# Building a network without backprop

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# Background/Motivation

- For learning neural networks, backpropagation is almost exclusively used
- Our goal: building a "network" without backprop
  - Using local search, SAT solving, etc.
  - Logic synthesis; input: goals and constraints, output: network structure
- [Chatterjee 2018] describes a scheme to build something similar to a neural network without backprop
  - Basic idea: lookup tables ("luts")
  - Some properties of neural networks are promiment in lut networks too

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# What is a lookup table (lut)?

| | $\boldsymbol{X}$ | | | $y$ |
|---|---|---|---|---|
| | Lives in water | Has eyes | Has limbs | Vertebrate |
| $\boldsymbol{x}_1$ | 0 | 1 | 1 | 1 |
| $\boldsymbol{x}_2$ | 1 | 1 | 0 | 1 |
| $\boldsymbol{x}_3$ | 1 | 0 | 0 | 0 |

Model for classification of animals into vertebrates/invertebrates

- We must binarize the features and labels
- We must limit the complexity
  - Suppose we have 784 columns $\rightarrow$ $2^{784} \propto 10^{236}$

# Preprocessing data

- MNIST dataset: 28x28 images of handwritten digits (0-9)
- We unroll the images: $28 \cdot 28 = 784$
- We map the values: $[0, 255] \rightarrow [0, 1]$
- We binarize the data using the operator $> 0.5$

- Labels: $y = 0$ (numbers 0-4), $y = 1$ (numbers 5-9)

- We end up with
  - Features $\boldsymbol{X}$: matrix of shape $(N, 784)$, boolean entries
  - Labels $y$: vector of shape $(N, )$, boolean entries

# A single lut

- Every example is an instance of a "bit pattern" (e.g. $\boldsymbol{x} = 10$) and has a label (e.g. $y = 1$)
- We need a classification for each bit pattern: $f(00) = ?$, $f(01) = ?$, $f(10) = ?$, $f(11) = ?$
- For each bit pattern in $\boldsymbol{X}$, we cound how many times $y = 0$ and $y = 1$

$$f(\text{bit pattern}) = \begin{cases} 0 & \text{if} \quad \sum\limits_{y=0} > \sum\limits_{y=1} \\ 1 & \text{if} \quad \sum\limits_{y=0} < \sum\limits_{y=1} \\ \text{rand}(0, 1) & \text{if} \quad \sum\limits_{y=0} = \sum\limits_{y=1} \end{cases}$$

# A single lut: example

Training set

| $\boldsymbol{X}$ | $y$ |
|---|---|
| 000 | 0 |
| 000 | 1 |
| 000 | 1 |
| 001 | 1 |
| 100 | 0 |
| 110 | 0 |
| 110 | 1 |

| bit pattern | $\sum_{y=0}$ | $\sum_{y=1}$ |
|---|---|---|
| 000 | 1 | 2 |
| 001 | 0 | 1 |
| 010 | 0 | 0 |
| 011 | 0 | 0 |
| 100 | 1 | 0 |
| 101 | 0 | 0 |
| 110 | 1 | 1 |
| 111 | 0 | 0 |

| bit pattern | $f$ |
|---|---|
| 000 | 1 |
| 001 | 1 |
| 010 | $0^*$ |
| 011 | $1^*$ |
| 100 | 0 |
| 101 | $1^*$ |
| 110 | $1^*$ |
| 111 | $0^*$ |

*: randomly chosen entries

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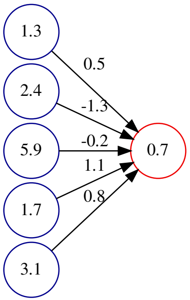
# A single neuron

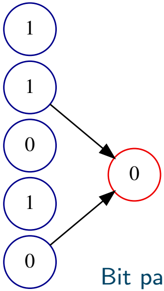
## Neural network




$z = 1.3 \cdot 0.5 - 2.4 \cdot 1.3 - 5.9 \cdot 0.2 + 1.7 \cdot 1.1 + 3.1 \cdot 0.8 = 0.7$

