

# Top-Down Parsing: LL Parsin

## Introduction to Top-Down Parsing

1. Previous lesson showed topdown parser that parses from left to right fulfills our syntactic checking needs for the compiler
  - This lesson covers LL(1) parsing
    - Top down, left to right
    - Can be automatically generated from its grammar
    - Automated scanners and parsers simplify the construction of the compiler front end

## LL(1) Parser Quiz

1. Which of the following characteristics are true for an LL(1) parser?
  - They perform backtracking
  - They perform leftmost derivation (true)
  - They perform one token look-ahead (true)
  - Not every context free language can be parsed by LL(1) parsers (true)

## LL(1) Parsing

1. Overview
  - Push start symbol onto stack
  - Replace non-terminal symbol on stack using grammar rules
  - If top of stack matches input token, both are to be discarded, mismatch is a syntax error
  - If, eventually, both stack and input string are empty then it is a successful parse

## Simple Example

1. The Grammar
  - $S \rightarrow (s) S \mid ''$
2. The Input
  - $()\$$
  - $\$$  means end of input
3. The Stack
  - $\$$
  - $\$S$  (Replace S with a rule from the grammar)
  - $\$S)S($  (Match the top of the stack with the next input character)
  - $\$S)S$  (Remove the matching symbol)
  - $\$S)$  (Pick S going to epsilon)
  - $\$S$  (Remove the matching symbol)
  - $\$$  (Pick S going to epsilon)

## LL(1) Parser

1. The top of the stack may contain tokens or non-terminals
2. How does the parser pick the right rule to match the input?
  - LL(1) parser is deterministic: rule for expansion selected by 1 token lookahead

## Parsing Table

M[N][T]	(	)	\$
S	$S \rightarrow (S) S$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$

Parsing Table

### Parse Table

1. Parse table is the brain of the parser
  - LL(1) parse table consists of a column for each token and a row for each non-terminal symbol
  - A grammar is LL(1) grammar if the associated LL(1) parsing table has at most one production rule in each table entry
  - LL(1) grammar is a proper subset of context-free grammar

### Table Construction

1. How to construct the parsing table if grammar is complex?
2. Grammar
  - $\text{exp} \rightarrow \text{term exp}'$
  - $\text{exp}' \rightarrow \text{addop term exp}' \mid ''$
  - $\text{addop} \rightarrow + \mid -$
  - $\text{term} \rightarrow \text{factor term}'$
  - $\text{term}' \rightarrow \text{mulop factor term}' \mid ''$
  - $\text{mulop} \rightarrow *$
  - $\text{factor} \rightarrow ( \text{exp} ) \mid \text{num}$

### Grammar Rules

1. The grammar must not be ambiguous
2. The grammar for LL(1) parsing must not be left recursive

### First Sets

1. First Set:
  - $X \rightarrow X_1 X_2 X_3 X_4 \dots X_n$
  - First set for a symbol (on the left hand side of a rule) is the set of tokens that we find beginning the right hand side of the rule
2. Let X be a grammar symbol (a terminal or nonterminal) or  $\epsilon$ . Then the set  $\text{First}(X)$  is defined as follows:
  - Continue to grow the first set until we find that a first of some  $X_k$  is not null

<b>X</b>	<b>First Set</b>
If X is a terminal or $\epsilon$	$\text{First}(X) = \{X\}$ .
If X is nonterminal, then for each production rule $X \rightarrow X_1X_2\dots X_n$	$\text{First}(X)$ contains $\text{First}(X_1) - \{\epsilon\}$ .
If for some $i < n$ , $\text{First}(X_1), \dots, \text{First}(X_i)$ all contain $\epsilon$	$\text{First}(X)$ contains $\text{First}(X_{i+1}) - \{\epsilon\}$
If $\text{First}(X_1), \dots, \text{First}(X_n)$ all contain $\epsilon$	$\text{First}(X)$ contains $\epsilon$

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First Sets

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### First Set Quiz

- Given the following grammar:
  - $S \rightarrow ABCDE$
  - $A \rightarrow a \mid \epsilon$
  - $B \rightarrow b \mid \epsilon$
  - $C \rightarrow c$
  - $D \rightarrow d \mid \epsilon$
  - $E \rightarrow e \mid \epsilon$
- Apply the following rules and find first sets for each of the non-terminals: S, A, B, C, D, and E. Show rules applied to find the sets for each non-terminal.

