB + AB = AB$, $B.B = B$

$= AB + AC + B(1+C)$

$= AB + AC + B$ | $1 + C = 1$

$= B + AB + AC$

$= B(1+A) + AC$ | $1 + A = 1$

$\boxed{Z = B + AC}$ ANS

(2) $Y = ABC + A\overline{B}C + AB\overline{C}$

$= AC(B+\overline{B}) + AB\overline{C}$

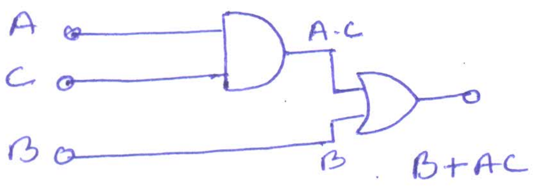
$= AC + AB\overline{C}$ | $B + \overline{B} = 1$

$= A(C + B\overline{C})$

$= A(C+B)$ | $C + B\overline{C} = C + B$

$\boxed{Y = A(B+C)}$ ANS

1



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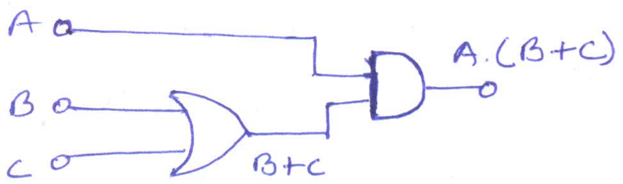


(3) $Z = AB + ABC + \bar{A}B + A\bar{B}C$

$= AB + \bar{A}B + ABC + A\bar{B}C$

$= B(A+\bar{A}) + AC(B+\bar{B})$

$\boxed{Z = B + AC}$

$A+\bar{A} = 1$
$B+\bar{B} = 1$

(4) $Z = AB + AC + B\bar{C}$

$= AB(C+\bar{C}) + AC + B\bar{C}$

$C+\bar{C} = 1$

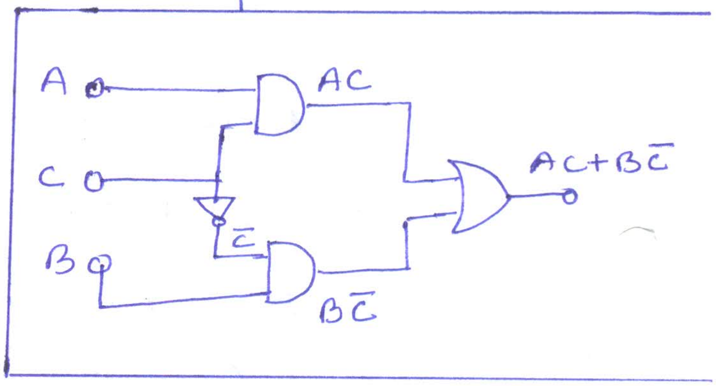
$= ABC + AB\bar{C} + AC + B\bar{C}$

$= ABC + AC + AB\bar{C} + B\bar{C}$

$= AC(B+1) + B\bar{C}(A+1)$

$A+1 = 1$
$B+1 = 1$

$\boxed{Z = AC + B\bar{C}}$ ANS



(5) $Z = (A+B)(\bar{A}+C)$

$= A\bar{A} + AC + \bar{A}B + BC$

$= AC + \bar{A}B + BC$

$A+\bar{A} = 1$

$= AC + \bar{A}B + (A+\bar{A})BC$

$= AC + \bar{A}B + ABC + \bar{A}BC$

$= AC + ABC + \bar{A}B + \bar{A}BC$

$1+B = 1$
$1+C = 1$

$= AC(1+B) + \bar{A}B(1+C)$

$\boxed{Z = AC + \bar{A}B}$ ANS

Realize the following using NAND gates.

