Q. $U_n = \sum (-1)^n \frac{n}{n^2+1}$

$|U_n| = \frac{n}{(n^2+1)}$ & $|U_{n+1}| = \frac{n+1}{(n+1)^2+1}$

$|U_n| - |U_{n+1}|$

$\frac{n}{(n^2)+1} - \frac{n+1}{(n+1)^2+1}$

$= \frac{n^2+n-1}{(n^2+1)((n+1)^2+1)} = +ve$

for n is positive

$|U_n| - |U_{n+1}| > 0 \Rightarrow$ positive

$U_n > U_{n+1}$

~~∴ series is conv~~

$\lim_{n\to\infty} |U_n| = \lim_{n\to\infty} \frac{n}{n^2+1} \quad \lim_{n\to\infty} \frac{n}{n^2(1+\frac{1}{n})}$

$\frac{1}{\infty} = 0$

∴ series is convergent

Q. $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \dots \dots (-1)^{n-1} \frac{1}{n\sqrt{n}}$

$|U_n| - |U_{n+1}|$ is $+ve$

$\lim_{n\to\infty} |U_n| = \lim_{n\to\infty} \frac{1}{n\sqrt{n}} = \frac{1}{\infty} = 0$

∴ convergent.

using Ratio test

① $\sum \frac{n^2}{3^n}$

② $\sum \frac{x^{n-1}}{3^n n}$

③ $\frac{1^2 2^2}{1!} + \frac{2^2 3^2}{2!} + \dots \frac{n^2(n+1)^2}{n!}$

② # Cauchy's Root test

If $\sum U_n$ is positive term series such that $\lim_{n\to\infty} (U_n)^{1/n} = k$ then

① if $k < 1$ the series is convergent

② if $k > 1$ the series is divergent

③ if $k = 1$ test fail.

① $U_n = \sum \left[\frac{n}{(n+1)}\right]^{n^2}$

Root's Test

$\lim_{n\to\infty} (U_n)^{1/n} = \lim_{n\to\infty} \left[\left(\frac{n}{n+1}\right)^{n^2}\right]^{1/n}$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$

$\lim_{n\to\infty} (n^{1/n}) = 1$

$\lim_{n\to\infty} \left[\frac{n}{n+1}\right]^n$

$\lim_{n\to\infty} \left[\frac{1}{\left(1+\frac{1}{n}\right)^n}\right] = \frac{1}{e} < 1$

∴ by root test given series is convergent

③ # Alternative series test [Lebnitz's rule for convergence]

A series in which terms are alternatively negatively is called alternative series

$u_1 - u_2 + u_3 - u_4 + u_5 - \dots$

condⁿ for convergency

① if each ~~test~~ term is numerically less than its preciding term.

② and $\lim_{n\to\infty} |u_n| = 0$

then series is convergent

otherwise TEST FAIL

$K = x \therefore$ if

$x < 1 \rightarrow$ convergent
$x > 1 \rightarrow$ divergent
$x = 1 \rightarrow$ test fail

Now put $x = 1$ in U_n

$$U_n = \frac{\sqrt{n}}{\sqrt{n^2+1}}$$

let take $V_n = \frac{1}{n^{1/2}}$

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} \sqrt{n} \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}}$$

$$= 1$$

Now $V_n = \frac{1}{n^{1/2}}$ then $p < 1 \therefore$ divergent

Now by comparision

U_n is also divergent

$\therefore$ when

$x = 1$, U_n = divergent.

for taking of V_n: take max. power of n common from Num. & Denom. and ~~solve~~ solve

$V_n = \frac{1}{n}$ by p-test

$p = 1$. V_n is divergent

by comparison test U_n is divergent

Q. $\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \dots + \frac{2n-1}{n(n+1)(n+2)}$

$U_n = \frac{2n-1}{n(n+1)(n+2)}$

Let $V_n = \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{U_n}{V_n} = \lim_{n\to\infty} \frac{(2n-1)\,n^2}{n(n+1)(n+2)}$

$\lim_{n\to\infty} \frac{\cancel{n}\left(2-\frac{1}{n}\right)\cancel{n^2}}{\cancel{n}\left(1+\frac{1}{n}\right)\cancel{n}\left(1+\frac{2}{n}\right)}$

$= \frac{2}{1} = 2$ finite

Check $V_n = \frac{1}{n^2}$ by p-test

$p = 2 > 1$ convergent.

