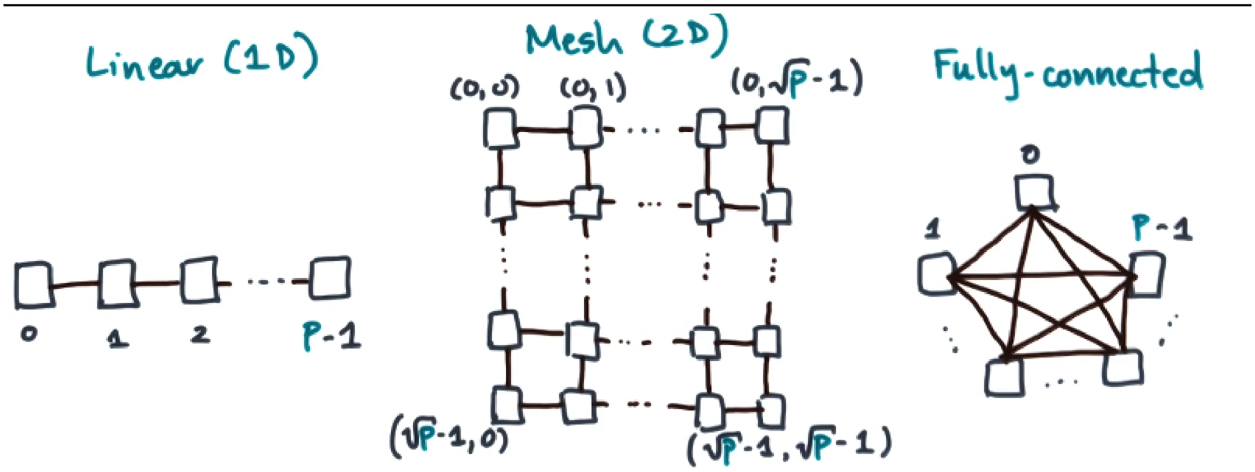
Topology

Introduction

1. Reasoning about distributed algorithms requires some consideration of the underlying network
    - If an algorithm was optimized for a fully-connected model, will it still perform well under a linear model?
2. Networks had become fast enough that we stopped worrying about the latency, leading to models like alpha-beta
    - However, as processors become faster and networks larger, we will need to consider the network topology once again
        - Begins to matter again at ~1 billion processors

Introduction to Network Models - Links and Diameter

1. Links (lambda)
    - Linear: P-1
    - Mesh: 2 * (sqrt(P)-1) * sqrt(P) ~ 2P
    - Fully-connected: P * (P-1) / 2 ~ P^2
    - Links are a proxy for cost; more links = more expensive
2. Diameter (delta) - Longest shortest path
    - Linear: P - 1
    - Mesh: 2 * (sqrt(P) - 1)
    - Fully-connected: 1
    - Proxy for the maximum distance any message must travel



Network Examples

Improve the Diameter of a Linear Network

1. Add a link (edge) to reduce the diamater of this network by half
    - Add the link between nodes 0 and 7

Improve the Diameter of a 2-D Mesh

1. Given a 2D mesh network with 16 nodes, where will adding links cut the diameter in half?
    - From (0,0) to (sqrt(P)-1,sqrt(P)-1) and (0,sqrt(P)-1) to (sqrt(P)-1,0)
    - A ring of links connecting (0,0) to (sqrt(P)-1,sqrt(P)-1) and (0,sqrt(P)-1) to (sqrt(P)-1,0)
    - Wraparound links from left to right, top to bottom

2. All three options reduce the diamater by a factor of (roughly) 2

Bisection (Band)Width

1. Bisection width: Minimum links to remove to cut the network in half
   - Equality is measured in number of nodes
2. For a linear network with 8 nodes, bisection width is 1
   - B(P) = 1
3. For a mesh network with 16 nodes, bisection width is 4
   - B(P) = sqrt(P) for a mesh
4. For a fully-connected network, there are roughly P^2/2 links
   - B(P) = P^2/4
5. Bisection width is important for an all-to-all personalized exchange
   - Send message from every node to every other node
6. Bisection bandwidth is link speed (Beta) times bisection width
   - If not all links have equal speed, look for a set of nodes that cuts the network in half and minimizes total bandwidth

