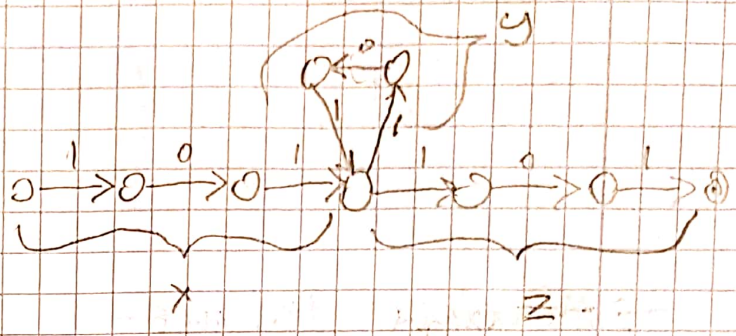
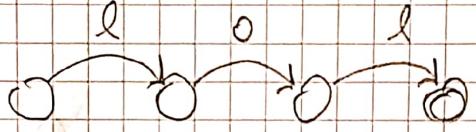
HW6

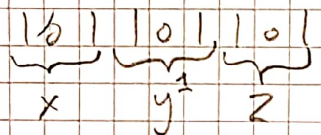
① $\Sigma = \{1, 0\}$



a) $\{1^n 0 1 \mid n \geq 0\}$



Finite strings are ALWAYS regular



This language is regular

b) $\{1^n 0 1^n \mid n \geq 0\} = L$

Suppose this language is regular and let $s = 1^p 0 1^p$, where p is the pumping length. As $s \in L$ and $|s| \geq p$, PL says that s divides into 3 parts, $s = xyz$, so that

1. $xy^i z \in L$ for all $i = 0, 1 \ldots$
2. $|y| > 0$
3. $|xy| \leq p$

$xy^0z = 0$ ← $|y| \neq 0$

$xy^1z = 1\,0\,1$

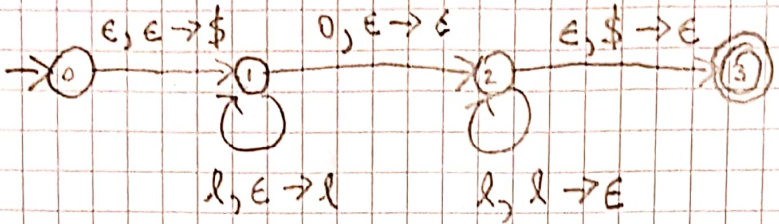
$xy^2z = 1\,1\,0\,1\,1$ $|xy| = 6$

$xy^3z = 1\,1\,1\,0\,1\,1\,1$ $p = 3$ $|xy| \nleq p$

x y z

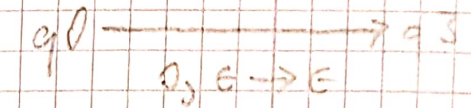
① b) cont. $\{1^n 0 1^n \mid n \geq 0\}$ $\Sigma = \{1, 0\}$

$\Gamma = \{\$, 1\}$



$\langle A \rangle ::= 1\langle A \rangle 1 \mid 0$

1 1 1 0 1 1 1



① c) $\{1 0^n 1 \mid n \geq 0\} = L$

Suppose that L is regular and let $s = 1 0^n 1$.

$s = xyz$, so that

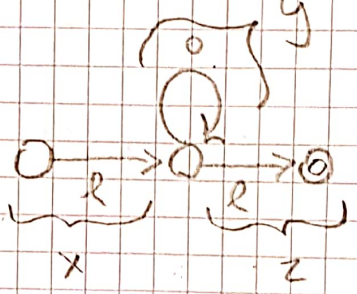
1. $xy^i z \in L$ for all $i = 0, 1, \ldots$
2. $|y| > 0$
3. $|xy| \leq p$

