[Title of thesis]

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Abstract

Hier die Kurzfassung in ganzen Sätzen.

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1 Introduction

We're constantly looking for materials with specific properties that can make our lives easier. To find a new material that fits our needs, we could look for it in nature. That means we'd go digging in the earth until we find something new, isolate it, take it to the lab, determine its properties and hope we found what we needed. This process is quite cumbersome and as technology gets more and more advanced, we'll find ourselves in need of materials that might not be naturally occurring in the first place. In that case we need to determine whether such materials could theoretically exist at all and if so, what are the requirements a compound needs to have in order to have a specific property? To find out, we conduct simulations which we're already doing very rigorously right now. However, when simulating quantum many-body systems, the Hilbert space scales dramatically and our conventional devices will fail.

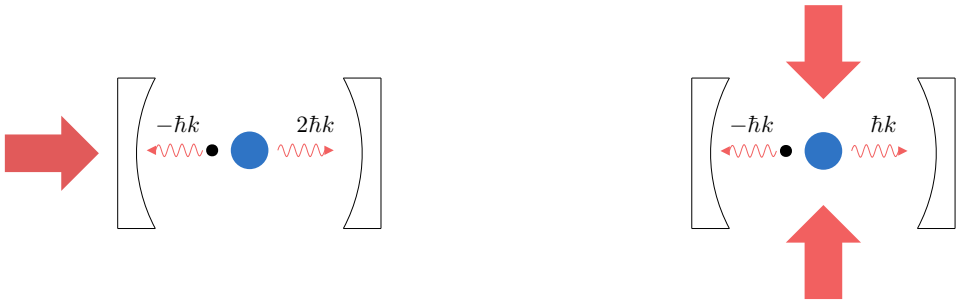
One solution to this is to use a quantum computer. Algorithms with exponential time on classical devices would operate then on polynomial time. There's just one problem: We don't have a sophisticated quantum computer yet and will likely not have one in the next 10-20 years, at least for our specific purposes. So rather then utilizing a general-purpose quantum computer, for now there's another approach that's more fruitful: We'll restrict ourselves to a specific problem by constructing a quantum system that simulates another quantum system. Such a system won't be able to simulate anything, just what we designed it for. This is called an *Analog Quantum Simulator*. With Analog Quantum Simulators, we can not only simulate properties of compounds, but any quantum problem that we wish to solve. A nice introduction to quantum simulation can be found at [1].

2 Cold Atoms in Optical Cavities

What we want to achieve is to order atoms in a lattice to simulate a solid. To prevent them from flying away, we have to fix them in space somehow. A great way to do this is to use a laser potential, e.g. two opposing laser beams. The atoms will localize at the antinodes of the laser potential to minimize the energy. We'll take that approach one step further and put the atoms in a cavity. That way the atoms themselves take part in creating an optical potential which confines them.

There are different types of cavities, such as the bow-tie, the ring and the Fabry-Perot cavity. We will utilize the Fabry-Perot cavity. There are two curved mirrors separated by a distance d. We shoot a laser with wavelength λ into the cavity, which can be done longitudinally or transversally, as illustrated in Figure 1. We choose d and λ, such that $d = n\lambda/2$, where $n \in \mathrm{N}$. The light along the cavity axis will be amplified which is what we want. We will work with a far red-detuned laser, that means the frequency of the laser ω_l is way smaller than the excitation frequency of atom ω_a. If we prevent the atom from being excited, a strong dipole force will build up and the atom will get strongly coupled to the cavity field. There is a fundamental difference how atoms scatter light in a transversally pumped cavity and in a longitudinally pumped cavity. In the longitudinal case, if a photon with momentum $\hbar k$ in x-direction bumps into an atom, it will recoil backward, having now a momentum $-\hbar k$. Momentum conversation thus requires the atom

to have now a momentum of $2\hbar k$. In the transversal case, there are two counter-propagating laser beams as to prevent kicking the atoms out of the cavity. If an atom scatters a transversally incoming photon along the cavity axis, it will now have a momentum of $\hbar k$ and the photon $-\hbar k$. We see that the fundamental difference between longitudinal and transversal pump is the momenta the atoms will be able to acquire. In the longitudinal case, there will only be momenta of $2n\hbar k$, where $n \in \mathrm{N}$, whereas for the transversal pump there are momenta of $\hbar k$.



(a) Longitudinal pumping. **(b)** Transversal pumping.

Figure 1: Longitudinal and transversal pumping. For longitudinal pumping, the atoms will acquire a momentum of $2\hbar k$ scattering a photon, whereas for transversal pumping there is momentum exchange of $\hbar k$.

3 Derivation of the Hamiltonian

For those wishing to refresh their knowledge of quantum mechanics, the introductory chapters of Fox's quantum optics book will be a great help [2]. In this section we'll derive the Hamiltonians being used for the simulation; one Hamiltonian for longitudinal pumping and one for transversal pumping. The Hamilton operator represents the total energy of a quantum system. We'll start with the Jaynes-Cummings Hamiltonian which describes the interaction of a two-level atom with a single mode of a cavity-field. We'll then modify the Hamiltonian according to our needs step by step. We'll tackle the crucial details and reference parts of the derivation which is not presented here.

