Introduction to Distributed Memory Models

Introduction

1. Consider computations that can't fit in the memory of a single computer or would take hundreds of years to finish using a single computer
    - Harness the collective power of many computers
    - Develop an abstract model to reason about computing across many computers
        - Network model
        - Distributed memory model
        - Message-passing model
        - Communicating Sequential Processes model
2. Message-passing model
    - Many computers collectively carrying out a computation that communicate by passing messages to each other

Simulating the Brain

1. Japanese researchers wrote a program to simulate 1% of the human brain
    - ~2 billion neurons and 10 trillion synapses
    - Synapses serve as a communication network, requiring 24 bytes of storage each
2. Suppose you have a computer with 16 GiB of RAM. How many are required to store the entire human brain?
    - 24 * 1e13 * 1e2 = 24e15 total memory
    - 24e15 / 16 * 2 ˆ 34 = 1400000 (1.4 million)
    - Sequoia (IBM supercomputer at LLNL) has ~19000 compute nodes in 2014

A Basic Model of Distributed Memory

1. Machine model: Collection of nodes connected by a network
    - Each node has a processor connected to a private memory
    - Source must put a message on the network to communicate
        - In shared memory, we read and write shared variables
2. Rules
    - Fully connected: Always a path between two nodes
    - Bidirectional links: Link can carry a message in both directions at the same time
    - A node can perform up to one send and one receive at a time
    - Cost to send/receive n words: Time to send n words is a + B * n
        - Cost to send a message is linear according to message size
        - Tmsg(n) = a + B * n where a is the latency [time] and B is inverse bandwidth [time/word]
    - K-way congestion reduces bandwidth -> a + B * n * k
        - Congestion is when messages are trying to use the same link at the same time
        - Cost is the same as if the beta term is serialized over the link

Pipelined Message Delivery

1. Consider a linear (1-D) network with P nodes:
    - Message prep: a [time]
    - Link time: t [time]
    - Number of words: n
2. How long does it take to send a message with n words, one word at a time?
    - n = 1: a + t(P-1)
    - n = 2: a + t(P-1) + t
    - n = 3: a + t(P-1) + 2t
    - n = a + t(P-2) + tn (alpha = a + t(P-2))

Getting a Feel for the Alpha Beta Model

1. Tmsg(n) = a + B * n
   - Tau = compute [time/op]
   - In practice, tau « B « a (1e-12, 1e-9, 1e-6)
2. Which are true?
   - Computation < communication, so avoid communication (true)
   - It's faster to send a few large messages than many small messages (true)
   - None of the above

Applying the Rules

1. Suppose you have a linear network with 8 nodes
   - Node 0 wants to send a message to node 2 at the same time that node 6 wants to send a message to node 3
2. How much time does it take for these messages to transmit?
   - The paths don't overlap, so the total time is a + B * n

Scenario 2 Quiz

1. Suppose you have a linear network with 8 nodes
   - Node 1 wants to send a message to node 6 at the same time that node 7 wants to send a message to node 4
2. How much time does it take for these messages to transmit?
   - a + b * n
   - I think there might be an error here; node 4 would be required to receive data from two messages at the same time, which violates the third rule

Scenario 3 Quiz

1. Suppose you have a linear network with 8 nodes
   - Node 1 wants to send a message to node 6 at the same time that node 4 wants to send a message to node 7
2. How much time does it take for these messages to transmit?
   - a + b * n * 2 because the messages are traveling in the same direction

Scenario 4 Quiz

1. Suppose you have a mesh network with 9 nodes arranged in a 3x3 grid
   - Node 0 wants to send a message to node 8 at the same time that node 4 wants to send a message to node 6
2. How much time does it take for these messages to transmit?
   - a + b * n
   - Assuming optimal pathing such that messages do not intersect

