

My, how you've grown:
A practical guide to modeling size transitions
for Integral Projection Model (IPM) applications

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Open Research Statement: Data are already published and publicly available, with those items properly cited in this submission. Three data sets are cited as data packages (Ochocki *et al.*, 2023; Winfield *et al.*, 2013b; Miller, 2020). Two other data sets are available in our Github repo, which also includes all of our code. The repo will be archived in a Zenodo package, with the DOI included at the end of the article, upon publication. During peer review, our data and code are available at https://github.com/texmiller/IPM_size_transitions.

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1 Abstract

2 Integral Projection Models (IPMs) are widely used for studying continuously size-structured populations.

3 IPMs require a growth sub-model that describes the probability of future size conditional on current size and

4 any covariates. Most IPM studies assume that this distribution is Gaussian, despite calls for non-Gaussian

5 models that accommodate skewness and excess kurtosis. We provide a general workflow for accommodating

6 non-Gaussian growth patterns while retaining important covariates and random effects. Our approach

7 emphasizes visual diagnostics from pilot Gaussian models and quantile-based metrics of skewness and kurtosis

8 that guide selection of a non-Gaussian alternative, if necessary. Across five case studies, skewness and excess

9 kurtosis were common features of growth data and non-Gaussian models consistently generated simulated data

10 that were more consistent with real data than pilot Gaussian models. However, effects of “improved” growth

11 modeling on IPM results were moderate to weak, and differed in direction or magnitude between different

12 outputs from the same model. Using tools not available when IPMs were first developed, it is now possible to fit

13 non-Gaussian models to growth data without sacrificing ecological complexity. Doing so, as guided by careful

14 interrogation of the data, will result in models that better represent the populations for which they are intended.

15 **1 Introduction**

16 Structured demographic models – matrix and integral projection models (MPMs and IPMs) – are powerful
17 tools for data-driven modeling of population and community dynamics. In contrast to MPMs for populations
18 with discrete structure (life stage, age class, etc.), IPMs (Easterling *et al.*, 2000) accommodate populations
19 structured by continuous state variables, most commonly size. A related innovation of the IPM framework
20 is its emphasis on regression-based modeling for parameter estimation, which often carries important
21 advantages for making the most of hard-won data (Ellner *et al.*, 2022).

22 A standard workflow allows ecologists to assemble an IPM from data using familiar regression
23 tools to describe growth, survival, reproduction, and other demographic transitions as functions of size
24 (Coulson, 2012; Ellner *et al.*, 2016). The relative ease of regression analyses, accommodating covariates
25 (e.g., environmental factors, experimental treatments) and complex variance structures (e.g., random effects,
26 correlated errors), has facilitated a growing IPM literature that examines how biotic or abiotic factors affect
27 population dynamics (e.g., Louthan *et al.*, 2022; Ozgul *et al.*, 2010) and explores the consequences of
28 demographic heterogeneity associated with spatial, temporal, and individual variation (e.g., Compagnoni *et al.*,
29 2016; Crone, 2016; Plard *et al.*, 2018). The vital rate regressions (or “sub-models”) are the bridge between
30 the individual-level data and the population-level model and its predictions; it is important to get those right.

31 Compared to other vital rates, growth is special. The survival and reproduction sub-models only
32 need to provide a single predicted value as functions of size (we use “size” as the name for whatever
33 continuous variable defines the population structure). But the growth model must specify the full probability
34 distribution of subsequent size conditional on initial size, defining the growth ‘kernel’ $G(z', z)$ that gives the
35 probability density of future size z' at time $t+1$ conditional on current size z at time t . Whenever survival
36 and reproduction are size-dependent, the entire distribution of size transitions can strongly influence IPM
37 predictions because it governs how frequently size changes are much greater or much lower than average.

38 Easterling et al. 2000 provided the original template for modeling size transitions in IPMs. They
39 first tried simple linear regression, assuming Normally distributed size changes with constant variance.
40 Because the residuals from this regression exhibited non-constant variance, they used a two-step approach
41 to estimate the size-dependence in mean squared residuals (better options soon became available, such
42 as the `lme` function in R). However, even after accounting for non-constant variance, growth data may
43 still be non-Normal. Size transitions are often skewed such that large decreases are more common than
44 large increases (Peterson *et al.*, 2019; Salguero-Gómez & Casper, 2010), or vice versa (Stubberud *et al.*,
45 2019). Size transitions may also exhibit excess kurtosis (“fat tails”), where extreme growth or shrinkage
46 is more common than predicted by the tails of the Normal distribution (Hérault *et al.*, 2011).

47 The observation that the Normal (or Gaussian) distribution may poorly describe size transitions in real
48 organisms has been made before, and several studies have emphasized that alternative distributions should be
49 explored (Easterling *et al.*, 2000; Peterson *et al.*, 2019; Rees *et al.*, 2014; Williams *et al.*, 2012). For example,
50 Peterson et al. 2019 showed that skewness in size transitions could be modeled through beta regression
51 on transformed data (for reasons we describe below, this approach also has some drawbacks), or by fitting
52 a skewed Normal distribution. They showed that incorporating skew could have important consequences for
53 model-based inferences, and concluded that “testing of alternative distributions for growth... [should] become
54 standard in the construction of size-structured population models.” Nonetheless, default use of Gaussian
55 growth distributions (often with non-constant variance) remains the standard practice. The general state-of-
56 the-art in the literature appears to remain where it was 20 or so years ago, using the default Gaussian model
57 without examining critically whether or not it actually describes the data well. We are guilty of this, ourselves.

58 The persistence of Gaussian growth models is understandable. Popular packages such as `lme4` (Bates
59 *et al.*, 2015), `mgcv` (Wood, 2017), and `MCMCglmm` (Hadfield *et al.*, 2010) make it easy to fit growth
60 models with potentially complex fixed- and random-effect structures, but the possible distributions of
61 continuous responses are limited, and default to Gaussian. Abandoning these convenient tools for the

62 sake of more flexible growth modeling means, it may seem, sacrificing the flexibility to model diverse
63 sources of demographic variation, some of which may be the motivation driving the study in the first place.

64 Our goal here is to present and illustrate a practical “recipe” that moves growth modeling past the
65 standards set over 20 years ago. Using software tools that are now readily accessible, ecologists can escape
66 the apparent trade-off between realistically modeling non-Gaussian size transitions and flexibly including
67 multiple covariates and random effects.¹ As with any recipe, users may need to make substitutions or
68 add ingredients to suit their needs. We emphasize graphical diagnostics for developing and evaluating
69 growth models, rather than a process centered on statistical tests or model selection. Through empirical
70 case studies we demonstrate how tools that were nonexistent or not readily available when IPMs first came
71 into use now make it straightforward and relatively easy to identify when the default model is a poor fit
72 to the data, and to then choose and fit a better growth model that is no harder to use in practice. We illustrate
73 our approach by revisiting three published case studies (and three additional case studies in the Supporting
74 Information), including examples from our own previous work. In each case, the Gaussian assumption
75 does not stand up to close scrutiny. We illustrate how we could have done better, and the consequences of
76 “doing better” for our ecological inferences. All analyses were carried out in R (R Core Team, 2022) version
77 4.0 or higher and may be reproduced from publicly available code and data (see *Data Availability Statement*).

78 **2 Flexible growth modeling**

79 The modeling process that we suggest runs as follows (Fig. 1):

80 **1. Fit a “pilot” model assuming a Gaussian distribution, but allowing for non-constant variance.** This
81 step is familiar to most IPM users, as it is the start and end of the standard approach. It may include model
82 selection to identify which treatment effects or environmental drivers affect the mean and/or variance of future
83 size. Non-constant variance is often fitted in a two-stage process, first fitting mean growth assuming constant

¹Our statements about software availability are based on what current software reliably delivers in our personal experience, not on what they promise.

84 variance, then doing a regression relating the squared residuals to initial size or the fitted mean of subsequent
85 size. Fitting mean and variance simultaneously as functions of initial size, as can be done with R packages
86 **mgev** and **nmle**, is advantageous when possible because incorrectly assuming constant variance can affect
87 model selection for the mean. We illustrate both one-step and two-step approaches in the case studies below.

88 Allowing non-constant variance removes the need for transforming the data to stabilize growth variance.
89 Transformation may still be useful if it does not create new problems such as making some state-fate
90 relationships highly nonlinear. In particular, log-transformation often reduces or eliminates heteroskedasticity
91 in growth data (Ellner *et al.*, 2016) and also helps avoid eviction at small sizes (Williams *et al.*, 2012).

92 The fitted mean and variance functions should be checked before going any further. If they are
93 perfectly correct, standardized residuals (residuals scaled by the standard deviation) will have zero mean
94 and unit variance overall, and will exhibit no trends in mean or variance with initial size or fitted mean
95 value. However, estimates of the mean and variance functions are somewhat smoothed because of the
96 inescapable bias-variance tradeoff, so scaled residuals will retain some variation in location and scale.
97 Given enough data, statistical tests will detect that variation. So instead, we take for granted the presence
98 of trends and assess their importance by fitting nonparametric spline regression models for residuals (trend
99 in mean) and absolute residuals (trend in variance) as a function of initial size or fitted value. The mean
100 and variance functions can be accepted if the regression curves for the scaled residuals are nearly flat.