3.1 The Jaynes-Cummings Hamiltonian

The Jaynes-Cummings model describes the interaction of a two-level atom with a single mode of a cavity field. The first appearance was in [3]. We'll restrict ourselves to one dimension and start with an atom (or a BEC) in an external potential:

$$H_0 = \frac{p^2}{2m} + V_{\mathrm{ext}}(x). \tag{1}$$

Now we place that atom in a cavity and it will interact with the cavity mode, creating more terms in our Hamiltonian that we have to consider. First, there's the energy of the field:

$$H_{\text{field}} = -\hbar\omega_{\text{c}} a^\dagger a, \tag{2}$$

where ω_{c} is the resonance frequency of the cavity and $a^\dagger$ and a are the creation and annihilation operators. Next, we'll add a term describing the atomic transitions:

$$H_{\text{transition}} = -\frac{1}{2}\hbar\omega_{\text{a}}\sigma_z, \tag{3}$$

where ω_{a} is the resonance frequency of the atom and σ_z is the Pauli z-matrix. The above term describes the atom being in the ground state or excited state, the transition energy is $1/2\hbar\omega_{\text{a}}$. The field-atom interaction we describe with following term:

$$H_{\text{interaction}} = \hbar g_0 \cos(kx)(\sigma^+ a + \sigma^- a^\dagger), \tag{4}$$

where g_0 is the coupling strength and σ^+ and σ^- are the raising and lowering operators. Finally, we'll add the term describing the pumping:

$$H_{\text{pump}} = \hbar\eta(ae^{i\omega_{\text{l}}t} + a^\dagger e^{-i\omega_{\text{l}}t}), \tag{5}$$

where η is the pumping strength and ω_{l} is the laser frequency. We now have the full Jaynes-Cummings Hamiltonian which is the sum of all terms above:

$$H_{\text{JC}} = \underbrace{p^2/2m}_{\text{atom}} + \underbrace{V_{\text{ext}}(x)}_{\text{external potential}} - \underbrace{1/2\hbar\omega_{\text{a}}\sigma_z}_{\text{atomic transitions}} - \underbrace{\hbar\omega_{\text{c}} a^\dagger a}_{\text{field}} + \underbrace{\hbar\eta(ae^{i\omega_{\text{l}}t} + a^\dagger e^{-i\omega_{\text{l}}t})}_{\text{pumping}} + \underbrace{\hbar g_0 \cos(kx)(\sigma^+ a + \sigma^- a^\dagger)}_{\text{field-atom interaction}}. \tag{6}$$

A more detailed derivation of the Jaynes-Cummings Hamiltonian (starting from Maxwell's equations and quantizing the cavity mode) can be found at [4]. At this point we should probably clarify that we have a composite system, i.e.

$$\psi_{\text{total}} = \psi_{\text{light}} \otimes \psi_{\text{atom}}. \tag{7}$$

In order to get rid of the explicit time-dependence, we transform the Hamiltonian to a frame rotating with ω_{l}. The Hamiltonian now reads:

$$H_{\text{JC}} = \frac{p^2}{2m} + V_{\text{ext}}(x) - \frac{1}{2}\hbar\Delta_{\text{a}}\sigma_z - \hbar\Delta_{\text{c}} a^\dagger a + \hbar\eta(a + a^\dagger) + \hbar g_0 \cos(kx)(\sigma^+ a + \sigma^- a^\dagger), \tag{8}$$

where $\Delta_{\text{a}} = \omega_{\text{l}} - \omega_{\text{a}}$ and $\Delta_{\text{c}} = \omega_{\text{l}} - \omega_{\text{c}}$.

3.2 Detuning

The derivation for the Hamiltonians for the following sections is taken from [5]. Now we derive heuristically a modified Hamiltonian. Going to the Heisenberg picture, we get:

$$\dot{a} = \frac{i}{\hbar}[H, a] = i\Delta_{\mathrm{c}} a - i\eta - ig_0 \cos(kx)\sigma^- . \tag{9}$$

Obviously, the kinetic energy and potential term vanish under the commutator. For the other terms:

$$a^\dagger a = N \qquad [N, a] = -a \tag{10}$$

$$(a + a^\dagger)a - a(a + a^\dagger) = aa + a^\dagger a - aa - aa^\dagger = 1 \tag{11}$$

$$\text{because we know:} \quad aa^\dagger = a^\dagger a + 1$$

$$[\sigma^+ a + \sigma^- a^\dagger, a] = \sigma^+ \underbrace{[a, a]}_{0} + \sigma^- \underbrace{[a^\dagger, a]}_{1} = \sigma^- \tag{12}$$