(6) $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}B + \bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}B(C+\bar{C}) + (A+\bar{A})\bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \underline{\bar{A}B\bar{C}} + \bar{A}BC + \underline{\bar{A}B\bar{C}} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$

$= \bar{A}\bar{B}(C+\bar{C}) + \bar{A}B(\bar{C}+C) + A\bar{B}\bar{C}$

$= \bar{A}\bar{B} + \bar{A}B + A\bar{B}\bar{C}$

$= \bar{A}(\bar{B}+B) + A\bar{B}\bar{C}$

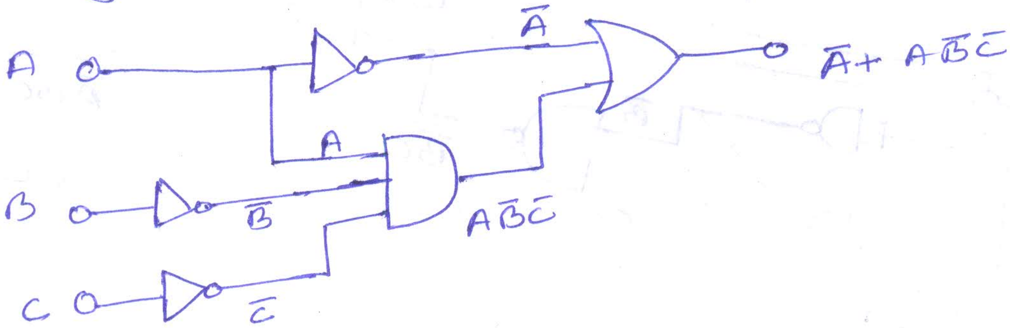
$\boxed{Z = \bar{A} + A\bar{B}\bar{C}}$ ANS

$\bar{A}B\bar{C} + \bar{A}B\bar{C} = \bar{A}B\bar{C}$

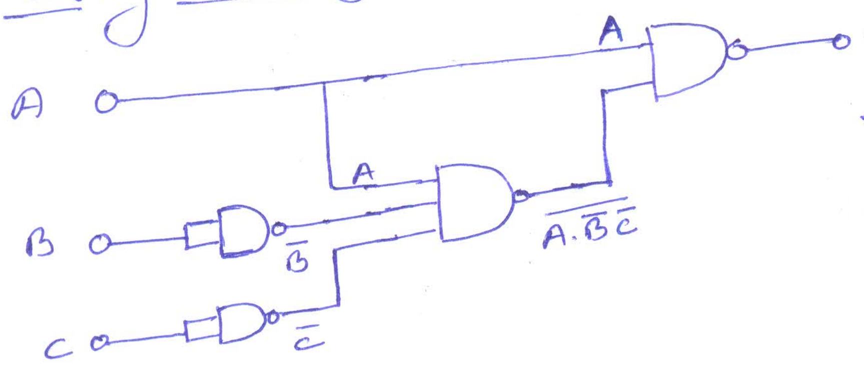
$B + \bar{B} = 1$

$C + \bar{C} = 1$

using gates



Using NAND gate



$= \overline{A \cdot \overline{A\bar{B}\bar{C}}}$

$= \overline{\bar{\bar{A}} \cdot \overline{A\bar{B}\bar{C}}}$

$= \overline{\overline{\bar{A} + A\bar{B}\bar{C}}}$

$= \bar{A} + A\bar{B}\bar{C}$

(7) $Y = \bar{A}BC + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

$= \bar{A}BC + \bar{B}\bar{C}(\bar{A} + A)$ $\quad | \ \bar{A} + A = 1$

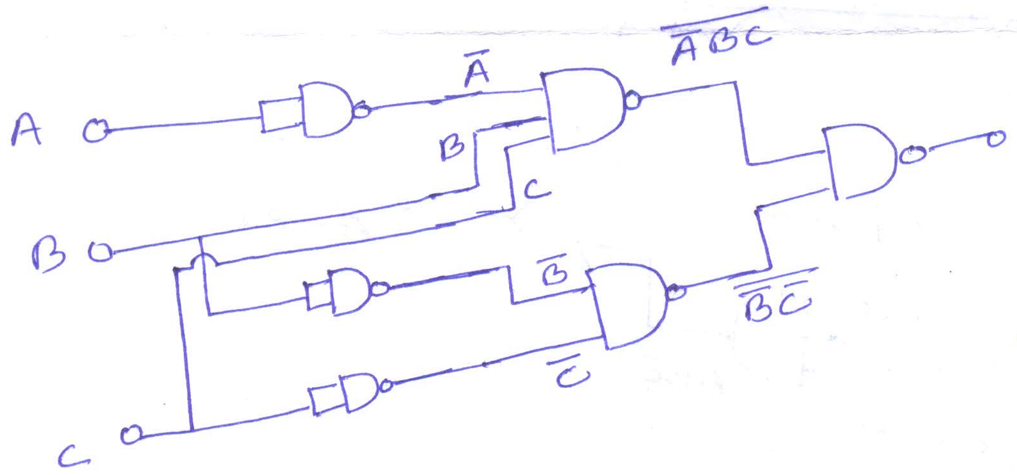
$$\boxed{Y = \bar{A}BC + \bar{B}\bar{C}}$$

