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CATASTROPHE THEORY AND ITS CRITICS

ABSTRACT. Catastrophe theory has been sharply criticized because it does not seem to have practical applications nor does it seem to allow us to increase our power over Nature. I want to 'rehabilitate' the theory by foregoing the controversy raised by scientists about its practical efficiency. After a short exposition of the theory's mathematical formalism and a detailed analysis of the main objections that have been raised against it, I argue that theory is not only to be judged on its practical 'results', which are in fact limited, but also on its epistemological and philosophical implications. Catastrophe theory indeed represents a real revolution in science: it announces the coming of a more theoretical, less practical, science, having more to do with understanding reality than with acting on it, and, from that point of view, it may be considered as the modern philosophy of Nature.

Catastrophe theory derives from the research of French mathematician René Thom into topology and differential analysis on the structural stability of differentiable maps. Its principles were expounded for the first time in his 1966 article 'A Dynamical Theory of Morphogenesis'.[1] It appeared internationally six years later, in 1972, in Thom's major book *Structural Stability and Morphogenesis* (written in 1968). The theory is mathematically connected with the works of Hassler Whitney on the singularities of differentiable maps (1955) and with those of Henri Poincaré and Alexander Andronov on the theory of the bifurcation of dynamical systems.

For a mathematical theory, catastrophe theory initially met with considerable success. It very quickly roused a great deal of interest in the international community. From the beginning, the theory was acclaimed as a real revolution in mathematics, comparable to the invention of differential and integral calculus in the seventeenth century. It provided a general method for studying discontinuous changes and qualitative jumps. It raised great hopes amongst the specialists of disciplines reputed to be unformalizable by traditional mathematical methods (psychology, ethology, sociology, etc.). This theory was to bring these disciplines into the much praised realm of accuracy. Mathematicians, led by Christopher Zeeman of Warwick University in England, have built catastrophic models to account for events as diverse as riots

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in jails, stock market crashes, the propagation of nerve impulses, and the treatment of anorexia. Carried away by his enthusiasm, Zeeman even imagined a 'catastrophe machine' based on rubber bands and paperclips so as to illustrate how the theory works.[2]

The success of catastrophe theory may have been aided, in fact, by the magical power of the word *catastrophe*,[3] which without doubt possesses something astonishing, and therefore appealing, in such a context. The use of this word may have led to a misunderstanding. Usually, a catastrophe prompts images of a sudden and dramatic event, a tragic upsetting of the order of things. However, this is not catastrophe theory. Rather, catastrophes have a very precise mathematical meaning. According to this theory, a catastrophe occurs when a continuous variation of causes produces a discontinuous variation of effects. In other words, the catastrophe is what upsets *causa aequat effectum*. It is linked with the central idea of discontinuity. When a function, for instance, presents a discontinuity at one point, this point is said to be catastrophic.

After such a positive reception, the theory underwent its first and, at the same time, strongest attacks.[4] Applied mathematicians, such as Raphael S. Zahler and Hector J. Sussman, pointed out the fact that

> the claims made for the theory [were] greatly exaggerated and that its accomplishments at least in the biological and social sciences, [were] insignificant . . . . Catastrophe theory is one of many attempts that have been made to deduce the world by thought alone . . . . An appealing dream for mathematicians, but a dream that cannot come true.[5]

Others, such as Stephen Smale, a Fields medallist like Thom, criticized the mathematical basis of the theory:

> Catastrophe theory puts two of the most fundamental ideas of modern mathematics together: the study of dynamical systems and the study of the singularities of maps. These two ideas together cover a very wide area, but catastrophe theory unites them in an arbitrary and forced way.[6]

Some physicists have added their voices to the chorus of critics, and have shown a certain amount of scepticism about the practical use of the theory. Jean-Marc Lévy-Leblond, for instance, wondered whether the applications of catastrophe theory to biology, linguistics, ethology, etc., would be nothing more than a "conceptual gimmick, something like an analogical facing and a mathematical alibi of disciplines in search of respectability".[7]

The quarrel between 'catastrophists' and their opponents now seems to be abating somewhat. Nowadays, applicability limits of the theory are better known. A sort of consensus has been established in the scientific community to accept the 'rigorous' applications of the theory in the domains permitting the elaboration of precise quantitative models, for instance, in physics or mechanics. However, opinions remain divided about the interest of the catastrophic models in fields where quantification is impossible (sociology, etc.).

Foregoing the controversy raised by scientists about the applicability limits of catastrophe theory, we shall examine its epistemological and philosophical implications. Indeed, catastrophe theory is not an ordinary scientific theory. It is not an experimental theory such as Newton's theory of gravitation, Maxwell's electromagnetism theory, or Einstein's general theory of relativity. Nor is it a mathematical theory *stricto sensu*, that is to say, a theory of mathematics. Of course it uses mathematics (for instance, the theory of the singularities of differentiable maps, the theory of the bifurcation of dynamical systems, theorems on the structure of attractors in qualitative dynamics, etc.), nevertheless, it is not a part of mathematics, for fundamentally it talks about the world and Nature. We would like to show here that the atopical and atypical character of catastrophe theory, which some may be tempted to interpret too quickly as a sign of non-scientificity, is really the sign of a break with the paradigm that governs contemporary science. Such a break is at the same time a return to the days when science and philosophy were not yet two antithetical worlds, as they are today. Thom reactivates an old idea of science, perhaps the only one which can be really acceptable – science that enables us to understand reality and yet does not aim at changing it. Paraphrasing a famous formula, we could define Thom's project thus: 'So far, scientists have done nothing but change the world in various ways, now we must interpret it'.[8]

First we shall give a brief summary of the mathematical formalism of catastrophe theory. Afterwards, we shall discuss the principles underlining its originality. Then we shall analyze the main criticisms levelled at it and Thom's replies. Lastly we shall speculate about its epistemological status and philosophical range.

1. THE MATHEMATICAL FORMALISM OF CATASTROPHE THEORY

Catastrophe theory may be considered as a general theory of morphogenesis, that is to say, as a theory of the creation and of the destruction of the forms of Nature. A form, according to Thom's analysis, spreads over a substratum whose phenomenological appearance varies according to the point being considered. If the properties of the substratum vary continuously, there is no morphology. To appear, a form demands a discontinuity in the phenomenological appearance of the substratum.[9] The substratum space of morphology is generally a domain of the usual space-time, an open set of the euclidian space R^4. It is in such a space that the forms of ordinary perception stand out. But in some cases, the support space of the morphology can be more complex.

For instance, in acoustics the substratum space is the functional space (of infinite dimension) which describes the vibrations of the air. Similarly, in atomic or subatomic physics, we have to substitute derivated spaces for the usual spaces, such as spaces of moments or Hilbert's spaces.[10]

In sociology, as the support of morphology, we have to consider a space of significant parameters, variable in number and sometimes difficult to specify.

In the substratum space, Thom distinguishes two types of points: those called 'regular', corresponding to the areas of continuity of the morphogenetic process, and those called 'catastrophic', where the phenomenological appearance of the substratum abruptly changes. In these points, the morphogenetical process exhibits a discontinuity. As Thom notices, the distinction between regular points (continuous in appearance), and catastrophic points (discontinuous), if it has a precise mathematical significance, empirically loses its acute nature since it depends on the accuracy of the means of observation used. A point that seems regular when macroscopically observed may be found to be catastrophic when regarded on a smaller scale, and vice versa. "Here we are facing an idealization with obvious limits",[11] Thom acknowledges. In fact, the main interest of this distinction lies in its very wide generality. The opposition of the continuous and discontinuous is indeed at the base of our naïve perception of things and the world. It shares the gestaltic distinction between the background (continuity) and the form (discontinuity). This distinction is also presupposed by most scientists and primarily by linguists, for whom the manner and the matter of an expression are opposite. It can be found in physics with the theory of shock waves, the theory of phase transitions, etc. With the two notions – regular points and catastrophic – Thom finally offers a general conceptual frame that includes every experimental morphology.

The great merit of catastrophe theory is to allow us to reach "a certain understanding of morphogenetical processes without having to use the special properties of the substratum of forms or the nature of acting forces".[12] In other words, Thom's theory explains the forms without considering the physical and chemical forces that gave birth to them. Instead of deducing the forms from underlying forces, above the substratum of the morphology Thom projects what is called in mathematics a *field of local dynamics*. To any point x of the substratum, he associates a dynamical system, dependent on x, whose role is to account for the qualitative properties of the phenomenological appearance of the substratum at this point.