The creation and annihilation operators ($a^\dagger$ and a) and the raising and lowering operators (σ^+ and σ^-) live in different Hilbert spaces and thus don't influence each other. A good reference for the commutator relation is [6]. The time-derivative for the raising operator reads:

$$\dot{\sigma}^+ = \frac{i}{\hbar}[H, \sigma^+] = \underbrace{-i\Delta_{\mathrm{a}}\sigma^+}_{(*)} + \underbrace{ig_0 \cos(kx)a^\dagger}_{(**)} . \tag{13}$$

For (*), we'll look at the matrix representation of the operators:

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{14}$$

We calculate the commutator relation $[\sigma_z, \sigma^+]$ explicitly:

$$[\sigma_z, \sigma^+] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} = 2\sigma^+ . \tag{15}$$

For (**), consider that $[\sigma^+, \sigma^-] = 1$. In our case, the pumping laser is far detuned from the atomic resonance frequency, i.e. $\Delta_{\mathrm{a}} = \omega_{\mathrm{l}} - \omega_{\mathrm{a}}$ is large. Thus the excitation probability of the atom is vanishing and we set $\dot{\sigma}^+ = 0$. We get:

$$\sigma^+ = \frac{g_0}{\Delta_{\mathrm{a}}} \cos(kx)a^\dagger \qquad\qquad \sigma^- = \frac{g_0}{\Delta_{\mathrm{a}}} \cos(kx)a. \tag{16}$$

Putting the above relation in equation 9, we get:

$$\dot{a} = -i\Delta_c a + \frac{ig_0}{\Delta_a}\cos(kx)a - i\eta. \tag{17}$$

We can thus make a guess of the effective Hamiltonian:

$$H_{\text{long}} = \frac{p^2}{2m} + V_{\text{ext}}(x) - \hbar\Delta_c a^\dagger a + \hbar\eta(a + a^\dagger) + \hbar U_0 \cos(kx)^2, \tag{18}$$

where we set $U_0 \coloneqq g_0^2/\Delta_a$. Note that because $H_{\text{long}} \propto \cos(kx)^2$, the Hamiltonian is $\lambda/2$-periodic. Later in the simulation program, we want to make sure all quantities are expressed with the recoil energy $E_r = \hbar\omega_r$, where $\omega_r = \hbar k^2/2m$ is the recoil frequency. Therefore we factor our E_r to see what we have to type into the program:

$$\begin{aligned} H_{\text{long}} = \hbar\omega_r\Bigg(\frac{1}{\hbar^2k^2}p^2 + \frac{1}{\hbar\omega_r}V_{\text{ext}}(x) - \frac{1}{\omega_r}\Delta_c a^\dagger a + \frac{1}{\omega_r}\eta(a + a^\dagger) + \\ + \frac{1}{\hbar\omega_r}U_0\cos(kx)^2 a^\dagger a\Bigg). \end{aligned} \tag{19}$$

In the simulation program, we will thus set $\hbar = 1$ and multiply each quantity by the preceding factors.

3.3 Transversal Pump

Now we want to focus our attention at a different case where the laser is incident transversally relative to the axis of the mirrors. The cavity mode will thus only be populated by photons which were scattered off the atoms. The Hamiltonian now reads:

$$\begin{aligned} H_{\text{trans}} = \frac{p^2}{2m} + V_{\text{ext}}(x) - \hbar\Delta_c a^\dagger a + \hbar\eta\cos(kx)\cos(kz)(a + a^\dagger) + \\ + \hbar\frac{\Omega^2}{\Delta_a}\cos(kz)^2 + \hbar U_0\cos(kx)^2 a^\dagger a, \end{aligned} \tag{20}$$

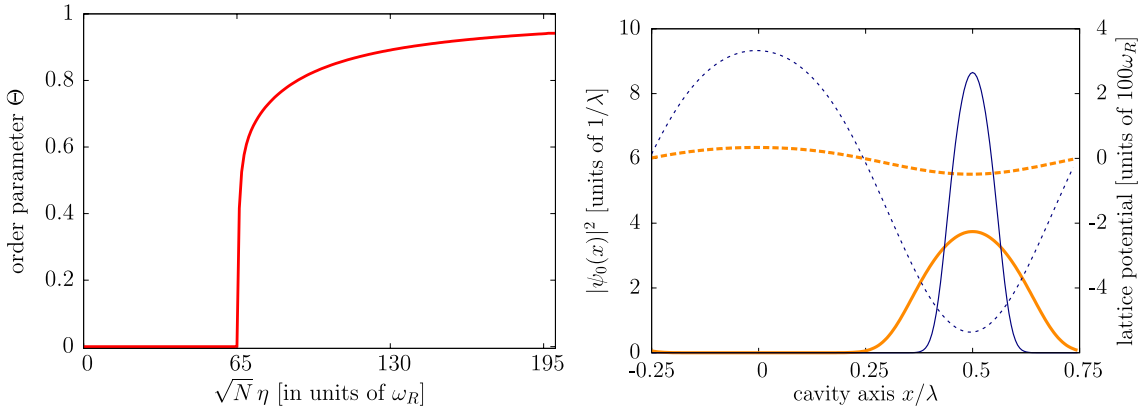
where Ω is the Rabi frequency. Let's look at the pump term a little more: When transversally pumping, there are two counter-propagating laser beams. We thus have to consider photons traveling in positive z-direction $\psi_{\text{photon}} \propto \exp(ikx)$ and negative z-direction $\psi_{\text{photon}} \propto \exp(-ikz)$. Combined we obtain the cosine term. For the x-direction, we can argue similarly: when an atom scatters a photon, we have to consider $\exp(ikx)$ and $\exp(-ikx)$. Here we only consider one dimension, so we set $z = 0$:

$$\begin{aligned} H_{\text{transv}} = \frac{p^2}{2m} + V_{\text{ext}}(x) - \hbar\Delta_c a^\dagger a + \hbar\eta\cos(kx)(a + a^\dagger) + \\ + \hbar U_0\cos(kx)^2 a^\dagger a. \end{aligned} \tag{21}$$

Note that because $H_{\text{trans}} \propto \cos(kx) + \cos(kx)^2$, the Hamiltonian is λ-periodic. As previously mentioned, keeping the right dimensionality in the simulation is very important. We define an *order parameter* Θ, which indicates whether the atoms are uniformly distributed or localized at potential minima:

$$\Theta := \langle\psi| \cos(kx) |\psi\rangle. \tag{22}$$

At $\Theta = 0$, there's a uniform distribution and at $\Theta = \pm 1$, the atoms are localized at even or odd antinodes. An analytic solution of Θ and the lattice potential can be seen in Figure 2. For transversal pumping, there's a clear critical pumping strength η_{crit}, at which self-organization starts taking place. For longitudinal pumping, Θ would increase starting from 0.



(a) Order parameter. **(b)** Lattice potential.

Figure 2: Order parameter and lattice potential for transversal pumping. Figures taken from [7].

4 Simulation

Having derived our Hamiltonians, we'll now set out to numerically simulate the systems with longitudinal and transversal pump each. In the following sections, we'll present the code necessary to simulate the quantum systems. However, any code to generate graphs will not be presented.

4.1 The Julia language and QuantumOptics.jl

Scientific computing requires high performance which low-level languages like C or Fortran can deliver. However, writing scripts in these languages can often be cumbersome. Julia is a programming language that combines the ease of use of high level languages and performance of low level languages [8]. Here we use Julia for our simulations with the framework QuantumOptics.jl [9]. The architecture of the functions of the package will not be discussed. A detailed documentation can be found at [10].

4.2 The Code

First, we'll add all the packages that we need:

```
using QuantumOptics, LinearAlgebra
```

The package `QuantumOptics` is the quantum simulation package mentioned earlier and `LinearAlgebra` is a package that comes with some useful functions like getting the diagonal entries of a matrix `diag()`. We'll set $k = 2\pi$, so that $\lambda = 1$. The recoil frequency we set $\omega_\mathrm{r} = 1$ and $\Delta_\mathrm{c} = -10\,\omega_\mathrm{r}$ and $U_0 = -1\,\omega_\mathrm{r}$:

```
k = 2*π
ωr = 1
Δc = -10 * ωr
U0 = -1 * ωr
```

We'll allow a maximum of $N = 16$ states. We'll confine the simulation spatially to $x_\mathrm{min} = 0$ and $x_\mathrm{max} = 1$. Setting a wider range would be redundant since the transversal Hamiltonian is λ-periodic. Thus, we could actually set $x_\mathrm{max} = 0.5$ for the $\lambda/2$-periodic longitudinal Hamiltonian, but we'll set same boundaries for both Hamiltonians for better comparability with the graphs. Usually, the step size is set to $\mathrm{Nsteps} = 2^n$, where $n \in \mathbb{N}$. Here, we set $\mathrm{Nsteps} = 64$ for a good compromise between simulation time and the look of the graphs.

```
N_cutoff = 16
xmin = 0
xmax = 1
Nsteps = 32
```

We define the bases, as well as the raising and lowering operators:

```
b_position = PositionBasis(xmin, xmax, Nsteps)
b_fock = FockBasis(N_cutoff)
p = momentum(b_position)
a = destroy(b_fock) ⊗ one(b_position)
ad = dagger(a)
```

The raising and lowering operators are a tensor product of the position and Fock basis. Note that in Julia it's possible to name variables with Greek symbols. In this case, the tensor product is defined with the symbol $\otimes$. We define the Hamiltonian and calculate the first three states with $\eta = 23\,\omega_\mathrm{r}$:

```
potential = x -> U0*cos(k*x)^2
H_int = (one(b_fock) ⊗ potentialoperator(b_position, potential))*ad*a
H_kin = (one(b_fock) ⊗ p^2) / k^2
H_cavity = -Δc*ad*a

function H(η)
    pump = x -> η*cos(k*x)
    H_pump = (one(b_fock) ⊗ potentialoperator(b_position, pump)) * (a + ad)
    return H_kin + dense(H_int) + H_pump + H_cavity
```

```
end

η = 23 * ωr
E, ψ_states = eigenstates((H(η) + dagger(H(η)))/2, 3)
```

