

Top-Down Parsing: LL Parsin

Introduction to Top-Down Parsing

1. Previous lesson showed topdown parser that parses from left to right fulfills our syntactic checking needs for the compiler
 - This lesson covers LL(1) parsing
 - Top down, left to right
 - Can be automatically generated from its grammar
 - Automated scanners and parsers simplify the construction of the compiler front end

LL(1) Parser Quiz

1. Which of the following characteristics are true for an LL(1) parser?
 - They perform backtracking
 - They perform leftmost derivation (true)
 - They perform one token look-ahead (true)
 - Not every context free language can be parsed by LL(1) parsers (true)

LL(1) Parsing

1. Overview
 - Push start symbol onto stack
 - Replace non-terminal symbol on stack using grammar rules
 - If top of stack matches input token, both are to be discarded, mismatch is a syntax error
 - If, eventually, both stack and input string are empty then it is a successful parse

Sample Example

1. The Grammar
 - $S \rightarrow (s) S \mid ''$
2. The Input
 - $()\$$
 - $\$$ means end of input
3. The Stack
 - $\$$
 - $\$S$ (Replace S with a rule from the grammar)
 - $\$S)S($ (Match the top of the stack with the next input character)
 - $\$S)S$ (Remove the matching symbol)
 - $\$S)$ (Pick S going to epsilon)
 - $\$S$ (Remove the matching symbol)
 - $\$$ (Pick S going to epsilon)

LL(1) Parser

1. The top of the stack may contain tokens or non-terminals
2. How does the parser pick the right rule to match the input?
 - LL(1) parser is deterministic: rule for expansion selected by 1 token lookahead

Parsing Table

M[N][T]	()	\$
S	$S \rightarrow (S) S$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$

Parsing Table

Parse Table

1. Parse table is the brain of the parser
 - LL(1) parse table consists of a column for each token and a row for each non-terminal symbol
 - A grammar is LL(1) grammar if the associated LL(1) parsing table has at most one production rule in each table entry
 - LL(1) grammar is a proper subset of context-free grammar

Table Construction

1. How to construct the parsing table if grammar is complex?
2. Grammar
 - $\text{exp} \rightarrow \text{term exp}'$
 - $\text{exp}' \rightarrow \text{addop term exp}' \mid ''$
 - $\text{addop} \rightarrow + \mid -$
 - $\text{term} \rightarrow \text{factor term}'$
 - $\text{term}' \rightarrow \text{mulop factor term}' \mid ''$
 - $\text{mulop} \rightarrow *$
 - $\text{factor} \rightarrow (\text{exp}) \mid \text{num}$

Grammar Rules

1. The grammar must not be ambiguous
2. The grammar for LL(1) parsing must not be left recursive

First Sets

1. First Set:
 - $X \rightarrow X_1 X_2 X_3 X_4 \dots X_n$
 - First set for a symbol (on the left hand side of a rule) is the set of tokens that we find beginning the right hand side of the rule
2. Let X be a grammar symbol (a terminal or nonterminal) or ϵ . Then the set $\text{First}(X)$ is defined as follows:
 - Continue to grow the first set until we find that a first of some X_k is not null

X	First Set
If X is a terminal or ϵ	$\text{First}(X) = \{X\}$.
If X is nonterminal, then for each production rule $X \rightarrow X_1X_2\dots X_n$	$\text{First}(X)$ contains $\text{First}(X_1) - \{\epsilon\}$.
If for some $i < n$, $\text{First}(X_1), \dots, \text{First}(X_i)$ all contain ϵ	$\text{First}(X)$ contains $\text{First}(X_{i+1}) - \{\epsilon\}$
If $\text{First}(X_1), \dots, \text{First}(X_n)$ all contain ϵ	$\text{First}(X)$ contains ϵ

First Sets

First Set Quiz

- Given the following grammar:
 - $S \rightarrow ABCDE$
 - $A \rightarrow a \mid \epsilon$
 - $B \rightarrow b \mid \epsilon$
 - $C \rightarrow c$
 - $D \rightarrow d \mid \epsilon$
 - $E \rightarrow e \mid \epsilon$
- Apply the following rules and find first sets for each of the non-terminals: S, A, B, C, D, and E. Show rules applied to find the sets for each non-terminal.