A dynamical system requires the datum of two elements: a space M (a manifold) that supports the dynamics, and a vector field X defined in this space. In catastrophe theory, the dynamics giving birth to the morphology do not work in the substratum space itself, but in an inner space that parametrizes the properties of the substratum at any of its points. To simplify things, Thom supposes that the inner space M, the support of the dynamics, is the same over all the substratum. The vector field associated with a point x of the substratum moves the point that parametrizes the state of the substratum in X within the inner space. Zeeman calls the inner space M the *behaviour space* or *state space*, and the space supporting the morphology the *control space*.

A dynamical system (M, X) generally, although not necessarily, exhibits attractors. An attractor is a subset of the manifold M, which may be reduced to a point but may be more complex (a curve, an area, etc.), towards which trajectories coming from all the points belonging to one of its neighbourhoods converge. We can imagine a dynamical system thanks to a geological metaphor. The manifold, the support of the dynamics, is the relief, and the trajectories of the dynamics are the slope lines. As for the attractors, they can be represented by the lakes that are the regions where all the water lines from a reservoir end up. An example of a dynamical system is the damped, heavy pendulum. The attractor of this system is the point of equilibrium of the pendulum. The attractors of local dynamics on the inner space M play the main part in morphogenesis in catastrophe theory. Indeed we may naturally suppose that at any point X of the substratum, the associated dynamic $X(x)$ lies near the attractor $c(x)$. The rest of the dynamic plays only an imaginary part, so that the observable phenomena of the system are determined by the sole presence of the attractor $c(x)$. The introduction of a 'fibre' dynamic above the substratum allows a precise mathematical meaning to be given to the notion of the catastrophe. A point x of the substratum will be called *catastrophic* if at this point the attractor of the dynamic $c(x)$ ceases to be structurally stable. In other words, the local vector field admits in x a discontinuity that corresponds to a discontinuity at the level of the observable phenomena in x. A point x of the substratum will, on the contrary, be called regular if the field $X(x)$, or equally if the attractor $c(x)$, continuously varies in the neighbourhood of x. The general theory of dynamical systems and of their attractors is far from being complete. This is why Thom does not go further than the study of dynamical systems of the simplest kind: gradient dynamical systems. These systems have the advantage of leading to a complete mathematical theory. They give birth to the elementary catastrophe theory whose fundamental results we shall consider shortly.

In the elementary case, the vector field $X(x)$ can be written in the form $X(x) = -\overrightarrow{\text{grad}}\, V$, where V is a differentiable map defined on M with real values. V is what is mathematically called a potential function. Moreover, Thom supposes that the manifold of the inner states M is a compact differentiable manifold, which means that the attractors are only constituted by *single isolated points*. In other words, the trajectories of gradient dynamical systems defined on a compact manifold converge towards points of equilibria that correspond to the potential minima. When the dynamic has reached one of these points of equilibrium, the system there remains stable. A very good example of the gradient dynamical system is provided by dissipative mechanical systems. These systems lose their energy (both kinetic and potential) during their motion because of friction, and this loss is not compensated by a new external energy. Dissipative systems always reach a point of equilibrium. With the elementary catastrophe theory, we suppose that the point that parametrizes in the inner space the state of the substratum in x, is represented by a minimum $c(x)$ of the associated potential function $V(x)$. If the minimum $c(x)$ is stable, the point x will be a regular point of the substratum. This means that the phenomenological appearance of the substratum varies continuously around the value $c(x)$. If, on the other hand, the minimum $c(x)$ is unstable, which happens when the potential $V(x)$ changes its topological type near to $c(x)$ because of slight disturbances, the point x will be a *catastrophic* point. In this case, the local minimum c is destroyed in the neighbourhood of x, usually as a result of a collision with a local maximum of $V(x)$. The point $c(x)$ ceases to be an attractor of inner dynamics at x. The state of the substratum suddenly jumps from c to c_1, where c_1 represents another attractor of the dynamic, such as $V_x(c_1) < V_x(c)$. It is said that the attractor c_1 has captured the attractor c. Thus, at the origin of any catastrophe, there is a situation of *instability*. An attractor of the dynamic bifurcates, which according to Thom provokes the catastrophe.[13] Unstable attractors correspond to critical degenerated points of the potential. Then Thom discovers, and here lies the main result, that only a small number of degenerated singularities (seven, exactly) can be at the origin of structurally stable morphologies in the ordinary space-time (R^4). Each of these morphologies corresponds to what Thom calls an elementary catastrophe. The seven elementary catastrophes which may happen in R^4 have been called in order of complexity: fold, cusp, swallow-tail, butterfly, hyperbolic umbilic, elliptic umbilic, parabolic umbilic. These names have been given because of the shape taken in every case by the manifold of the equilibria, that is to say, the set of the minima of the potentials in the product space $R^4 \times R^n$ (where R^n denotes the behaviour space). The morphologies in the substratum space (the only ones that can be observed in practice) are obtained by the projection on the substratum space of the set of the points S of the manifold of the equilibria where the tangent plane is vertical, that is to say, parallel to R^n. Generally speaking, one can show that the number of elementary catastrophes on a given space depends solely on the dimension of this space. The dimension of the substratum space sets boundaries for the topological complexity of the catastrophes that can occur there. Geometry governs topology. These are extremely strong results, which, in spite of their apparent simplicity, require deep and difficult mathematical theorems that we cannot possibly develop here. For that, the reader should refer to the works of Thom, or those, for example, of Tim Poston and Ian Stewart,[14] which are less complex. Now, we shall inquire into the epistemological status of Thom's theory so as to bring out its extreme originality.

2. THE ORIGINALITY OF CATASTROPHE THEORY

The originality of catastrophe theory, compared to the whole of contemporary science, comes firstly from the subject itself. Thom's theory, as a model of the morphogenesis, is a theory of discontinuities. Conversely, classical science, in its analysis, prefers continuous phenomena. The best part of the continuous phenomena, which produces as a consequence the practically never-ending scorn of scientists of morphology, especially in physics and chemistry, can be explained in several ways. Firstly, it comes from the very difficulty we have in formalizing mathematically discontinuous phenomena.[15] We only have to consider at what disadvantage a theoretical physicist is when facing a discontinuity, for instance, a shock wave in hydrodynamics. "If, by throwing a stone into a pond, you wish to understand what is happening, you had much better do the experiment and film it, than try to make the theory out of it. The best specialist of Navier–Stokes's equation would certainly be unable to tell you more".[16] The other explanation of the best part of the continuous phenomena is that, so as to influence the world, a scientist has to foretell the evolution of phenomena, that is to say, from the former state of a process, to predict its future state at any time. The paradigmatical method here consists of determining the way in which the phenomenon evolves locally during an infinitesimal period, then in integrating these local evolutions into a global evolution. For this purpose, science possesses an outstanding efficient tool, differential and integral calculus, developed in the seventeenth century. The use of these methods of differential analysis unavoidably results in the smoothing out of studied phenomena and the elimination of discontinuities from the domain where science researches. Discontinuity is either ignored or artificially reduced to continuity submitted to very quick variations, which means it is considered as a kind of 'extreme case' of continuity itself. We must note that the generalization of these methods of calculus, whose efficacy cannot be denied, at least within some limits, consequently puts aside certain branches of mathematics, such as geometry and topology, which are less highly thought of in the study of natural phenomena. It is precisely in these disciplines discarded by traditional science that catastrophe theory lies.

Secondly, the originality of Thom's theory is that of its method, that is to say, its way of approaching and understanding reality. Classical scientific explanation is *reductionist*. The most typical example of the reductionist explanation is the atomistic one whose principle is shown by Thom in the following manner:

Positions and velocities of n atoms are described by a moving point in an euclidian space R^{6n} of $6n$ dimensions (x_j); the laws of interaction between those particles allow a system of differential equations to be written: $dx_i/dt = X_i(x_j)$, whose integration gives the temporal evolution of the studied system.[17]

Unfortunately, the generally great number of equations to be solved (let us consider a mole of gas $N = 6 \times 10^{23}$ (Avogadro number), hence a system with 36×10^{23} equations) rapidly renders the reductionist programme unrealizable. Only a statistical tool allows an advance towards its realization, but only in completely isolated and simple cases (e.g., the theory of perfect gas in statistical mechanics). But then, as Thom remarks cleverly, "there is no longer morphology".[18] The form has slowly faded amidst elementary phenomena that are supposed to explain it. Let us also note that the privilege of the reductionist method in contemporary science bases itself on a myth that Thom says "is unfortunately still very much alive",[19] the myth of the belief in the "fundamental character of the physics of the infinitely small".[20]

