

# Constraint-based Analysis

## Introduction

1. Constraint-based analysis
  - Follows a declarative program
    - Express what the analysis computes instead of how it computes it
    - Concerned with the specification of the analysis instead of the implementation
  - Specification involves constraints over program facts
  - Implementation involves solving these constraints using an off-the-shelf constraint solver
2. Advantages
  - Simplifies design and understanding of analysis
  - Allows rapid prototyping
  - Continuous performance improvement in constraints solvers
  - Datalog: Constraint programming language

## Constraint-based Analysis Motivation

1. Motivation
  - Designing an efficient program analysis is challenging:
    - Program analysis = Specification + Implementation
    - Specification is what, implementation is how
  - Null pointer analysis
    - Specification: No null pointer is dereferenced along any path in the program
    - Implementation: Many design choices
      - \* Forward vs backward traversal
      - \* Symbolic vs explicit representation

## What is Constraint-based Analysis?

1. Constraint-based analysis: Analysis designer defines the specification of the program analysis using a constraint language
  - Constraint solver automates the implementation of the analysis

## Benefits of Constraint-based Analysis

1. Benefits
  - Separates analysis specification from implementation
    - Analysis writer can focus on “what” rather than “how”
  - Yields natural program specifications
    - Constraints are usually local, whose conjunctions capture global properties
  - Enables sophisticated analysis implementations
    - Leverage powerful, off-the-shelf solvers

## Specification and Implementation Quiz

1. Consider a dataflow analysis such as live variables analysis. If one expresses it as a constraint-based analysis, one must still decide:
  - The order in which statements should be processed (false)
  - What the gen and kill sets for each kind of statement are (true)
  - In what language to implement the chaotic iteration algorithm (false)
  - Whether to take intersection or union at merge points (true)

## Outline of the Lesson

1. A constraint language: Datalog
  - Two static analyses in Datalog:
    - Intra-procedural analysis: computing reaching definitions
    - Inter-procedural analysis: computing points-to information

## A Constraint Language: Datalog

1. Datalog
  - A declarative logic programming language
  - Not Turing-complete: subset of Prolog, or SQL with recursion
    - Efficient algorithms to evaluate Datalog programs
  - Originated as query language for deductive databases
  - Later applied in many other domains: software analysis, data mining, networking, security, knowledge representation, cloud-computing, ...
  - Many implementations: Logicblox, bddbldb, IRIS, Paddle, ...

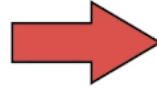
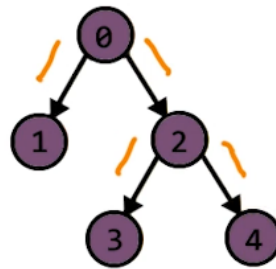
## Syntax of Datalog: Example

1. Example: Graph Reachability
  - Find all pairs of nodes that are connected in a graph
  - Need to define form of input, form of output, and rules of inference
  - A relation is similar to a table in a database
  - A tuple in a relation is similar to a row in a table
  - Rules of inference: Deductive rules that hold universally (i.e., variables like  $x, y, z$  can be replaced by any constant)
    - Specify “if ... then ...” logic
2. Input Relations:
  - $\text{edge}(n:N, m:N)$
  - $n$  and  $m$  are nodes in the set of all nodes  $N$
3. Output Relations:
  - $\text{path}(n:N, m:N)$
4. Rules:
  - $\text{path}(x, x).$ 
    - (if TRUE,) there is a path from each node to itself
  - $\text{path}(x, z) \text{ :- } \text{path}(x, y), \text{edge}(y, z).$ 
    - If there is a path from node  $x$  to  $y$ , and there is an edge from  $y$  to  $z$ , then there is a path from  $x$  to  $z$ .
    - Hypothesis on the left, conclusion on the right
    - Rules separated by commas are logically ANDed together

Input Relations:  
 $\text{edge}(n:N, m:N)$

Output Relations:  
 $\text{path}(n:N, m:N)$

Rules:  
 $\text{path}(x, x).$   
 $\text{path}(x, z) :- \text{path}(x, y), \text{edge}(y, z).$



