Number System

Main Idea for Number System

Counting and representation of quantity Magnitude

↳ Initially we learnt uniary Number System

Complex Calculation uses decimal (Base-10)

↳ Uniary → 111 → 3

11111 → 5

$5\times10^2 \; 3\times10^1 \; 4\times10^0$

(534) Σ digit x P.V

5×10^2 3×10 4×1

↳ from early age the idea of Counting from uniary and then extended to decimal.

Number System

- Real life
  - Unary (Base-1)
  - Decimal (Base-10)
- Computer Science
  - Binary (Base-2)
  - Octal (Base-8)
  - Hexadecimal (Base-16)

Unary - 0

Decimal - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Binary - 0, 1

Octal - 0, 1, 2, 3, 4, 5, 6, 7

Hexadecimal - 0 to 9, A, B, C, D, E, F

Base of Number System or Radix

(Representing the Value of Number System)

$(0,1,2 \ldots, r-1)_r$ → r → radix (no. of digits in a particular Number system)

For Ex:- $r = 1$

$(0)_1$ → Total Symbols are 1

$r = 10$

$(0,1,2, \ldots 9)_{10}$ → Total Symbols = 10

$(0,1,2,3,4)_5$ → Total symbols = 5

$(985)_7$ → wrong number

Base 7

So symbols will be $(0,1,2,3,4,5,6)_7$

$(345)_7$

$3 \times 7^2 \quad 4 \times 7^1 \quad 5 \times 7^0$

~~Comp~~ Computer is digital or deals with (HIGH or LOW), (YES OR NO), so we started with base 2.

$r = 2$ → $(0,1)_2$

101

$1 \times 2^2 \quad 1 \times 2^1 \quad 1 \times 2^0$

$4 + 0 + 1 = 5$

↳ Base 2 is tedious & time consuming

↳ So combining of bits started like

3 in groups → Octal

4 in groups → Hexadecimal

(for long calculation)

Base-16 → $(0, \text{---} \, 15)_{16}$

(0, 1, ... 9, A, B, C, D, E, F)

A → 10, B → 11, C → 12, D → 13, E → 14, F → 15

$\underbrace{01}\underbrace{010}\underbrace{1011}$ → octal

283

10 16

$\underbrace{1110}\underbrace{1111}$ → Hexadecimal

Base Conversion

$(\quad)_x \rightarrow (\quad)_y$

$(\quad)_{10}$

So main idea will be any base to any base

1st convert it base (x) base 10 & base 10 to base y

Find out the base or radix

a) Determine the possible unknown base of a Relation

Ex:- $\sqrt{22} = 6$

Maximum value of digit = 6

$\boxed{\text{base} \geq 7}$

Process

i) Convert both sides to decimal

$$(\sqrt{22})_b = 6_b$$

$(10)_2 \xrightarrow{\text{decimal}} 1 \times 2^1 + 1 \times 2^0 = 2 + 1 = 3$

(binary to decimal)

$$(\sqrt{\overset{1}{2}\overset{0}{2}})_b = \overset{0}{6}_b$$

Σ digit x b (Place Value)

$$\sqrt{2 \times b^0 + 2 \times b^1} = 6 \times b^0$$

$$\sqrt{2 + 2b} = 6$$

Squaring both sides

$$2 + 2b = 36$$

$$2b = 34$$

$$b = \frac{34}{2} = 17$$

base is 17