101 **2. Use graphical diagnostics to identify if and how the standardized residuals deviate from Gaussian,**
102 **and to choose a more appropriate distribution.** If the Gaussian growth model is valid, the standardized
103 residuals should be Gaussian with zero skewness or excess kurtosis. Growth data may deviate from this in many
104 ways, and the nature of the deviations can guide the search for a better distribution. Tests such as the D'Agostino
105 test of skewness (D'Agostino, 1970) and the Anscombe-Glynn test of kurtosis (Anscombe & Glynn, 1983)
106 can be used to diagnose whether the standardized residuals, in aggregate, deviate from normality (Komsta &

¹⁰⁷ Novomestky, 2015). However, the aggregate distribution may be misleading if skewness or kurtosis vary with
¹⁰⁸ size or other covariates. Skewness changing from positive at small sizes to negative at large sizes might produce
¹⁰⁹ zero overall skewness, but really requires a distribution that can allow both positive and negative skew, such
¹¹⁰ as the skewed Normal or Johnson S_U distributions. Alternatively, growth data may exhibit leptokurtosis (in
¹¹¹ which case the t distribution may be a good choice) or may shift from platykurtosis to leptokurtosis depending
¹¹² on initial size (in which case the power exponential distribution may be a good choice). It is therefore essential
¹¹³ to visualize trends in distribution properties with respect to either initial size, or expected future size for models
¹¹⁴ with multiple covariates. Fig. 1 includes guidance on how the skew and kurtosis properties of the standardized
¹¹⁵ residuals suggest options for an appropriate growth distribution. In our case studies we exploit the many
¹¹⁶ distributions in the **gamlss** R package (Stasinopoulos *et al.*, 2007), but other distribution families can be used.

¹¹⁷ **3. Refit the growth model using the chosen distribution.** In models with multiple covariates and/or random
¹¹⁸ effects, each potentially affecting several distribution parameters, “refit the model” could entail a massive
¹¹⁹ model selection process to identify the “best” non-Gaussian model. With so many options, model uncertainty
¹²⁰ may be overwhelming and over-fitting becomes a significant risk even when precautions against it are taken.

¹²¹ We therefore argue for adopting a more modest goal: remedy the defects evident in the standardized
¹²² residuals of the Gaussian model. This recommendation is based on the finding that parameter estimation
¹²³ using Gaussian regression models is generally robust to deviations from normality of the residuals (Scheipl et al.,
¹²⁴ 2020). That is, the fitted mean of the Gaussian model (as a function of covariates) is probably a
¹²⁵ very good approximation for the fitted mean in the corresponding non-Gaussian model — and if it is
¹²⁶ not, the next step in the modeling process will catch that. The functional forms for skew and kurtosis
¹²⁷ of the non-Gaussian model can be guided by the qualitative features of the graphical diagnostics (e.g.,
¹²⁸ that skewness switches from positive to negative with increasing size). As we demonstrate below, the
¹²⁹ mean and standard deviation functions can often be carried over exactly from the pilot Gaussian model.

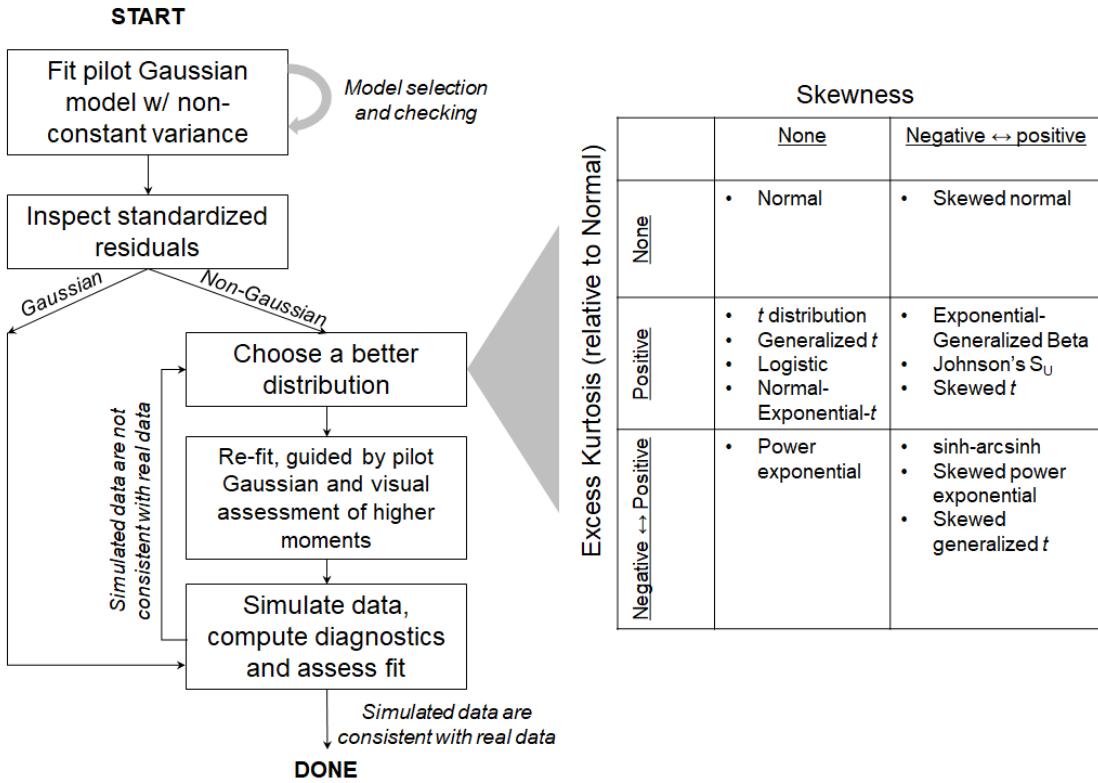


Figure 1: Recommended steps in growth modeling (left) and guide to common non-Gaussian distributions of size x for $x \in \mathbb{R}$ that can accommodate different combinations of skewness and kurtosis (right). All of these distributions (often including multiple versions or parameterizations of each) are available in the R package **gamlss.dist**, except for the skewed generalized t , which is available in the package **sgt** (Davis, 2015).

- 130 **4. Evaluate the final growth model through graphical diagnostics comparing simulated and real**
- 131 **growth data.** A good model will generate simulated data that look like the real data. Again, it is important
- 132 to inspect the properties of simulated data as a function of initial size, fitted mean, or other covariates
- 133 rather than examining the aggregate distribution. We again suggest below graphical diagnostics, based
- 134 mainly on quantiles, that can be used to compare simulated with real growth data. If the simulated data do
- 135 not correspond well with the real data, alternative or more flexible distribution families should be considered,
- 136 or more complex functions relating distribution parameters to size and other covariates.

¹³⁷ **3 How should skewness and kurtosis be measured?**

¹³⁸ Non-Gaussian growth modeling requires scrutinizing the skewness and kurtosis of standardized residuals,
¹³⁹ so measurement of these properties warrants attention. The standard measures are based on the third and
¹⁴⁰ fourth central moments, respectively, of the distribution: skewness = m_3/σ^3 , excess kurtosis = $m_4/\sigma^4 - 3$
¹⁴¹ where $m_k = \mathbb{E}(X - \bar{X})^k$ is the k^{th} central moment of a random variable X and σ^2 is the variance (second
¹⁴² central moment). A Gaussian distribution has zero skewness and zero excess kurtosis.

¹⁴³ The standard measures are simple and easy to use, but they have poor sampling properties. Because
¹⁴⁴ the measures involve high powers of data values, a few outliers can produce very inaccurate estimates. Figure
¹⁴⁵ 2 shows a simulated example, where the underlying data are samples of 200 values from a t distribution
¹⁴⁶ with 8 degrees of freedom, repeated 5000 times; the true skew is 0, and the true excess kurtosis is 1.5.
¹⁴⁷ The distance between the largest and smallest estimates (indicated by the dotted red vertical lines), relative
¹⁴⁸ to the distance between the 5th and 95th percentiles, shows the broad extent of extreme values that can
¹⁴⁹ occur even with a large sample, especially for kurtosis.

¹⁵⁰ We therefore recommend nonparametric (NP) measures of skewness and kurtosis that are based on
¹⁵¹ quantiles and thus are less sensitive to a few extreme values. Let q_α denote the α quantile of a distribution
¹⁵² or sample (e.g., $q_{0.05}$ is the 5th percentile). For any $0 < \alpha < 0.5$, a quantile-based measure of skewness
¹⁵³ is given by (McGillivray, 1986)

$$\text{NP Skewness} = \frac{q_\alpha + q_{1-\alpha} - 2q_{0.5}}{q_{1-\alpha} - q_\alpha}. \quad (1)$$

¹⁵⁵ NP Skewness measures the asymmetry between the tails of the distribution above and below the median.
¹⁵⁶ The size of the upper tail can be measured (for any $0 < \alpha < 0.5$) by $\tau_U = q_{1-\alpha} - q_{0.5}$; for $\alpha = 0.05$ this is the
¹⁵⁷ difference between the 95th percentile and the median. The lower tail size is $\tau_L = q_{0.5} - q_\alpha$. The definition