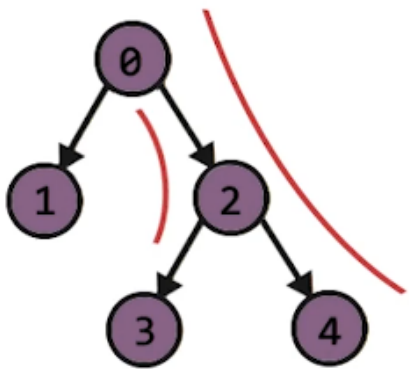
edge	
n	m
0	1
0	2
2	3
2	4

Datalog Syntax

## Semantics of Datalog: Example

### 1. Example: Graph Reachability

- Start with empty path relation
- Apply each rule, growing the path relation with each application
- Stop when the path relation stops growing
- If there are multiple rules, the order in which the rules are applied does not matter
- The result is the “least” solution; the smallest solution that satisfies all the rules



### Input Tuples:

$\text{edge}(0, 1), \text{edge}(0, 2), \text{edge}(2, 3),$   
 $\text{edge}(2, 4)$

### Output Tuples:

$\text{path}(0, 0), \text{path}(1, 1), \text{path}(2, 2),$   
 $\text{path}(3, 3), \text{path}(4, 4), \text{path}(0, 1),$   
 $\text{path}(0, 2), \text{path}(2, 3), \text{path}(2, 4),$   
 $\text{path}(0, 3), \text{path}(0, 4)$

Datalog Semantics

## Computation Using Datalog

1. Check each of the below Datalog rules which correctly computes the relation `scc`: `scc(n1, n2)` if and only if `n2` is reachable from `n1` AND `n1` is reachable from `n2`
  - `scc(n1, n2) :- edge(n1, n2), edge(n2, n1).` (false)
  - `scc(n1, n2) :- path(n1, n2), path(n2, n1).` (true)
  - `scc(n1, n2) :- path(n1, n3), path(n3, n2), path(n2, n4), path(n4, n1).` (true)
  - `scc(n1, n2) :- edge(n1, n3), edge(n2, n3).` (false)
  - First rule is not incorrect, but won't return non-adjacent nodes

## Reaching Definitions Analysis

1. Specification of reaching definitions analysis:
  - $OUT[n] = (IN[n] - KILL[n]) \cup GEN[n]$ 
    - Input relations:
      - \* `kill(n:N, d:D)`
      - \* `gen(n:N, d:D)`
      - \* `next(n:N, m:N)`
  - $IN[n] = \bigcup OUT[n']$  where `n'` is predecessors(`n`)
    - Output relations:
      - \* `in(n:N, d:D)`
      - \* `out(n:N, d:D)`
  - Rules:
    - `out(n, d) :- gen(n, d).`
    - `out(n, d) :- in(n, d), !kill(n, d).`
    - `in(m, d) :- out(n, d), next(n, m).`
  - `n` are statements
  - `d` are definitions

## Reaching Definitions Analysis 2

# Reaching Definitions Analysis in Datalog

Input Relations:

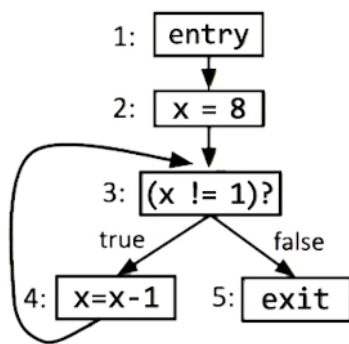
`kill(n:N, d:D)`  
`gen(n:N, d:D)`  
`next(n:N, m:N)`

Output Relations:

`in(n:N, d:D)`  
`out(n:N, d:D)`

Rules:

`out(n, d) :- gen(n, d).`  
`out(n, d) :- in(n, d), !kill(n, d).`  
`in(m, d) :- out(n, d), next(n, m).`



Input Tuples:

`kill(4, 2),`  
`gen(2, 2), gen(4, 4),`  
`next(1, 2), next(2, 3),`  
`next(3, 4), next(3, 5),`  
`next(4, 3)`

Output Tuples:

`in(3, 2), in(3, 4), in(4, 2),`  
`in(4, 4), in(5, 2), in(5, 4),`  
`out(2, 2), out(3, 2), out(3, 4),`  
`out(4, 2), out(4, 4), out(5, 2),`  
`out(5, 4)`

