CS 663 : Digital Image Processing : Assignment 5

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Question 2

Question:
Consider a 1D image (for example, a single row from a 2D image). You know that given such an image, computing its gradients is trivial. An inquisitive student frames this as a convolution problem to yield $g = h * f$ where g is the gradient image (in 1D), h is the convolution kernel to represent the gradient operation, and f is the original 1D image. The student tries to develop a method to determine f given g and h. What are the fundamental difficulties he/she will face in this task? Justify your answer. You may assume appropriate boundary conditions. Now consider that you are given the gradients of a 2D image in the X and Y directions, and you wish to determine the original image. What are the difficulties you will face in this task? Justify your answer. Again, you may assume appropriate boundary conditions.

Solution:

Here, we first assume that the gradients that we have are not noisy.

For the 1D Image of size N, it is given that $g = h * f$,
where g is the gradient image, h is the convolution kernel & f is the original image

Since image gradients are computed discretely using $g(x) = f(x+1) - f(x)$ for $x \in \{1, 2, 3, ..., N\}$, the convolution kernel h for computing the gradient image can be given by [-1 1]

Our objective is given g & h, we need to obtain f. We can proceed using the technique of applying Discrete Fourier Transform on the equation & using the Fourier Shift Theorem.

$$G(u) = F(u)(e^{\frac{2\pi uj}{N}} - 1)$$
$$\implies F(u) = \frac{G(u)}{e^{\frac{2\pi uj}{N}} - 1}$$

Thus, when $e^{\frac{2\pi uj}{N}} = 1$, the denominator becomes 0 & $F(u)$ becomes undefined. This is particularly the case when $u = 0$. Thus, we cannot recover the DC component of the original 1D image using this method & must resort to other methods of estimating the DC component [Eg: Using Boundary Conditions, etc]

In the case of a 2D Image of size $N \times N$, a similar problem as above is observed while trying to use DFT to obtain $f(x, y)$ from $g(x, y)$ & $h(x, y)$.

Let $f_x(x, y)$ & $f_y(x, y)$ be the gradients of $f(x, y)$ in the spatial domain, with $F_x(u, v)$ & $F_y(u, v)$ as the corresponding Fourier domain representations.

In the Fourier Domain, we have:

$$F_x(u,v) = F(u,v)(e^{\frac{2\pi uj}{N}} - 1)$$
$$F_y(u,v) = F(u,v)(e^{\frac{2\pi vj}{N}} - 1)$$

Problems similar to the 1D case arise at $u = 0$ for F_x and at $v = 0$ for F_y. Even of we know both, the intersection $(u = 0, v = 0)$ will still remain unknown. Thus, we will again have to estimate the DC component using some other means.
This is a big difficulty with this approach.

Moreover, in our analysis till now, we have assume gradients to be noise-free. But in reality, this may not be the case. The noisy gradients will pose greater problems in the reconstruction of the original image (both 1D & 2D).