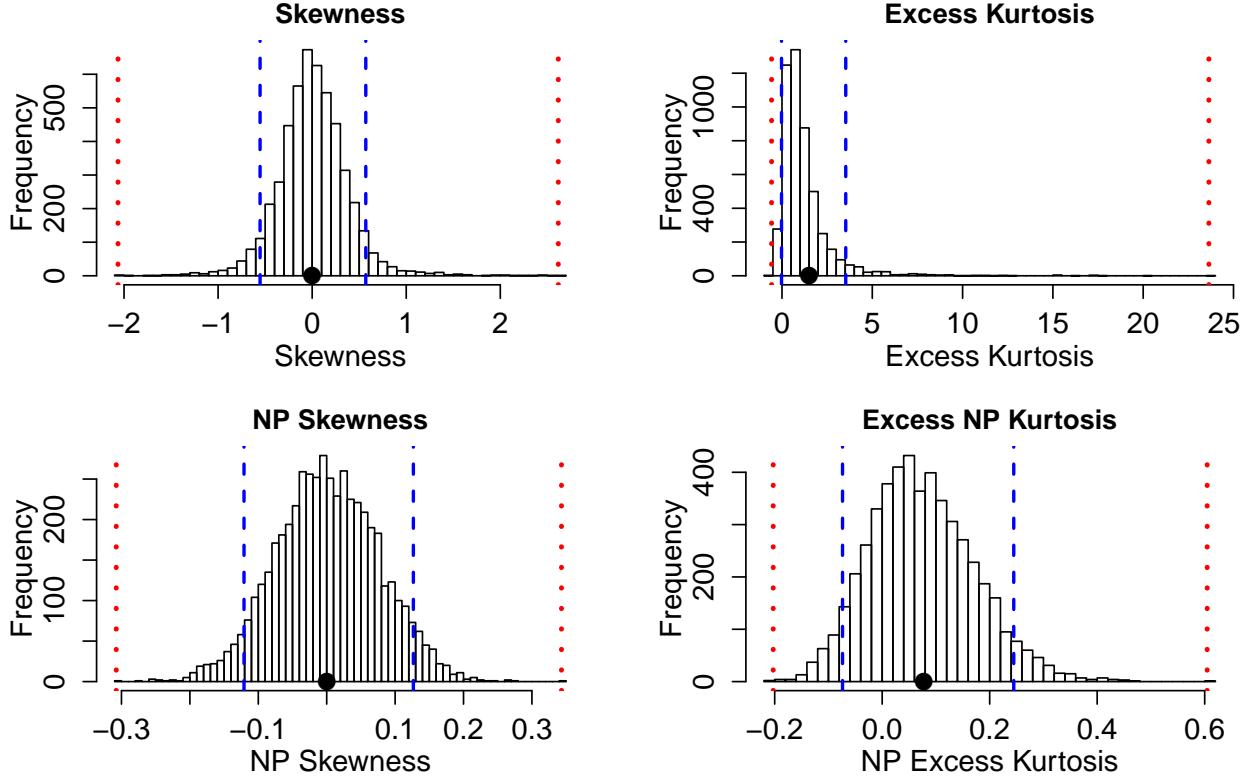


Figure 2: Histograms of skewness and kurtosis estimates using moment-based definitions (top two panels), compared with the nonparametric measures based on quantiles (bottom two panels). Note the very large differences in scale. Histograms are based on 5000 replicate draws of a sample of 200 independent values, from a t distribution with 8 degrees of freedom. Dotted red vertical lines mark the minimum and maximum of sample estimates, and dashed blue lines show the 5th and 95th percentiles. The true value is indicated by a black dot on the x -axis. Figure drawn by script `NPmoments.R`

158 above is equivalent to

$$159 \quad \text{NP Skewness} = \frac{\tau_U - \tau_L}{(\tau_U + \tau_L)}. \quad (2)$$

160 An NP Skewness of ± 0.2 says that the difference in tail sizes is 20% of their total. The range of possible
 161 values is -1 to 1. Both $\alpha=0.25$ (sometimes called “Kelly’s skewness”) and $\alpha=0.1$ (“Bowley’s skewness”)
 162 are common choices. We used $\alpha=0.1$.

163 An analogous quantile-based measure of kurtosis (Jones *et al.*, 2011) is

$$164 \quad \text{NP Kurtosis} = \frac{q_{1-\alpha} - q_\alpha}{q_{0.75} - q_{0.25}}. \quad (3)$$

165 For $\alpha=0.05$, NP Kurtosis is the difference between the 95th and 5th percentiles, relative to the interquartile
 166 range. To facilitate interpretation, we scale NP Kurtosis relative to its value for Gaussian distribution,
 167 and subtract 1 so that the value for a Gaussian is zero. We call this “NP Excess Kurtosis”. A value of ± 0.2
 168 means that the tails are on average 20% heavier than those of a Gaussian with the same interquartile range.
 169 We calculate NP Kurtosis using $\alpha=0.05$, to focus on the tail edges, but again this is somewhat arbitrary.

170 Figure 2C,D illustrate how, applied to the same simulated samples, the nonparametric measures
 171 produce a smaller fraction of highly inaccurate estimates caused by a few extreme values. Also note that,
 172 in contrast to the moment-based measures, numerically small values of the nonparametric measures (e.g., 0.1
 173 or 0.2) should not be disregarded, because both measures are scaled so that a value of 1 indicates extremely
 174 large departures from a Gaussian distribution.

175 Using quantile-based measures carries the added value that quantile regression can be used to estimate
 176 how they vary with initial size or expected future size. In the examples below, we use the **qgam** package
 177 (Fasiolo *et al.*, 2020) to fit spline quantile regression models, which accommodate nonlinear size-dependence
 178 in skewness and kurtosis. One risk of spline regression is that fitted quantiles may be excessively “wiggly”
 179 without constraints on their complexity; with realistic amounts of data, we can hope to estimate broad trends
 180 in distribution shape, but not fine-scale variation. In the examples below, we limit complexity by fitting
 181 splines with $k=4$ basis functions unless otherwise noted. Parametric quantile regression is also an option.

182 For consistency we also use quantile-based measures of mean and standard deviation when comparing
 183 real and simulated data, and use quantile regression to visualize their trends. Specifically, following Wan
 184 *et al.* (2014),

$$185 \text{NP Mean} = \frac{q_{0.25} + q_{0.5} + q_{0.75}}{3}, \quad \text{NP SD} = \frac{q_{0.75} - q_{0.25}}{1.35}. \quad (4)$$

186 **4 Case study: lichen, *Vulpicida pinastri***

187 We begin with a simple example where current size is the only predictor of future size. Growth data for
188 the epiphytic lichen *Vulpicida pinastri* were analyzed first by Shriver et al. 2012 and again by Peterson
189 et al. 2019 in their study of skewed growth distributions. We therefore had an *a priori* expectation of
190 deviation from normality. The data set includes 1,542 inter-annual transitions in thallus area (cm^2) observed
191 from 2004 to 2009 in Kennicott Valley, AK. Shriver et al. 2012 used a mixture distribution that separated
192 “normal growth or shrinkage” from “extreme shrinkage”. We aimed to fit a single growth model that could
193 realistically accommodate both types of size transition without requiring *ad hoc* decisions about which
194 observations of shrinkage were “extreme” or not.

195 With initial size as the only predictor, a convenient way to fit a Gaussian model with non-constant
196 variance is the `gam` function in **mgcv** library (Wood, 2017) using the `gaulss` family. Following a bit
197 of model selection, we fit the mean and standard deviation of future size as second-order polynomials
198 of current size², then calculated the scaled residuals from the fitted mean and standard deviation. Here,
199 the first argument to `gam()` is a two-element list that defines the linear predictors for mean and sd:

```
200 # d is the data frame; t0,t1 are initial & final thallus area, respectively  
201 fitGAU <- gam(list(t1~t0 + I(t0^2), ~t0 + I(t0^2)), data=d, family=gaulss())  
202 d$fitted_mean = predict(fitGAU, type="response")[,1]  
203 d$fitted_sd <- 1/predict(fitGAU, type="response")[,2]  
204 d$scaledResids=residuals(fitGAU, type="response")/d$fitted_sd
```

205 The data and fitted mean and standard deviation are shown in Fig. 3A, and the corresponding diagnostic
206 plots are in Fig. 4A,B. Our diagnostic plots are similar to plots made by R’s `plot.lm` function, except
207 that we use spline regression to allow data-driven choice of curve smoothness, and use absolute residuals

²`gam()` is most commonly used to fit smooth splines (`s()`) for predictor variables, but it can also fit parametric regressions.

208 (rather than their square roots) so that the standard deviation of the regression curve is on the same scale
209 as the residuals. The spline curves are not exactly flat – their standard deviations, given above each panel,
210 are positive – but the trends are much too small to be worth fixing.

211 Quantile regression on the scaled residuals generates the skewness and kurtosis diagnostics shown
212 in Fig. 3B. As expected based on previous analyses, the graphical analysis of the standardized residuals
213 indicates negative skew, especially at larger sizes (Fig. 3B). We also find positive excess kurtosis for all sizes.

214 We turned to the Johnson's *S-U* (JSU) distribution for improvement. The JSU is a four-parameter
215 leptokurtic distribution allowing positive or negative skew, with the convenient property that its location
216 and scale parameters `mu` and `sigma` are the mean and standard deviation, respectively, which greatly
217 facilitates the transition from a pilot Gaussian model. JSU is not available in any standard linear or additive
218 modeling packages, to our knowledge. But that is not a barrier because we can write a likelihood function
219 using the `dJSU()` function in the **gamlss.dist** package. Following the best-fit Gaussian model, we defined
220 `mu` and `sigma` of the JSU as quadratic polynomials of initial size and, based on Fig. 3B) we define
221 the skewness parameter `nu` as a linear function of size and kurtosis parameter `tau` as a positive constant.
222 The likelihood function therefore has nine parameters to estimate. We fit the model using the **maxLik**
223 package³ with starting coefficient values for `mu` and `sigma` based on the pilot Gaussian model:

```
224 ## define function that returns the JSU negative log-likelihood  
225 LogLikJSU=function(pars){  
226   dJSU(t1, mu=pars[1]+pars[2]*t0+pars[3]*t0^2,  
227   sigma=exp(pars[4]+pars[5]*t0+pars[6]*t0^2),  
228   nu = pars[7]+pars[8]*t0, tau = exp(pars[9]), log=TRUE)  
229 }
```

³We chose **maxLik** because it offers the BHHH optimization method, which works well for non-Gaussian likelihoods in our experience.