Collective Operations - Part 1

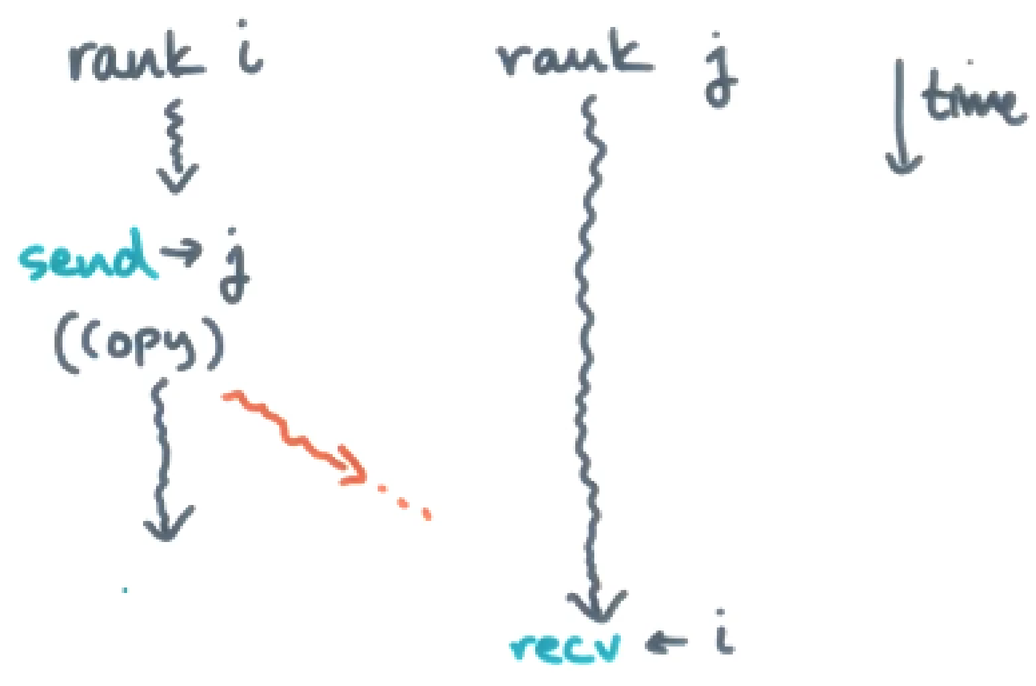
1. Tree-based reduce
   - Odd ranks send value to even ranks
   - Continue sending odd of the remaining ranks to the evens until only the zeroth node remains
     - It will contain the total sum
   - You can tell which nodes should be sending by their least significant bit
     - If LSB is 1, it should send. Otherwise, it should receive.

Point to Point

1. Sequential pseudocode
   - Single-program, multiple data (SPMD)
   - Running copy = process
     - Rank is ID of running process (unique)
     - P is the number of running processes
   - Asynchronous send
     - handle <- sendAsync(buf[1:n] -> dest)
     - Return does NOT imply buf[:] has been sent. Do not modify "buf"
   - Asynchronous receive
     - handle <- recvAsync(buf[1:n] -> src)
     - Return does NOT imply buf[:] has been received
   - Blocking wait
     - wait(handle1,. . .)
     - wait(*) waits for all pending sends and receives

Point to Point Completion Semantics

1. Return implies corresponding "buf" is available for reuse
   - => delivered, for recvAsync()
   - => not much, for sendAsync()
2. Two-sided messaging: Every send must have a matching receive



Message Paths

Send and Receive in Action

1. RANK = 1
   - text[1:5] = ' ' // empty

    - sendAsync('hello' -> 2)
    - waitAll()
    - recvAsync(text[:] <- 2)
    - waitAll()
2. RANK = 2
    - text[1:5] = ' ' // empty
    - sendAsync('world' -> 1)
    - waitAll()
    - recvAsync(text[:] <- 1)
    - waitAll()
3. What is the value of text[:] when the processes complete?
    - hello on rank 1, world on 2
    - world on rank 1, hello on 2
    - Either is possible
    - Neither is possible
    - Processes don't complete (true, this causes deadlock)

Send and Receive Revisited

1. The initial values of six nodes are as follows:
    - Rank 0: x = 6
    - Rank 1: x = 6
    - Rank 2: x = 7
    - Rank 3: x = 3
    - Rank 4: x = 8
    - Rank 5: x = 4
2. What is the final state of the x values after the following pseudocode completes?

```
for i <- 0 to P-1 do
    sendAsync(x -> (RANK+1) % P)
    recvAsync(y -> (RANK+P-1) % P)
    waitAll()
    swap(x,y)
```

```
* initial: 6, 6, 7, 3, 8, 4
* i = 0:   4, 6, 6, 7, 3, 8
* i = 1:   8, 4, 6, 6, 7, 3
* i = 2:   3, 8, 4, 6, 6, 7
* i = 3:   7, 3, 8, 4, 6, 6
* i = 4:   6, 7, 3, 8, 4, 6
* i = 5:   6, 6, 7, 3, 8, 4
```