for realization using NAND gates

$$Y = \overline{\overline{\bar{A}BC + \bar{B}\bar{C}}}$$

$$= \overline{\overline{\bar{A}BC} \cdot \overline{\bar{B}\bar{C}}}$$

$$= \overline{\overline{\bar{A}BC} \cdot \overline{\bar{B}\bar{C}}}$$



$= \overline{\overline{\bar{A}BC} \cdot \overline{\bar{B}\bar{C}}}$

$= \overline{\overline{\bar{A}BC + \bar{B}\bar{C}}}$

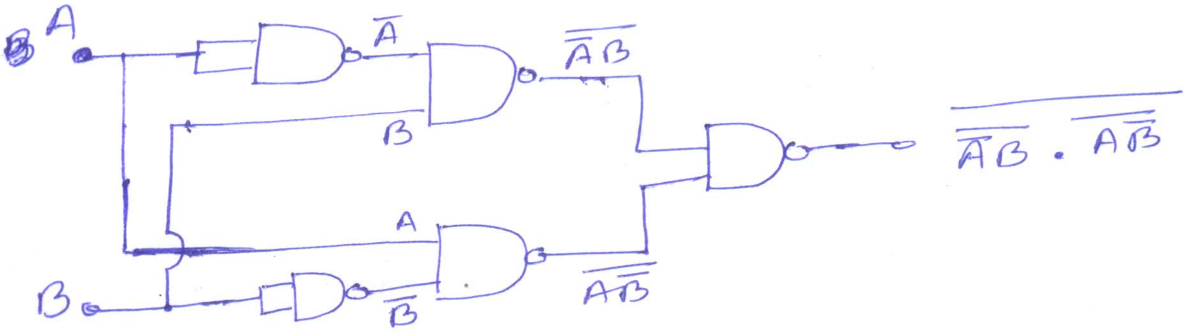
$= \bar{A}BC + \bar{B}\bar{C}$

(8)

$$Y = A\bar{B} + \bar{A}B = \text{XOR Gate using NAND}$$

$$= \overline{\overline{A\bar{B} + \bar{A}B}}$$

$$= \overline{\overline{A\bar{B}} \cdot \overline{\bar{A}B}}$$

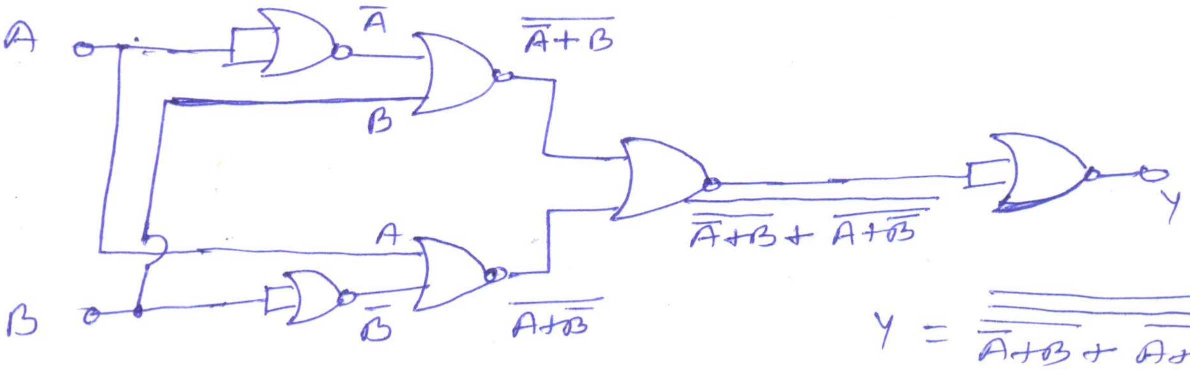


(9) ~~using~~ XOR using NOR gates

$$Y = A\bar{B} + \bar{A}B = \overline{\overline{A\bar{B}}} + \overline{\overline{\bar{A}B}}$$