"We must reject as illusionary this primitive and almost cannibalistic conception of knowledge which believes that the understanding of something demands that it first be broken up into pieces".[21] Catastrophe theory, standing against reductionism, nowadays considered as a principle of scientific explanation, does not break the studied form into a set of atomic entities, but takes it at the level of organization where it appears. It is a "hermeneutic theory as it endeavours, facing any experimental data, to build the simplest mathematical object capable of generating them".[22] Catastrophe theory interprets the data (the morphologies) 'phenomenologically', that is to say, without trying to derive them from inner, hidden, underlying processes. As it is, it saves the autonomy of the macroscopic, and preserves its means of intelligibility. Thom calls the approach of catastrophe theory 'structural', referring to the methods in use in the social sciences and especially in linguistics. In such an approach,

> we take the morphology X in itself, at a certain level of organization, as a combination of morphogenetic fields. We try to reduce the arbitrary character of the description of the corpus bringing its regularities and its hidden symmetries into evidence. We aim to generate the morphology axiomatically with a small number of forms liable to give birth to new forms through spatial aggregation.[23]

The structural approach does not seek to reach an elementary level of organization in the substratum space, but, above it, draws a new space that is both mathematical and abstract in character. More precisely, above the morphology, it builds a geometric being, which Thom calls *logos*, after Heraclitus, that is to say, in elementary catastrophe theory, a potential organizer. "Catastrophe theory", Thom says, supposes that the things we see are but images and that to reach the being itself, we must multiply the substratum space by an auxiliary one, and, in this product space, we must define the simplest being that gives, by projection, its origin to the studied morphology.[24]

This doubling of the concrete morphological space, the place of appearing, with a mathematical abstract space, no doubt makes us think of Plato. Thom moreover compares the morphologies appearing in the substratum space to the shadows of the cave in Plato's *Republic* VII. Just as to explain the shadows of the cave, that is to understand the laws which govern their apparition, we must get out of the space (the wall) in which they appear, and turn to the exterior, towards real things that are transported outside and lit up by a fire; in the same way, to account for morphologies, we must, according to catastrophe theory, leave the substratum space where they appear and go towards an algebraic being that gives birth to them. Thus we may rightly talk about a Thomian Platonism: at the origin of forms, mere images, we must place an ideal world occupied by mathematic beings, the world of *logoi*, of potential organizers of the elementary catastrophe theory.

Here we must point out that the structural method belonging to Thom's theory, although basically non-reductionist, is not incompatible with the reductionist approach in the study of natural phenomena. Thom does not demand the exclusivity of this method. But, the advantage of the structural method is that it remains reliable when the reductionist one does not work, especially when the number of parameters governing a morphology is so great that it defies ordinary quantitative analysis, such as in those familiar phenomena that are "cracks in an old wall, the shape of a cloud, the fall of a dead leaf, the foam of a glass of beer".[25] We cannot do a quantitative analysis of these phenomena: Who indeed could precisely and accurately foretell the trajectory followed by a dead leaf falling from a tree? Here quantitative analysis must give way to a less ambitious approach, such as is allowed by catastrophe theory, that may explain the forms revealing the *logoi* that govern them and account of course not for their precise quantitative characteristic but for their local qualitative determinations, and so permits one to predict their evolution to a certain extent. Thom's studies can and must be compared here with those of Poincaré, who, in another domain, that of celestial mechanics, used a qualitative analysis of motion instead of a quantitative one when the latter proved ineffective. Thus in the problem of three bodies (e.g., determine the positions of three bodies submitted to the sole forces of gravitational attraction at the instant t, knowing their positions and their velocities at the initial instant), Poincaré, after having shown that the solution could not be discovered, although it existed and was unique, invented a method that allowed the form of trajectories to be determined. The result was a new discipline: qualitative dynamics. Catastrophe theory, in a way, may be considered the heir to this new science, at least because of its method. It is to physics and traditional science, as a rule, what qualitative dynamics is to classical dynamics.

Being both a morphological and a structural theory, catastrophe theory is also geometrical. It explains empirical morphologies by geometrical structures – the set of catastrophes – that generate the studied morphology by a projection onto the substratum space. The importance given by Thom's theory to the geometrical notion of form in the explanation of natural phenomena deserves notice. Indeed, since Newton through the end of the nineteenth century, the paradigm that prevailed in the scientific explanation was the mechanism. "I am never satisfied", Lord Kelvin used to say, "if I have not been able to make a mechanical model of the object; if I can make a mechanical model, I understand. As long as I cannot make a mechanical model, I do not understand".[26] Elsewhere, Kelvin uses the same idea: "It seems to me that the true meaning of the question: do we or do we not understand a particular subject in physics is: can we make a corresponding mechanical model?".[27] In a classical mechanical model, forces, be they inert, at a distance or in contact, play a central role. They allow the equation of motion to be written, and the position of equilibrium of the system to be calculated. The main role played by the concept of force, and the correlative relegation of the old Platonic-Aristotelian concept of form in the understanding of the behaviour of material systems, is due, Thom considers, to two main reasons. Firstly, force is a notion we can easily and instinctively understand, thanks to the experience of muscular effort. He says: "[I]f force was given an ontological status deeper than the one of form, it is probably due to a kind of simple anthropomorphism coming from the fact that we act upon outer objects through the forces we apply with our muscles".[28] Force is a concept that comes from biology. Secondly, force is the power that acts and allows us to act. Drawing the evolution of a system from the intervention of forces satisfies our desire for efficiency beforehand and is motivated by it. To be aware of the forces is indeed, at least in principle, to be able to act on the future of the system. As to the form, it is a quite different point of view. It brings intelligibility and rationality with it. Force, an anthropomorphic and simplifying concept, for it practically reduces a being to a vector, is in fact an obscure notion. This was clearly noted in the eighteenth century by Lagrange and then by Hamilton, the two main advocates of analytic mechanics, who tried to eliminate force from the equations of dynamics, wherein it introduced an element of irrationality. They achieved this aim by proposing an analysis of motion different from that developed by Newton. According to their point of view, trajectories are not determined by forces acting on the body in motion, but by the structure of a mathematical function or of a quadratic form called the Hamiltonian of the system. The two interpretations of the motion in terms of forces (Newton) or forms (Hamilton) are in fact interchangeable, because one can go from one to another by canonical procedure.[29] If one prefers the point of view of action on reality, one will doubtlessly choose explanation by forces. But if one puts intelligiblity and rationality above all, one will prefer interpretation by form. Catastrophe theory, which explains by forms and not by forces, surely sides with intelligibility and with rationality.

Being morphological, structural, and geometrical, Thom's theory is also classificatory: the ideas of class and classification play an important part, and in a twofold way. Firstly, the aim of the theory, in its elementary form, consists of identifying and then classifying all the algebraic-geometrical singularities that may give birth to structurally stable morphologies in the space R^4. Thom establishes, as he himself says, "the catalogue of all the possible constructions of stable entities"[30] of all the *logoi* that command morphogenesis in the space-time. These mathematical beings somehow form the a priori that governs all the empirical morphologies, at least locally. But catastrophe theory is classificatory in a second sense, that is to say, in considering the type of explanation it sets to work. Explaining a morphology means deriving it from an archetypal mathematical structure. In this formalism, explanation is given by classifying, that is to say, by pointing out a membership of a genus. "The model of catastrophes", Thom says,

> is at the same time much less and much more than a scientific theory. One must consider it as a language, as a method which allows classification and systematization of empirical data, and which offers those phenomena the beginnings of an explanation which makes them intelligible.[31]

The cataloguing and the classifying operation make us think of the procedures of eighteenth-century taxonomists (Linné, etc.) rather than of the analytic methods of modern science. It is not, however, a mere coincidence because, for Thom, as for the taxonomists, Nature has a meaning, it speaks its own language. Things have an intrinsic meaning contained in their *logos* that analysis must discover. This attitude goes against modern conventionalism. It is motivated by the same desire we have just been discussing, the desire to understand reality and to expose its intelligibility. As for modern science, it does not aim at understanding essences, but at establishing laws allowing for calculus and efficient prediction.

Finally, the originality of catastrophe theory lies in the practically boundless expanse of the domain it is supposed to cover. Thom's theory has a practically universal range. It can model all sorts of phenomena belonging to quite diverse sectors of reality. From this point of view, one may consider it as a modern resurgence of the old dream of the 'mathesis universalis'. It breaks with specialization, which today is the characteristic of scientific work. "Thom", Jean Petitot writes,

> is nowadays the center of a network of heterogenical speeches (from the tectonic of strata to the dynamics of revolution, from embryogenesis to symbolic structures), which interdisciplinarity never succeeded in putting together. And there, maybe, lies the main interest of catastrophe theory: it brings about new links between disciplines, it delocalizes thought, it asks long forgotten questions again, it sets together the pieces of ontology, divided, broken by the techno-industrial development.[32]

The theoretical basis of the universality of catastrophe theory is the principle of the independence of form from the substratum that sets analogies free. Morphogenesis does not depend on the nature of acting forces, but on geometrical and topological constraints. A similar form may appear on different support. "Catastrophe theory could be seen as the first and fairly general systematization of analogy since Aristotle".[33] Analogy can be expressed mathematically in terms of isomorphism: two empirical morphologies are similar when they come from the same *logos*, the same potential organizer. One single form may be seen in different registers of reality. Thus, catastrophe theory creates links between phenomena as diverse as phase transitions, riots in jails, cellular differentiation, etc. It opens horizontal passages between scientific disciplines, physics and sociology, biology and linguistics. It struggles against the scattering of knowledge by discovering the fundamental structural unity of Nature itself.