Improve the Bisection of a 2-D Mesh

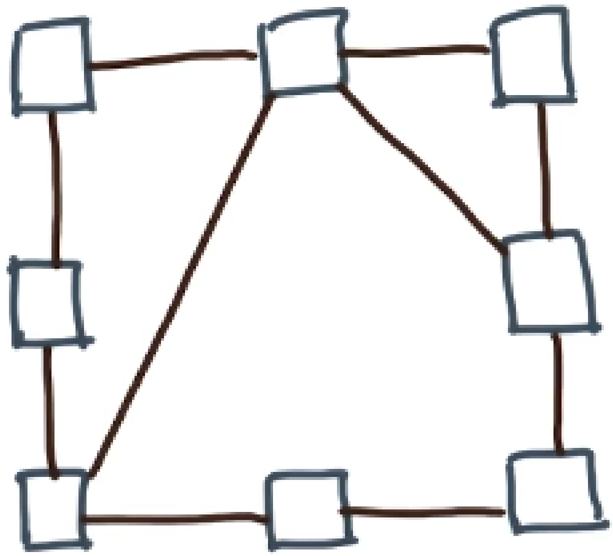
1. Given a 2D mesh network with 16 nodes, where will adding links double the bisection width?
   - From (0,0) to (sqrt(P)-1,sqrt(P)-1) and (0,sqrt(P)-1) to (sqrt(P)-1,0)
   - A ring of links connecting (0,0) to (sqrt(P)-1,sqrt(P)-1) and (0,sqrt(P)-1) to (sqrt(P)-1,0)
   - Wraparound links from left to right, top to bottom
2. Only the third option doubles the bisection width

Some Other Network Topologies

1. Tree: Compute nodes are leaves of the tree, carrier nodes don't do any actual computation (typically more links at higher levels of the tree to improve BW)
   - Links: P
   - Diameter: log(P)
   - Bisection: 1
2. d-dimensional mesh or torus: P^(1/d) nodes per dimension (extension of a mesh to higher dimensions)
   - Links: dP
   - Diameter: d * P^(1/d) / 2
   - Bisection: 2 * P^((d-1)/d)
   - Many of the world's top supercomputers use low-dimensional toroidal networks
3. Hypercube: log(P) dimensional torus
   - Links: P * log(P)
   - Diamater: log(P)
   - Bandwidth: P/2
   - Much more expensive in terms of number of wires, but lower diameter and larger bisection width
   - Build a hypercube by copying the topology of the previous dimension and connecting corresponding nodes

Diameter and Bisection

1. Consider the following network:

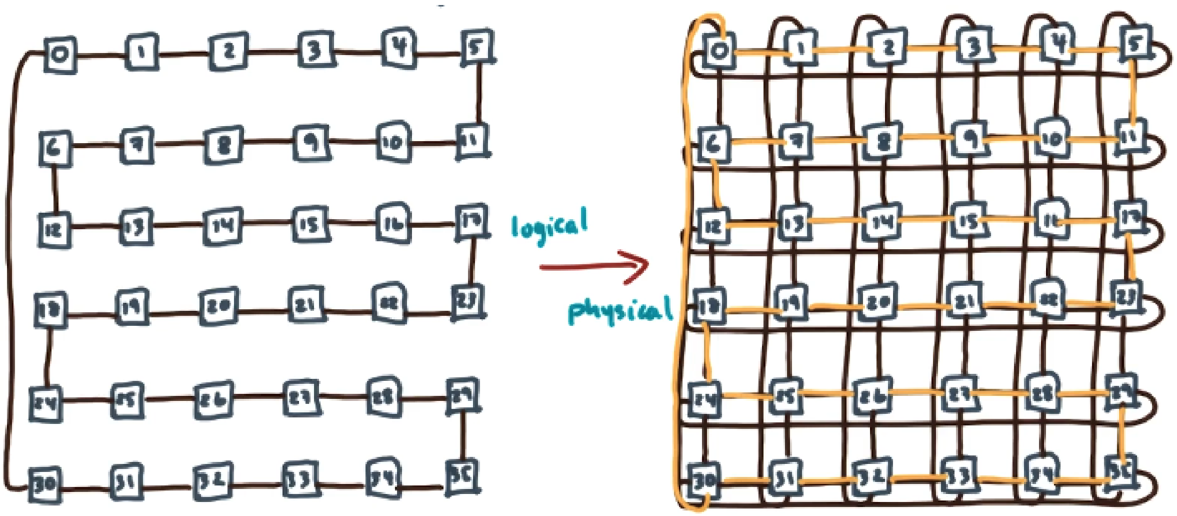


Network Examples

2. Answer the following questions:
   - How many nodes does it have?
     - P = 8
   - How many links does it have?
     - Links = 10
   - What's its diameter?
     - Diameter = 3
   - What's its bisection width?
     - Bisection width = 3