$$= \overline{\bar{A} + \bar{\bar{B}}} + \overline{\bar{\bar{A}} + B}$$

$$= \overline{\bar{A} + B} + \overline{A + \bar{B}}$$



$$Y = \overline{\overline{\overline{\bar{A}+B} + \overline{A+\bar{B}}}}$$

$$Y = \overline{\bar{A}+B} + \overline{A+\bar{B}}$$

Standard Representations of logical functions :

1. Sum of Products (SOP)

Ex. (i) $AB + \bar{A}B + A\bar{B}$

(ii) $AB + \bar{B}CD + AB\bar{C}$

2. Product of Sum (POS)

Ex. (i) $(A+\bar{B})(A+B+C)(B+C)$

(ii) $(\bar{A}+C)(\bar{B}+A)(A+\bar{B}+\bar{C})$

Canonical forms :-

When each term of a logical expression contains all literals, it is called a canonical form of the logical variables. If each term in SOP and POS forms contains all the literals then these are known as canonical or standard SOP and POS forms respectively.

Each individual term in standard SOP form is called minterm and in standard POS form as maxterm.

Exp (1) Convert $A + A\bar{B}$ into standard SOP form.

$$A + A\bar{B} = A(B+\bar{B}) + A\bar{B}$$

$$= AB + A\bar{B} + A\bar{B}$$

$$= \underbrace{AB + A\bar{B}}_{\text{minterm}}$$

(ii) Convert $(A+B)(B+C)$ into standard POS form.

$$(A+B)(B+C) = (A+B+C\bar{C})(B+C+A\bar{A})$$

$$= (A+B+C)(A+B+\bar{C})(B+C+A)(B+C+\bar{A})$$

$$\boxed{\because A+BC = (A+B)(A+C)}$$

same

$$= (A+B+C)(A+B+\bar{C})(A+B+C)(\bar{A}+B+C)$$

$$= (A+B+C)(A+B+\bar{C})(\bar{A}+B+C) \qquad [\because A \cdot A = A]$$

$\downarrow$

maxterm

Minterm and Maxterm :-

$$Y = ABCD + A\bar{B}CD + A\bar{B}\bar{C}D + AB\bar{C}D$$

above expression can be expressed in the form of minterm.

$$Y = \underbrace{ABCD}_{1111} + \underbrace{A\bar{B}CD}_{1011} + \underbrace{A\bar{B}\bar{C}D}_{1001} + \underbrace{AB\bar{C}D}_{1101}$$

$1111 \rightarrow 15 \rightarrow m_{15}$

$1011 \rightarrow 11 \rightarrow m_{11}$

$1001 \rightarrow 9 \rightarrow m_{9}$

$1101 \rightarrow 13 \rightarrow m_{13}$

$\cancel{Y = \Sigma m}$

$$\boxed{Y(A,B,C,D) = \Sigma m(9, 11, 13, 15)}$$

Then the corresponding maxterm form is

$$Y(A,B,C,D) = \Pi M(0,1,2,3,4,5,6,7,8,10,12,14)$$

Maxterm convertion can be done by given method.

$$Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)$$

$$\begin{array}{cccc}
\underbrace{0\quad 0\quad 1} & \underbrace{0\quad 1\quad \phi} & \underbrace{1\quad 0\quad 0} & \underbrace{0\quad 1\quad 0} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
1 & 3 & 4 & 2 \\
\downarrow & \downarrow & \downarrow & \downarrow \\
M_1 & M_3 & M_4 & M_2
\end{array}$$

$$Y(A,B,C) = \prod M(1,2,3,4)$$

then minterm form is

$$Y(A,B,C) = \sum m(0,5,6,7)$$

Karnaugh Maps

(1) $Y(A, B, C) = \sum m(0, 2, 4, 5, 6)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0 — 1	1	3	2 — 1
A	4 — 1	5 — 1	7	6 — 1

