Combinations and Permutations

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1 Introduction

Define the set A where $|A| = n$. Obviously, every element is unique, by the definition of a set.

1.1 Product rule of counting

We will assume the product rule of counting.

Theorem 1 *Say we have a collection of* r *sets,* A_1*,* A_2*, to* A_r *with respective cardinalities of* n_1*,* n_2*, to* n_r*. Suppose we have to successively choose an element from each set in order. A particular sequence of items chosen from the* r *sets is a* composite outcome*. The number of distinct composite items, where order matters, is*

$$\prod_{i=1}^{r} n_i \tag{1}$$

2 Permutations

Order matters.

2.1 With repetition

2.1.1 Problem

Work out how many ways there are of choosing r elements from A, and we can choose the same element multiple times.

2.1.2 Solution

The first time we choose an element, we have n choices. It is the same for all r choices we make. By theorem 1, the total number of outcomes is n^r.

2.2 Without repetition

2.2.1 Problem

Work out how many ways there are of choosing r elements from A, and we cannot choose the same element twice.

2.2.2 Solution

At the first step, we will have n choices. At the successive step, if it exists, we will have $n-1$ choices. This continues until there are $n-r+1$ choices. By theorem 1, the answer is

$$n \times (n-1) \times \ldots \times (n-r+1) = \frac{n!}{(n-r)!} \tag{2}$$

3 Combinations

Order doesn't matter.

3.1 Without repetition

3.1.1 Problem

Say we can create sets by choosing r elements from A, and we cannot choose the same element twice. Work out how many distinct sets can be created in this way.

3.1.2 Solution

The set of the distinct sets (call this B) that can be created in this way is a subset of the distinct sequences that can be created in section 2.1 because order doesn't matter.

The number of ways of organising r items is $r!$, by theorem 1. Therefore, there are $r!$ different sequences in the problem in section 2.1 for every unordered sequence in B. This means that

$$|B| = \frac{n!}{(n-r)!} \times \frac{1}{r!} \tag{3}$$

3.2 With repetition

3.2.1 Problem

Work out how many ways there are of choosing r elements from A, and we can choose the same element multiple times. The order of the sequence does not matter.

3.2.2 Solution

The first time we choose an element, we have n choices. It is the same for all r choices we make. By theorem 1, the total number of outcomes is n^r.