

CS663: HW5

Q.01] $g_1 = f_1 + h_2^* f_2$;
 $G_1 = F_1 + H_2 \cdot F_2$

$g_2 = f_2 + h_1^* f_1$
 $G_2 = F_2 + H_1 \cdot F_1$

Given g_1, g_2 ;
 Assuming h_1, h_2 known
 (Blur kernels)

Solving for F_1 & F_2

$$F_1(\mu) = \frac{G_1 - H_2 \cdot G_2}{1 - H_1 \cdot H_2} \Rightarrow f_1(x) = \text{ifft2}(F_1(\mu))$$

$$F_2(\mu) = \frac{G_2 - H_1 \cdot G_1}{1 - H_1 \cdot H_2} \Rightarrow f_2(x) = \text{ifft2}(F_2(\mu))$$

Evident problem whenever $H_1(\mu) \cdot H_2(\mu) = 1$.

$\therefore h_1, h_2$ are blur kernels

$$\Rightarrow \int_{-\infty}^{\infty} h_1(x) dx = 1 \text{ and } \int_{-\infty}^{\infty} h_2(x) dx = 1$$

Now we observe,

$$H_1(\mu) = \int_{-\infty}^{\infty} h_1(x) \cdot e^{-j\mu x} dx$$

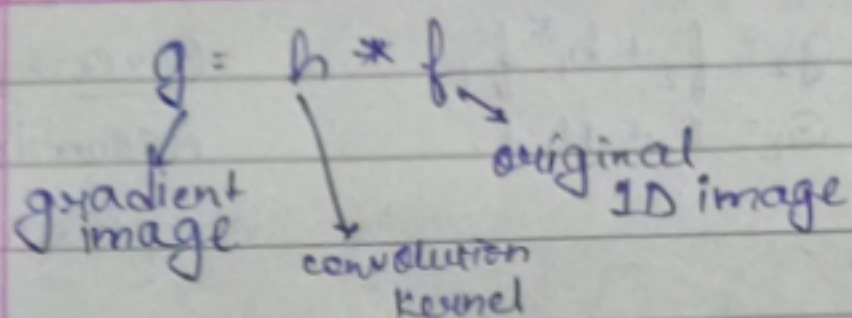
$$H_2(\mu) = \int_{-\infty}^{\infty} h_2(x) \cdot e^{-j\mu x} dx$$

For $\mu = 0$, $\Rightarrow H_1\left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right) = H_2\left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right) = 1$

$\Rightarrow H_1(0) \cdot H_2(0) = 1 \Rightarrow F_1\left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right) \text{ \& } F_2(0)$ will be undefined

\therefore We cannot recover the DC component

Q.02]



Fundamental problem:

Presence of noise in image.

If we can neglect that, there is another problem \rightarrow

h can simply be $[-1 \ 1] \Rightarrow g(x) = f(x+1) - f(x)$

Here let $x \in [1, N]$

To obtain f from g and h , we'll make use of Discrete Fourier Transform,

$$G(\mu) = F(\mu) \cdot e^{j\frac{2\pi\mu}{N}} - F(\mu) \quad [\text{By Shift Thm}]$$

$$\Rightarrow F(\mu) = \frac{G(\mu)}{e^{j\frac{2\pi\mu}{N}} - 1} \quad \text{When } \mu=0, \text{ denominator becomes } 0$$

We cannot recover the DC component of original image ~~by~~ ^{by} this ~~same~~ method.

In case of 2D image, the same problem as above is observed. If we consider $f(x, y) \rightarrow F(\mu, \nu)$

$$\Rightarrow F_x(\mu, \nu) = (e^{j\frac{2\pi\mu}{N}} - 1) \cdot F(\mu, \nu) \Rightarrow \text{Problem at } \mu=0$$

$$F_y(\mu, \nu) = (e^{j\frac{2\pi\nu}{N}} - 1) \cdot F(\mu, \nu) \Rightarrow \text{Problem at } \nu=0$$

Even if we ~~had~~ knew both, the intersection ($\mu=0, \nu=0$) still remains unknown, i.e., DC component information can't be obtained.

On the hand, using the integration method, integration across rows will be inconsistent with answer from integration across columns.