$\boxed{Y = \bar{C} + A\bar{B}}$ Ans

(2) $Y(A, B, C, D) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0 — 1	1 — 1	3	2 — 1
$\bar{A}B$	4 — 1	5 — 1	7	6 — 1
AB	12 — 1	13 — 1	15	14 — 1
$A\bar{B}$	8 — 1	9 — 1	11	10

$Y = \bar{C} + \bar{A}\bar{D} + B\bar{D}$

Ans

③ $Y(A,B,C,D) = \sum m(0,2,4,6,7,8,10)$

$+ \sum d(12,14,15)$

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: 1	1:	3:	2: 1
$\bar{A}B$	4: 1	5:	7: 1	6: 1
AB	12: X	13:	15: X	14: X
$A\bar{B}$	8: 1	9:	11:	10: 1

$$\boxed{Y = \bar{D} + BC}$$ ANS.

(4) $Y(A,B,C,D) = \Sigma m(0,1,2,12,13) + \Sigma d(8,9,10)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: 1	1: 1	3:	2: 1
$\bar{A}B$	4:	5:	7:	6:
AB	12: 1	13: 1	15:	14:
$A\bar{B}$	8: X	9: X	11:	10: X

$$\boxed{Y = A\bar{C} + \cancel{\bar{B}\bar{C}} + \bar{A}\bar{B}\bar{D}}$$ ANS

Redundant group

Problems on K-map

1) $Y = \Sigma m(1, 2, 3, 7) = B\bar{C} + AB + A\bar{C}$

2) $Y = \Sigma m(0, 3, 5, 6) = \bar{A}(\bar{B}\bar{C} + BC) + A(B\bar{C} + \bar{B}C)$

3) $Y = \Sigma m(0, 1, 3, 5, 6, 7) =$ [redacted] $\bar{A}\bar{B} + AB +$ [redacted] C

4) $Y = \Sigma m(0, 1, 2, 3, 4, 6) = \bar{B} + \bar{C}$

5) $Y = \Sigma m(0, 1, 2, 4, 6) = \bar{B} + \bar{A}\bar{C}$

6) $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C = \bar{A}B + \bar{A}\bar{C} + A\bar{B}C$

7) $Y = \Sigma m(1, 5, 7, 9, 11, 13, 15) = \bar{C}D + BD + AD$

8) $Y = \Sigma m(1, 3, 5, 9, 11, 13) = D(\bar{B} + \bar{C})$

9) $Y = \Sigma m(1, 3, 4, 5, 7, 9, 11, 13, 15) = Y = \bar{A}B\bar{C} + D$

10) $Y = \Sigma m(1, 2, 9, 10, 11, 14, 15) = \bar{B}(\bar{C}D + C\bar{D}) + AC$

11) $Y = \Sigma m(4, 5, 8, 9, 11, 12, 13, 15) = \bar{C}(A + B) + AD$

Redundant Group

12) $Y = \Sigma m(1, 5, 6, 7, 11, 12, 13, 15) = \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + ACD$

13) $Y = \Sigma m(0, 1, 2, 5, 13, 15) = \bar{A}\bar{B}\bar{D} + ABD + \bar{A}\bar{C}D$

14) $Y = \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + A\bar{C} + \bar{B}$

$= \bar{B} + A\bar{C} + \bar{A}C\bar{D}$

Don't care Cadn

15) $Y = \Sigma m(1, 3, 7, 11, 15) + d(0, 2, 5) = CD + \bar{A}\bar{B}$

16) $Y = \Sigma m(1, 3, 11, 15) + d(0, 2, 5, 8, 14) = CD + \bar{A}\bar{B}$

17) $Y = \Sigma m(0, 1, 5, 9, 13, 14, 15) + d(3, 4, 7, 10, 11)$

$= D + (AC + \bar{A}\bar{C})$