Mappings and Congestion

1. Scatter/gather implementations were designed to avoid contention for links
   - If we design an algorithm for a linear network then try to run it on a different topology, how well would it work?
     - Need to map between the networks
   - If we map a ring to a 2D torus, we observe that the edges of the torus are a superset of the edges of the ring
     - This means if there's no contention on the ring, there will be no contention on the torus either
     - The opposite isn't necessarily true; there might be contention on the ring even if there wasn't on the torus
2. Congestion: Maximum number of logical edges that map to a given physical edge
   - Congestion (ring to torus) = 1
   - Congestion (torus to ring) >= 6



Mapping a Ring to a Torus

2-D to 1-D Congestion

1. Consider the mapping from a 2D torus (logical) to a 1D ring (physical)
2. What is the congestion of this mapping?
   - sqrt(P) + 2 (two comes from the wraparound edges)

A Lower Bound on Congestion

1. Consider two hypothetical networks: one logical, one physical
   - Suppose we find a bisection in the physical network
     - Bx is the number of physical edges cut
   - Bisecting the physical network implies a cut in the logical network as well
     - L is the number of logical edges cut
   - Congestion (C) >= L / Bl
   - L >= Bl
   - C >= L / Bx >= Bl / Bx
2. Congestion for ring to 2D torus
   - Bx = 2
   - Bl = 2 * sqrt(P)
   - C >= Bl / Bx = sqrt(P)
     - Truth: C = sqrt(P) + 2

Congestion Lower Bounds

1. For which pair of networks might a congestion-free mapping be possible? (logical -> physical)
   - fully-connected -> hypercube
   - hypercube -> butterfly (true)
   - butterfly -> 3D torus
   - complete binary tree -> ring (true)
   - none of these

Exploiting Higher Dimensions

1. Consider an allgather primitive
   - Tree-based: T(n,P) = alpha * log(P) + Bn

- Bucketing: T(n,P) = alpha * P + Bn

2. Can we do better if the underlying topology is a 2D mesh instead of ring?
    - First, do allgathers along each row
        - Trow(m * sqrt(P); sqrt(P)) = a * sqrt(P) + B * m * sqrt(P)
    - Then, perform allgathers of entire rows along each column
        - Tcol(n;sqrt(P)) = a * sqrt(P) + Bn
    - Trow + Tcol = 2 * a * sqrt(P) + B * m * (P + sqrt(P))
        - Alpha now scales with sqrt(P), B is basically optimal

2D Broadcast

1. Consider a broadcast on a mesh
    - Scheme 1:
        - Tree-based broadcast in each row
        - Tree-based broadcast in each col
    - Scheme 2:
        - Scatter in all rows
        - Scatter in all cols
        - Bucket allgather in cols
        - Bucket allgather in rows
2. Which scheme sends fewer messages?
    - Scheme 1: Alpha is proportional to log(P) for a tree-based approach while bucket-allgather is proportional to P

All to All Personalized Exchange

1. Consider a distributed matrix transpose where each node has a column of data
    - Every node wants to send data of size m
        - n = m * P
    - Distance the ith message needs to travel is min(i, P-i) if we assume a ring buffer
        - Average distance = sum(min(i,P-i))/(P-1) = P/4
    - Traffic volume = P * m * (P-1) * (P/4)
    - Total bandwidth = P / B
    - Time = Volume / Speed >= B * n * (P-1) / 4
2. Algorithm for all-to-all exchange: Each node sends all the data that should be distributed each timestep (circshift)
    - Round i: send m(P - i) words
    - Total time = (a + B * m * (P/2)) * (P - 1) = a * (P-1) + B * n * (P-1)/2

All to All in Higher Dimensions

1. Which network has the best chance to reduce the asymptotic running time to O(a * log(P) + B * n * log(P))
    - Complete binary tree
    - d-dimensional torus
    - Hypercube
    - Fully-connected
2. All-to-all is intrinsically bisection limited; only hypercube and fully- connected networks have linear bisection widths or better

Conclusion

1. Idea of congestion: Concept lets you design for one topology and then estimate whether it will map well or poorly to another

2. Exploit high-dimensional networks: There are algorithmic scalability gains from using a 3D mesh network instead of 2D