LMMSE

1 MRC

The mean and the variance of estimated channel are $\hat{H}$ and $\hat{H}_v$. For a system,

$$y = Hx + z$$

the estimated original signal is

$$\hat{x} = diag(\hat{H}^H \hat{H})^{-1} \hat{H}^H y = D_{\hat{H}}^{-1} \hat{H}^H y$$

The error is

$$\begin{aligned} e &= \hat{x} - x \\ &= D_{\hat{H}}^{-1} \hat{H}^H (Hx + z) - x \\ &= D_{\hat{H}}^{-1} \hat{H}^H (\hat{H}x + Hx - \hat{H}x + z) - x \\ &= D_{\hat{H}}^{-1} \hat{H}^H \hat{H}x + D_{\hat{H}}^{-1} \hat{H}^H (H - \hat{H})x + D_{\hat{H}}^{-1} \hat{H}^H z - x \\ &= D_{\hat{H}}^{-1} \hat{H}^H \hat{H}x + D_{\hat{H}}^{-1} \hat{H}^H \Delta Hx + D_{\hat{H}}^{-1} \hat{H}^H z - x \end{aligned}$$

The expectation of the loss is

$$\begin{aligned} E(e) &= E\{\hat{x} - x\} \\ &= E\{D_{\hat{H}}^{-1} D_{\hat{H}} x - x\} + E\{D_{\hat{H}}^{-1} \hat{H}^H \Delta Hx\} + E\{D_{\hat{H}}^{-1} \hat{H}^H z\} \\ &= D_{\hat{H}}^{-1} \hat{H}^H E\{\Delta Hx\} + D_{\hat{H}}^{-1} \hat{H}^H E\{z\} \end{aligned}$$

1.1 Variance

Here, we set $G = D_{\hat{H}}^{-1} \hat{H}^H$ and $P = G\hat{H} - I$

$$\begin{aligned} cov(\hat{x}) &= E\{(\hat{x} - E\{\hat{x}\})(\hat{x} - E\{\hat{x}\})^H\} \\ &= E\{(\hat{x} - x)(\hat{x} - x)^H\} \\ &= E\{ee^H\} \\ &= E\{[(G\hat{H} - I)x + G\Delta Hx + Gz][(G\hat{H} - I)x + G\Delta Hx + Gz]^H\} \end{aligned}$$