## Live Variables Analysis

1. Complete the rules for live variables analysis:
  - Input relations:
    - $\text{kill}(n:N, v:V)$
    - $\text{gen}(n:N, v:V)$
    - $\text{next}(n:N, m:N)$
  - Output relations:
    - $\text{in}(n:N, v:V)$
    - $\text{out}(n:N, v:V)$
  - Rules:
    - $\text{in}(n, v) \text{ :- } \text{gen}(n, v).$
    - $\text{in}(n, v) \text{ :- } \text{out}(n, v), \text{!kill}(n, v).$
    - $\text{out}(n, v) \text{ :- } \text{in}(m, v), \text{next}(n, m).$

## Pointer Analysis in Datalog

1. Consider a flow-insensitive may-alias analysis for a simple language:
  - (function body)
    - $f(v) \{s_1, \dots, s_n\}$
  - (statement)
    - $s ::= v = \text{new } h \mid v = u \mid \text{return } u \mid v = f(u)$
  - (pointer variable)
    - $u, v$
  - (allocation site label)
    - $h$
  - (function name)
    - $f$
  - Performs weak updates, not strong updates

## Intra-procedural Pointer Analysis

1. Input Relations:
  - $\text{new}(v:V, h:H)$
  - $\text{assign}(h:H, u:V)$
2. Output Relations:
  - $\text{points}(v:V, h:H)$
3. Rules:
  - $\text{points}(v, h) \text{ :- } \text{new}(v, h).$
  - $\text{points}(v, h) \text{ :- } \text{assign}(v, u), \text{points}(u, h).$

## Querying Pointer Analysis

1. Assume we allow function calls now
  - Parameter passing and return statements can be treated as copy assignments

```
x = new h1;
y = f(x); // v = x; u = v; y = u;

f(v) {
    u = v;
    return u;
}
```

2. Need to add input relations and rules for function calls and return statements
  - Input Relations:

- new(v:V, h:H)
- assign(h:H, u:V)
- arg(f:F, v:V) where F is set of all function calls
- ret(f:F, u:V)
- call(y:V, f:F, x:V)
- Output Relations:
  - points(v:V, h:H)
- Rules:
  - points(v, h) :- new(v, h).
  - points(v, h) :- assign(v, u), points(u, h).
  - points(v, h) :- call(⟦, f, x), arg(f, v), points(x, h).
    - \* Underscore is a wildcard
  - points(y, h) :- call(y, f, ⟦), ret(f, u), points(u, h).

## Context Sensitivity

1. Check each of the below Datalog programs which correctly computes the relation mustNotAlias: mustNotAlias(u,v) if and only if u and v do no alias in any run of the program.
  - Rule 1 (false; need to check every allocation site)
    - mustNotAlias(u, v) :- points(u, h1), points(v, h2), h1 != h2.
  - Rule 2 (true)
    - mayAlias(u, v) :- points(u, h), points(v, h).
    - mustNotAlias(u, v) :- !mayAlias(u, v).
  - Rule 3 (false; wildcard means they may be pointing to different sites)
    - mayAlias(u, v) :- points(u, ⟦), points(v, ⟦)
    - mustNotAlias(u, v) :- mayAlias(u, v).
  - Rule 4 (true)
    - common(u, v, h) :- points(u, h), points(v, h).
    - mayAlias(u, v) :- common(u, v, ⟦).
    - mustNotAlias(u, v) :- !mayAlias(u, v).