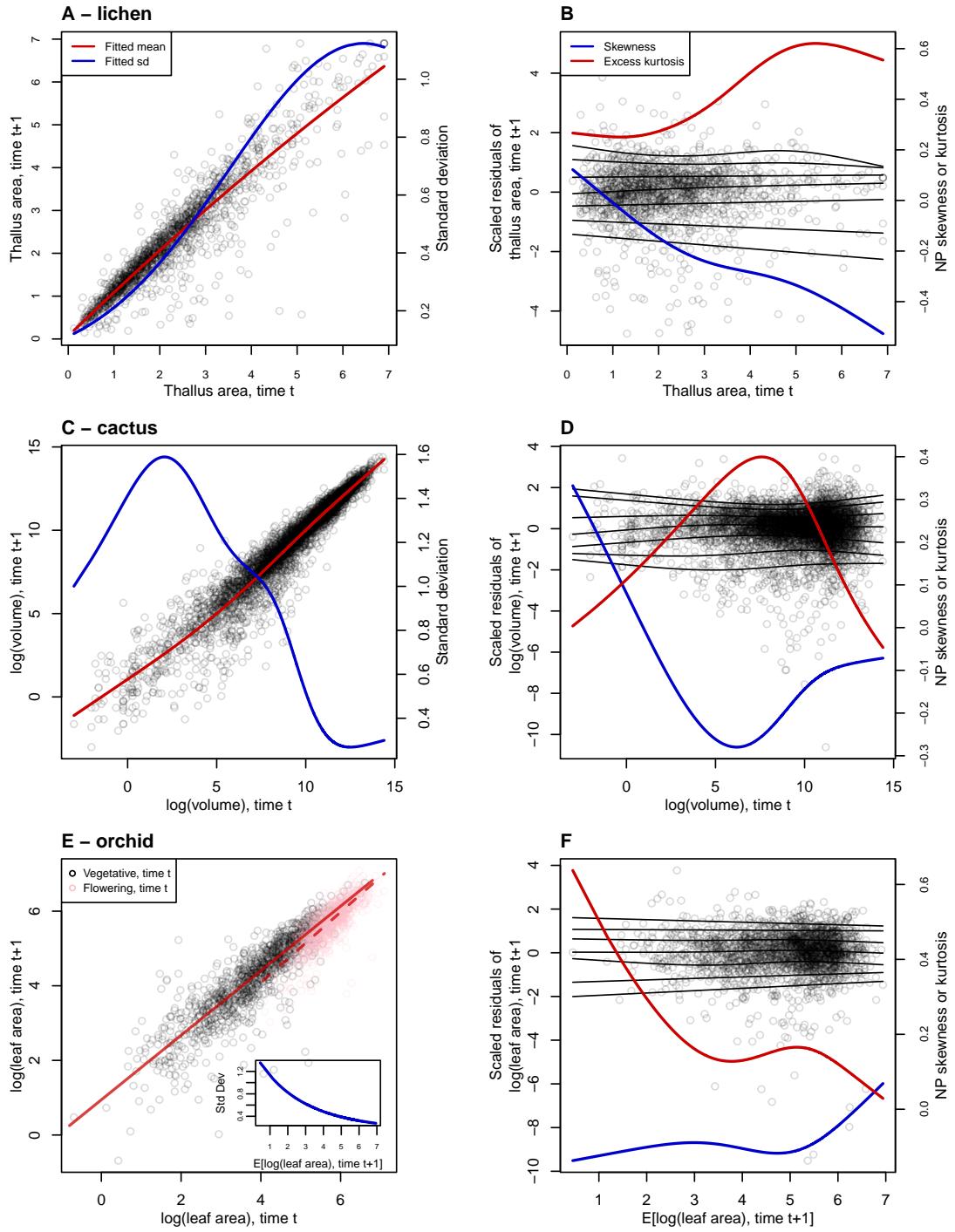


Figure 3: Best Gaussian models and diagnostics of standardized residuals for lichen (*Vulpicida pinastri*) **A,B**, cactus (*Cylindriopuntia imbricata*) **C,D**, and orchid (*Orchis purpurea*) **E,F** case studies. **A,C**, fitted mean (red) and standard deviation (blue) of size at time $t+1$ conditional on initial size at time t . **E**, fitted means for plants that were vegetative (solid line) or flowering (dashed line) at the start of the census interval and standard deviation as a function of the fitted mean (inset). **B,D,F** Quantile regressions of scaled residuals (lines show 5%, 10%, 25%, 50%, 75%, 90%, and 95% quantiles) and non-parametric measures of skewness (blue) and excess kurtosis (red) derived from them. In **B,D** scaled residuals are shown with respect to initial size and in **F** they are shown with respect to fitted values. Figure made by script `crosspp_growth.R`.

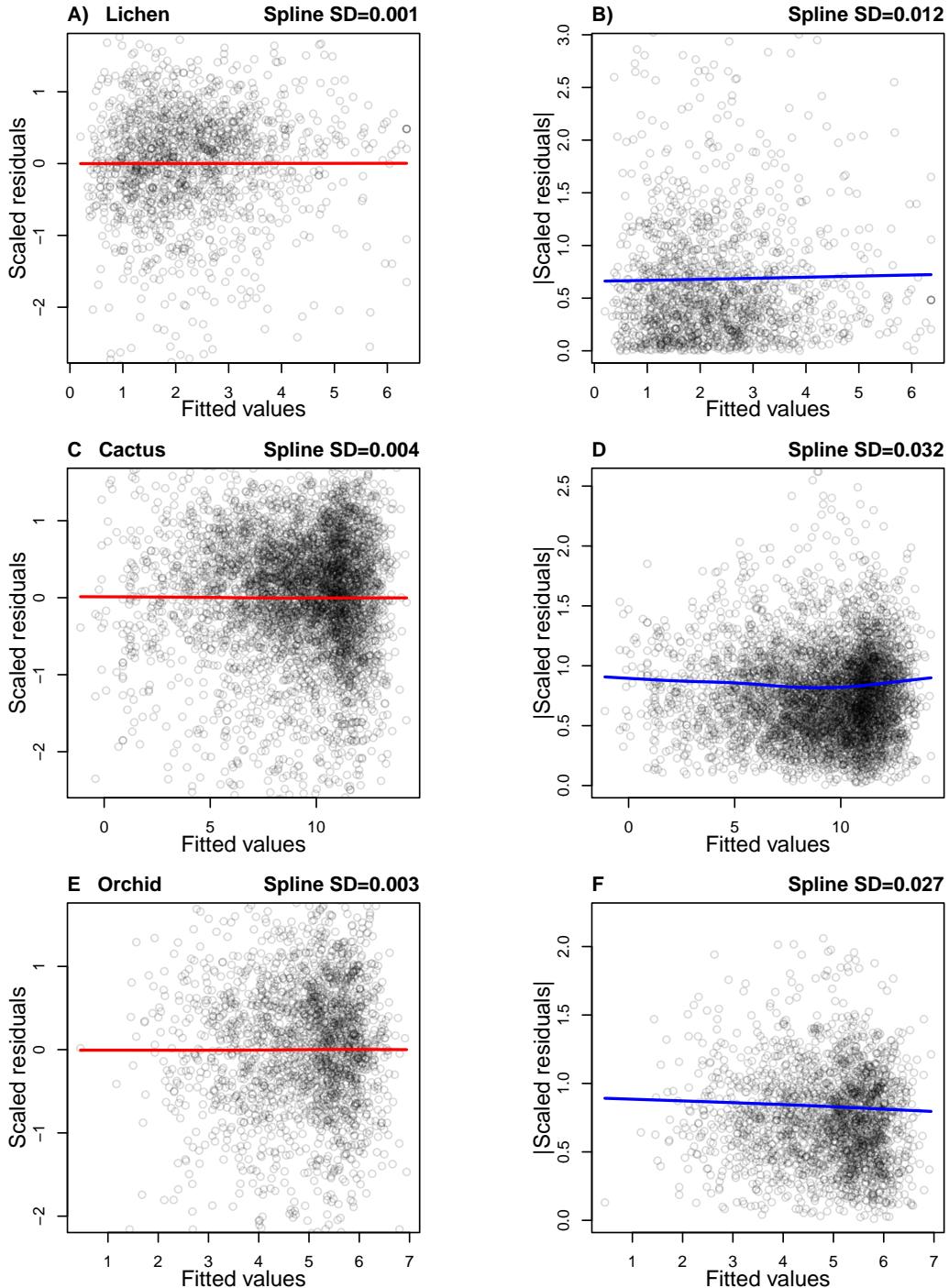


Figure 4: Diagnostic plot for trends in the mean (left column) or variance (right column) of scaled residuals from a pilot Gaussian model, for the lichen (*Vulpicida pinastri*) **A,B**, cactus *Cylindropuntia imbricata* **C,D**, and orchid *Orchis purpurea* **E,F** case studies. In **A,C,E** the standardized residuals are plotted, and in **B,D,F** the absolute values of standardized residuals, as functions of fitted mean subsequent size values. The solid curves are cubic splines (R function `smooth.spline`) fitted by generalized cross-validation with a modest over-penalization of model degrees of freedom to prevent overfitting (`penalty=1.4` as recommended by Gu (2013)). The numbers appearing above each panel are the standard deviation of the values on the spline regression curve, evaluated at all of the fitted values. Figure made by script `crossssp_diagnose_pilot.R`.

```

230 ## starting parameter values
231 p0<-c(coef(fitGAU)[1:6],0,0,0)
232 ## fit with maxlik, adding some noise to starting values
233 outJSU=maxLik(logLik=LogLikJSU,start=p0*exp(0.2*rnorm(length(p0))),
234 method="BHHH",control=list(iterlim=5000,printLevel=2),finalHessian=FALSE);

```

235 Simulating data from the fitted JSU model indicates a compelling improvement over the best Gaussian
236 model, not only in skewness and kurtosis (Fig. 5C-D) but also the nonparametric standard deviation (5B).
237 Note, in these data simulation figures Gaussian and non-Gaussian data are offset by an arbitrary amount
238 to more easily visualize their correspondence to the real data (black lines in Fig. 5).

