Threshold

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We denote the received signal as $\eta_{k,l}$ where k and l are the Doppler index and the delay index respectively. Also, the noise power is $1+\sigma^2$. We assume a threshold ρ_0 that the power lower then the threshold is presumed to be noise. Please note that the threshold is believable enough that $p(\frac{|\eta_{k,l}|^2}{1+\sigma^2} \leq \rho_0) \geq 99.99\%$. Now, we need to simplify $|\eta_{k,l}|^2$

$$\begin{aligned}
|\eta_{k,l}|^2 &= \frac{1}{N_b}\sum |\eta_{k,l,n_b}|^2 \\
\frac{|\eta_{k,l}|^2}{(1+\sigma^2)/2} &= \frac{1}{N_b}\sum \frac{|\eta_{k,l,n_b}|^2}{(1+\sigma^2)/2} \\
N_b\frac{|\eta_{k,l}|^2}{(1+\sigma^2)/2} &= \sum \frac{|\eta_{k,l,n_b}|^2}{(1+\sigma^2)/2} \\
N_b\frac{|\eta_{k,l}|^2}{(1+\sigma^2)/2} &= \sum \frac{real(\eta_{k,l,n_b})^2}{\sigma^2/2} + \frac{real(\eta_{k,l,n_b})^2}{(1+\sigma^2)/2} \\
2N_b\frac{|\eta_{k,l}|^2}{1+\sigma^2} &= \sum \frac{real(\eta_{k,l,n_b})^2}{(1+\sigma^2)/2} + \frac{real(\eta_{k,l,n_b})^2}{(1+\sigma^2)/2}
\end{aligned} \tag{1}$$

Therefore, $2N_b\frac{|\eta_{k,l}|^2}{1+\sigma^2}$ is a chi-square distribution at the freedom of $2\cdot N_b$ where N_b is the block number. (1) can be rewritten as,

$$\begin{aligned}
p(\frac{|\eta_{k,l}|^2}{1+\sigma^2} \leq \rho_0) \geq 99.99\% \\
p(2N_b\frac{|\eta_{k,l}|^2}{1+\sigma^2} \leq 2N_b\rho_0) \geq 99.99\%
\end{aligned} \tag{2}$$

Therefore,

$$\begin{aligned}
2N_b\rho_0 \geq chi2inv(0.9999, 2N_b) \\
\rho_0 \geq chi2inv(0.9999, 2N_b)/2N_b
\end{aligned} \tag{3}$$

Therefore, we assume the signal is not noise is over 99.99%, i.e.,

$$p(|\eta_{k,l}|^2) \geq (1+\sigma^2)\rho_0) \geq 99.99\% \tag{4}$$

Under this assumption, we suppose the pilot power is E_s and the pilot power on a certain path is $\frac{E_s}{p}$. Therefore, we assume that the probability that the signal is a path is over 99.99%, i.e.,.

$$\begin{aligned}
p(\frac{|\eta_{k,l}|^2}{E_s/p+1+\sigma^2} \geq \frac{1+\sigma^2}{E_s/p+1+\sigma^2}\rho_0) \geq 99.99\% \\
p(\frac{|\eta_{k,l}|^2}{E_s/p+1+\sigma^2} \leq \frac{1+\sigma^2}{E_s/p+1+\sigma^2}\rho_0) \leq 1-99.99\%
\end{aligned} \tag{5}$$

Therefore,

$$\begin{aligned}
2N_b\frac{1+\sigma^2}{E_s/p+1+\sigma^2}\rho_0 &\leq chi2inv(1-99.99\%,2N_b)\\
chi2inv(1-99.99\%,2N_b)\frac{E_s/p+1+\sigma^2}{2N_b(1+\sigma^2)} &\geq \rho_0\\
chi2inv(1-99.99\%,2N_b)\frac{E_s/p+1+\sigma^2}{2N_b(1+\sigma^2)} \geq \rho_0 &\geq chi2inv(0.9999,2N_b)/2N_b\\
chi2inv(1-99.99\%,2N_b)\frac{E_s/p+1+\sigma^2}{2N_b(1+\sigma^2)} &\geq chi2inv(0.9999,2N_b)/2N_b\\
E_s/p+1+\sigma^2 &\geq \frac{chi2inv(0.9999,2N_b)}{chi2inv(1-99.99\%,2N_b)}(1+\sigma^2)\\
E_s/p &\geq (\frac{chi2inv(0.9999,2N_b)}{chi2inv(1-99.99\%,2N_b)}-1)(1+\sigma^2)\\
E_s &\geq p(\frac{chi2inv(0.9999,2N_b)}{chi2inv(1-99.99\%,2N_b)}-1)(1+\sigma^2)\\
\frac{E_s}{1+\sigma^2} &\geq p(\frac{chi2inv(0.9999,2N_b)}{chi2inv(1-99.99\%,2N_b)}-1)\\
SINR_p &\geq p(\frac{chi2inv(0.9999,2N_b)}{chi2inv(1-0.9999,2N_b)}-1)
\end{aligned} \tag{6}$$