3. CRITICISM OF CATASTROPHE THEORY

After being welcomed quite favorably by nearly all members of the scientific community, even by those who were to be its most severe opponents, such as Sussman, catastrophe theory found itself at the centre of a great controversy. Different criticisms were made concerning various aspects (mathematical basis, method, efficiency, etc.). Here we shall discuss some of those that will enable us to understand better, *a contrario*, the real aim and status of the theory. We shall leave aside purely mathematical objections, simply because addressing these would take us into technical developments that we cannot possibly tackle here. Moreover, these objections are somewhat marginal to the very heart of the theory itself. Epistemological objections seem much more radical, thus we shall now devote our study to them.

These objections fall into two main themes. The first series of criticisms consisted in questioning the scientific character of catastrophe theory because it is not experimental in nature (which Thom himself willingly acknowledges). Catastrophe theory does not come from experience, nor can experience confirm it in any way. The empirical confirmation of a catastrophic model does not prove the model to be true as it is, but, at most, that the model may rightly be used to formalize the considered phenomenon. The reason why experience cannot confirm the models of catastrophe theory really lies in the nature of these models. Indeed they are fundamentally qualitative, which means that their precise quantitative characteristics, when an attempt is made to apply them, are necessarily fixed by experience. Thus, it is not surprising if a model is in agreement with the experience, since the model has been built precisely to agree with it. "If the process studied", Thom says, "is whole within a chreode C, what can experience add if not the confirmation of the stability of the chreode?".[34] Finally, catastrophic models are ad hoc models. Thom's theory does not satisfy positivist criterion, according to which any scientific theory has to be confirmed or at least verified by experience, and cannot be considered, from this point of view, as truly scientific.

But neither can experience invalidate catastrophe theory. If predictions drawn from a model are in contradiction with experience, it does not put the theory itself in doubt, but shows that the model does not fit the studied morphology or that it has not been rightly chosen from the catastrophic arsenal. For instance, studied morphology might be ruled by a singularity from a higher degree than that of the model (and thus we must change the model and use, for instance, the model of the butterfly instead of that of the cusp); or morphology may not even be formalizable in the frame of the elementary catastrophe theory, and this is what Thom calls a 'generalized catastrophe' (i.e., a catastrophe whose dynamics are more complicated and do not come from a potential). As experience cannot invalidate catastrophe theory, it does not satisfy the Popperian rule that says a scientific theory must be falsifiable, and, as a result, catastrophe theory is not scientific. It is on the same level as psychoanalysis or magic, which can always render seemingly incompatible facts compatible with their own explanatory system. So, in psychoanalysis, the failure of a treatment may always be seen as a symptom of resistance on the part of the patient, and, far from invalidating the theory, confirms it. In the same way, "if a magic act or process", Thom says, "is of no avail, public opinion can always explain this sort of failure through local reasons: the efficiency of such an act was reduced or completely nullified because of a legal flaw, the officiant's inexperience, an anti-spell by the victim or by the enemy".[35] Catastrophe theory escapes the ordeal of experimentation in the same way and, just as the above mentioned practices, is nothing more than a pseudo-science.

The second series of criticism, which follows from the first, discusses the problem of the practical efficiency of the theory. It presents three main objections.

One, catastrophe theory has no real predictive power. Ivar Ekeland says that

> it does not provide any knowledge a priori before the experiment: even if we possess a good dissipative system, and have three parameters, we cannot know if there will be a swallowtail, an elliptic or hyperbolic umbilic, cusps, folds, nothing at all, or anything else before having tried – or calculated, if we know the potential. Even if it is a swallowtail, the theory is unable to predict its position, its measurements, and even its accurate shape.[36]

Also, and above all, as a rule catastrophic models have not allowed new facts to be discovered, which experimentation may later confirm. Thom opposes two types of applications of catastrophe theory: firstly, 'rigorous' applications, which form what he calls a 'hard theorization', and which are built on precise quantitative laws, as in physics or mechanics. "What catastrophe theory permits in that case", Thom says, "is a quick qualitative interpretation of the global behaviour of solutions and of their singularities. Of course, in that case, a precise quantitative calculus is in principle possible, and hence the model has predictive abilities".[37] In spite of this optimistic statement, Thom acknowledges that, until now, there has existed only one example of a catastrophic model that has permitted an effective prediction – that established in geometric optics, which allowed new shapes of caustics to be predicted before experimental evidence proved their existence. "But even in this case", Thom says, "we are dealing with such obvious and *a priori* predictable phenomena that we are not greatly surprised".[38] The other 'rigorous' applications of the theory did not permit any discovery. Their main result was to formalize well-known facts, most of them ancient, in another way (the transition of phases, the veiling of Euler, the stability of ships). The second type of applications that Thom opposes may be found in biology and in the social sciences, and gives birth to what the mathematician calls a 'soft theorization'. "The models of the second type", Thom says,

> start with an empirical morphology which must be interpreted. We then build a field of differential systems on a control space, and try to make the observed morphology coincide with the catastrophic set of the model. As a rule, these differential systems are defined to a differentiable equivalence, and, afterwards, they do not allow quantitative prediction. Their interest is to provide a global view of the situation, interpreted through terms of a conflict of regimes.[39]

Catastrophic models in biological and social sciences only allow qualitative predictions. Moreover, these predictions, as shown by Sussman and Zahler, are generally trivial, and have only a limited practical range (when they are not wholly wrong). This is not really surprising, because, as Thom notices, "for a model to lead to good possibilities of prediction, and then of action, the model must be quantitative".[40] "A merely qualitative prediction", he adds,

> with no scale of dates or places, is of practically no interest. I may predict, in all certainty, that any political system, in any society whatsoever, will end. If I am incapable of saying when (even imprecisely) my prediction will only be a triviality.[41]

Similarly, if I know that the behaviour of a morphology obeys a catastrophic model whose type is well authenticated but I cannot give the place of the sets of catastrophes precisely, any prediction I can make will necessarily be banal (of the type: a frightened dog will end up attacking if it is made angry, in the model of dog aggressiveness). It is always possible to localize and to quantify a catastrophic model of the second type, but localization very often lies on the data of a series of special experiments, and cannot not be extrapolated. Quantification, in such a case, is necessarily a posteriori, and does not allow the anticipation of the results of the experiment. Generally speaking, the models issued by catastrophe theory have a very weak heuristic power, inferior in any case to the power of classical quantitative models based on well-known theories (mechanical models, for instance). Thom is of course aware of this practical shortcoming of his theory.[42]

Two, catastrophe theory exhibits another shortcoming: it is qualitative. It classifies the singularities (the *logoi*) and describes the structure of the catastrophic sets (the sets of bifurcation) derived from them; but it does not give any information about their precise quantitative characteristics (dimensions, localization above the support space, etc.). Positivists concluded that the elementary catastrophe theory was not scientific because science is quantitative. A famous formula created by Ernest Rutherford reflects fairly well the scorn shown by traditional scientists towards the qualitative science: "[T]he qualitative agreement of a theory with experiment only shows a rough quantitative agreement. Qualitative is nothing but poor quantitative".[43]

Lastly, the Thomian theory was reproached for being fundamentally local. But what is practically interesting, so Sussman and Zahler argue, is to know the global behaviour of phenomena. Although many mathematical properties are merely local, such as the continuity or the differentiability of a function, the theorem used in practical applications deals with global properties or allows us to draw, from local properties, a lesson on the global behaviour of the system.[44] Catastrophe theory, except in very particular cases, does not allow this passage from the local to the global. "From the local knowledge that the only singularities of a curve are folds, nothing global can be inferred about it".[45] The locality of catastrophe theory is enough, according to Sussman and Zahler, to "invalidate most of its uses and, in particular, the claim that catastrophe theory has something significant to say about discontinuous behavior".[46] For instance, let us suppose that the theory had predicted the existence of a cusp in the neighbourhood of a point P. It is then sure that the two sheets, the upper one and the lower, of the cusp will start diverging from the point P. But as the neighbourhood U may be very small, there is no reason why, outside of U, the distance between the two sheets should significantly increase and reach a certain amount of perceptibility, and the corresponding qualitative jump on the substratum will not be perceptible with a macroscopic observation. "The *mathematical* jump that appears in catastrophe theory models", Sussman and Zahler conclude, "is a *local* phenomenon, which has little in common with observed jumps".[47]

All the criticisms (of the non-experimental aspect of the theory, its very limited predictive power, its qualitative nature, its locality) finally converge on the same theme: the practical inefficiency of catastrophe theory. Catastrophic models permit neither prediction nor, consequently, any efficient action on the real. Catastrophe theory does not say anything about the world. It is an abstract construction cut off from reality, a reality of which we cannot even judge its falsehood since it escapes the control of experimentation. From the positivist point of view, it is a hollow speculation, ultimately of little interest. Thom is of course aware of these criticisms. He has discussed them and in some cases helped to lead to their expression. He did not try to avoid them; on the contrary, he gave long and precise replies to the objections. Through these answers, which exceed the narrow frame in which criticisms have been developed, is revealed what seems to be the actual dimension and deep ambition of catastrophe theory.