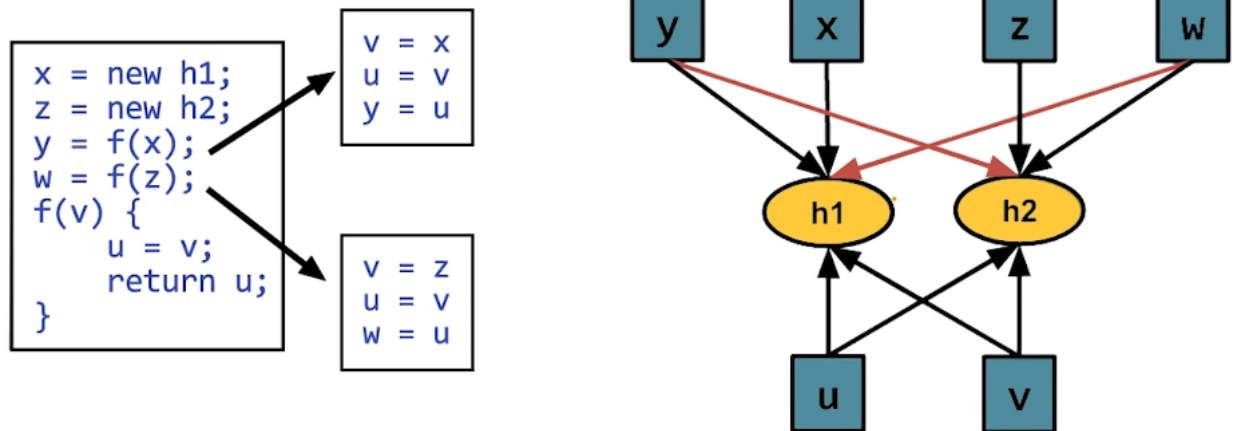
## Context Sensitivity

1. Current analysis targets pointer aliasing across function calls
  - Results in a loss of precision

```

x = new h1;
y = new h2;
y = f(x);
w = f(z);
f(v) {
    u = v;
    return u;
}

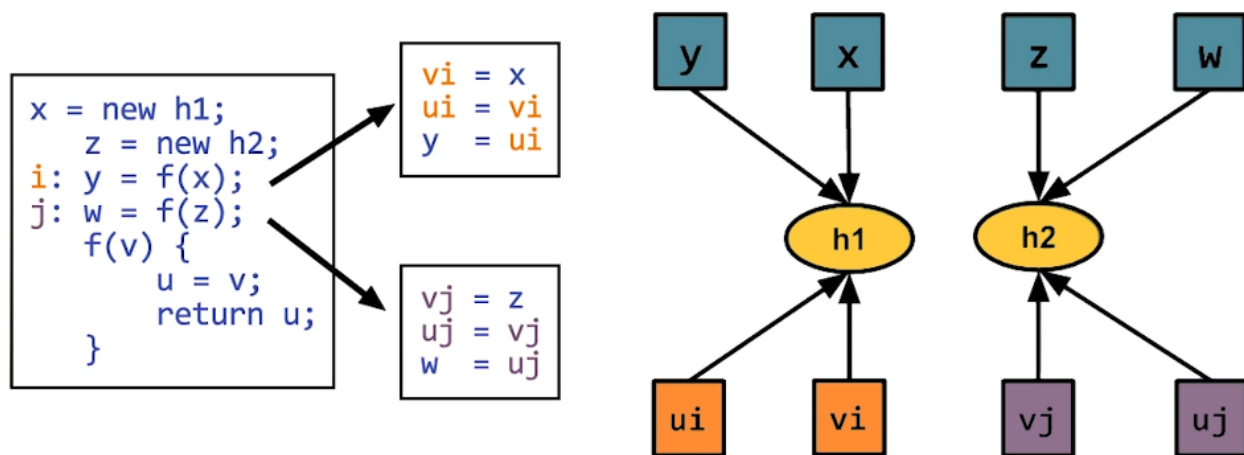
```



Context Insensitivity

## Cloning-based Inter-procedural Analysis

1. Cloning achieves context sensitivity by reproducing the bodies of the procedure inline with distinguished variable names
  - Cloning depth improves precision at the cost of scalability
    - Cost becomes exponential with depth of call stack



Context Sensitivity

## What About Recursion?

1. Need infinite cloning depth to differentiate the points-to sets of x,y and w,z

```
x = new h1;
y = new h2;
y = f(x);
w = f(z);
```

```
f(v) {
```

```

    if (*)
        v = f(v);
    return v;
}

```

## Summary-based Inter-procedural Analysis

1. Summary-based Approach
  - Use the incoming program states to differentiate calls to the same procedure
    - Same incoming program states yield same outgoing program states for a give procedure
  - As precise as cloning-based analysis with infinite cloning depth

## Other Constraint Languages

Constraint Language	Problem Expressed	Example Solvers
Datalog	Least solution of deductive inference rules	LogixBlox, bddbdb
SAT	Boolean satisfiability problem	MiniSat, Glucose
MaxSAT	SAT extended with optimization	open-wbo, SAT4j
SMT	Satisfiability modulo theories problem	Z3, Yices
MaxSMT	SMT extended with optimization	Z3

## Conclusion

1. Constraint-based analysis and its benefits
2. The Datalog constraint language
3. How to express static analyses in Datalog
  - Analysis logic == constraints in Datalog
  - Analysis inputs and outputs == relations of tuples
4. Context-insensitive and context-sensitive inter-procedural analysis