All to One Reduce Pseudocode

1. Assume P = 2 ˆ k

```
let s = local value
bitmask <- 1
while bitmask < P do
    partner <- rank ^ bitmask
    if rank & bitmask then
        sendAsync(s -> partner)
        waitAll()
        break
    else
        recvAsync(t <- partner)
```

```
        waitAll()
        s <- s + t
    bitmask <- (bitmask << 1)
if rank = 0
    print(s)
```

All to One Reduce Pseudocode Quiz

1. Fix the pseudocode to work if P is not a power of 2
   - Only senders drop out
   - Senders have 1 at the bitmask position

```
let s = local value
bitmask <- 1
while bitmask < P do
    partner <- rank ^ bitmask
    if rank & bitmask then
        sendAsync(s -> partner)
        waitAll()
        break
    elseif partner < P
        recvAsync(t <- partner)
        waitAll()
        s <- s + t
    bitmask <- (bitmask << 1)
if rank = 0
    print(s)
```

Vector Reductions

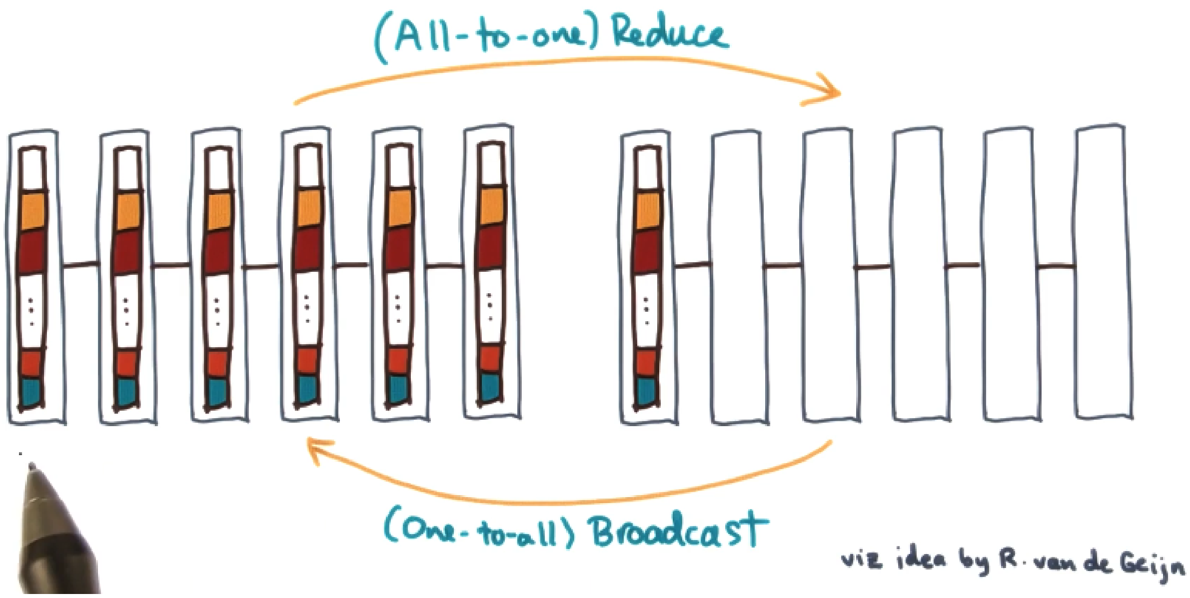
1. Vector reductions means applying the operation element-wise to a vector
   - Instead of sending a scalar, send a vector (sendAsync(s[:] -> partner)

Vector Reductions Quiz

1. What is the time to do a vector reduction?
   - a + Bn
   - a * logP + Bn
   - a + B * n * logP
   - (a + Bn) * logP (true)