4. THE THOMIAN CRITICISM OF POSITIVISM

Thom's answers to the criticism of his theory go much further than a mere defence of a threatened theory. They lead to a real questioning of contemporary scientific thought itself. They shake the convictions of the opponents of the theory and, even further, a complete scientific tradition. They question contemporary positivist epistemology. They unsettle a certain number of certitudes, and, last but not least, *a priori* give scientific research itself new horizons.

Catastrophe theory is not considered scientific because it is not experimental. Thom, the geometrician, is quite aware of this, but denies that it can be an argument against the scientificity of his models. Moreover, he notices, the strict application of such a criterion to evaluate the scientific character of a discipline would ipso facto take away branches of knowledge generally reputed as being scientific. He says:

> [T]o demand of any scientific discipline that it be controlled by means of action, of experimentation automatically excludes from the scientific field the sciences of the past, such as paleontology and history, and all those where experimentation is impossible, for instance as it is still the case for astronomy.[48]

A great number of sciences do not and cannot practice experimentation either because their object of study is of a past epoch (history, etc.), or because of spatial distance (astronomy), or "for ethical reasons (some psychological or social phenomena)".[49] These sciences cannot create in a laboratory the morphologies they study, as physics or chemistry do. They are reduced to pure observation. Of course, one could reply that these sciences, even if they are not experimental in the classical sense of the term, are nevertheless considered as *scientific* precisely because they rely on concrete, duly authenticated, and universally controllable facts. But it is precisely this empirical base that catastrophe theory lacks. In fact, Thom's argumentation does not aim to prove the scientificity of catastrophe theory (the objection 'that is not science' is thoroughly ridiculous), but rather aims to show how it is possible and even necessary to separate the two ideas of 'scientificity' and 'efficient action on the real'. A discipline can be perfectly scientific without increasing our possibilities of intervening in Nature. Sciences of pure observation describe and explain the morphologies they study without being able to change them or a fortiori to create them.

In response to the accusation that catastrophe theory would be incapable of non-trivial prediction, Thom shows, revealing the conditions of the possibility of scientific predictions, that they are really so restrictive that it would be totally absurd and in vain to demand of any scientific theory that it be able to lead to precise quantitative predictions. There are fields where prediction is impossible by nature. "I think that, to a wide extent", Thom says,

> there will always be two types of science: those allowing effective predictions to be made, even precise quantitative predictions, and which nowadays seem limited to mechanics and physics, and those where you cannot quantitatively foretell, but where you will have to use classifications of a qualitative or topological character.[50]

Thom perceives the prediction as an operation where we plunge the past into the future. When we foretell a phenomenon, we extend the knowledge we have of it beyond the domain where this knowledge has been acquired. We suppose that what has been seen in the past will be seen again in the future if similar conditions present themselves. In other words, we extend a function defined on a certain space-time field to a wider one. Thus prediction implies a passage from local to global. But in mathematics there exists a canonical process, practically the only one, that precisely allows this passage from local to global: it is the analytic extension of a function. Analytic extension is a procedure by whose means a germ of an analytic function, defined by its Taylor series at one point, spreads to the whole domain of existence of the function. All quantitative predictions in science are, as a last resort, based on the use of analytic functions. "Pragmatically efficient models", Thom says, "allowing prediction, imply the analyticity of the functions and of the solutions of the temporal evolution. This consequently demands that the substratum space on which we work be provided with a natural analytic structure".[51] We understand therefore why the models built in physics allow quantitative predictions so precise that they have even been said to have an 'unreasonable effectiveness'. It is only due to the natural analytic structure of usual space-time on which they are defined. This fact is mathematically expressed by saying that the space-time of physics is comparable to a "Lie group of continuous equivalences (euclidian, galilean, lorentzian groups)...".[52] We also understand why the 'miracle' of physics is isolated: a great number, and even most, of the substratum spaces have no analytic structure, and "it is not easy to extend extrapolation processes where there is no natural analytic support".[53] "The scientific domain where we can build exact quantitative models is much more restrained than generally believed. It is a small halo around fundamental physics, whose boundaries become more and more imprecise as statistical considerations come into play".[54] As most catastrophic models are not built on a substratum space having a natural analytic structure (second-type applications), it is inevitable that they do not yield precise quantitative predictions. If we want to build models leading to predictions, it is essentially so that we can transform the real. "In action, there is always an aim beyond the phenomenon, since we are always looking to realize what does not present itself spontaneously".[55] The ability to predict increases our power over the world. Thom's theory is not predictive, apart from very particular cases. It explains morphologies, but does not allow us to act on them. From this point of view, we must break the link between theory and practice that a short-sighted epistemology would be tempted to establish. Theory is not necessarily followed by practice, and, equally, practice is not necessarily lead by theory. We can very easily understand a situation without being able to modify it. Thom shows this by giving the example of the gentleman who, surprised by a flood, finds refuge on a roof only to watch the water engulf him.[56] The unfortunate man understands perfectly what is happening to him, however, he cannot change his surroundings. Conversely, one may act successfully and yet be quite unable to give a theoretical basis for such a success. "Without exaggeration", Thom affirms, "we could nearly say that the whole of contemporary medicine is a proof thereof. Indeed cases are rare where the working of a medicine could be satisfactorily explained on the 'fundamental' level of molecular biology".[57]

The third criticism levelled at Thom's theory is about the qualitative aspect of its models. To this criticism, Thom replies that you cannot introduce some quantitative where it does not exist, as is the case in social sciences, in ethology, in psychology, etc. Here Thom would quite agree with Sussman and Zahler's objections when they reproach Zeeman for devoting himself through his models to an abusive and illusory quantification. Thom says that the models built by Zeeman "do not lend themselves to a quantitative treatment. If you want to make them quantitative by force, you obtain but a spurious quantification, as wrote Sussman and Zahler".[58] Moreover, Thom remarks that "any quantitative model presupposes a qualitative division of reality".[59] Any scientific subject spreads within a specific qualitative domain that determines the nature of its problems and its methods. Therefore, there is no pure quantitative, but any quantitative is always the quantitative of a qualitatively determined reality. This breaking down of the real into scientific sectors, "which our perceptive senses pass on to us nearly unconsciously"[60] and which any scientist uses, is not absolutely evident. We never ask ourselves where it comes from, and science itself does not understand it, either. Sciences have a priori implicit knowledge of their respective domains of investigation, but they do not try, except by accident, to explain this knowledge, nor to determine its essence and the specificity of the reality whose properties they study. Biology, for instance, does not question the essence of the biological, and moreover it cannot possibly succeed in explaining it. The essence of biology is nothing biological; we can never observe it through a microscope like a cell or a molecule. Thus sciences are forced to accept the thematical division of reality that presides over their development. "Under such conditions", Thom asks, "perhaps it would be of great interest to question this decomposition and to settle it in the frame of a general abstract theory, rather than accepting it as an irreducible datum of reality".[61] This general and abstract theory is nothing but catastrophe theory, which can, as a general theory of forms, throw light on the nature and specificity of the geometrical and topological complexity of morphologies particular to every science. Instead of rejecting it as a qualitative theory, positivists should on the contrary be pleased about its existence, for, by bringing about the beginning of intelligibility, it allows the 'qualitative a priori' to be clarified, wherein any science spreads its quantitative models and whose structural essence remained until now unquestioned and misunderstood.