239 To understand the practical consequences of improved growth modeling, we assembled the remainder
240 of the lichen IPM following Shriver et al. 2012. The asymptotic population growth rate λ based on Gaussian
241 growth differs from the JSU growth model by about 1% annual population growth (Table 1), in line with
242 results of Peterson et al. 2019. However, even this modest difference can lead to biased estimates of extinction
243 risk from the Gaussian model, particularly over longer time horizons (Fig. 6). We also explored differences in
244 other life history metrics (Table 1) using functions from Hernández *et al.* (2024). For example, the JSU growth
245 model predicts values for mean lifespan, mean lifetime reproductive success, and generation time that are 15–
246 25% lower than the Gaussian growth model. In this case study, properly modeling non-normal size transitions
247 – which was easy to do with a few extra lines of code – can influence ecological inferences, at least based on
248 point estimates. However, Table 1 also provides bias-corrected, bootstrapped confidence intervals (Diciccio &
249 Efron, 1996), and these are heavily overlapping between the Gaussian and JSU models for all life history traits,
250 suggesting that effects of “improved” growth modeling are small relative to our uncertainty in model parameters.

251 One could argue that this example was a convenient “straw man” to disqualify Gaussian growth,
252 because it was recognized by the original and subsequent analysts that size transitions are strongly skewed

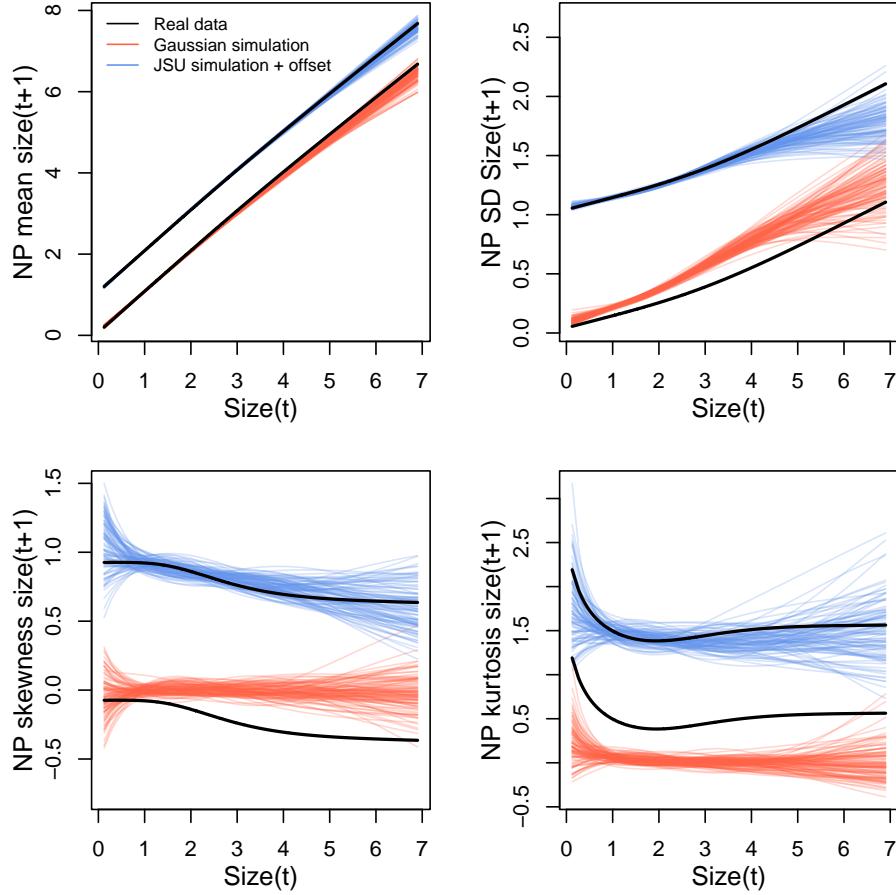


Figure 5: Comparisons among real lichen data and data simulated from Gaussian and JSU growth models for NP mean, NP standard deviation, NP skewness, and NP excess kurtosis of future size conditional on current size. Colored lines show 100 simulated data sets from the fitted Gaussian (red) or JSU (blue) growth models. Thick black line shows the real data. Gaussian and JSU data are offset by one unit and the real data line is duplicated with a one-unit offset for ease of visualization. Figure made by script `Vuplicida_IPMs.R`.

253 (Peterson *et al.*, 2019; Shriver *et al.*, 2012). In all remaining case studies, including those in the Supporting

254 Information, we re-examine growth data that were modeled as Gaussian in the original published analysis.

255 5 Case study: tree cholla cactus, *Cylindriopuntia imbricata*

256 The next case study, focused on the tree cholla cactus *Cylindriopuntia imbricata* at the Sevilleta Long-Term

257 Ecological Research site in central New Mexico, adds a new feature to the simple size-dependent regressions

258 in the previous study: random effects associated with temporal (year) and spatial (plot) environmental

259 heterogeneity. This long-term study was initiated in 2004 and different subsets of the data have been analyzed

Table 1: Life history attributes derived from IPM kernels that included Gaussian or “improved” growth sub-models for six case studies. The improved distributions were JSU (lichen, creosote), SHASH (cactus, pike, coral), and skewed t (orchid). Pike, creosote and coral case studies are presented in the Supporting Information. The original coral case study assumed an open population with constant recruitment from a large source region, so some life history attributes cannot be computed from the published model. Values in parenthesis are 95% bootstrap confidence intervals, specifically the bias-corrected (BC) bootstrap confidence intervals Diciccio & Efron (1996). Table can be reproduced from scripts `crosspp_growth.R`, `Vulpicida_boot.R`, `Akumal_corals_boot.R`.

Species	Growth model	λ	Lifespan	Lifetime reproductive output	Age at reproduction	Generation time
Lichen (<i>V. pinastri</i>)	Gaussian	1.01 (0.99, 1.04)	6.4 (3.6, 11.1)	1.4 (0.5, 3.1)	6.5 (5.7, 7.3)	40.8 (30.5, 57.4)
	Improved	1.00 (0.98, 1.03)	5.4 (3.1, 9.7)	1 (0.4, 2.4)	6.4 (5.4, 7.3)	36.6 (27.5, 48.6)
Cactus (<i>C. imbricata</i>)	Gaussian	0.994 (0.99, 0.996)	6.11 (3.66, 8.63)	21.8 (8.27, 49.4)	17.6 (1.75, 22.7)	189 (131, 266)
	Improved	0.993 (0.991, 0.998)	5.38 (3.34, 16.3)	13.4 (5.72, 251)	20.3 (1.21, 22.2)	179 (133, 298)
Orchid (<i>O. purpurea</i>)	Gaussian	1.09 (1.08, 1.1)	1.08 (1.06, 1.11)	20.0 (12.6, 31.0)	5.07 (4.78, 5.31)	104 (73.1, 150)
	Improved	1.09 (1.08, 1.1)	1.08 (1.06, 1.1)	19.3 (12.0, 29.9)	5.03 (4.75, 5.3)	100.7 (71.0, 145.0)
Pike (<i>E. Lucius</i>)	Gaussian	1.62 (1.35, 1.89)	1.2 (1.09, 1.35)	5.75 (2.9, 9.7)	1.09 (1.03, 1.18)	4.96 (4.26, 5.84)
	Improved	1.62 (1.35, 1.88)	1.2 (1.09, 1.35)	5.76 (2.91, 9.73)	1.09 (1.03, 1.18)	4.94 (4.30, 5.84)
Creosote (<i>L. tridentata</i>)	Gaussian	1.033 (1.029, 1.04)	4.52×10^6 (2.14×10^5 , 1.82×10^8)	3.19×10^5 (1.27 $\times 10^4$, 1.24×10^7)	32.7 (29.2, 36.0)	5.27×10^6 (2.50 $\times 10^5$, 1.95×10^8)
	Improved	1.034 (1.03, 1.04)	3.26×10^5 (1.98×10^3 , 1.66×10^7)	2.31×10^4 (5.83 $\times 10^2$, 1.27×10^6)	32.8 (29.3, 36.0)	3.7×10^5 (2.63 $\times 10^3$, 1.93×10^7)
Coral (<i>G. ventalina</i>)	Gaussian	—	17.3 (11.9, 24.3)	—	10.5 (9.3, 11.8)	31.6 (28.3, 36.7)
	Improved	—	17.5 (12.1, 24.3)	—	10.7 (9.4, 12.2)	30.9 (27.4, 35.3)