If we want to plot the wave function, we'll have to extract the position part of the composite basis. We can do that with the command `ptrace()`. We thus obtain a matrix whose diagonal entries are the complex values of the wave function:

```
pos_dense = ptrace(ψ_states[1], 1)
density = diag(pos_dense.data)
```

Likewise, by changing the second argument of `ptrace()` to 2, we trace out the position basis. The diagonal entries of the obtained matrix represents the photon number distribution:

```
photon_dense = ptrace(ψ_states[1], 2)
probab = diag(photon_dense.data)
```

We can calculate the expected photon number as follows:

```
ada_exp = expect(ad*a, ψ_states[1])
```

We can also calculate the momentum distribution which is the Fourier transform of the position distribution. The function `transform()` performs a Fourier transform in the background:

```
b_momentum = MomentumBasis(b_position)
Tpx = transform(b_momentum, b_position)

pos_dense = ptrace(ψ_states[1], 1)
states_p = Tpx * pos_dense
density_p = diag(states_p.data)
```

Now let's tackle the transversal pump. The bases are the same as before. However, we have to define different Hamiltonians:

```
potential = x -> U0*cos(k*x)^2
H_int = (one(b_fock) ⊗ potentialoperator(b_position, potential))*ad*a
H_kin = (one(b_fock) ⊗ p^2) / k^2
H_cavity = -Δc*ad*a

function H(η)
    pump = x -> η*cos(k*x)
    H_pump = (one(b_fock) ⊗ potentialoperator(b_position, pump)) * (a + ad)
    return H_kin + dense(H_int) + H_pump + H_cavity
end

η = 10 * ωr
E, ψ_states = eigenstates((H(η) + dagger(H(η)))/2, 3)
```

To visualize the degree of self-organization, we'll take a look at the photon state. The Husimi Q representation is a way of visualizing a wave function. It's defined as follows:

$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle, \tag{23}$$

where α is the state we want to visualize and ρ is the density operator:

$$\rho = |\psi\rangle\langle\psi|. \tag{24}$$

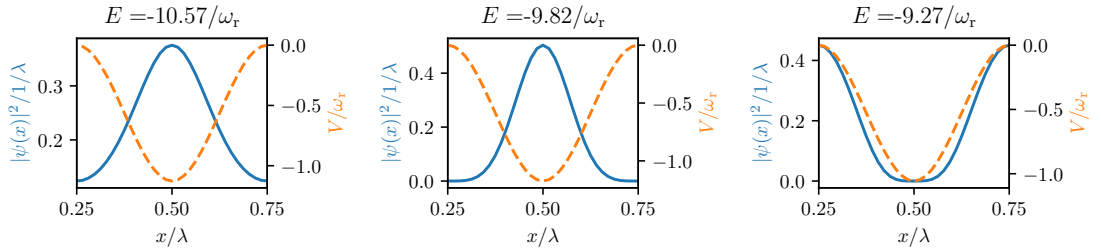
In Julia with QuantumOptics.jl, we can use the command `qfunc()`:

```
bdr = 6
xvec = [-bdr:.1:bdr;]
yvec = [-bdr:.1:bdr;]
photon_dense = ptrace(ψ_states[1], 2)
grid = qfunc(photon_dense, xvec, yvec)
```

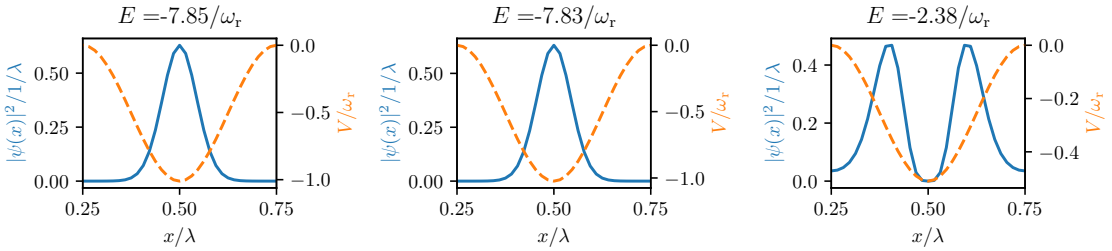
The variable `bdr` was set heuristically for plotting. The maximum value of `qfunc` with respect to the origin gives us the previously defined order parameter Θ.

5 Results and Discussion

The longitudinal and transversal spatial wave function densities for $\eta = 10\,\omega_\mathrm{r}$ are depicted in Figure 3. The leftmost state is the first eigenstate, the second eigenstate is in the middle and the third on the right. Both for longitudinal and transversal pump, because of equal contributions of a and $a^\dagger$, the spatial densities are $\lambda/2$-periodic. The energy is the lowest of the first eigenstate and increasing with each one. Each peak of the ground state density being located at the potential minima is in accordance with our expectations. However, these graphs don't give us enough insight into the physical processes of the system. For that purpose, looking at the momentum space is more fruitful.