As for the criticism of locality, Thom instead sees an argument in favour of his theory. He reminds us that locality and intelligibility go together in science. Introducing a non-local action, an action at a distance, into the explanation of a phenomenon is not really satisfactory from a strictly theoretical point of view; for a non-local action, propagating itself with an infinite velocity and manifesting itself simultaneously at several points in space, is incomprehensible.[62] The demand for locality may seem surprising on first sight, for many great scientific successes have been linked with the formalization and the exploitation of seemingly non-local actions. This is obviously the case for Newton's theory of gravitation, which supposes that one body exerts a force on a distant body whose force is equal to $F = kmm'/r^2$. In electromagnetism, the Coulombian interaction of two electric charges $F = kqq'/r^2$, and the one of a magnet on a current $F = idl \wedge B$, are both actions at a distance. As for quantum mechanics, "when we try to formulate it classically in terms of hidden parameters, it necessarily introduces actions at distance with a velocity greater than that of the light (Bell's theorem)".[63] However, in spite of the scientific success they represent and of their unquestionable practical efficiency, these theories are more like magic, according to Thom (in magic, the same person may be at two different places at the same time: a witch can be both a man sleeping in his hut and a tiger hunting in the forest quite far away), than science, even if it is true that we are dealing with a magic with narrowly controlled manifestations. "The objections that critics addressed to Newton, in his day, remain for me still valuable today. Even nowadays, some are trying to find gravitational waves".[64] Newton was of course aware of the difficulties created by his theory of gravitation. "It is utterly inconceivable", he said, "that inanimate brute matter, without the mediation of some immaterial being, should operate upon and affect matter without mutual contact".

That gravity should be innate, inherent and essential to matter so that one body may act upon another through a *vacuum*, without the mediation of anything else, by and through which their action and force may be conveyed, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it.[65]

In spite of these reservations, it was in this way, in the form of an action at a distance, spreading instantaneously across the void, that Newton understood gravitation, as did in fact his immediate successors. This conception, a little shocking to his contemporaries, ended up being accepted as true, because nobody trying to show a prop used as an aid for gravitation ever succeeded, and finally the conception became accepted as being natural. In the eighteenth and nineteenth centuries, all forces were considered as actions at a distance. Only at the end of the nineteenth century and at the beginning of the twentieth did scientists begin to object to this concept of action at a distance and try to localize the non-local theory previously developed. In fact, all of the main conceptual progresses made during the last hundred years have developed in this direction. Einstein's theory of general relativity localizes Newton's theory of gravitation. According to general relativity, gravity is no longer considered as a force exerted by one material body on another, but comes from the space-time curve. Bodies fall, not because they are attracted by others, but because they follow geodesics of the space-time that have a Riemannian structure. The trajectories of the motion are no longer determined by forces, but by geometrical constraints. The matter modifies the structure of the space-time wherein it lies. And, above all, this modification of the space-time curve does not happen instantaneously, but at the speed of light. Gravitation is a wave that spreads locally, gradually, from place to place. In the same way, J. C. Maxwell has localized electromagnetism. Generally speaking, the localization of a non-local theory increases the intelligibility of our representation of the real. However, we must admit that its practical interest is generally fairly limited. Indeed, in most applications, the approximations provided by the corresponding non-local theory are sufficient.

From this point of view, in science there is a certain antinomy between "practice" and "theory". If science allows a practice, it is due to its controlling non-local elements which seem "magic". The explanation by a local contact, if it eliminates all magic appearances, does not create any new possibilities of action. Catastrophe theory, anti-magic par excellence, emphasizes the principle of locality: that is probably the reason why it offers hardly any possibilities of new practices.[66]

We come now to the major objection to catastrophe theory, that which sums all the others – the reproach of practical inefficiency. Thom does not contest it and would be the first to admit that his theory is unable to increase our possibility of action on reality.[67] Indeed, we generally misunderstand what Thom calls "the relatively fast degeneration of the possibilities of the mathematical tool in science",[68] as soon as we go far from the domain where mathematics reigns: fundamental physics and mechanics. When we leave that domain, where mathematics is not only an instrument but also plays a main part in the determination of the scientific object itself, we enter the domain of approximation. Theoretical laws deduced from pure physics are not enough in themselves to formalize and to calculate the temporal evolution of systems. We must add ad hoc hypotheses, generally inferred by empirical considerations. The efficiency of the mathematical tool is deteriorating in applied mechanics and fluid dynamics. The situation is even worse in biology, where mathematics is only used to build local models that have a limited practical range (the circulation of blood in the arteries, for instance). This deterioration is at its peak in ethology, psychology, and social sciences, where the number of parameters to be taken into account is so high that it is impossible or illusory to try a precise quantitative modelling. Thus mathematics is of no use but to provide "statistics recipes", Thom says, "whose legitimacy itself is dubious".[69] This reduction of the efficiency of the mathematical tool in science reflects a reality of which "few scientists are conscious, (and this small number prefer to cover it with Noah's pious veil), that is the extreme theoretical misery of most sciences".[70] Here we touch the central theme of Thomian epistemological analysis: science itself giving up its fundamental task of 'theorizing' reality.[71] This renunciation by science of its essential theoretical vocation happened in the time of Galileo and Newton. With Galileo and Newton, we gain accuracy in the description of the motion of material bodies by eliminating the use of hidden forces, *entelechies*, but we loose sight of any knowledge of the cause of the motion. Science becomes descriptive and ceases to be explanatory. It tells us 'how', but not 'why' phenomena occur. Newton mathematically describes, with the law of gravity and the second law of dynamics, the way in which celestial bodies move, but does not give the reason for it. The law of gravity is a functional relation that allows for calculus and prediction, but it is not a final explanation of reality. The nature and the true origin of gravitation is unknown, and Newton did not want to give the least hypothesis for it.[72] He shows that the law of gravitation in k/r^2

allows him to find Kepler's law and to account for other phenomena, but he confesses its inability to explain why two masses attract each other precisely according to that law and why one exerts a force on the other that is inversely proportional to the square of their distance. Since Newton, many have tried to derive the law of gravitation from other mechanisms, but none of these attempts succeeded without predicting at the same time other phenomena that do not exist. Gravity allows a good formalization of motion, but its cause remains mysterious, even now, to the point that we would admire he who could succeed in explaining the law as much as we admire Newton. Through the changing of Galileo–Newtonian paradigm, we no doubt gain in efficiency, but we lose in understanding. Science tends to become a set of recipes that work, and that we apply, without our really knowing why they work.[73] Dealing with the Newtonian "*hypotheses non fingo*", science today, for the most part, confines itself to research and to the proper setting of technical procedures that allow it to act on reality and to predict its evolution, without bothering too much to give a satisfactory theoretical basis. Hence the current proliferation of experimental researches "with no other aim", Thom says, "but to add a minuscule and quickly forgotten contribution to the universal knowledge".[74] Thom's analysis reminds us of Heidegger's, for whom, as we know, technique was not an avatar nor a by-product of modern science, but the element in which it spreads and that determines it in its own essence. To confirm that the views of the scientist and the philosopher are alike, it is sufficient to quote three statements where Heidegger notes the fundamental features that characterize modern science:

> 1°. Modern science is based on the essence of the technique. 2°. The very essence of technique is nothing technical. 3°. The essence of technique is not simply the work of man, which a human superiority and sovereignty could master through a fitting moral attitude.[75]

Moreover, Thom often refers to Heidegger and to the fairly cold judgement he makes about contemporary science.

> Contemporary science is now dealing with general investigation (description) and, concerning experimentation, with something which is scarcely more than an active form of odd jobs. In this way, the condemnation of Heidegger in 1929: "science does not think" was justified, four years after quantum mechanics which was already a superb illustration of the first failure.[76]

Can catastrophe theory invalidate Heidegger's icy condemnation and represent the counter-example of an endeavour to fill precisely the gap that separates science from thought? Heidegger would certainly reply no, because catastrophe theory cannot, any more than science in general, understand the being as being nor the ontological difference. Thom's reply would probably be different inasmuch as catastrophe theory, as we shall see, rediscovers the theoretical, and even philosophical, dimension that has been lost to view by contemporary science.

