Building a network without backprop

Bernhard Gstrein

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  - Basic idea: lookup tables ("luts")

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  - Basic idea: lookup tables (“luts”)
  - Some properties of neural networks are promiment in lut networks too

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What is a lookup table (lut)?

	$\boldsymbol{X}$			y
	Lives in water	Has eyes	Has limbs	Vertebrate
$\boldsymbol{x}_1$	0	1	1	1
$\boldsymbol{x}_2$	1	1	0	1
$\boldsymbol{x}_3$	1	0	0	0

Model for classification of animals into vertebrates/invertebrates

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Model for classification of animals into vertebrates/invertebrates

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  - Suppose we have 784 columns $\rightarrow$ $2^{784} \propto 10^{236}$

Preprocessing data

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  - Features $\boldsymbol{X}$: matrix of shape $(N, 784)$, boolean entries
  - Labels y: vector of shape $(N,)$, boolean entries

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$$f(\text{bit pattern}) = \begin{cases} 0 & \text{if} \quad \sum\limits_{y=0} > \sum\limits_{y=1} \\ 1 & \text{if} \quad \sum\limits_{y=0} < \sum\limits_{y=1} \\ \text{rand}(0,1) & \text{if} \quad \sum\limits_{y=0} = \sum\limits_{y=1} \end{cases}$$

A single lut: example

Training set

$\boldsymbol{X}$	y
000	0
000	1
000	1
001	1
100	0
110	0
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bit pattern	$\sum_{y=0}$	$\sum_{y=1}$
000	1	2
001	0	1
010	0	0
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101	0	0
110	1	1
111	0	0

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bit pattern	f
000	1
001	1
010	0*
011	1*
100	0
101	1*
110	1*
111	0*

*: randomly chosen entries

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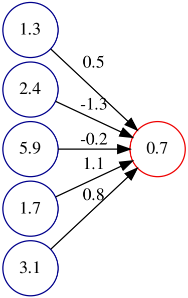
A single neuron

Neural network



$z = 1.3 \cdot 0.5 - 2.4 \cdot 1.3 - 5.9 \cdot 0.2 + 1.7 \cdot 1.1 + 3.1 \cdot 0.8 = 0.7$

$f(z) = \text{ReLU}(z) = \max(0, z) = 0.7$

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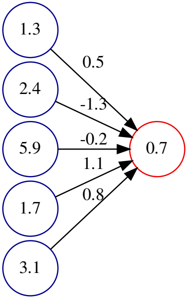

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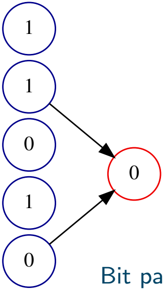
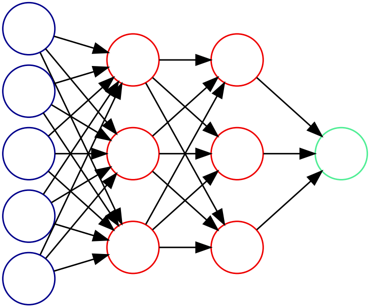
Lut network



Bit pattern	f
00	0
01	1
10	0
11	1

Neural network - lut network: comparison

Neural network



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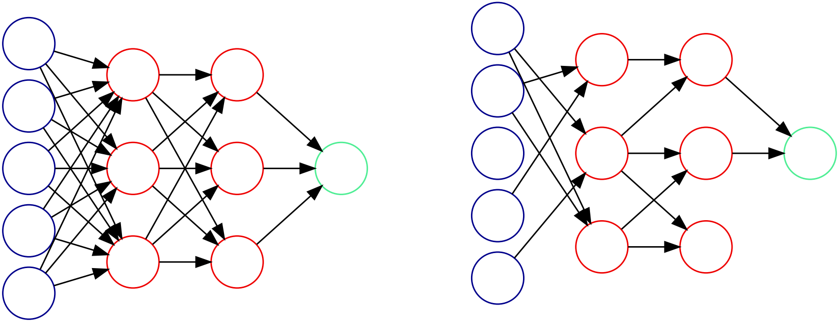

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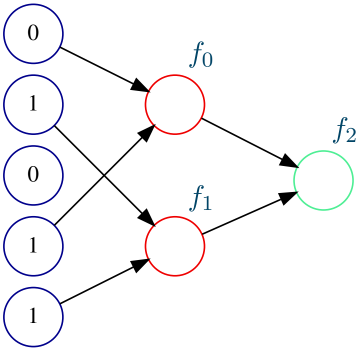

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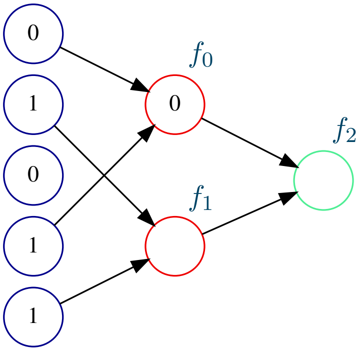

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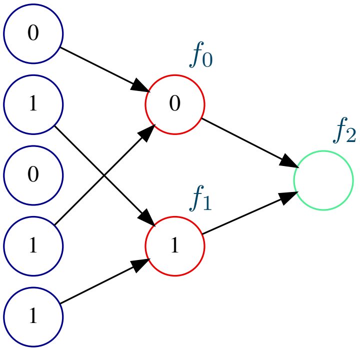

