SOLUTION OF LINEAR EQUATION:

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right] \rightarrow \text{NON HOMOGENEOUS}$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B$$

$$[A \vdots B] = \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

if $\rho(A \vdots B) = \rho(A) =$ no. of unknown] UNIQUE

$\rho(A \vdots B) = \rho(A) <$ no. of unk.] INFINITE

$\rho(A|B) \neq \rho(A) \rightarrow$ NO SOLN:
NON CONSISTENT.

HOMOGENEOUS LINEAR EQUATION → always have a solution

$$AX = 0$$

if $\rho(A)$

= no. of unknown		< no. of unknown
unique | trival sol^n	let k and find value ←	infinite no. of non trival sol^n
each sol^n is zero		

By performing elementary row and column transformation any non zero matrix A can be reduced to any one of the matrix following forms called as normal form of A.

(a) I_r

(c) $\left[\begin{array}{c} I_r \\ \hline 0 \end{array}\right]$

(b) $[I_r \mid 0]$

(d) $\left[\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right]$

Then $\rho(A) = r$

I is identity matrix.

<u>linear dependance and independence of vectors:</u>

① vectors $x_1, x_2 --- x_n$ are said to be linearly dependent if there exists n scaler s.t $(\lambda_1 \lambda_2 -- \lambda_n)$] not all zero

$\lambda_1 x_1 + \lambda_2 x_2 + --- + \lambda_n x_n = 0$

② ______________ linearly independent ______________ (all zero)

$\lambda_1 x_1 + \lambda_2 x_2 + -- + \lambda_n x_n = 0.$

if $\lambda_1 = \lambda_2 = \lambda_3$ ∴ L.I

$\lambda_1 \neq \lambda_2 \neq \lambda_3$ ∴ L.D

SOLUTION OF LINEAR EQUATION:

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right] \rightarrow \text{NON HOMOGENEOUS}$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B$$

$$[A \vdots B] = \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

if $\delta(A \vdots B) = \delta(A) = \text{no. of unknown}$] UNIQUE

$\delta(A \vdots B) = \delta(A) < \text{no. of unk.}$] INFINITE

$\delta(A \vdots B) \neq \delta(A)$ $\rightarrow$ NO SOLN: NON CONSISTENT.

HOMOGENEOUS LINEAR EQUATION $\rightarrow$ always have a solution

$$AX = 0$$

if $\delta(A)$

= no. of unknown unique | trival sol^n each sol^n is zero	let k and find value $\leftarrow$	< no. of unknown infinite no. of non trival sol^n

NORMAL FORM $\rightarrow$ conical form. (correction above "conical": "on")

By performing elementary row and column transformation any non zero matrix A can be reduced to any one of the matrix following forms called as normal form of A.

(a) I_r

(b) $\left[\, I_r \,\vdots\, 0 \,\right]$

(c) $\left[\begin{array}{c} I_r \\ \hline 0 \end{array}\right]$

(d) $\left[\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right]$

Then $\rho(A) = r$

I is identity matrix.

Linear dependence and independence of vectors:

(1) Vectors $x_1, x_2, \ldots, x_n$ are said to be linearly dependent if there exists n scalar s.t

$\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n = 0$ $\left.\begin{array}{l}(\lambda_1\, \lambda_2 \cdots \lambda_n)\end{array}\right]$ not all zero

(2) ——— linearly independent ——— (all zero)

$\lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n = 0.$

if $\lambda_1 = \lambda_2 = \lambda_3$: L.I

$\lambda_1 \neq \lambda_2 \neq \lambda_3$: L.D.

2^{nd} Method:

if vectors are given like $x_1 = [1,1,0]$
$x_2 = [1,0,1]$
$x_3 = \{0,1,1\}$

$$A = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

find rank ; if rank = no. of vectors (ie 3 here)

$\therefore$ L.I

if rank < no. of vectors

$\therefore$ L.D

Eigen values: If A is a square matrix of order n, we can form the matrix $A - \lambda I$ where λ is a scaler and I is an identity matrix.

$|A - \lambda I| x_1 = 0$

$\downarrow$

get characteristic eq^n.

$\downarrow$

find value of λ $\rightarrow$ eigen values

put eigen values $\lambda_i = 1, 2, 3$

$[A - \lambda_1 I][x_1] = 0$

get $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ values

eigen vector

Eigen vector cannot be 0.

for repeated eigen values:

eg $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$\therefore (2-\lambda)(2-\lambda)(1-\lambda) - (1-\lambda) = 0$

$(1-\lambda)\left[(2-\lambda)^2 - 1\right] = 0$

$1-\lambda = 0$ | $(2-\lambda)^2 - 1 = 0$

$\lambda = 1$ | $\therefore \lambda = 1, 3$

$\lambda = 1, 1, 3$ → repeated value

$[A - 1\lambda]X = 0$ → $[A - 3\lambda]X_3 = 0$

X_1 X_2

ek bar x_3 ko 0 lelo | ek bar x_2 ko 0 lelo

but eigen vectors should not be same.

sq. matrix A is
→ of a matrix A is the reduction of A to a diagonal form D

→ DIAGONALISATION

$$\boxed{D = P^{-1} A P}$$

P = Modal Matrix
D = Spectral Matrix of A

$$\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

diagonal elements will be eigen values.

Matrix A will only be diagonalizable only if its vectors are L.I

vectors = rank of A.

eigen vectors vertically

$P = [X_1 \quad X_2 \quad X_3]$

Get P^{-1} → $\frac{\text{adj } P}{|P|}$

$$\boxed{P^{-1} A P = D}$$

Cayley-Hamilton Theorem:

Every square matrix satisfies its own characteristic equation.