(1) D' ALEMBERT'S RATIO TEST

If $\sum U_n$ is a positive term series such that

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = k$ then

series is convergent if $k < 1$

series is divergent if $k > 1$

if $k = 1$ TEST FAIL

Q. test the convergence

$U_n = \sum \frac{n^2}{2^n}$

Ratio test

$U_{n+1} = \frac{(n+1)^2}{2^{n+1}}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{(n+1)^2}{2^{n+1}} \times \frac{2^n}{n^2}$

$= \frac{\cancel{n^2}\left(1+\frac{1}{n}\right)^2 \times \cancel{2^n}}{\cancel{2^n}\times 2\,\cancel{n^2}}$

$= \frac{1+\frac{1}{n}}{2} = \frac{1}{2} < 1$

Series { finite no. of terms are finite
infinite no. of terms are infinite

NOTE

(1) The sum of first n terms of a series are denoted by

$$S_n = \sum u_n$$

concept of convergence, divergence & oscillation

(i) $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum u_n$ = finite (given series is convergent

(ii) $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum u_n$ = infinite (given series is divergent

(iii) $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum u_n$ = not unique (given series is oscillatory

Q. $1+2+3 \dots n$

$u_n = n$

$S_n = \sum u_n$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \sum u_n = \infty$

divergent

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots \frac{1}{2^n}$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2^n} = \frac{1}{\infty} = 0$

series is convergent

p-Series Test

The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} \dots \infty$ is

convergent if $p > 1$
divergent $p \leq 1$

Comparison Test

If two positive series $\sum u_n$ & $\sum v_n$ be such that

$\lim_{n\to\infty} \frac{u_n}{v_n}$ = finite then both series u_n & v_n are converges or diverges together

Q. Test the series

$U_n = \sum \frac{1}{n+10}$

by comparison test
take $v_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{U_n}{V_n} = \frac{n}{n+10} = \frac{n}{n}\left(\frac{1}{1+10/n}\right)$

$= \lim_{n\to\infty} \frac{1}{1+10/n} = 1$ finite

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$$\lim_{x\to 0} \frac{x^3-y^3}{x^2+y^2} = \frac{0}{0}$$

put $y = mx$

$$\lim_{x\to 0} \frac{x^3-m^3x^3}{x^2+m^2x^2} = \frac{x^3(1-m^3)}{x^2(1+m^2)}$$

$$\lim_{x\to 0} x\left(\frac{1-m^3}{1+m^2}\right) = 0$$

along $y = mx^2$

$$\lim_{x\to 0} \frac{x^3-m^3x^6}{x^2+m^2x^4} = \frac{x^3(1-m^3x^3)}{x^2(1+m^2x^2)}$$

$$\lim_{x\to 0} \frac{x(1-m^3x^2)}{1+m^2x^2}$$

$= 0$

lim of $f(x,y)$ is exist at $(0,0)$

$$\lim_{xy\to(00)} f(xy) = 0$$

and $f(00) = 0$

$\therefore$ continuous function.

Sequence series

SEQUENCE

It is a successive term or number formed according to some definite rule

$3, 5, 7, \ldots \ldots (2n+1)$

u_n, n^{th} term

if a sequence tend to limit l then we write $\boxed{\lim_{n\to\infty} u_n = l}$

If limit of sequence is finite we say sequence is convergent otherwise divergent (not finite)

Ex $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots \ldots \frac{1}{n^2}$

$U_n = \frac{1}{n^2}$

$\lim U_n = \lim_{n\to\infty} \frac{1}{n^2} = 0$ finite

this sequence is convergent.

SERIES

A series is the sum of sequence

let $u_1, u_2, \ldots u_n$ be a sequence

then expression

$u_1 + u_2 + \ldots \ldots u_n$ is series

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Q. Evaluate $\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2 y}{x^4+y^2}$

$\lim_{x\to 0} \frac{x^2(0)}{x^4+0} = \frac{0}{x^4} = 0$

$\lim_{y\to 0} \frac{0(y)}{0+y^4} = \frac{0}{y^4} = 0$

along $y = mx$

put $y = mx$

$\lim_{x\to 0} \frac{x^3 m}{x^4+m^2x^2} = \frac{x^2(mx)}{x^2(x^2+m^2)}$

$= \frac{0}{m^2} = 0$

along $y = mx^2$

put $y = mx^2$

$\lim_{x\to 0} \frac{x^4 m}{x^4+m^2x^4} \quad \lim_{x\to 0} \frac{m}{1+m^2} = f_4$

