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CS663 : HW5

Q.01] $g_1 = f_1 + h_2 * f_2$; $g_2 = f_2 + h_1 * f_1$ Given: g_1, g_2;

$G_1 = F_1 + H_2 \cdot F_2$ $G_2 = F_2 + H_1 \cdot F_1$ Assuming h_1, h_2 known (Blur kernels)

Solving for F_1 & F_2

$$F_1(\mu) = \frac{G_1 - H_2 \cdot G_2}{1 - H_1 \cdot H_2} \Rightarrow f_1(x) = \text{ifft2}(F_1(\mu))$$

$$F_2(\mu) = \frac{G_2 - H_1 \cdot G_1}{1 - H_1 \cdot H_2} \Rightarrow f_2(x) = \text{ifft2}(F_2(\mu))$$

Evident problem whenever $H_1(\mu) \cdot H_2(\mu) = 1$.

$\because$ h_1, h_2 are blur kernels

$$\Rightarrow \int_{-\infty}^{\infty} h_1(x)dx = 1 \text{ and } \int_{-\infty}^{\infty} h_2(x)dx = 1$$

Now we observe,

$$H_1(\mu) = \int_{-\infty}^{\infty} h_1(x) \cdot e^{-j\mu x} dx \qquad H_2(\mu) = \int_{-\infty}^{\infty} h_2(x) \cdot e^{-j\mu x} dx$$

For $\mu = 0$, $\Rightarrow H_1(0) = H_2(0) = 1$

$\Rightarrow H_1(0) \cdot H_2(0) = 1 \Rightarrow F_1(0)$ & $F_2(0)$ will be undefined

$\therefore$ We cannot recover the DC component

Q.02] $g = h * f$

gradient image — g; convolution kernel — h; original 1D image — f

Fundamental problem:
Presence of noise in image.
If we can neglect that, there is another problem →

h can simply be $[-1 \;\; 1]$ ⇒ $g(x) = f(x+1) - f(x)$
Here let $x \in [1, N]$
To obtain f from g and h, we'll make use of Discrete Fourier Transform,

$$G(\mu) = F(\mu) \cdot e^{j\frac{2\pi\mu}{N}} - F(\mu) \quad \text{[By Shift Thm]}$$

$$\Rightarrow F(\mu) = \frac{G(\mu)}{e^{j\frac{2\pi\mu}{N}} - 1}$$

When $\mu = 0$, denominator becomes 0

We cannot recover the DC component of original image by this ~~same~~ method.

In case of 2D image, the same problem as above is observed. If we consider $f(x,y) \rightarrow F(\mu, \nu)$

$$\Rightarrow F_x(\mu,\nu) = (e^{\frac{j2\pi\mu}{N}} - 1) \cdot F(\mu,\nu) \Rightarrow \text{Problem at } \mu = 0$$

$$F_y(\mu,\nu) = (e^{\frac{j2\pi\nu}{N}} - 1) \cdot F(\mu,\nu) \Rightarrow \text{Problem at } \nu = 0$$

Even if we ~~kn~~ knew both, the intersection $(\mu = 0, \nu = 0)$ still remains unknown, i.e., DC component information can't be obtained.

On the hand, using the integration method, integration across rows will be inconsistent with answer from integration across columns.