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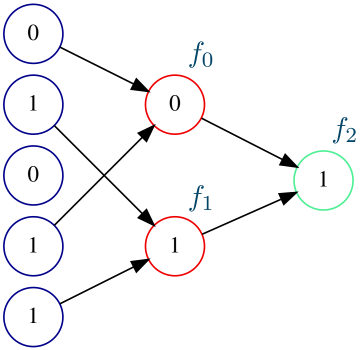
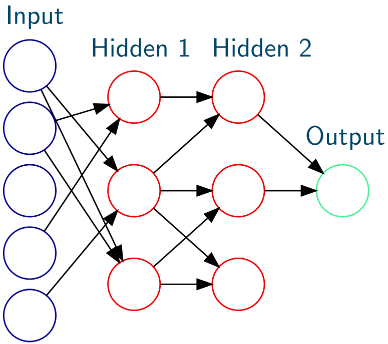


Training a network of luts

- Lut network: layers are trained successively



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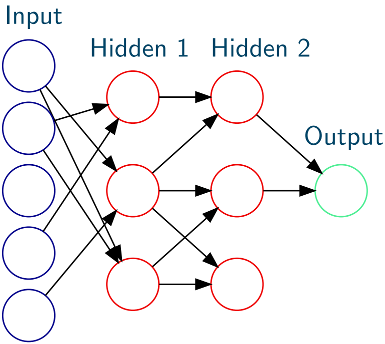
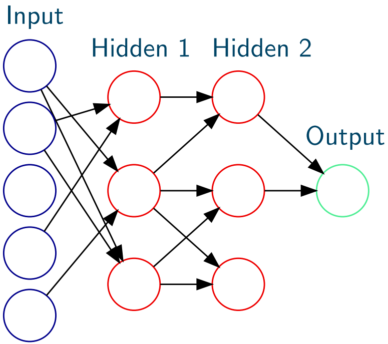


Training a network of luts

- Lut network: layers are trained successively
- Random choice of k columns for each lut
- Label vector is **always** y



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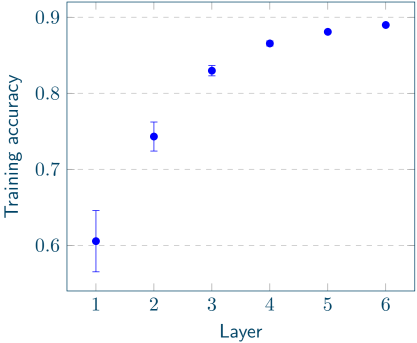
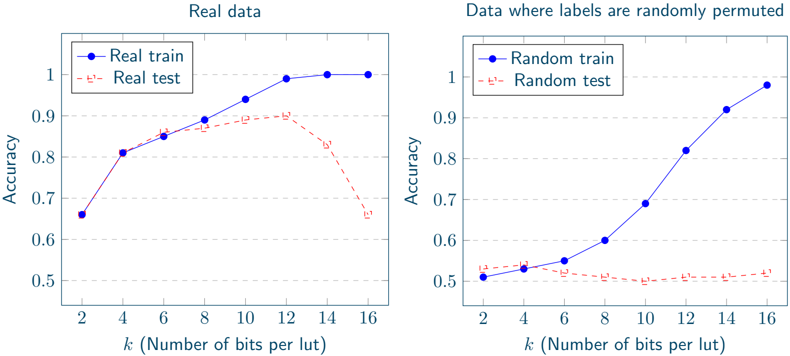
- Accuracy on held-out set: 0.87
- Results significantly above 0.5

Network of luts: depth improves performance



Training accuracy by layer for a network of 8-input lookup tables on Binary-MNIST. Each layer has 1024 luts except the last one which has only 1. Total height of error bars are two standard deviations.

Network of luts



Effect of varying lookup table size on Binary-MNIST. There are 5 hidden layers with 1024 luts per layer.

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- **The lut network is built without backprop and is able to perform non-trivial tasks**

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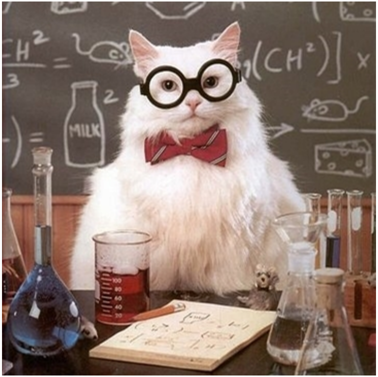
- Should we continue with luts?
- Building an AIG using local search/SAT solving sounds more interesting



Thank you for your attention!

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References

Chatterjee, Satrajit (2018). "Learning and memorization". In: *International Conference on Machine Learning*. PMLR, pp. 755–763.

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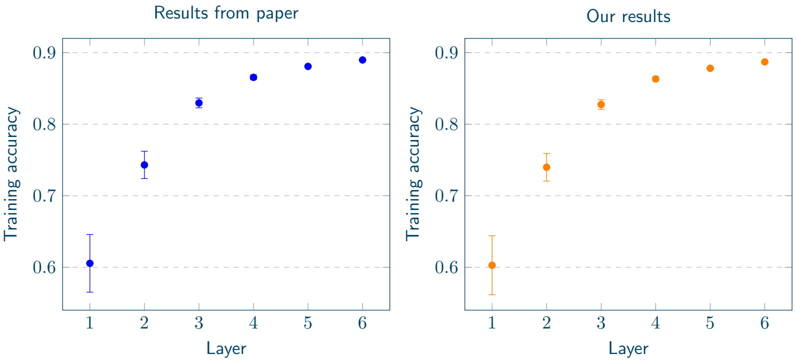
Depth improves performance: Our results



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