$f_1 = f_2 = f_3 \neq f_4$

$\therefore$ limit does not exist

Continuity

A function $f(x,y)$ is said to be continous at a point (a,b) if

$\lim_{xy\to ab} f(xy) = f(a,b)$

or

$|f(xy) - f(ab)| < \epsilon \quad \forall \quad |x-a| < \delta$ & $|y-b| < \delta$

NOTE :-

if the limit of a function at (ab) exist and the value of a function at (ab)

$f(xy)$ is continuous at (a,b)

Q. Test the function $f(x,y) = \frac{x^3-y^3}{x^2+y^2}$, $(xy) \neq (0,0)$

0, $(x,y) = (0,0)$

is continous at $(0,0)$

Q. Find $\lim_{(x,y)\to(1,2)} x^2+2y$

$$\lim_{\substack{x\to 1\\ y\to 2}} x^2+2y = \lim_{y\to 2} 1+2y$$
$$= 1+2\times 2$$
$$= 5$$

$$\lim_{x,y\to(1,1)} \left(\frac{xy}{x^2+y^2}\right) = \frac{0}{0}$$

Let $y = mx$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2 m}{x^2+m^2x^2} = \lim_{x\to 0}\left(\frac{m}{1+m^2}\right)$$

$\frac{m}{1+m^2}$ at $m=-1$

$f_1 = \frac{-1}{2}$

at $m = 1$

$f = \frac{1}{2}$

$f_1 \neq f$

$\therefore$ limit does not exist

$y = mx^2$ $\quad y = x^2$

$$\frac{x\cdot mx^2}{x^2+mx^4} = \frac{mx^3}{x^2+mx^4} = \frac{m}{1+}$$

$\frac{xy\,x^3}{x^2+y^4x^4}$

x^4+y^2

$y^2 = x^2$

$x^2 = y^2$

$y = x$

$y = -x$

$(1\,2)$

$y = mx$

$y =$

or

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limit for two variable

A function f is said to tends to limit 'l' as a point $(x,y) \to (a,b)$ if every arbitrary small positive number ε

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such that $|f(x-y) - l| < \varepsilon$

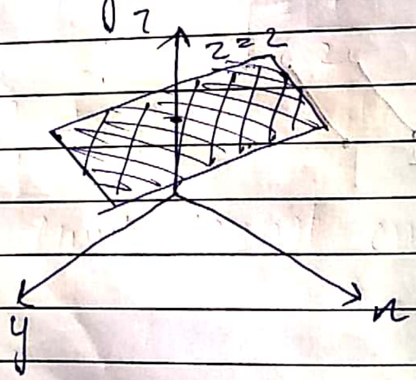
$|x-a| \le \delta$, $|y-b| \le \delta$

or

$$\boxed{\lim_{(x,y)\to(a,b)} f(x,y) = l}$$

or $$\boxed{\lim_{\substack{x\to a \\ y\to b}} f(x,y) = l}$$

existance of limit



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LIMITS AND CONTINUITY for two variable

Limits for one variable

Let a function $y = f(x)$ is defined at every point in the neighbourhood of point 'a' if x approches to a then value of function $f(x) \to$ 'l' (any scalar) we

limit of a func. is l at $x = a$

$$\boxed{\lim_{x \to a} f(x) = l}$$

or

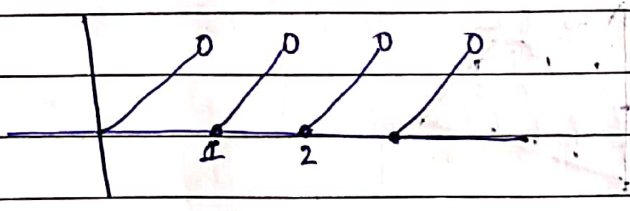
$$|f(x) - l| < \varepsilon \;\; \forall \;\; |x - a| < \delta$$

where δ & ε lies b/w 0 & 1

Existing of a limit of function

LHL = RHL

$$f^{-}(a) = f^{+}(a)$$



$f(1) = 0$