5. CATASTROPHE THEORY: A NEW PHILOSOPHY OF NATURE

The practical infirmity of catastrophe theory, which stands out against the efficiency of contemporary science, is in effect everything but a sign of the weakness, the imperfection, or the immaturity of the theory. Thom, rather than minimizing it, claims credit for it. "Really", Jean Largeault says, "Thom is not angry that his theory does not lend itself to quantitative applications. What he praises more is the freedom, the speculation and the qualitative comprehension of the studied process".[77] Catastrophe theory has a further ambition in addition to quantitative prediction or action on the real. Its aim is not, supposing it is possible, to establish empirical laws ruling the appearances of morphologies. Neither does it pretend to propose to research new ideas of experimentation, as has been said of it. Its aim is not practical but theoretical. Thom does not simply provide a description of the morphologies he studies, but also tries to understand and explain them. Catastrophe theory explains the morphologies by building, above the substratum space where they appear, the *logos*, that is to say, the simplest algebraic-geometrical being able to generate them. Its aim is neither to predict nor to produce new morphologies, but to base existing morphologies in the 'being', that is, to draw them from a mathematical being that would logically and ontologically come first. Here Thom rehabilitates an old theme rejected by positivist epistemology, according to which 'all knowledge is knowledge by causes'. Catastrophe theory explains empirical morphologies by returning to the causes that gave birth to them, of course not to the efficient causes that act in the same space as the studied morphology, but to formal causes that spread in another space (the inner space or behaviour space) and that are the only really rational ones. Through its demand for intelligibility, through its will not to be satisfied with the 'how' and to search systematically for the 'why' of things, through its call to base the appearance in the being, through its pretention to universality, catastrophe theory possesses an essential philosophical dimension. Since it searches for the principles of reality, of the world of experience beyond (*meta* in Greek) reality or Nature or physics (in the *logoi*), it can be considered as mathematical meta-physics. Thom himself regards it as a philosophy of Nature. His is a return to the days when science and philosophy were not two antithetical terms.

Through his care to explain and understand and his refusal to confine himself to 'the recipe that works', Thom feels close to philosophers such as Descartes and Leibniz, nearer in any case to Descartes than to Newton. "Descartes", he says,

> with his whirls, his hooked atoms, explained everything and calculated nothing. Newton, with the law of gravity in $1/r^2$ calculated everything and explained nothing. History proved Newton to be right and relegated cartesian constructions to the level of free imaginations and museum exhibits. To be sure, the newtonian point of view is quite right as regard efficacy, possibilities of prediction and therefore of action on phenomena . . . . But minds anxious to understand will never have that scornful attitude towards qualitative and descriptive theories, from the presocratics to Descartes.[78]

Because of his taste for pure knowledge, for free speculation that makes him place contemplation higher than action, "an enormous boldness in a century where research is subject to the necessity of efficiency and to administrative routines",[79] Thom feels above all close to the Greeks. Firstly, close to Plato, whose philosophy of Nature, in the *Timaeus*, through the part played by geometry, evokes the Thomian reduction of space-time morphologies to a finite set of algebraic-geometrical structures.[80] Thom also has a link with Aristotle or, more precisely, with Aristotelianism, from which catastrophe theory takes up a certain number of themes that it interprets quite freely.

> As regards a being or an object we can classically distinguish its existence, its *Da-sein*, the fact that the being occupies a certain portion of the space-time, and its essence, that is to say, the totality of its aspects and qualities. The materialist attitude, traditional in science, consists of saying that existence precedes essence (in fact existence implies essence); catastrophe theory model in morphogenesis goes against this formula since it presupposes that, to a certain extent, existence is determined by essence, the whole set of the qualities of a being. Here we can see a return of the aristotelian scheme of hylomorphismus: the matter aspiring to the form. This underlying idealism explains to a large extent the reticence of biologists when facing, for instance, the embryological models of catastrophe theory.[81]

The matter, which is never absolutely formless, cannot take just any form. Geometrical and topological constraints are imposed, which limit the complexity of morphologies that can develop therein. Thus, in the usual space-time, the most complex morphologies that can be found are, typically, according to the inner-space dimension, butterflies (dimension 1) or umbilics (dimension 2). Catastrophe theory refers to another Aristotelian theme, the distinction between act and power, which enabled the Stagirite to solve the aporia of motion (How can the same become another?). Thom gives this distinction a mathematical significance through the concept of the universal unfolding of a function. The notion of the universal unfolding of a singularity

in a way rehabilitates and gives a new interest to the aristotelian couple power/act. Any unstable situation is the source of indetermination: [the universal unfolding allows] any possible actualisations of virtualities contained in a situation which we mathematically represent with a singularity.[82]

The growth of morphology in the substratum space corresponds to the progressive actualization of potentialities contained in a degenerated singularity. Universal unfolding explains or develops what a singularity implies or envelops inside itself. Other chapters of catastrophe theory evoke Aristotelianism, such as the theory of the categories (*logoi*) or the theory of analogy. But the thinkers with whom Thom feels most affinity are unquestionably the Presocratics, notably Heraclitus.[83] Often Thom sets a link between his own theory and the catastrophist conception of the world it implies, and the doctrines of the first Greek thinkers.

Our models attribute all morphogenesis to conflict, a struggle between two or more attractors. This is the 2500 years old idea of the first presocratic philosophers, Anaximander and Heraclitus. These thinkers have been accused of primitive confusionism because they used a vocabulary with human and social origins such as conflict and injustice to explain the appearance of the physical world. I think that they were far from wrong because they had the following fundamentally valid intuition: *the dynamical situations governing the evolution of natural phenomena are basically the same as those governing man and societies. Thus the use of anthropomorphic words in physics is profoundly justified.*[84]

Catastrophe theory, which considers any form as being the result of a conflict, a struggle between attractors sharing a same substratum space, brings about a dialectical and Heraclitean conception of the universe. Other traits link Thom to the Presocratics: firstly, the inscription of man in Nature. There is nothing less Thomian indeed than anthropocentrism or subjectivism. Then the refusal of any transcendence. Archetypal *logoi* are not transcendent realities, like Plato's ideas, but they are immanent in the morphologies that they determine. The causality of algebraic-geometrical structures works within the support-space of morphogenesis. The matter is not given its form by an external principle, but forms itself by obeying the constraints imposed on it by the hidden complexity it possesses. A third feature brings Thom close to the Presocratics. That is his 'anti-globalism'. Thom denies the possibility of a human knowledge that would succeed in englobing the whole of reality, as the Hegelian system does for instance.

If it was effectively possible to integrate all the local models in an immense synthesis, man would be right in saying that he knows the "ultimate nature of reality" since there would exist no other more global model than that one. But I personally think that it is an exorbitant pretention. Quite likely the era of big cosmic synthesis finally came to an end with general relativity, and it seems doubtful and probably of little use to try to reopen it.[85]

Our knowledge can only work locally. To maintain the contrary and to aim at reaching the whole of reality would be a form of immoderation. The Presocratics, and especially Heraclitus, did not think otherwise, reminding us that 'Nature enjoys being hidden'. This means that the appearance (the non-being) is an essential component of the being. Nature always conserves a part of its mystery so that we could never reveal the being or the real in its globality. The wise man is not he who undertakes to make this obscure part of nature disappear, but he who, after having acknowledged its fundamental necessity, preserves it as it is. With his demand of locality, Thom would certainly agree with the Delphic sentence that provides one of the most magnificent and famous expressions of it: 'Know yourself'. This formula has indeed nothing to do with any affirmation of the power of reflection – in spite of all the modern interpretations to this effect – but first and foremost means: 'Know your own limits, acknowledge the fact that you are mortal, within a precarious world whose meaning is partly veiled'. Knowledge does not aim to enable us to escape our end, but to plunge us into it. Thom himself does not say otherwise: "Perhaps thanks to catastrophe theory, we shall be able to demonstrate the inescapable character of some catastrophes, such as illness and death. Knowledge will not necessarily be a promise of success or survival: it may just as easily be the certainty of our failure, of our end".[86] Catastrophe theory does not enable us to escape our fate, but, if it is contemplative, it is enough to assure happiness.

