* This is continuation of yesterday's calculations. For reasons given there wish to 'grade a polynomial ring' so that number of terms in each grade is 2^m.

π m = 0   0 = 0   1 'generator'.

~~π m = 2   2 =~~

π m = 1   Need 2 generators, — and —.

π m = 2   2 = 1 + 1.   Already have ||, ||, ||.   3
                              Need one more ∠?
                                                                ‾1‾
                                                                ‾4‾

π m = 3   3 = 1 + 1 + 1 |||...||| 4
                3 = 1 + 2 | × ∠↗   2
                3 = 3   irreducible   2   ‾8‾
                              ↙↗ with central band
                                  | ie. | or |.

π m = 4 - see 2 Jan Ⓒ p 2.
          4 = 1 + 1 + 1 + 1   5
          4 = 2 + 1 + 1   3
          4 = 2 + 2   1
          4 = 3 + 1   2 × 2 = 4
                                                          ‾13‾

          4 = 4 expecting 3 possibilities.
          The 'middle belt' has 'length' 2.
          Expect ||, ||, || to give 3   3 + 13 = ‾16‾

π m = 5 - see 2 Jan Ⓓ p 1.
          5 = 1 + 1 + 1 + 1 + 1   6
          5 = 2 + 1 + 1 + 1   4
          5 = 2 + 2 + 1   2
          5 = 3 + 1 + 1   2 × 3 = 6
          5 = 3 + 2   2
          5 = 4 + 1   3 × 2 = ‾6‾
                                                    ‾26‾

          Thus 5 = 5 expecting 6 possibilities.

π (cont) For m = 5 belt has path length 3.
          In m = 4 and 5 I'm puzzled as to how
          to realize these extra m = m items
                    in the belt.

¶ I found an error 2 Jan ⑤ p3, which I will now collect.

¶ m=6

$$6 = 1 + 1 + 1 + 1 + 1 + 1 \qquad 7$$
$$6 = 2 + \quad 1 + 1 + 1 + 1 \qquad 5$$
$$6 = 2 + 2 \qquad + 1 + 1 \qquad 3$$
$$6 = 2 + 2 + 2 \qquad\qquad 1 \qquad \overline{16}$$

¶ (cont)

$$6 = 3 + 1 + 1 + 1 \qquad 2 \times 4 = 8$$
$$6 = 3 + 2 + 1 \qquad 2 + 2 = 4$$
$$6 = 3 + 3 \qquad\qquad (2 + 2 = 4) \text{— The error.}$$

same reason as $1+1 \leadsto 3$. $\leadsto$ 3 $\overline{15}$

¶ (cont)

$$6 = 4 + 1 + 1 \qquad 3 \times 3 = 9$$
$$6 = 4 + 2 \qquad\qquad 3 \qquad \overline{12}$$

¶ (cont) $6 = 5 + 1$ $6 + 2$ $\overline{12}$

¶ Totals.

$$16 + 15 + 12 + 12 = 55$$

Want $64 = 55 + 9$.

¶ So obtain

0	1	2	3	4	5	6
2	1	2	3	6	6	9

¶ This A100 A001037, degree n irreducible polynomials over GF(2) as first result of 9. Also binary Lyndon words, dimensions of Free Lie Algebras.

¶ OEIS gives nice formula

$$2^n = \sum_{d \mid n} d \times a(d)$$

¶ Problem   Does this formula agree with our algorithm.

¶ Problem   Write down proof of this formula.