$xy^0z =$ 1 1

$xy^1z =$ 1 0 1

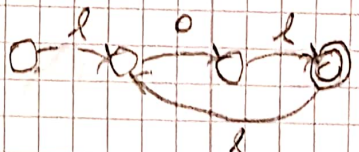
$xy^2z =$ 1 0 0 1

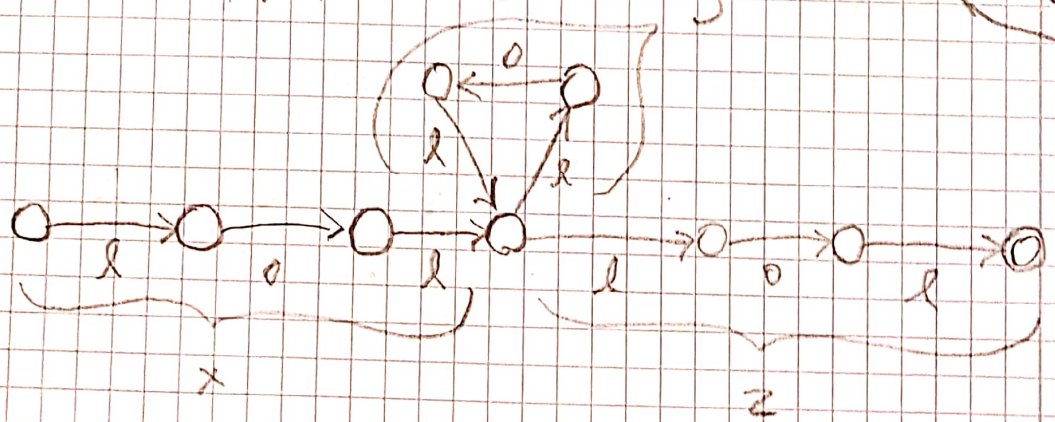
$xy^3z =$ 1 0 0 0 1 (x = 1, y = 0 0 0, z = 1)



Language is regular

① d) $\{(1 0 1)^n \mid n \geq 0\}$





1 0 1 | 1 0 1 | 1 0 1 | 1 0 1 | 1 0 1

x y z

REGULAR

② $F = \{ a^i, b^j, c^k \mid i,j,k \geq 0, \text{if } i = 1 \text{ then } j = k \}$