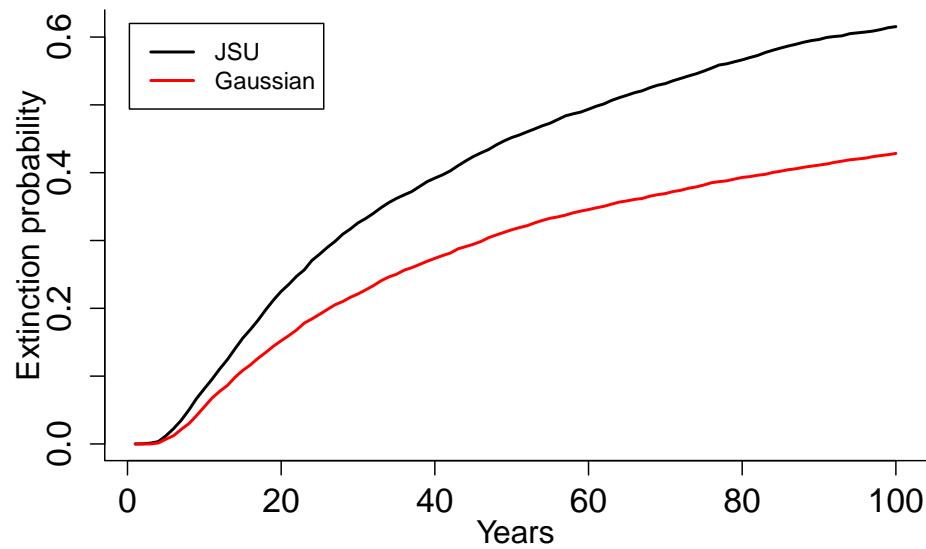


Figure 6: Extinction risk estimated from individual-based simulation of IPMs based on Gaussian and Johnson’s S-U (JSU) growth distributions. Figure made by script `Vulpicida_IPMs.R`.

260 in various IPM studies, all using Gaussian growth kernels (Compagnoni *et al.*, 2016; Czachura & Miller, 2020;
261 Elderd & Miller, 2016; Miller *et al.*, 2009; Ohm & Miller, 2014). In fact, Elderd and Miller 2016 presented
262 a Gaussian growth model as an example of a well fit growth function, based on an overall distribution of
263 residuals that appeared Gaussian and posterior predictive checks (PPCs) of a Bayesian model that suggested
264 consistency between the real data and data simulated from the fitted model (Fig. 4 in (Elderd & Miller,
265 2016)). While PPCs and the associated “Bayesian P-value” are popular diagnostic tools, they are often too
266 conservative (Conn *et al.*, 2018; Zhang, 2014), failing to reject marginally bad models even though they are
267 very effective in rejecting terrible models. The choice of discrepancy function (the statistic used to compare
268 real and simulated data) can also be limiting: in our previous work, we used a discrepancy function focused
269 on variance (the sum of squared residuals), creating a blind spot for poor modeling of higher moments.

270 The data includes 4844 size transition observations from 929 individuals spanning 13 transition years
271 (2004–2018) and 11 spatial replicates (three spatial blocks in years 2004–2008 and eight 30m-by-30m plots
272 in years 2009–2018). The data are provided in Miller (2020). Following previous studies, we quantified
273 size as the natural logarithm of plant volume (cm^3), derived from height and width measurements.

274 We begin growth modeling, as above, with a generalized additive model with the mean and standard
275 deviation of size in year $t+1$ modeled as smooth function of size in year t , with random intercepts for
276 year and plot and assuming normally-distributed residuals:

```
277 # t0 and t1 are initial and final log(volume), respectively  
278 fitGAU <- gam(list(t1 ~ s(t0,k=4) + s(plot,bs="re") + s(year,bs="re"),  
279 ~ s(t0,k=6)), data=caactus, family=gaulss())
```

280 Note that here we fitted the standard deviation function with $k=6$ basis functions rather than our default
281 of $k=4$ because, in a preliminary analysis, we found a moderate variance trend in the standardized residuals

282 using $k = 4$, suggesting a need for greater flexibility. With $k = 6$, spline regression detected essentially
283 no trend in the mean of the resulting standardized residuals (Fig. 4C,D).

284 The growth variance is estimated to peak at small to medium sizes (Fig. 3C). The standardized
285 residuals show clear signals of negative skew and positive excess kurtosis across most of the size distribution,
286 but strongest in the middle (Fig. 3D). We therefore need a distribution family allowing negative skew
287 and positive excess kurtosis, both of which may be negligible at some sizes. We first tried Johnson's S_U and
288 then the skewed t distributions, which provided some improvements but there were still visible discrepancies
289 between simulated and real data. We next turned to the SHASH distribution, which allows a greater range
290 of kurtosis for a given amount of skew, and vice versa (Jones & Pewsey (2009); Supporting Information
291 S.1). This flexibility proved necessary to generate simulated data that compared favorably to the real data,
292 so we proceeded with the SHASH. Conveniently, SHASH is available as an **mgcv** family, allowing for
293 flexible size-dependence in skewness and kurtosis without having to select specific size-dependent functions.

294 Here, the first argument to `gam()` is now a four-element list specifying the linear predictors for
295 the four parameters of the SHASH distribution.

```
296 fit_shash <- gam(list(t1 ~ s(t0,k=4) +  
297   s(plot,bs="re") + s(year_t,bs="re"), # location  
298   ~ s(t0,k=4), # log-scale  
299   ~ s(t0,k=4), # skewness  
300   ~ s(t0,k=4)), # log-kurtosis  
301   data = cactus, family = shash,optimizer = "efs")
```

302 Data simulated from the SHASH model compared favorably to the real data (Fig. S-11). Similar to
303 the lichen case study, we see that correctly modeling skewness and kurtosis improved estimation of the
304 nonparametric mean and standard deviation (Fig. S-11A,B), yielding a growth model that is truer to the data.

We next explored how improved growth modeling influenced IPM results. The λ values predicted by Gaussian and SHASH growth functions, corresponding to the average plot and year, were nearly identical (Table 1) but we could also leverage structure of the study design to quantify demographic variance associated with temporal and spatial heterogeneity. We used the fitted random effects from the vital rate models to estimate the asymptotic growth rate for each year (λ_t), centered on the average plot, and for each plot (λ_p), centered on the average year. Estimates of λ_t from the Gaussian growth model were often greater than estimates from the SHASH growth model, particularly in some of the harshest years (Fig. 7A), and therefore the Gaussian model predicted lower temporal variance in fitness ($SD(\lambda_{t(Gaussian)}) = 0.04$, $SD(\lambda_{t(SHASH)}) = 0.048$). Plot-to-plot variation was more similar between the two models ($SD(\lambda_{p(Gaussian)}) = 0.0026$, $SD(\lambda_{p(SHASH)}) = 0.0028$), although spatial variation in fitness was much lower than temporal variation (Fig. 7B). The difference in temporal variance would suggest that Gaussian growth modeling would predict a higher stochastic growth rate λ_S , because temporal variance has a negative effect on λ_S . However, the stochastic growth rate from the Gaussian growth model ($\lambda_S = 0.992$) was nearly identical to that of the SHASH growth model ($\lambda_S = 0.991$). This is likely because temporal fluctuations in vital rates, which is where the SHASH growth model would make a difference, have a weaker influence on λ_S than the temporal fluctuations in size structure that they generate (Compagnoni *et al.*, 2016; Ellis & Crone, 2013). The SHASH and Gaussian growth models predicted small differences in other life history traits but, as in the lichen case study, these differences were small relative to the uncertainty captured by bootstrapped confidence intervals (Table 1). Thus, in this case study, modeling non-Gaussian size transitions with a Gaussian growth model may or may not influence IPM results depending on the target of the analysis.

6 Case study: lady orchid, *Orchis purpurea*

Our final case study examines selection on life history strategies in the lady orchid *Orchis purpurea*. In a prior study, Miller *et al.* 2012 analyzed how costs of reproduction (flowering or not in year t) affected

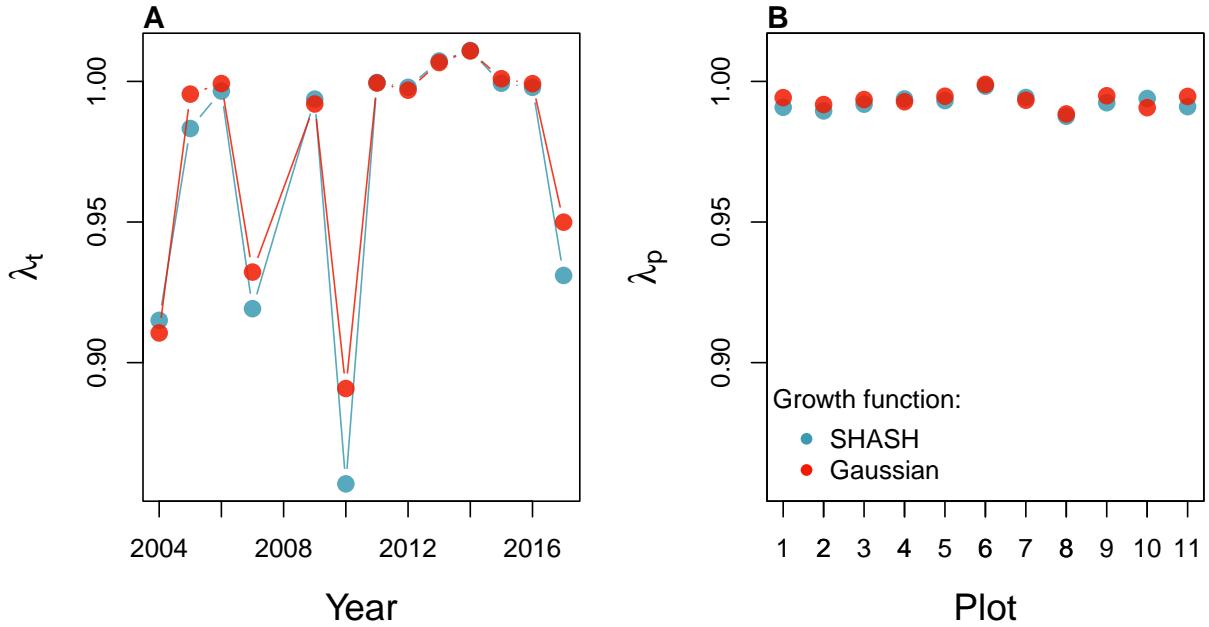


Figure 7: Temporal (A) and spatial (B) heterogeneity in fitness for the tree cholla cactus (*Cylindropuntia imbricata*) predicted by IPMs using Gaussian or SHASH growth models. Figure made by script `cactus_growth_modeling_qgam.R`.

