Abstract Algebra

Free lecture videos by a Harvard professor on abstract algebra

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Algebra is the language of modern mathematics. This course introduces students to that language through a study of groups, group actions, vector spaces, linear algebra, and the theory of fields.

The lectures videos

The recorded lectures are from the Harvard Faculty of Arts and Sciences course Mathematics 122, which was offered as an online course at the Extension School.

The Quicktime and MP3 formats are available for download, or you can play the Flash version directly. Each week has 3 lectures that are 50 minutes each.

Review of linear algebra

Groups. Examples of groups. Basic properties and constructions.

- Video/Audio
  - Introduction to the course; Review: Linear algebra; Definition of groups
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Administrative notes; Generalities on groups; Symmetric groups on n letters; A stabilizer subgroup; The subgroups of Z; Cyclic subgroups gen by element
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - The story so far; Isomorphisms; Homomorphisms; Images
    - Quicktime
    - Flash video
    - MP3 (audio only)

Permutations

Cosets, Z/nZ.

- Video/Audio
  - Review, kernels, normality; Examples; Centers and inner autos
    - Quicktime
    - Flash video

- MP3 (audio only)

- Equivalence relations; Cosets; Examples
  - Quicktime
  - Flash video
  - MP3 (audio only)

- Congruence mod n; (Z/nZ)*
  - Quicktime
  - Flash video
  - MP3 (audio only)

Quotient groups, first isomorphism theorem

Abstract fields, abstract vectorspaces. Construction and invariants of vectorspaces.

- Video/Audio
  - Quotients
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - More on quotients; Vectorspaces
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - Continued
    - Quicktime
    - Flash video
    - MP3 (audio only)

Abstract linear operators and how to calculate with them

Properties and construction of operators.

- Video/Audio
  - Bases and vectorspaces; Matrices and linear transfs
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - Bases; Matrices
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - Eigenvalues and eigenvectors
    - Quicktime
    - Flash video
    - MP3 (audio only)

- Review for midterm; Orthogonal group
  - Quicktime
  - Flash video
  - MP3 (audio only)

Orthogonal groups

- Video/Audio
  - Orthogonal group & geometry
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Finite groups of motions
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Discrete groups of motions
    - Quicktime
    - Flash video
    - MP3 (audio only)

Isometrics of plane figures

Cyclic and dihedral groups. Finite and discrete subgroups of symmetry groups.

- Video/Audio
  - Discrete groups of motions; Abstract group actions
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Group actions
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Continued
    - Quicktime
    - Flash video
    - MP3 (audio only)

Group actions

Basic properties and constructions. Groups acting on themselves by left multiplication. Groups acting on themselves by conjugation.

- Video/Audio
  - Part 1

- Quicktime
- Flash video
- MP3 (audio only)

- Part 2
  - Quicktime
  - Flash video
  - MP3 (audio only)

- Part 3
  - Quicktime
  - Flash video
  - MP3 (audio only)

A5 and the symmetries of an icosahedron

Sylow theorems. Study of permutation groups.

- Video/Audio
  - Alternating group structure
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - Rings
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - Continued
    - Quicktime
    - Flash video
    - MP3 (audio only)

Rings

Examples of rings. Basic properties and constructions.

- Video/Audio
  - Part 1
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - Part 2
    - Quicktime
    - Flash video
    - MP3 (audio only)

  - Part 3
    - Quicktime

- Flash video
- MP3 (audio only)

Extensions of rings

Quotient rings. Integral domains, fields of fractions.

- Video/Audio
  - Flash video
  - MP3 (audio only)

Special lecture

- Video/Audio
  - Part 1
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Part 2
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Part 3
    - Quicktime
    - Flash video
    - MP3 (audio only)

Euclidean domains, PIDs, UFDs

Gauss' lemma. Eisenstein's criterion. Algebraic integers.

- Video/Audio
  - Part 1
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Part 2
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Part 3
    - Quicktime
    - Flash video
    - MP3 (audio only)

Structure of ring of integers in a quadratic field

Dedekind domains. Ideal class groups.

- Video/Audio
  - Part 1
    - Quicktime
    - Flash video
    - MP3 (audio only)

Wrap-up

- Video/Audio
  - Part 1
    - Quicktime
    - Flash video
    - MP3 (audio only)
  - Part 2
    - Quicktime
    - Flash video
    - MP3 (audio only)

Class Materials

- Syllabus
- Notes
  - Notes 1
  - Notes 2
  - Notes 3
  - Notes 4
  - Notes 5
  - Notes 6
  - Notes 7
  - Notes 8
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- Notes 36

- Problem Sets
  - Problem Set 1
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  - Problem Set 5
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  - Problem Set 34

- Problem Set 35
- Problem Set 36

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