Cauchy = Euler Homogeneous Equation

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots a_n y = X$$

put $x = e^z$ $\qquad z = \log x$

$$x^n D^n y + a_1 x^{n-1} D^{n-1} y + \cdots a_n y = X$$

Put ;

$$x D y = D_1 y$$

$$x^2 D^2 y = D_1 (D_1 - 1) y$$

$$x^3 D^3 y = D_1 (D_1 - 1)(D_1 - 2) y$$

Example :

$$x^2 y_2 + y = 3x^2$$

$$D_1 (D_1 - 1) y + y = 3e^{2z}$$

put $x^2 = e^{2z}$ $\quad$ as $x = e^z$

$$[D_1 (D_1 - 1) + 1] y = 3e^{2z}$$

For C.F

$$D_1 (D_1 - 1) + 1 = 0$$

$$D_1^2 - D_1 + 1 = 0$$

$$D_1 = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}\, i}{2}$$

$$C.F = e^{\frac{1}{2}z}\left[c_1 \cos \frac{\sqrt{3}}{2} z + c_2 \sin \frac{\sqrt{3}}{2} z\right]$$

$$C.F = e^{\frac{1}{2}\log x}\left[c_1 \cos\left(\frac{\sqrt{3}}{2} \log x\right) + c_2 \sin\left(\frac{\sqrt{3}}{2} \log x\right)\right]$$

$$C.F = \sqrt{x}\left[c_1 \cos\left(\frac{\sqrt{3}}{2} \log x\right) + c_2 \sin\left(\frac{\sqrt{3}}{2} \log x\right)\right]$$

For P.I

$$P.I = \frac{3}{[D_1 (D_1 - 1) + 1]} e^{2z}$$

$$= \frac{3}{2(1) + 1} e^{2z} = \frac{3e^{2z}}{3} = e^{2\log x}$$

$$= x^2$$