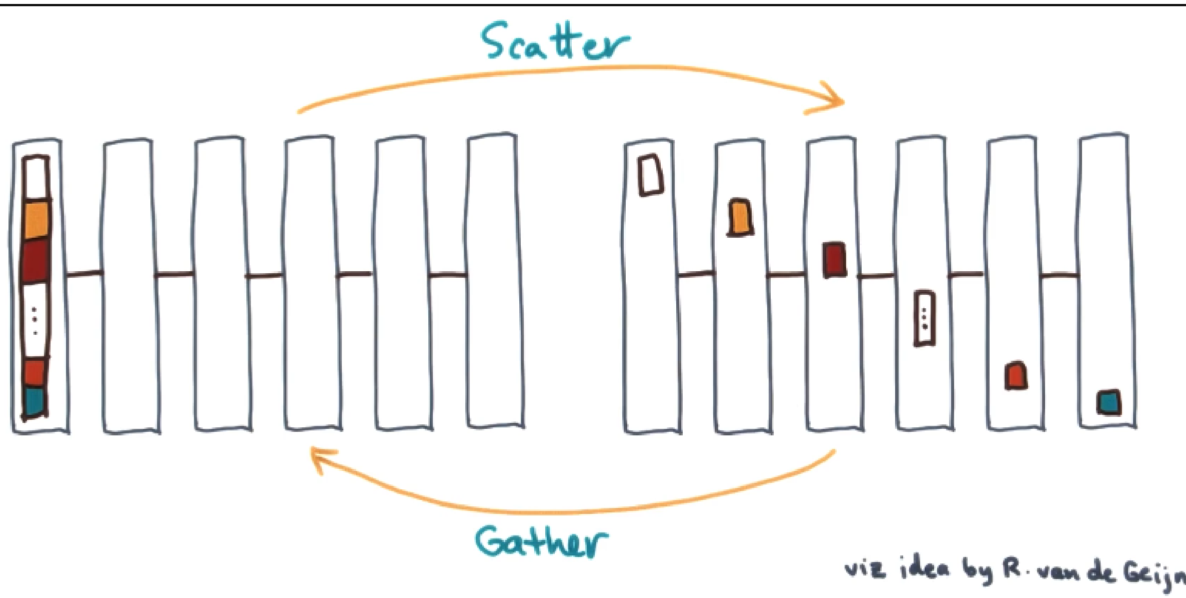
More Collectives

1. Corollary to a reduce is a one-to-all broadcast
   - One processor has all the data initially and wants to send a copy to all other processors
   - Reduce and broadcast are duals



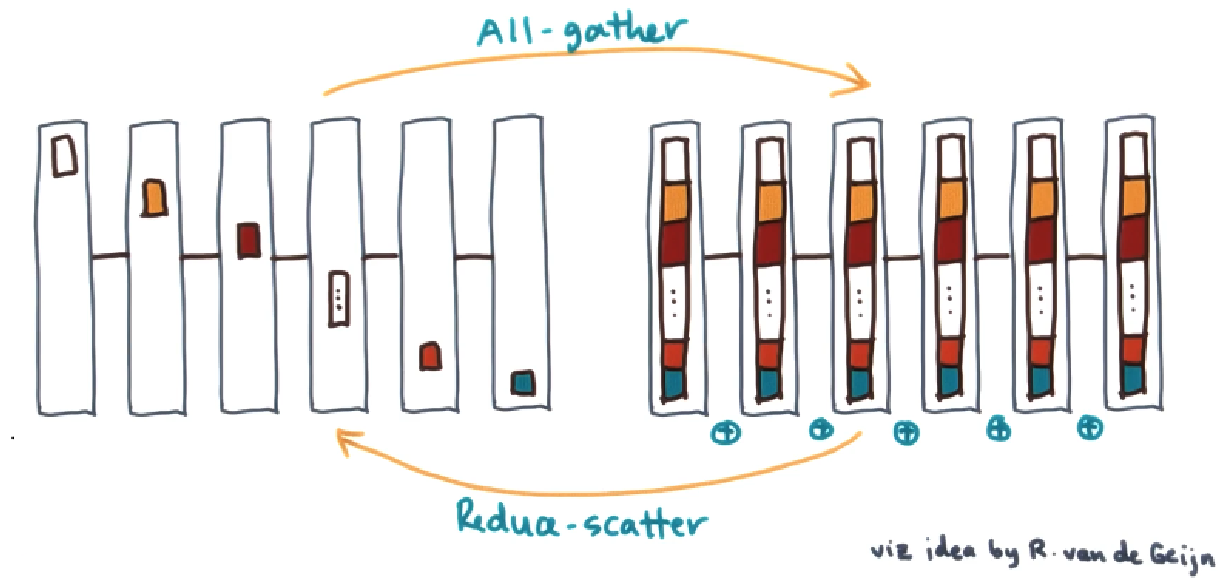
Reduce-Broadcast

2. Scatter sends a piece of its data to each of the other processors
    - The dual to a scatter is a gather



Scatter-Gather

3. All-gather: Similar to a gather, but instead of only the root having all of the data, each node contains all of the data
    - Dual is a reduce-scatter
        - All processes contain a vector of data
        - They globally reduce the vector using some sort of vector-reduce
        - Result is distributed to all processes