Production	Rule Applied	First Set
$S \rightarrow ABCDE$	3	$\{a, b, c\}$
$A \rightarrow a/\epsilon$	2, 4	$\{a, \epsilon\}$
$B \rightarrow b/\epsilon$	2, 4	$\{b, \epsilon\}$
$C \rightarrow c$	2	$\{c\}$
$D \rightarrow d/\epsilon$	2, 4	$\{d, \epsilon\}$
$E \rightarrow e/\epsilon$	2, 4	$\{e, \epsilon\}$

First Sets Quiz

First Set Algorithm

1. Algorithm:

```
for each nonterminal X do First(X) := {}
while there are changes to any First(X) do
  for each production rule X -> X1X2...Xn do
    k := 1;
    while k <= n do
      First(X) = First(X) U (First(Xk) - {ε})
      if ε is not in First(Xk) then break;
      k := k + 1;
    if (k > n) then First(X) = First(X) U {ε}
```

2. Real world example:

- stmt -> if-stmt | other
- if-stmt -> if (exp) stmt else-part
- else-part -> else stmt | ε
- exp -> 0 | 1
- Tokens are if, else, other, 0, 1

3. First sets

- First(stmt) = {other} U First{if-stmt} = {other, if}
- First{if-stmt} = {if}
- First{else-part} = {else, ε}
- First{exp} = {0, 1}

First Set Example Quiz

1. Given the following grammar, determine the first sets.

- E -> T X
- X -> + E
- X -> ε
- T -> int Y
- T -> (E)
- Y -> * T
- Y -> ε

Symbol	First Set
((
))
+	+
*	*
int	int
Y	$\epsilon, *$
X	$\epsilon, +$
T	int, (
E	int, (

First Sets Quiz

First Set Quiz 2

- Given the following grammar, determine the first sets.
 - $E \rightarrow TE'$
 - $E' \rightarrow +TE'$
 - $E' \rightarrow e$
 - $T \rightarrow FT'$
 - $T' \rightarrow *FT'$
 - $T' \rightarrow e$
 - $F \rightarrow a$
 - $F \rightarrow b$

First Set for:

E	$\{a, b\}$
E'	$\{\epsilon, +\}$
T	$\{a, b\}$
T'	$\{\epsilon, *\}$
F	$\{a, b\}$

First Sets Quiz

Follow Sets Part 1

- What is a follow set?
 - Follow set of A is those symbols which will follow after A and is used to determine if a rule such as $A \rightarrow e$ should be invoked to remove the A to expose the tokens that follow A for matching them.
- Given a nonterminal A, the set Follow(A) is defined as:
 - If A is start symbol, then \$ is in Follow(A)
 - If there is a production rule $B \rightarrow a A B'$, then Follow(A) contains $\text{First}(B') - \{e\}$
 - If there is a production rule $B \rightarrow a A B'$ and B' is nullable, then Follow(A) contains Follow(B)
 - Notes:
 - \$ is needed to indicate end of string
 - e is never a member of Follow set

Follow Sets Part 2

- Construction:

```
for each nonterminal X do
    Follow(X) := {$} for start symbol or {} for others

while there are changes to any Follow(X) do
    for each production rule  $X \rightarrow X_1 X_2 \dots X_n$  do
        for each  $X_i$  that is a nonterminal do
            Follow( $X_i$ ) = Follow( $X_i$ ) U ( $\text{First}(X_{i+1} \dots X_n) - \{e\}$ )
            if e is in  $\text{First}(X_{i+1} \dots X_n)$  then
                Follow( $X_i$ ) = Follow( $X_i$ ) U Follow(X)
```

Follow Sets Part 3

1. Grammar:

- $\text{stmt} \rightarrow \text{if-stmt} \mid \text{other}$
- $\text{if-stmt} \rightarrow \text{if (exp) stmt else-part}$
- $\text{else-part} \rightarrow \text{else stmt} \mid \text{e}$
- $\text{exp} \rightarrow 0 \mid 1$

2. Example:

- $\text{Follow}(\text{exp}) = \{\}$
- $\text{Follow}(\text{else-part}) = \text{Follow}(\text{if-stmt}) = \text{Follow}(\text{stmt})$
- $\text{Follow}(\text{stmt}) = \{\$\} \cup \text{First}(\text{else-part}) - \{\text{e}\} \cup \text{Follow}(\text{if-stmt}) = \{\$, \text{else}\}$

Follow Set Quiz

1. Given the following grammar:

- $S \rightarrow \text{ABCDE}$
- $A \rightarrow \text{a} \mid ''$
- $B \rightarrow \text{b} \mid ''$
- $C \rightarrow \text{c}$
- $D \rightarrow \text{d} \mid ''$
- $E \rightarrow \text{e} \mid ''$