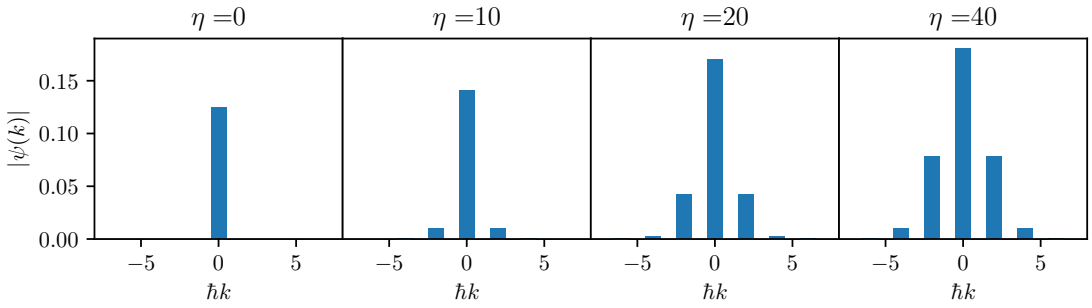
(a) Longitudinal.



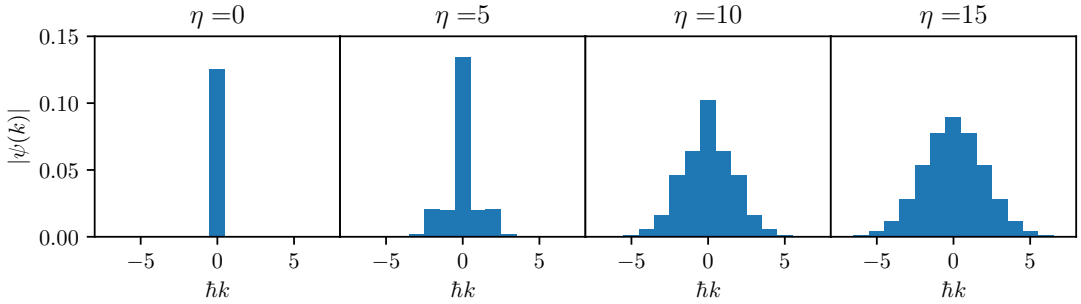
(b) Transversal.

Figure 3: Longitudinal and transversal wave function densities for $\eta = 10\,\omega_r$. The first eigenstate is on the left, the second in the middle and the third on the right.

The momentum distribution for different values of η can be seen in Figure 4. Here we can see much more of the actual physics of the system. At $\eta = 0$, i.e. when the laser is off, there's only a peak at 0, meaning the atoms have no momentum. When we start pumping, we get other peaks than 0. Now the atoms do have momentum. For longitudinal pumping, there's always a gap between each peak, which is not the case for transversal pumping. Take a look again at figure 1. When we pump longitudinally, a photon is only able to transfer a momentum of $2\hbar k$ because of momentum conservation. Thus we only observe peaks at $2n\hbar k$, where $n \in \mathbb{N}$. For transversal pumping, the same processes of photons transferring momenta of $2\hbar k$ are happening, but now we also have a momentum transfer of $\hbar k$ when a transversally incoming photon is being scattered into the cavity. Naturally, the more we pump, the more outer momenta we will get.



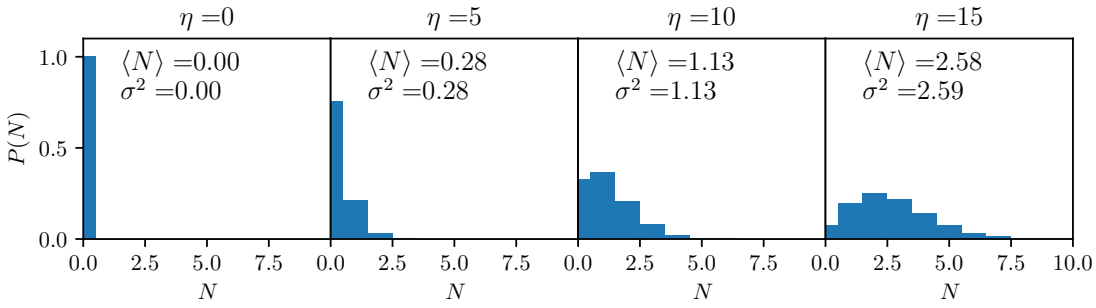
(a) Longitudinal.