Power of a Matrix by diagonalisation:

$D = P^{-1} A P$

$$\boxed{A^n = P D^n P^{-1}}$$

$\therefore$ eg $\boxed{A^4 = P D^4 P^{-1}}$

D^4 will be $\begin{bmatrix} \lambda_1^4 & 0 & 0 \\ 0 & \lambda_2^4 & 0 \\ 0 & 0 & \lambda_3^4 \end{bmatrix}$

Please bta de.

Adj [A] =

Q. $x+y+z=-3$

$3x+y-2z=-2$

$2x+4y+7z=7$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$$

$$\delta(A|B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & 7 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 2 & 5 & 13 \end{array}\right]$$

$R_3 \to R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & 0 & 20 \end{array}\right]$$

$R_2 \to R_2 / -2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 5/2 & -7/2 \\ 0 & 0 & 0 & 20 \end{array}\right]$$

$\delta(A) = 2$

$\delta(A|B) = 3$

$\delta(A) \neq \delta(A|B) =$ No sol^n

Cayley - Hamilton Theorem:

Every square matrix satisfies its own characteristic equation.

Power of a Matrix by diagonalisation:

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$\therefore$ eg $\boxed{A^4 = P D^4 P^{-1}}$ → D^4 will be $\begin{bmatrix} \lambda_1^4 & 0 & 0 \\ 0 & \lambda_2^4 & 0 \\ 0 & 0 & \lambda_3^4 \end{bmatrix}$

Please bta de.

Adj [A] =

Q. $x + y + z = -3$

$3x + y - 2z = -2$

$2x + 4y + 7z = 7$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} -3 \\ -2 \\ 7 \end{bmatrix}$$

$$\delta(A|B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & 7 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 2 & 5 & 13 \end{array}\right]$$

$R_3 \to R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & 0 & 20 \end{array}\right]$$

$R_2 \to R_2 / -2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 1 & 5/2 & -7/2 \\ 0 & 0 & 0 & 20 \end{array}\right]$$

$\delta(A) = 2$

$\delta(A|B) = 3$

$\delta(A) \neq \delta(A|B) =$ No sol^n

Q. $4x - 2y + 6z = 8 \rightarrow 2x - y + 3z = 4$

$x + y - 3z = -1$

$15x - 3y + 9z = 21 \rightarrow 5x - y + 3z = 7$

$$[A|B] = \left[\begin{array}{ccc|c} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 1 & 1 & -3 & -1 \\ 5 & -1 & 3 & 7 \end{array}\right]$$

$R_1 \Leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 3 & 7 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 5R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & -3 & 9 & 6 \\ 0 & -6 & 18 & 12 \end{array}\right]$$

$R_2 \rightarrow R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right.$$

$-6 - 2(-3)$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & -1 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$

$\rho(A \mid B) = 2$

$\therefore \rho(A) = \rho(A \mid B) = 2 < 3$

$\therefore$ infinite sol^n

$x_1 + x_2 - 3x_3 = -1$

$x_2 - 3x_3 = -2$

$x + y - 3z = -1$ let $x_3 = 0$

$y - 3z = -2$

let $z = k$

$y = 3k - 2$ let $k = 1$

$= 1$ $z = 1$

$(xi + yj + zk)$

$\therefore x = -1 + 3z - y$

$= -1 + 3 - 1$

$= 3 - 2$

$= 1$

$1 + 1 + 1 = 3$

$\therefore X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Q. $x_1 + 3x_2 + 2x_3 = 0$
$2x_1 - x_2 + 3x_3 = 0$
$3x_1 - 5x_2 + 4x_3 = 0$
$x_1 + 17x_2 + 4x_3 = 0$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\delta(A) =$ $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$-14 - 2(-7)$
$-14 + 14$

$R_4 \to R_4 + R_3$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2/-7$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1/7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\delta(A) = 2 < 3$ $\therefore$ infinite sol

$\therefore x$

$x_1 + 3x_2 + 2x_3 = 0$

$x_2 + \frac{x_3}{7} = 0$

let $x_2 = k$

$\frac{x_3}{7} = -k$ let $k = \frac{1}{7}$

$x_3 = -7k$

Q. $x_1 = (2,2,1)^T$
$x_2 = (1,3,1)^T$
$x_3 = (1,2,2)^T$

$$x_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad x_3 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Let $\lambda_1, \lambda_2, \lambda_3$ be three scalers.

$\therefore$ $$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$$

$$\lambda_1 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + \lambda_3 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = 0$$

$$2\lambda_1 + \lambda_2 + \lambda_3 = 0$$
$$2\lambda_1 + 3\lambda_2 + 2\lambda_3 = 0$$
$$\lambda_1 + \lambda_2 + 2\lambda_3 = 0$$

$$\frac{\lambda_1}{2-3} = \frac{\lambda_2}{(4-2)} = \frac{\lambda_3}{6-2} = k$$

$$\frac{\lambda_1}{-1} = \frac{\lambda_2}{-2} = \frac{\lambda_3}{4} = k$$

$$\lambda_1 = -k$$
$$\lambda_2 = -2k$$
$$\lambda_3 = 4k$$

Let $k = -1$

$\therefore$ $\lambda_1 = 1$ $\qquad$ $\lambda_1 \neq \lambda_2 \neq \lambda_3$ $\quad \therefore$ L.D

$\lambda_2 = +2$

$\lambda_3 = -4$