CS 663 : Digital Image Processing : Assignment 1

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Question 3: Bi-Histogram Equalization

(a) Since $I(x)$ has a continuous domain, histogram equalization over any part of the histogram $h(x)$ will result in a *uniform distribution* over the entire intensity range of $[0, 1]$.

Histogram $h(I)$ is split into two histograms $h_1(I)$ (over the domain $[0, a]$) and $h_2(I)$ (over the domain $(a, 1]$) , for some arbitrary $a(0, 1)$

Let mass of histogram $h_1(I)$ in $[0, a]$ be $\alpha \implies$ mass of histogram $h_2(I)$ in $(a, 1]$ is $(1 - \alpha)$

It is said that masses of $h_1(I)$ & $h_2(I)$ are preserved after performing equalization.

Within $[0, a]$, after equalization of $h_1(I)$, the resulting uniform histogram $h_1'(I)$ is s.t $P(I) = \frac{\alpha}{a}$
Within $(a, 1]$, after equalization of $h_2(I)$, the resulting uniform histogram $h_2'(I)$ is s.t $P(I) = \frac{1-\alpha}{1-a}$

$$E[I] = \int_0^1 IP(I)dI = \frac{\alpha}{a}\frac{I^2}{2}\bigg|_0^a + \left(\frac{1-\alpha}{1-a}\right)\frac{I^2}{2}\bigg|_a^1$$

$$E[I] = \frac{\alpha a}{2} + \frac{(1-\alpha)(1+a)}{2}$$

$$\therefore E[I] = \frac{1-\alpha+a}{2}$$

(b) Given, a is the median intensity $\implies \alpha = 0.5$

From the expression obtained in (a), it is evident that:

$$E[I] = \frac{0.5 + a}{2}$$

(c)
- **Scenario:**
  Bi-histogram equalization could be used in situations where final image brightness needs to be close to the original image brightness, while still bringing out the details within the image and utilizing the full range of intensities.
- **Reasons:**
  Simple Histogram Equalization causes the final mean to become close to 0.5, which in some ways distorts the original brightness. Bi-histogram equalization ensures final mean is close to initial mean a. This can help preserving, at least to some extent, the overall brightness of the scene, preventing unnatural / out-of-context output images, e.g., in the television industry.

(d) **Example Image to Demonstrate the Bi-Histogram** Equalization

(a) Original Image (b) Histogram Equalized Image (c) Bi-Histogram Equalized Image

Figure 1: Comparison of Histogram Equalized and Bi-Histogram Equalized Images

The original image has $Mean = 0.078$ and $median = 0.071$.

The histogram equalized image has a $Mean = 0.4984$ $(almost\ equal\ to\ 0.5)$, whereas the bi-histogram equalized image (using $a = Median$) has a $Mean = 0.409$ (more towards the mean of the original image).