allGather-reduceScatter

A Pseudocode API for Collectives

1. Suppose every processor has a private array of size n
    - reduce(Alocal[1:n], root)
        - Must be executed on all processors
    - broadcast(Alocal[1:n], root)
    - gather(In[1:m], Out[1:m][1:P], root)
        - Out is only valid on the root processor
        - n = m * P
    - scatter(In[1:m][1:P], root, Out[1:m])
    - allGather(In[1:m], Out[1:m][1:P])
    - reduceScatter(In[1:m][1:P], Out[1:m])
2. Reshaping
    - reshape(A[1:m][1:n]) -> A[1:m * n]
    - reshape(A[1:mi * n]) -> A[1:m][1:n]
    - Column-major by convention

All Gather - From Building Blocks

1. Implement an allGather using reduce, broadcast, scatter, gather, reshape

```
gather(In, Out, root)
broadcast(reshape(Out), root)
```

Collective Lower Bounds

1. T(n) = (a + B * n) * log(P)
    - On a linear network, if a node can only send and receive one message at a time, we require at least log(P) rounds of communication
        - Therefore, a * log(P) is optimal
    - Each process has n words of data and must send all n words
        - T(n) = (a + B * n) * log(P) >= n * (P-1) words
        - If all nodes send their data simultaneously, the lower bound on time is n * B

    - This suggests the tree-based scheme is sending too much data by a factor of log(P)
2. Lower bound for all collectives:
    - T(n) = O(a * log(P) + B * n)

All Gather Quiz

1. If we implement allGather using the gather/broadcast approach and gather and broadcast both achieve the lower bound, is allGather optimal?
    - Yes; a constant number of optimal primitives is still optimal

Implement Scatter Quiz

1. Consider the following pseudocode:

```
scatter(In[1:m][1:P], root, Out[1:m])
    if RANK == root then
        for i != root do
            sendAsync(In[:][i], i)
    else
        recvAsync(Out[:], root)
    waitAll()
```

2. How much communication time does this algorithm need?
    - a + B * n
    - a * log(P) + B * m
    - (a + B * m) * log(P)
    - a * P + B * m
    - (a + B * m) * P (true)

Implementing Scatter and Gather - Part 2

1. Instead of the naive implementation that scales linearly with P, we need a different approach
    - Instead, split the data in half and send it to another node
    - Continue splitting in half at each node until the data has propagated to all nodes
2. What is the communication complexity?
    - Iteration i: ni = n / 2 ^ i
    - T(i) = a + B * ni = a + B * n / 2 ^ i
    - T(n) = sum(Ti) from 1 to log(P)
        - a * log(P) + B * n * (P-1) / P
        - This is the lower bound with respect to latency and bandwidth

When to Use Tree-Based Reduce

1. When is the tree-based scheme okay? (T(n) = a * log(P) + B * n * log(P))
    - B * n « a (true)
    - a * B « n
    - log(P) « P
    - n is "small" (true)
    - inverse bandwidth » latency

What's Wrong with Tree-Based Reduce?

1. What causes the B term to be suboptimal in a tree-based reduce?
    - There is redundant communication; each round sends the same data

Bucketing Algorithm for Collectives

1. Bandwidth term encourages every process sending data at each round
    - This results in P-1 communication steps
    - T(n = m * P) = (a + Bn/P)(P-1) ~ aP + Bn
        - Suboptimal with respect to the alpha term
        - This is okay if n/P « a/B

Bandwidth Optimal Broadcast

1. Give a bandwidth-optimal algorithm for broadcast:
    - Assuming allGather uses bucketing

```
broadcast(A[1:m*P], root)
    let B[1:m][1:P] <- reshape(A)
    T[1:m] = temp array
    scatter(B[1:m][1:P], root, T[1:m])
    allGather(T[1:m], B[1:m][1:P], root)
    A <- reshape(B)
```

All Reduce

1. All-reduce is similar to reduce, but the answer is on all processors
2. Which pair of collectives can be combined to obtain a bandwidth-optimal implementation of allReduce?
    - Scatter
    - Gather
    - reduceScatter (true)
    - allGather (true)

Conclusion

1. How do we think about efficiency in terms of communication and computation?
2. Message-passing model thoughts:
    - Who/how many processes
    - When/how processes communicate
    - How processes are connected
3. Open research question: Is there a framework for developing efficient algorithms independent of the network?