<b>Production</b>	<b>Rule Applied</b>	<b>First Set</b>
$S \rightarrow ABCDE$	3	$\{a, b, c\}$
$A \rightarrow a/\epsilon$	2, 4	$\{a, \epsilon\}$
$B \rightarrow b/\epsilon$	2, 4	$\{b, \epsilon\}$
$C \rightarrow c$	2	$\{c\}$
$D \rightarrow d/\epsilon$	2, 4	$\{d, \epsilon\}$
$E \rightarrow e/\epsilon$	2, 4	$\{e, \epsilon\}$

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First Sets Quiz

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## First Set Algorithm

1. Algorithm:

```
for each nonterminal X do First(X) := {}
while there are changes to any First(X) do
  for each production rule X -> X1X2...Xn do
    k := 1;
    while k <= n do
      First(X) = First(X) U (First(Xk) - {ε})
      if ε is not in First(Xk) then break;
      k := k + 1;
    if (k > n) then First(X) = First(X) U {ε}
```

2. Real world example:

- stmt -> if-stmt | other
- if-stmt -> if (exp) stmt else-part
- else-part -> else stmt | ε
- exp -> 0 | 1
- Tokens are if, else, other, 0, 1

3. First sets

- First(stmt) = {other} U First{if-stmt} = {other, if}
- First{if-stmt} = {if}
- First{else-part} = {else, ε}
- First{exp} = {0, 1}

## First Set Example Quiz

1. Given the following grammar, determine the first sets.

- E -> T X
- X -> + E
- X -> ε
- T -> int Y
- T -> ( E )
- Y -> \* T
- Y -> ε

Symbol	First Set
(	(
)	)
+	+
*	*
int	int
Y	$\epsilon, *$
X	$\epsilon, +$
T	int, (
E	int, (

First Sets Quiz

### First Set Quiz 2

- Given the following grammar, determine the first sets.
  - $E \rightarrow TE'$
  - $E' \rightarrow +TE'$
  - $E' \rightarrow e$
  - $T \rightarrow FT'$
  - $T' \rightarrow *FT'$
  - $T' \rightarrow e$
  - $F \rightarrow a$
  - $F \rightarrow b$

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## First Set for:

E	$\{a, b\}$
E'	$\{\epsilon, +\}$
T	$\{a, b\}$
T'	$\{\epsilon, *\}$
F	$\{a, b\}$

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First Sets Quiz

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### Follow Sets Part 1

- What is a follow set?
  - Follow set of A is those symbols which will follow after A and is used to determine if a rule such as  $A \rightarrow e$  should be invoked to remove the A to expose the tokens that follow A for matching them.
- Given a nonterminal A, the set Follow(A) is defined as:
  - If A is start symbol, then \$ is in Follow(A)
  - If there is a production rule  $B \rightarrow a A B'$ , then Follow(A) contains  $\text{First}(B') - \{e\}$
  - If there is a production rule  $B \rightarrow a A B'$  and B' is nullable, then Follow(A) contains Follow(B)
  - Notes:
    - \$ is needed to indicate end of string
    - e is never a member of Follow set

### Follow Sets Part 2

- Construction:

```
for each nonterminal X do
  Follow(X) := {$} for start symbol or {} for others

while there are changes to any Follow(X) do
  for each production rule  $X \rightarrow X_1 X_2 \dots X_n$  do
    for each  $X_i$  that is a nonterminal do
      Follow( $X_i$ ) = Follow( $X_i$ ) U ( $\text{First}(X_{i+1} \dots X_n) - \{e\}$ )
      if e is in  $\text{First}(X_{i+1} \dots X_n)$  then
        Follow( $X_i$ ) = Follow( $X_i$ ) U Follow(X)
```

## Follow Sets Part 3

1. Grammar:

- $\text{stmt} \rightarrow \text{if-stmt} \mid \text{other}$
- $\text{if-stmt} \rightarrow \text{if (exp) stmt else-part}$
- $\text{else-part} \rightarrow \text{else stmt} \mid \text{e}$
- $\text{exp} \rightarrow 0 \mid 1$

2. Example:

- $\text{Follow}(\text{exp}) = \{\}$
- $\text{Follow}(\text{else-part}) = \text{Follow}(\text{if-stmt}) = \text{Follow}(\text{stmt})$
- $\text{Follow}(\text{stmt}) = \{\$\} \cup \text{First}(\text{else-part}) - \{\text{e}\} \cup \text{Follow}(\text{if-stmt}) = \{\$, \text{else}\}$

## Follow Set Quiz

1. Given the following grammar:

- $S \rightarrow \text{ABCDE}$
- $A \rightarrow \text{a} \mid ''$
- $B \rightarrow \text{b} \mid ''$
- $C \rightarrow \text{c}$
- $D \rightarrow \text{d} \mid ''$
- $E \rightarrow \text{e} \mid ''$