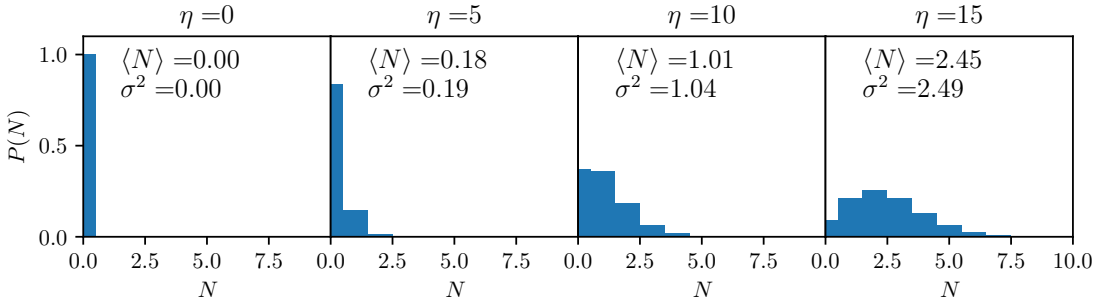
(b) Transversal.

Figure 4: Longitudinal and transversal momentum distributions. For longitudinal pump, there are only momenta of $2n\hbar k$ because longitudinal scattering processes only allow momenta transfer of $2\hbar k$. For transversal pump, there is no such restriction and we have momenta of $n\hbar k$.

Having looked at the atom part of the composite system, let's take a look at the photon part. The photon number distribution for different values of η can be seen in Figure 5. The mean and variance are pretty much the same, thus we have a Poisson distribution.



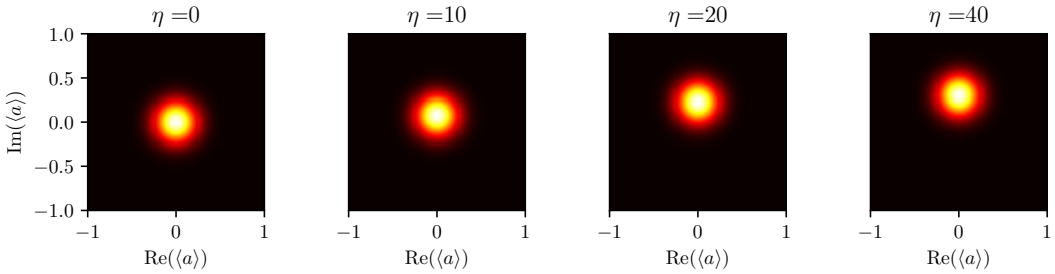
(a) Longitudinal.



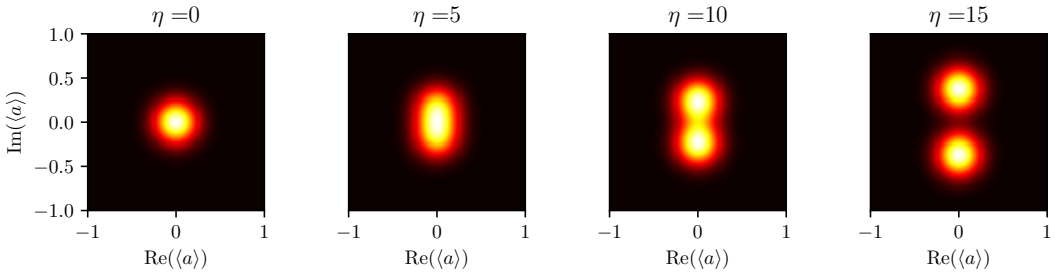
(b) Transversal.

Figure 5: Longitudinal and transversal photon distributions. Since the mean and the variance are the same, we have a Poisson distribution.

The Husimi Q representation of both longitudinal and transversal pump can be seen in Figure 6.



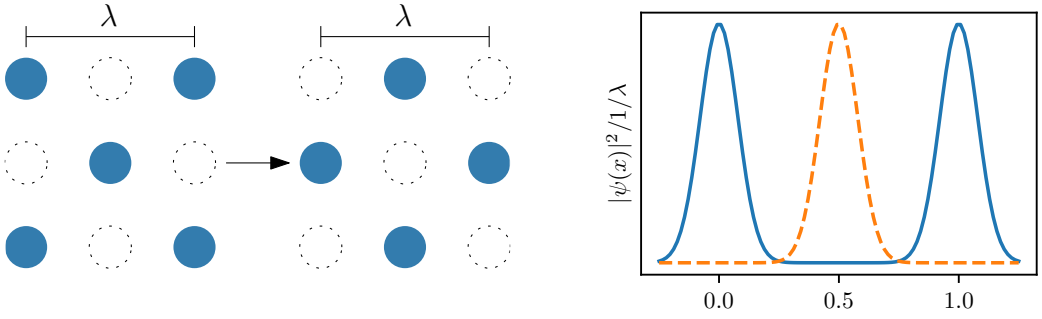
(a) Longitudinal.



(b) Transversal.

Figure 6: Husimi Q representation of longitudinal and transversal wave functions.