$i\ j\ k = 0 \ \ldots$

$i = 1$
$j = 1$
$k = 1$ — a b c

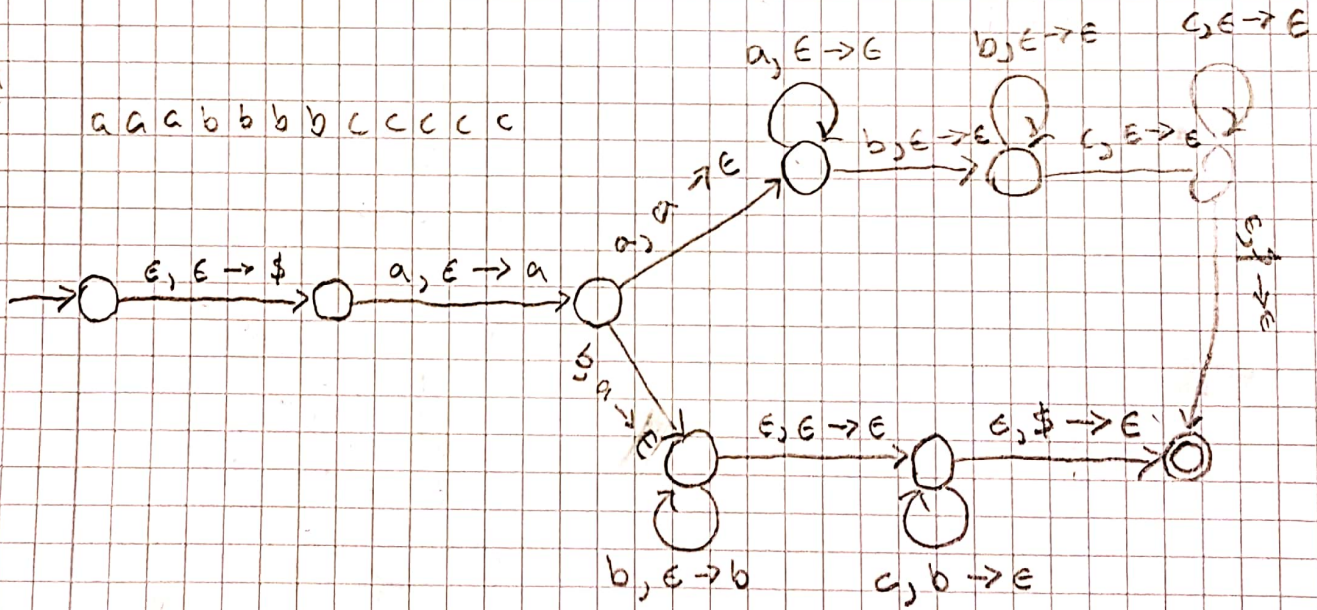
$i = 1$
$j = 2$
$k = 2$ — a b b c c

$i = 1$
$j = 3$
$k = 3$ — a b b b c c c

$i = 2$
$j = 2$
$k = 1$ — a a b c

$i = 2$
$j = 3$
$k = 1$ — a a b b b c

$i = 3$
$j = 4$
$k = 5$ — a a a b b b b c c c c c



(3) 

$C = A/B = \{w \mid wx \in A \text{ for some } x \in B\}$

$A = \{1^n 0 1^n \mid n \geq 0\}$

$B = \{1 0 1 \mid n \geq 0\}$

$s = wx$

$wx/x = w$

$A n = 3$, $\underbrace{1\,0\,1\,1\,0\,1}_{w}\ \underbrace{1\,0\,1}_{x}$ $\qquad A/B = 1\,0\,1\,1\,0\,1$

u $\underbrace{1\,1\,1}_{w}\ \underbrace{1\,0\,1}_{x}\ \underbrace{1\,1\,1}_{w}$

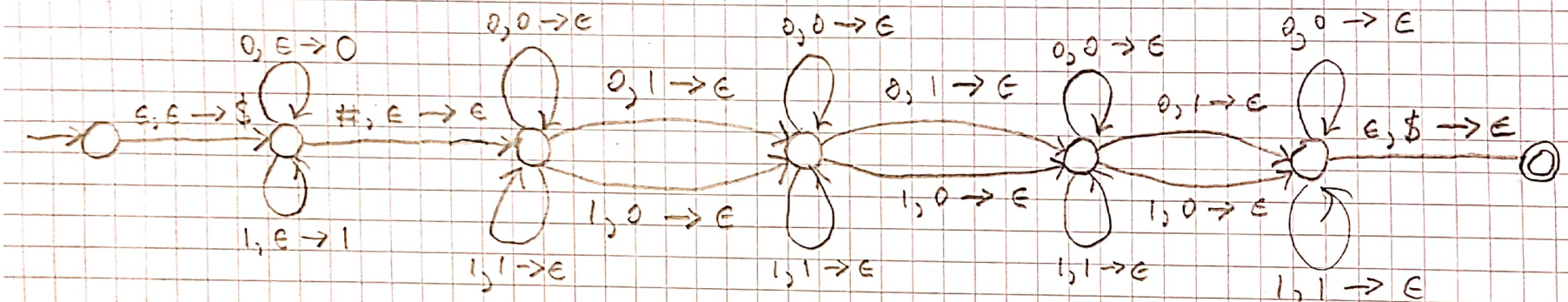
$A = \{0^n 1^n \mid n \geq 0\}$

$B = \{(1\,1)^n \mid n \geq 0\}$

wx

$\underbrace{0\,0\,0\,0}_{w}\ \underbrace{1\,1\,1\,1}_{x}$ $\qquad A/B = 0\,0\,0\,0$

According to wikipedia "the quotient A/B of A by a regular language B is closed under this operation."



0/1, 0/1 → ε

x x x x

0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 -
0 0 - - 0 0 - - 0 0 - - 0 0 - -
0 0 0 0 - - - - 0 0 0 0 - - - -
0 0 0 0 0 0 0 0 - - - - - - - -

0 1 0 0 0 0 0 1 0