Linear differential with Constant Coefficient

General form : $\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_n y = X$

$$D^n + a_1 D^{n-1} + \cdots + a_n y = X$$

$a_1, a_2, a_3 \ldots = a_n$ = Constant

X = Constant / func. of x

Symbolic Representation : $\boxed{f(D)y = X}$

Solution : y = Complementary factor (CF) + Particular Integral (P.I)

* Complementary factor : To find the complementary factor, first we have to make R.H.S = 0 ($f(D)y = 0$).

Then, write auxillary Equation $f(D) = 0$

Solve for roots.

Check nature of roots.

Nature of roots	C.F
Real root:	
(i) If one real root	$c_1 e^{mx}$
(ii) If two real roots	$c_1 e^{m_1 x} + c_2 e^{m_2 x}$
(iii) If three real roots	$c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$
(iv) Two real and repeated roots	$(c_1 + c_2 x) e^{mx}$
(v) Three real & repeated roots	$(c_1 + c_2 x + c_3 x^2) e^{mx}$
Imaginary Roots (Complex roots)	
(i) One pair of Complex roots $(\alpha \pm i\beta)$	$e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$
(ii) Two pair of (complex roots repeated) $(\alpha \pm i\beta)(\alpha \pm i\beta)$	$e^{\alpha x}[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x]$