2. Find the follow sets:

Rule	First Set	Follow Set
$S \rightarrow \text{ABCDE}$	$\{\text{a}, \text{b}, \text{c}\}$	$\{\$\}$
$A \rightarrow \text{a}/\epsilon$	$\{\text{a}, \epsilon\}$	$\{\text{b}, \text{c}\}$
$B \rightarrow \text{b}/\epsilon$	$\{\text{b}, \epsilon\}$	$\{\text{c}\}$
$C \rightarrow \text{c}$	$\{\text{c}\}$	$\{\text{d}, \text{e}, \$\}$
$D \rightarrow \text{d}/\epsilon$	$\{\text{d}, \epsilon\}$	$\{\text{e}, \$\}$
$E \rightarrow \text{e}/\epsilon$	$\{\text{e}, \epsilon\}$	$\{\$\}$

Follow Sets Quiz

## Follow Set Example Quiz

1. Given the following grammar, determine the follow sets.

- $E \rightarrow \text{T X}$

- $X \rightarrow + E$
- $X \rightarrow e$
- $T \rightarrow \text{int } Y$
- $T \rightarrow ( E )$
- $Y \rightarrow * T$
- $Y \rightarrow e$

Symbol	Follow Sets
(	N/A
)	N/A
+	N/A
*	N/A
int	N/A
Y	), \$, +
X	), \$
T	), \$, +
E	), \$

*A detailed solution to this answer is in the recommended reading for this lesson*

#### Follow Sets Quiz

### Parsing Tables

- Repeat the following two steps for each nonterminal A and production choice  $A \rightarrow a$ 
  - For each token a in  $\text{First}(a)$ , add  $A \rightarrow a$  to the entry  $M[A, a]$
  - If e is in  $\text{First}(a)$ , for each element a in  $\text{Follow}(A)$  (a token or \$), add  $A \rightarrow a$  to the entry  $M[A, a]$

### Complete Example

- Grammar
  - $\text{exp} \rightarrow \text{term exp}'$
  - $\text{exp}' \rightarrow \text{addop term exp}' \mid ''$



- addop  $\rightarrow + \mid -$
  - term  $\rightarrow \text{factor term}'$
  - term'  $\rightarrow \text{mulop factor term}' \mid ''$
  - mulop  $\rightarrow *$
  - factor  $\rightarrow ( \text{exp} ) \mid \text{num}$
2. First sets
    - $\text{First}(\text{exp}) = \{ ( \text{number} \}$
    - $\text{First}(\text{exp}') = \{ + - \text{e} \}$
    - $\text{First}(\text{addop}) = \{ + - \}$
    - $\text{First}(\text{term}) = \{ ( \text{number} \}$
    - $\text{First}(\text{term}') = \{ * \text{e} \}$
    - $\text{First}(\text{mulop}) = \{ * \}$
    - $\text{First}(\text{factor}) = \{ ( \text{number} \}$
  3. Follow sets
    - $\text{Follow}(\text{exp}) = \{ \$ ) \}$
    - $\text{Follow}(\text{exp}') = \{ \$ \}$
    - $\text{Follow}(\text{addop}) = \{ ( \text{number} \}$
    - $\text{Follow}(\text{term}) = \{ + - \$ ) \}$
    - $\text{Follow}(\text{term}') = \{ + - \$ ) \}$
    - $\text{Follow}(\text{mulop}) = \{ ( \text{number} \}$
    - $\text{Follow}(\text{factor}) = \{ * + - \$ ) \}$
  4. Predict sets
    - $\text{Predict}(A \rightarrow a) = \text{First}(a)$  if  $\text{First}(a)$  does not contain epsilon
    - $\text{Predict}(A \rightarrow a) = \text{First}(a) - \{\epsilon\} \cup \text{Follow}(A)$  otherwise
  5. How to generate parsing table from predict sets?
    - If a token  $t$  appears in  $\text{Predict}(A \rightarrow a)$ , put rule  $A \rightarrow a$  in entry  $M[A][t]$

$M[N][T]$	(	number	)	+	-	*	\$
exp	exp $\rightarrow$ term exp'	exp $\rightarrow$ term exp'					
exp'			exp' $\rightarrow$ $\epsilon$	exp' $\rightarrow$ addop term exp'	exp' $\rightarrow$ addop term exp'		exp' $\rightarrow$ $\epsilon$
addop				addop $\rightarrow$ +	addop $\rightarrow$ -		

Final Parse Table Part 1

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M[N][T]	(	number	)	+	-	*	\$
term	term $\rightarrow$ factor term'	term $\rightarrow$ factor term'					
term'			term' $\rightarrow \epsilon$	term' $\rightarrow \epsilon$	term' $\rightarrow \epsilon$	term' $\rightarrow$ mulop factor term'	term' $\rightarrow \epsilon$

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Final Parse Table Part 2

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M[N][T]	(	number	)	+	-	*	\$
mulop						mulop $\rightarrow$ *	
factor	factor $\rightarrow$ ( exp )	factor $\rightarrow$ number					

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Final Parse Table Part 3

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