# Threshold

## Threshold

We denote the received signal as $\eta_{k,l}$ where $k$ and $l$ are the Doppler index and the delay index respectively. Also, the noise power is $1+\sigma^2$. We assume a threshold $\rho_0$ that the power lower then the threshold is presumed to be noise. Please note that the threshold is believable enough that $p(\frac{|\eta_{k,l}|^2}{1+\sigma^2} \leq \rho_0) \geq 99.99\%$. Now, we need to simplify $|\eta_{k,l}|^2$

$$
\begin{aligned}
|\eta_{k,l}|^2 &= \frac{1}{N_b} \sum |\eta_{k,l,n_b}|^2 \\
\frac{|\eta_{k,l}|^2}{(1+\sigma^2)/2} &= \frac{1}{N_b} \sum \frac{|\eta_{k,l,n_b}|^2}{(1+\sigma^2)/2} \\
N_b \frac{|\eta_{k,l}|^2}{(1+\sigma^2)/2} &= \sum \frac{|\eta_{k,l,n_b}|^2}{(1+\sigma^2)/2} \\
N_b \frac{|\eta_{k,l}|^2}{(1+\sigma^2)/2} &= \sum \frac{real(\eta_{k,l,n_b})^2}{\sigma^2/2} + \frac{real(\eta_{k,l,n_b})^2}{(1+\sigma^2)/2} \\
2N_b \frac{|\eta_{k,l}|^2}{1+\sigma^2} &= \sum \frac{real(\eta_{k,l,n_b})^2}{(1+\sigma^2)/2} + \frac{real(\eta_{k,l,n_b})^2}{(1+\sigma^2)/2}
\end{aligned}
\tag{1}
$$

Therefore, $2N_b \frac{|\eta_{k,l}|^2}{1+\sigma^2}$ is a chi-square distribution at the freedom of $2 \cdot N_b$ where $N_b$ is the block number. (1) can be rewritten as,

$$
\begin{aligned}
p(\frac{|\eta_{k,l}|^2}{1+\sigma^2} \leq \rho_0) \geq 99.99\% \\
p(2N_b \frac{|\eta_{k,l}|^2}{1+\sigma^2} \leq 2N_b \rho_0) \geq 99.99\%
\end{aligned}
\tag{2}
$$

Therefore,

$$
\begin{aligned}
2N_b \rho_0 &= chi2inv(0.9999) \\
\rho_0 &= chi2inv(0.9999)/2N_b
\end{aligned}
\tag{3}
$$

Also, we suppose the pilot power is $E_s$ and the pilot power on a certain path is $\frac{E_s}{p}$, i.e.,

$$
\begin{aligned}
p(\frac{|\eta_{k,l}|^2}{E_s/p+1+\sigma^2} \geq \rho_1) \geq 99.99\% \\
p(\frac{|\eta_{k,l}|^2}{E_s/p+1+\sigma^2} \leq \rho_1) \leq 1 - 99.99\% \\
p(\frac{2N_b |\eta_{k,l}|^2}{E_s/p+1+\sigma^2} \leq 2N_b \rho_1) \leq 1 - 99.99\%
\end{aligned}
\tag{4}
$$

where $\rho_1 = chi2inv(1-0.9999)/2N_b$.

Therefore,

$$
\begin{aligned}
\rho_0(1+\sigma^2) \leq |\eta_{k,l}|^2 &\leq \rho_1(1+\sigma^2+E_s/p) \\
\rho_0(1+\sigma^2) &\leq \rho_1(1+\sigma^2+E_s/p) \\
\frac{\rho_0}{\rho_1}(1+\sigma^2) &\leq (1+\sigma^2+E_s/p) \\
(\frac{\rho_0}{\rho_1}-1)(1+\sigma^2) &\leq E_s/p \\
p(\frac{\rho_0}{\rho_1}-1)(1+\sigma^2) &\leq E_s \\
\frac{E_s}{1+\sigma^2} &\geq p(\frac{\rho_0}{\rho_1}-1) \\
SINR_p &\geq p(\frac{\rho_0}{\rho_1}-1) \\
SINR_p &\geq p(\frac{\rho_0/2N_b}{\rho_1/2N_b}-1) \\
SINR_p &\geq p(\frac{chi2inv(0.9999)}{chi2inv(1-0.9999)}-1)
\end{aligned}
\tag{5}
$$