2. Find the follow sets:

Rule	First Set	Follow Set
$S \rightarrow \text{ABCDE}$	$\{\text{a}, \text{b}, \text{c}\}$	$\{\$\}$
$A \rightarrow \text{a}/\epsilon$	$\{\text{a}, \epsilon\}$	$\{\text{b}, \text{c}\}$
$B \rightarrow \text{b}/\epsilon$	$\{\text{b}, \epsilon\}$	$\{\text{c}\}$
$C \rightarrow \text{c}$	$\{\text{c}\}$	$\{\text{d}, \text{e}, \$\}$
$D \rightarrow \text{d}/\epsilon$	$\{\text{d}, \epsilon\}$	$\{\text{e}, \$\}$
$E \rightarrow \text{e}/\epsilon$	$\{\text{e}, \epsilon\}$	$\{\$\}$

Follow Sets Quiz

Follow Set Example Quiz

1. Given the following grammar, determine the follow sets.

- $E \rightarrow \text{T X}$

- $X \rightarrow + E$
- $X \rightarrow e$
- $T \rightarrow \text{int } Y$
- $T \rightarrow (E)$
- $Y \rightarrow * T$
- $Y \rightarrow e$

Symbol	Follow Sets
(N/A
)	N/A
+	N/A
*	N/A
int	N/A
Y), \$, +
X), \$
T), \$, +
E), \$

A detailed solution to this answer is in the recommended reading for this lesson

Follow Sets Quiz

Parsing Tables

1. Repeat the following two steps for each nonterminal A and production choice $A \rightarrow a$
 - For each token a in $\text{First}(a)$, add $A \rightarrow a$ to the entry $M[A, a]$
 - If e is in $\text{First}(a)$, for each element a in $\text{Follow}(A)$ (a token or \$), add $A \rightarrow a$ to the entry $M[A, a]$

Complete Example

1. Grammar
 - $\text{exp} \rightarrow \text{term exp}'$
 - $\text{exp}' \rightarrow \text{addop term exp}' \mid ''$

- addop $\rightarrow + \mid -$
 - term $\rightarrow \text{factor term}'$
 - term' $\rightarrow \text{mulop factor term}' \mid ''$
 - mulop $\rightarrow *$
 - factor $\rightarrow (\text{exp}) \mid \text{num}$
2. First sets
- $\text{First}(\text{exp}) = \{ (\text{number} \}$
 - $\text{First}(\text{exp}') = \{ + - e \}$
 - $\text{First}(\text{addop}) = \{ + - \}$
 - $\text{First}(\text{term}) = \{ (\text{number} \}$
 - $\text{First}(\text{term}') = \{ * e \}$
 - $\text{First}(\text{mulop}) = \{ * \}$
 - $\text{First}(\text{factor}) = \{ (\text{number} \}$
3. Follow sets
- $\text{Follow}(\text{exp}) = \{ \$) \}$
 - $\text{Follow}(\text{exp}') = \{ \$ \}$
 - $\text{Follow}(\text{addop}) = \{ (\text{number} \}$
 - $\text{Follow}(\text{term}) = \{ + - \$) \}$
 - $\text{Follow}(\text{term}') = \{ + - \$) \}$
 - $\text{Follow}(\text{mulop}) = \{ (\text{number} \}$
 - $\text{Follow}(\text{factor}) = \{ * + - \$) \}$
4. Predict sets
- $\text{Predict}(A \rightarrow a) = \text{First}(a)$ if $\text{First}(a)$ does not contain epsilon
 - $\text{Predict}(A \rightarrow a) = \text{First}(a) - \{e\} \cup \text{Follow}(A)$ otherwise
5. How to generate parsing table from predict sets?
- If a token t appears in $\text{Predict}(A \rightarrow a)$, put rule $A \rightarrow a$ in entry $M[A] [t]$

$M[N][T]$	(number)	+	-	*	\$
exp	exp \rightarrow term exp'	exp \rightarrow term exp'					
exp'			exp' \rightarrow ϵ	exp' \rightarrow addop term exp'	exp' \rightarrow addop term exp'		exp' \rightarrow ϵ
addop				addop \rightarrow +	addop \rightarrow -		

Final Parse Table Part 1

M[N][T]	(number)	+	-	*	\$
term	term \rightarrow factor term'	term \rightarrow factor term'					
term'			term' $\rightarrow \epsilon$	term' $\rightarrow \epsilon$	term' $\rightarrow \epsilon$	term' \rightarrow mulop factor term'	term' $\rightarrow \epsilon$

Final Parse Table Part 2

M[N][T]	(number)	+	-	*	\$
mulop						mulop \rightarrow *	
factor	factor \rightarrow (exp)	factor \rightarrow number					

Final Parse Table Part 3