Thom's references to the Occidental philosophical tradition and more particularly to the Presocratics are incontestably seductive, even if they seem surprising in the mathematical context of catastrophe theory, where there is the question of dynamic systems, attractors, singularities, etc. They differ in any case from

> the haughty indifference, even from the sovereign contempt, as that openly exhibited by some famous bourbakist, with whom the high mathematical society consider philosophical questions, without mentioning ideological quarrels. For such people, mathematics constitutes their own end and has become necessary and sufficient for any thought coming under that name. Thus philosophers have to turn to it to learn the definitive language of reason.[87]

Thom's philosophical references contrast with this mathematical elitism and allow us to perceive the possibility of drawing science nearer to philosophy. However, they also raise questions, one of fundamental importance to us being the pertinence of an interpretation of philosophical doctrines in the light of a mathematical theory, even a 'qualitative' one like catastrophe theory. "All the fundamental intuitions in morphogenesis", Thom says, "are already found in Heraclitus. My only contribution is to have placed them again within a geometrical and dynamical frame allowing them one day to be analyzed quantitatively".[88] Thom confesses here, as regards fundamental intuitions, his debt to the doctrines of one of the first Greek thinkers: the dialectical conception of the world he proposes is a geometrization of that sketched out by Heraclitus more than 2500 years ago. According to Thom, such a geometrization allows some precision to be brought "to the magnificent but a little vague views of the first presocratics".[89] It may even allow the speculative doctrines of these thinkers to be saved from a supposedly unavoidable decline, for it diminishes "the necessarily naïve and imprecise character of the images they use".[90] The philosophical interest of such a geometrization does not seem incontestable. Indeed, the imprecise character that Thom deplores in the philosophical notions is really due to the fact that these notions are fundamentally overdetermined: there is always more, in the main statements of thinkers, than we notice at first sight. Thus it is impossible to translate philosophical notions without reducing them and, at the same time, without missing their true meaning in a mathematical language that, by definition, would suppress all ambiguity. Perhaps it is not coincidental that the Leibnizian dream of Characteristica Universalis (algebraic transcription of concepts and formalization of reasoning), for a while entertained by Thom, broke down. Of course, we could try to justify this Thomian undertaking by considering it not as a translation but as an interpretation of philosophical doctrines in the light of mathematics. But then the status of a mathematical interpretation of philosophy becomes problematic. Indeed the mathematical interpretation of a philosophical text (or another one), that is its 'mathematization', is not like any other interpretation. Mathematical symbols do not mean anything in themselves, and the mathematical formalization, far from being an exegesis, can itself be nothing but a transcription. Being required to transcribe, mathematical formalization must naturally be as accurate as possible. But how can it be so, since mathematical formalism is a language without any depth, and is therefore unable to restore the semantic wealth of the philosophical concepts it transcribes? The following example will show how great the difficulty is: Which mathematical equivalent can be found for the word 'being'? To find one, 'being' should have only one meaning, not only in everyday language, but also in philosophical language, which is obviously not the case. In reality, the contribution of catastrophe theory does not lie there. It does not lie in some geometrization of the meaning, which could only be hypothetical, but rather in what would be the 'philosophical becoming' of science itself. Catastrophe theory shows the possibility, and even the necessity, of a less practical and more theoretical, less technical and more philosophical science. Thom's approach reminds us of another quite independent one, effected by the contemporary thinker, Heidegger. Indeed, Heidegger also denounces the hold of technical and calculating ideas over contemporary thought; he, too, refers to the Presocratics as thinkers still moving in the nowadays buried realm of the truth of being. And, in the same way that Heidegger demands a more thinking thought, Thom, similarly pleads for a more scientific science, that is, a more philosophical one.[91] Catastrophe theory opens the way. It is a scientific theory and at the same time a philosophy, not of course in the sense of a body of doctrinal propositions, but as it possesses a fundamentally philosophical dimension and places contemplation higher than action, theory higher than practice. Only the future will tell us whether Thom's work is but the late offshoot of a bygone period when science and philosophy went hand in hand, or if, on the contrary, as I would like to believe, it opens a new era where science, becoming once again conscious of its fundamental theoretical mission, will no longer be content only to predict and calculate reality, but will also endeavour to understand it.

NOTES

[1] Cf. Thom, René: 1980a, *Modèles Mathématiques de la Morphogenèse*, Bourgois, Paris, pp. 9–23.
[2] Cf. Zeeman, Christopher: 1980, *Catastrophe Theory – Selected Papers, 1972–1977*, Addison-Wesley, Reading, pp. 8–23.
[3] Lévy-Leblond, Jean-Marc: 1977, 'Des Mathématiques Catastrophiques', *Critique* **33**, p. 439.
[4] Thom, René: 1977, *Stabilité Structurelle et Morphogenèse*, InterEditions, Paris, p. XI.
[5] Zahler, Raphael, and Hector Sussman: 1977, 'Claims and Accomplishments of Applied Catastrophe Theory', *Nature* **269**, pp. 759–63.
[6] In Woodcock, A., and M. Davis: 1984, *La Théorie des Catastrophes*, L'Age d'homme, Lausanne, p. 75.
[7] Lévy-Leblond (1977, p. 437).
[8] Ibid., p. 438.
[9] Thom (1977, p. 9).
[10] Thom, René: 1974a, *Modèles Mathématiques de la Morphogenèse*, coll. 10/18, n. 887, U.G.E., Paris, p. 9.
[11] Ibid., p. 10.
[12] Thom (1980a, p. 10).
[13] Thom (1977, p. 46).
[14] Cf. Poston, Tim, and Ian Stewart: 1978, *Catastrophe Theory and its Applications*, Pitnam, London.
[15] Thom (1980a, p. 12).
[16] Ibid.
[17] Thom, René: 1974b, 'La Science malgré tout . . .', in *Encyclopaedia Universalis, Organum 1974*, Encyclopaedia Universalis, Paris, p. 8.
[18] Ibid., p. 9.
[19] Thom, René: 1982, 'La Science en Crise?', *Le Débat* **18**, p. 39.
[20] Ibid.
[21] Thom (1977, p. 158).
[22] Thom, René: 1983, *Paraboles et Catastrophes*, Flammarion, Paris, p. 66.
[23] Thom (1974a, pp. 21–22).
[24] Thom (1983, p. 85).
[25] Thom (1977, p. 10).
[26] Thomson, W.: 1884, *Lectures on Molecular Dynamics and the Wave Theory of Light*, Johns Hopkins University, Baltimore, p. 270.
[27] Ibid., p. 131.
[28] Thom (1983, p. 112).
[29] Ibid.
[30] Thom (1980a, p. 298).

[31] Thom 1980b: 'La Théorie des Catastrophes: État Présent et Perspectives', in Zeeman (1980), p. 615.
[32] Petitot, Jean: 1978, 'La Théorie des Catastrophes, Un Événement et un Avènement', in *Encyclopaedia Universalis, Universalia 1978*, Encyclopaedia Universalis, Paris, p. 202.
[33] Thom (1983, p. 135).
[34] Thom (1977, p. 325; see also 1980a, p. 17).
[35] Thom (1983, p. 99).
[36] Ekeland, Ivar: 1984, *Le Calcul, L'Imprévu*, Seuil, Paris, p. 122.
[37] Thom (1980, p. 101).
[38] Thom (1983, p. 102).
[39] Thom (1980a, p. 101).
[40] Ibid., p. 117.
[41] Ibid., p. 116.
[42] Thom (1983, p. 102).
[43] See Thom (1977, p. 4; 1980a, pp. 116ff.).
[44] Sussman, Hector, and Raphael Zahler: 1978, 'Catastrophe Theory as Applied to the Social and Biological Sciences: A Critique', *Synthese* **37**, p. 200.
[45] Ibid.
[46] Ibid.
[47] Ibid., p. 201.
[48] Thom (1983, p. 13).
[49] Ibid., p. 8.
[50] Ibid., p. 93.
[51] Ibid., p. 116.
[52] Cf. Thom (1980a, p. 117).
[53] Thom (1983, p. 93).
[54] Thom (1980a, p. 120).
[55] Ibid., p. 115.
[56] Ibid., pp. 114–15.
[57] Ibid., p. 115.
[58] Thom (1983, p. 48).
[59] Thom (1977, p. 325).
[60] Ibid.
[61] Ibid.
[62] Thom (1983, p. 123).
[63] Thom (1980a, p. 137).
[64] Thom (1983, p. 93).
[65] Letter to Richard Bentley of 25 February 1692, in Samuel Horsley (ed.): 1785, *Isaaci Newtoni Opera quae existant Omnia*, IV, London, p. 438.
[66] Thom (1980a, p. 137).
[67] Ibid., p. 112.
[68] Ibid., p. 113.
[69] Ibid.
[70] Thom (1974b, p. 7).
[71] Ibid., p. 6.
[72] Newton, Isaac: 1726, *Philosophiae Naturalis Principia Mathematica*, Scholium Generale.

[73] Thom (1983, pp. 33–34).
[74] Thom (1974b, p. 7).
[75] Heidegger, Martin: 1959, *Qu'appelle-t-on Penser?*, PUF, Paris, p. 235.
[76] Thom (1982, p. 39).
[77] Largeault, Jean: 1984, *Philosophie de la Nature 1984*, Université Paris XII, Val de Marne, Créteil, p. 115, n. 25.
[78] Thom (1977, pp. 5–6).
[79] Largeault (1984, p. 143).
[80] Ekeland (1984, pp. 124–25).
[81] Thom (1980a, p. 87).
[82] Thom (1983, p. 70).
[83] Thom (1980a, p. 167, n. 1).
[84] Thom (1977, p. 327).
[85] Ibid., p. 7.
[86] Thom (1980b, p. 621).
[87] Lévy-Leblond (1977, p. 43).
[88] Thom (1977, p. 10).
[89] Ibid., p. 6.
[90] Ibid.
[91] Thom (1974b, p. 10).

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