328 growth from year t to $t+1$. The two growth kernels for flowering and non-flowering were then used in
 329 an IPM to quantify the optimal flowering size that balances the benefits of waiting to flower at larger sizes
 330 against the greater risk of death before flowering. The original study assumed Gaussian size transitions
 331 with non-constant variance depending on initial size. Here we re-visit that analysis to derive improved
 332 growth kernels. We use this case study to illustrate several new elements and challenges, including modeling
 333 skewness and kurtosis as functions of expected future size.

334 The data, originated by Dr. Hans Jacquemyn, come from 368 plants in a Belgian population censused
 335 annually from 2003 through 2011. Here we use data only from the “light” habitat in the original study.
 336 We used the natural logarithm of total leaf area as the size variable in the IPM.

337 As a variation on software, we fitted the pilot Gaussian model using the `lmer` function in the **lme4**
 338 package, as in the original study. We fit three candidate linear models that included fixed effects of size
 339 in year t (model 1), additive effects of size and flowering status in year t (model 2), or an interaction between
 340 size and flowering (model 3), all including random intercepts for year. The interaction model was strongly

341 favored ($\Delta AIC = 10.5$). Unlike our previous case studies, here we have multiple fixed effects (initial size
342 and flowering status) that may influence the variance of future size. In cases such as this it is convenient
343 to model variance as a function of expected future size, rather than initial size as we did with the lichens
344 and cacti. The expected (or “fitted”) values reflect the combined influence of all fixed and random effects,
345 and therefore implicitly account for multiple sources of variation in the variance.

346 Models where error variance is a function of fitted values cannot be fitted directly with `lme4` (nor in
347 the `mgcv` functions for generalized additive models). But it can still be done with `lmer` through an iterative
348 re-weighting approach, as follows. In `lmer`, weights w_i can be used to indicate that the observations y_i
349 have error variance proportional to $1/w_i^2$. The iterative steps are as follows, and code that executes these
350 steps is in `orchid_growth_modeling.R`.

- 351 1. Fit the expected value assuming Gaussian-distributed residuals with constant variance.
352 2. Fit the standard deviation of the residuals as a function of the corresponding fitted value.
353 3. Re-fit the model, with weights equal to the inverse of the standard deviation estimated in step 2.

354 We iterated steps 2 and 3 until the root mean square change in weights was below 10^{-6} . This is not elegant,
355 but it works and converges quickly. In step 2, we modeled the log of the standard deviation (because standard
356 deviations cannot be negative) as a quadratic polynomial in the fitted mean. In exploratory analyses we found
357 that the quadratic term was necessary to fit the standard deviation. We did this for all candidate models and,
358 for a fair AIC comparison, we then re-fit all candidate models with the weights estimated from the top model.

359 The updated model selection continued to favor the size \times flowering interaction model (3), but now
360 with a weaker improvement over the next-best model ($\Delta AIC = 6.7$). The fitted mean (a function of initial
361 size and flowering status) and fitted standard deviation (a function of the fitted mean) are shown in Fig.
362 3E. Spline regression found no trend in the mean of the resulting standardized residuals, and only small
363 variation in the variance (Fig. 4E,F).

364 The best Gaussian model indicated a growth cost associated with flowering at the start of the census
365 interval and a decline in growth variance with increasing expected values (Fig. 3E). The standardized
366 residuals indicated negative skewness (10–20% difference in tail weight) and excess kurtosis (10–40% fatter
367 than Gaussian) across much of the size distribution but both negligible at large expected sizes (Fig. 3F).

368 As possible improvements, we explored the skewed *t* and JSU distributions, both leptokurtic
369 distributions with flexible skewness. Based on comparisons between real and simulated data we were
370 happier with the skewed *t*, which we fit with a custom likelihood function similar to the JSU growth model
371 for the lichen data. However, rather than re-fitting all parameters of the skewed *t* model, as we did with
372 the lichen JSU, we built a “hybrid” likelihood function that uses the fitted mean and standard deviation from
373 the best Gaussian model, and estimates parameters that control skewness and kurtosis as linear functions
374 of expected future size. This is easy because the **gamlss.dist** package provides a parameterization of the
375 skewed *t* in which the location parameter μ is the mean and scale parameter σ is the standard deviation
376 (Rigby *et al.*, 2019). The hybrid likelihood looks like this:

```
377 ## GAU_fitted and GAU_sd are mean & standard deviation from the best Gaussian.  
378 SSTLogLik=function(pars){  
379   dSST(log_area_t1,  
380   mu=GAU_fitted, sigma=GAU_sd,  
381   nu = exp(pars[1] + pars[2]*GAU_fitted),  
382   tau = exp(pars[3] + pars[4]*GAU_fitted)+2, log=TRUE)  
383 }  
384 p0<-c(0,0,0,0) ## default starting parameters  
385 SSTout=maxLik(logLik=SSTLogLik,start=p0) ## fit with maxLik
```

386 Based on diagnostics of the standardized residuals, parameters that control skewness and kurtosis are defined as
387 linear functions of the mean (note that the `tau` parameter uses a $\log(x-2)$ link function). This approach relies
388 on the robustness of fitted Gaussian models to deviations from normality, which implies that the fitted mean
389 and variance from a Gaussian model are good approximations for the mean and variance of the corresponding
390 non-Gaussian model. If one is skeptical of this approach, it is possible to simultaneously re-fit all parameters of
391 the skewed t . However, recall that unlike the lichen case study, the pilot Gaussian model here includes random
392 year effects, and the expected values getting passed into dSST account for this source of variation. Estimating
393 random effects “from scratch” with a custom likelihood model is possible (we provide guidance on doing this
394 with a “shrinkage” approach, in Supporting Information S.2), but generally should not be necessary. Instead, a
395 key advantage of the hybrid approach is retention of the fitted random effects and associated variance compo-
396 nents, which get shuttled from the Gaussian model into the non-Gaussian model without any fuss (though it was
397 critical to use a parameterization of the skewed t for which `mu` is the mean and `sigma` is the standard deviation).
398 And, if this approach does not “work” (i.e., deviations from normality biased the fitted values of the Gaussian
399 model) one would quickly find out when comparing simulated with real data. In this case, size transition
400 data simulated from this model corresponded favorably to the real data, much better than the pilot Gaussian
401 model, including improvements in the standard deviation, skewness, and kurtosis of future size (Fig. S-12).

402 Finally, we used the improved growth model to revisit key results of the original study. Miller et
403 al. (2012) used the orchid IPM to estimate the evolutionarily stable strategy (ESS) as the mean size at
404 flowering that maximizes lifetime reproductive success (R_0), given the constraint that flowering when
405 small reduces growth and thus elevates mortality risk. Repeating that analysis here, we found that improved
406 growth modeling has virtually no influence on predictions for optimal life history strategies (Fig. 8). ESS
407 flowering sizes were nearly identical between IPMs with Gaussian vs skewed t growth models, and both
408 aligned well with the observed mean flowering size (dashed vertical line in Fig. 8). **Similarly, there were**

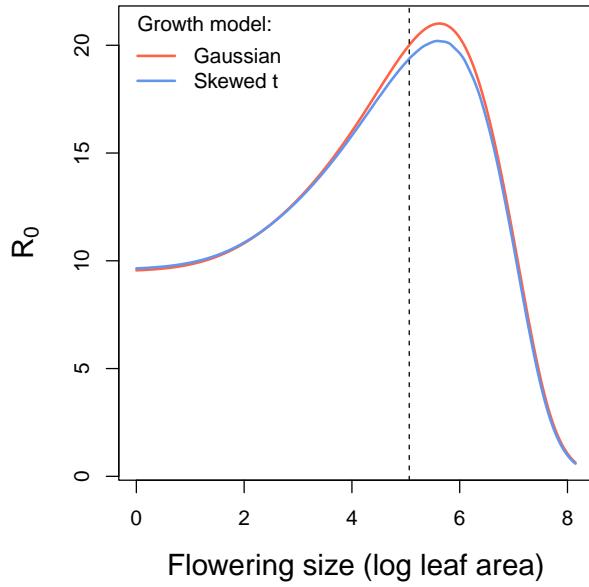


Figure 8: Orchid life history results from IPMs using Gaussian or skewed t growth models. Lifetime reproductive success (R_0) is shown as a function of mean size of flowering. Dashed vertical line shows the observed mean flowering size.