$f(z) = \mathsf{ReLU}(z) = \mathsf{max}(0, z) = 0.7$

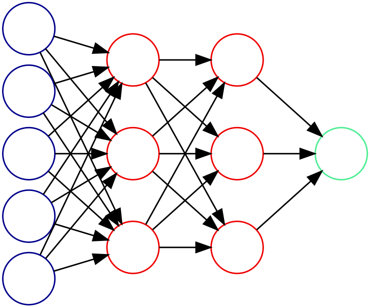
## Lut network



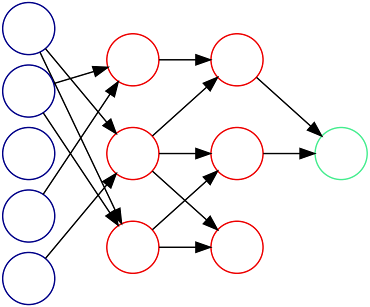

| Bit pattern | $f$ |
|---|---|
| 00 | 0 |
| 01 | 1 |
| 10 | 0 |
| 11 | 1 |

# Neural network - lut network: comparison

Neural network

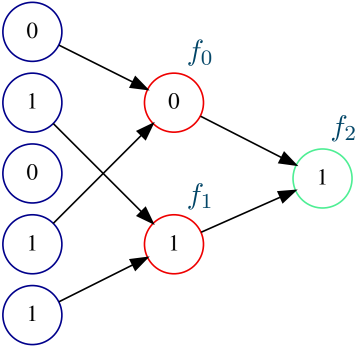


Lut network



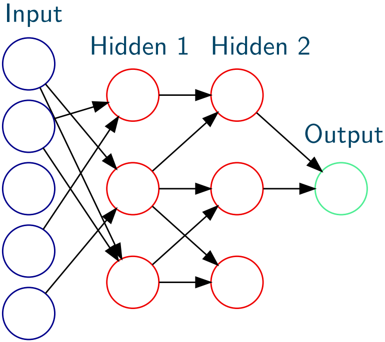
# Lut network: example

| bit pattern | $f_0$ | $f_1$ | $f_2$ |
|---|---|---|---|
| 00 | 1 | 1 | 1 |
| 01 | 0 | 0 | 1 |
| 10 | 1 | 0 | 0 |
| 11 | 0 | 1 | 0 |



# Training a network of luts

- Lut network: layers are trained successively
- Random choice of $k$ columns for each lut
- Label vector is **always** $y$



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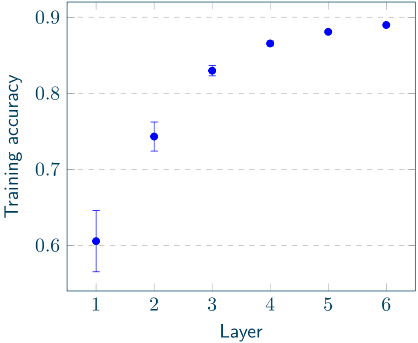
# First experiment

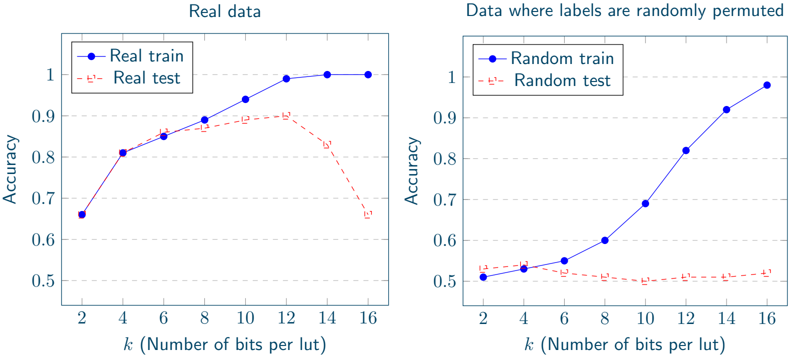
- Network with 5 hidden layers of 1024 luts and 1 lut in the output layer
- Each lut takes 8 inputs
- Training accuracy: 0.89
- Accuracy on held-out set: 0.87
- Results significantly above 0.5

# Network of luts: depth improves performance




Training accuracy by layer for a network of 8-input lookup tables on Binary-MNIST. Each layer has 1024 luts except the last one which has only 1. Total height of error bars are two standard deviations.

# Network of luts




Effect of varying lookup table size on Binary-MNIST. There are 5 hidden layers with 1024 luts per layer.

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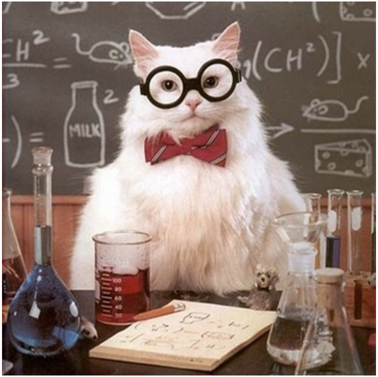
# Conclusion

- In the paper, there is also:
  - More experiments with other datasets and other tasks
  - Comparison of luts to other predictive models
- Neural networks and lut networks share some properties:
  - Depth improves performance
  - There is generalization on real data
  - Random data can be memorized
- **The lut network is built without backprop and is able to perform non-trivial tasks**

# Outlook

- We already wrote code that implements lut networks
  - Recreate paper results for practical work?
- Should we continue with luts?
- Building an AIG using local search/SAT solving sounds more interesting




# Thank you for your attention!

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# References

Chatterjee, Satrajit (2018). "Learning and memorization". In: *International Conference on Machine Learning*. PMLR, pp. 755–763.

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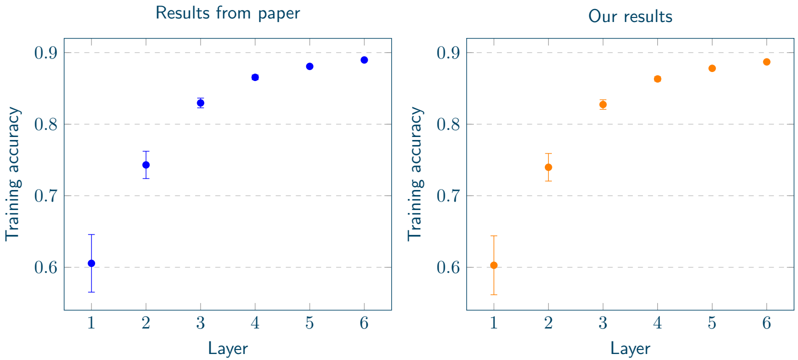
# Depth improves performance: Our results




Training accuracy by layer for a network of 8-input lookup tables on Binary-MNIST. Each layer has 1024 luts except the last one which has only 1. Total height of error bars are two standard deviations. We can see that our results closely match the results from the paper.