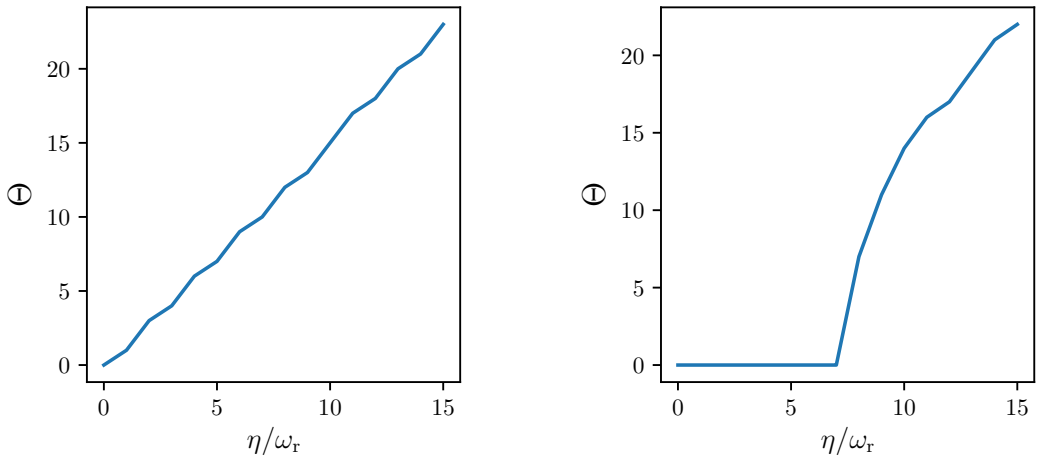
At $\eta = 0$, we see for both cases a blob at the center. Actually, it'd be just a point, but there's a quantum uncertainty. For transversal pump, we can see that two blobs separate with increasing pump strength. If we were to measure the system experimentally, we'd only obtain one blob since measuring means breaking the symmetry. The superposition is also reflected in the fact that the graph of Figure 3 with transversal pumping is $\lambda/2$-periodic. Actually, it should be λ-periodic. Figure 7 illustrates the superposition and the lattice of the atoms.



(a) Lattice. **(b)** Densities.

Figure 7: Lattice and superposition of densities.

We can break the symmetry artificially by only looking at one half of the graph. The order parameter Θ can be seen in Figure 8. For longitudinal pumping, there's no threshold at which order sets in rapidly, but a linear increase. For transversal pumping, we can clearly observe such a threshold.



(a) Longitudinal. **(b)** Transversal.

Figure 8: Order parameter Θ as a function of the pumping strength η.

The more we pump, the more photons will appear. We have to take that into account by raising the maximum amount of allowed photon states N_{cutoff}. Raising N_{cutoff} results in longer simulation times, however if we don't do so, our results become faulty. Take a look at Figure 9 which depicts the photon number distributions for different values of N_{cutoff} at $\eta = 40\,\omega_r$. For our parameters, $40\,\omega_r$ is a relatively high value for η and we thus would expect a high average photon number which cannot be the case if limit N_{cutoff} to 8. To check the validity of our results, i.e. if N_{cutoff} is set high enough, we can look at the standard deviation. For a Poisson distribution, the mean has to be the same as the standard deviation which is not the case if we set the cutoff too low.

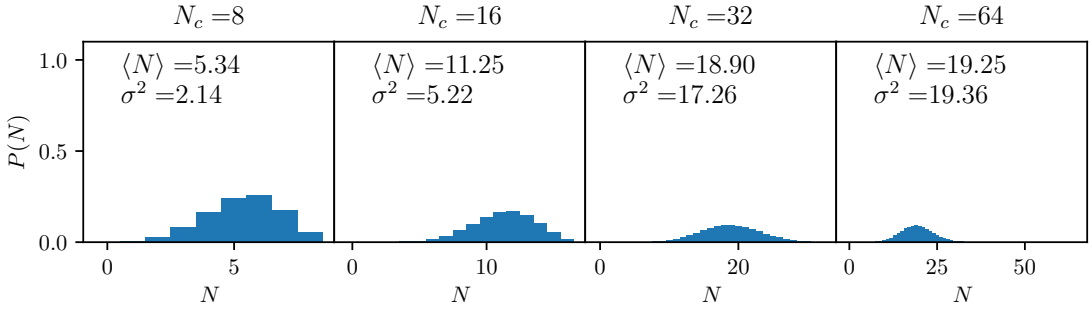


Figure 9: Photon number distributions for different values of N_{cutoff} at $\eta = 40\,\omega_r$. If we don't set N_{cutoff} sufficiently high, we get bogus results. A quick sanity check is to compare the mean with the variance. For a Poisson distribution, they have to be the same.

6 Outlook/Current research

Quantum simulation is a research field which is expanding rapidly. For those wishing to explore some of the recent advancements, [1] is part of a dossier with mini-reviews.

7 Conclusion

We demonstrated self-ordering in a longitudinal and transversally pumped cavity. Starting from the Jaynes-Cummings model, we modified the Hamiltonian for large detuning in a high-finesse cavity. We implemented simulations in QuantumOptics.jl, a package written in Julia. Our results meet our prior expectations and match with existing literature.

8 References

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