409 very small differences between growth functions in other metrics of orchid life history and, again, these
 410 differences were overwhelmed by uncertainty associated with parameter estimation (Table 1).

411 7 Discussion

412 Much of the appeal of IPMs has stemmed from their embrace of continuous size structure through regression-
 413 based approaches, and the potentially complex fixed- and random-effect structures that those approaches allow.
 414 Using familiar statistical tools and with relatively few parameters to estimate, IPM users can incorporate
 415 important sources of variation in demography and interrogate their influence on ecological and evolutionary
 416 dynamics. With this opportunity comes the burden of getting it right: an IPM is only as good as the statistical
 417 sub-models for the underlying data. The growth sub-model is the trickiest part because it defines a distribution
 418 of future size conditional on current size. Distributions have many properties – “moments” – and a good
 419 growth model should recapitulate the properties of real size transitions. The default assumption of Normally
 420 distributed size transitions, employed overwhelmingly across 20+ years of IPM studies, is an arbitrary historical
 421 precedent. In our case studies and, we suspect, more broadly, skewness and excess kurtosis were common

422 features of size transitions. Our most important message is that the assumption of normally-distributed size
423 transitions can easily be abandoned, and a more inquisitive process of growth modeling should take its place.

424 We have attempted to lay out what that process should look like, emphasizing visual diagnostics
425 to characterize how data deviate from Gaussian. One implication of relying on visual diagnostics is that
426 goodness of fit is in the eye of the beholder. This empowers IPM users to make informed choices, but it is not
427 very prescriptive; we have not suggested any hard rules for choosing among distributions, only that a good
428 growth model should generate data that look like the real thing. Alternatively, model selection could be used
429 to identify best-fitting growth distributions and best-fitting functions for higher moments. However, model
430 selection among growth distributions with 3-5 parameters, each of which may be functions of multiple state
431 variables or fitted values, can quickly explode in complexity, and we are not convinced it is worth the trouble.

432 Our work follows the important contribution of Peterson et al. 2019, who were similarly motivated
433 by inadequacy of the Gaussian model but arrived at different recommendations. These authors developed
434 a creative approach in which size data are transformed onto a [0,1] scale and size transitions on that scale
435 are modeled using beta regression. The beta distribution can accommodate positive, negative, or zero skew,
436 potentially varying with size, so the Peterson et al. approach is a flexible option for skewed growth data.
437 However, beta regression also has some limitations: common beta regression packages do not fit random
438 effects (e.g., **betareg** (Cribari-Neto & Zeileis, 2010)) or do not do so reliably (in our experience **gamlss**
439 regressions with random effects are numerically unstable); and the two-parameter beta distribution does not
440 allow skewness and kurtosis to be fitted independently. Additionally, the initial transformation onto [0,1] scale
441 requires estimating extreme quantiles of the growth distribution (e.g., 0.01 and 0.99) as a function of initial size.
442 In our experience those quantile estimates can be very sensitive to how size-dependence is modeled, and model
443 selection is challenging for extreme quantiles where data are (by definition) very sparse. Rather than picking
444 one distribution as a new default, users can leverage the vast arsenal of continuous probability distributions
445 – all at one's fingertips with a few lines of code – so that the data and their particular deviations from

446 normality can guide the choice of a better distribution. It is also possible to use mixtures of multiple growth
447 distributions, as done by Shriver et al. 2012 to model “normal” and “extreme” types of lichen shrinkage. In
448 re-analyzing that data set, we found that a single, flexible distribution (Johnson’s *S-U*) could recapitulate the
449 observed size transitions, but there may be other cases where mixture distributions are preferable or necessary.

450 In all of our case studies, non-Gaussian growth models always yielded more satisfying fits to size
451 transition data than the Gaussian models published in those papers. However, to our relief, none of these
452 re-analyses yielded a “gotcha” result that overturned results of the original study. In fact, in this small
453 sampling of case studies, improved growth modeling had weak to modest effects on IPM results, similar in
454 magnitude to the results of Peterson *et al.* (2019), and for most species and life history metrics these effects
455 were overwhelmed by the uncertainty associated with parameter estimation (Table 1). For some case studies,
456 one might argue that non-Gaussian modeling was not worth the trouble – only it was almost no trouble
457 at all, and we could not have known whether or not a non-Gaussian model would have made a difference
458 before fitting it. Even where Gaussian and improved growth models differed in IPM results, we do not know
459 “true” values to compare them against, only that one describes the data better than another, and a closer
460 match to the data does not necessarily translate to better predictive ability due to the bias-variance trade-off.

461 We caution against taking too much comfort in weak effects of “improved” growth modeling; in other
462 scenarios the choice of growth distribution could be more consequential. We focused on life history metrics
463 such as mean lifespan and mean lifetime reproductive success (Table 1). It is possible that higher moments
464 of those traits (e.g., variance or skewness of lifespan and lifetime reproduction) are more sensitive to the
465 tails of the growth distribution. It is also worth noting that most of our case studies focused on perennial
466 life histories (perennial plants and lichens) characterized by relatively slow growth, heavy losses during
467 recruitment, and high survival once established, and these species all had mean lifespans between one and six
468 years and generation times on the order of decades. Life histories such as these may be relatively robust to
469 subtle features of the growth kernel. In the Supporting Information we present three additional case studies

470 that broaden our life history coverage, including pike (*Esox lucius*), a fish with a generation time of four
471 to five years and creosotebush (*Larrea tridentata*), a desert shrub that is virtually immortal once established.
472 Life history metrics from these “fast” and “slow” populations were no more sensitive to improved growth
473 modeling than those of the perennial plants and lichens (Table 1). More systematic comparative analyses
474 may provide insight into which types of species and life histories are more likely to exhibit strong skewness
475 and kurtosis, and which demographic quantities are more or less sensitive to these features of size transition.

476 Our case studies illustrate a diversity of software packages and computational approaches, to reflect the
477 diversity of preferences and habits that the community of IPM analysts bring to their own problems. We like
478 spline generalized additive models (gams) for their flexibility and for **mgcv**’s numerous options for distribution
479 families and overall speed and reliability. However, there are some applications for which classical parametric
480 regression would be preferable because the coefficients carry biological meaning. For example, regression
481 coefficients may be targets of natural selection (Rees & Ellner, 2016) and may combine to influence traits
482 of interest such as the expected size at flowering (e.g. in Fig. 8A), a function of the intercept and slope of the
483 size-dependent flowering function (Metcalf *et al.*, 2003). Some potentially useful distributions are not available
484 in linear modeling software packages, but that should not be a barrier to their use: as in several of our case
485 studies, custom likelihood functions allow non-Gaussian models without sacrificing the complex, multi-level
486 features that one might be accustomed to fitting in **lme4**, for example. Bayesian analysis may further broaden
487 the options for non-Gaussian candidate distributions and may help estimate hard-to-fit parameters through the
488 brute force of sampling algorithms. Bayesian analysis also provides a natural way to propagate uncertainty from
489 vital rate sub-models to full model predictions (Elderd & Miller, 2016), as an alternative to our bootstrapping
490 approach. However, as of this writing, most of the non-Gaussian distributions that we have discussed are
491 not available in popular Bayesian software packages such as Stan or JAGS. While user-defined distributions
492 can always be coded from scratch, this may be a significant technical barrier for many IPM analysts.

493 From the outset there have been concerns about “how well these methods [IPM growth kernels]
494 can deal with different patterns of growth, stasis, and shrinkage” (Morris & Doak, 2002, p. 200), compared
495 to “binning” methods that use observed transition frequencies between user-defined size classes as the
496 transition probabilities in a (possibly large) matrix model (Doak *et al.*, 2021). The non-Gaussian models that
497 we have considered here are not a panacea. For example, none of them allow bimodal growth, such as might
498 occur if herbivore- or pathogen-attached individuals experience rapid tissue loss. When the shape of the
499 growth distribution is nearly the same for all initial sizes, a nonparametric IPM growth kernel can be defined
500 from a kernel density estimate for scaled residuals (Ellner *et al.*, 2016, p. 288). Outside that special situation,
501 nonparametric approaches require choosing multiple smoothing parameters, which is very challenging.
502 We are currently exploring whether “targeted learning” approaches developed for causal inference (van der
503 Laan & Rose, 2011) can be used to circumvent smoothing parameter selection. Targeted learning starts
504 with a pilot model and updates it iteratively to achieve unbiased estimates and valid confidence intervals for
505 a particular “target” quantity, such as λ or mean lifespan. Preliminary results suggest that targeted learning
506 with a deliberately under-smoothed pilot model works well for complex growth patterns (Zhou & Hooker,
507 2024). But nonparametric methods are data-hungry, so when departures from Gaussian are quantitative
508 rather than qualitative, parametric modeling as developed here will make more efficient use of limited data.

509 Conclusion

510 Gaussian-distributed size transitions are probably the exception in nature, not the rule, yet two decades
511 of IPM studies have relied overwhelmingly on Gaussian growth models. Using tools not available when
512 IPMs were first developed, it should often be possible now to make major improvements over a Gaussian
513 model, without worrying about finding the “best” alternative. By generating predicted size transitions
514 that are truer to the data, IPM analysts can narrow the gap between model and nature.

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518 idea to use quantile regression instead of binning to estimate trends in skewness and kurtosis. Ali Campbell
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520 **Authorship statement:** All authors discussed all aspects of the research and contributed to developing
521 methods, analyzing data, and writing and revising the paper.

522 **Conflict of interest statement:** The authors have none to declare.

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