CS 663 : Digital Image Processing : Assignment 5

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Question 1

Question:
Suppose you are standing in a well-illuminated room with a large window, and you take a picture of the scene outside. The window undesirably acts as a semi-reflecting surface, and hence the picture will contain a reflection of the scene inside the room, besides the scene outside. While solutions exist for separating the two components from a single picture, here you will look at a simpler-to-solve version of this problem where you would take two pictures. The first picture g_1 is taken by adjusting your camera lens so that the scene outside (f_1) is in focus (we will assume that the scene outside has negligible depth variation when compared to the distance from the camera, and so it makes sense to say that the entire scene outside is in focus), and the reflection off the window surface (f_2) will now be defocussed or blurred. This can be written as $g_1 = f_1 + h_2 * f_2$ where h_2 stands for the blur kernel that acted on f_2. The second picture g_2 is taken by focusing the camera onto the surface of the window, with the scene outside being defocussed. This can be written as $g_2 = h_1 * f_1 + f_2$ where h_1 is the blur kernel acting on f_1. Given g_1 and g_2, and assuming h_1 and h_2 are known, your task is to derive a formula to determine f_1 and f_2. Note that we are making the simplifying assumption that there was no relative motion between the camera and the scene outside while the two pictures were being acquired, and that there were no changes whatsoever to the scene outside or inside. Even with all these assumptions, you will notice something inherently problematic about the formula you will derive. What is it?

Solution:
Given g_1 & g_2 such that $g_1 = f_1 + h_2 * f_2$ & $g_2 = h_1 * f_1 + f_2$ where h_1 & h_2 are the blur kernels acting respectively on f_1 & f_2.

In the Fourier Domain:

$$G_1 = F_1 + H_2 \cdot F_2$$

$$G_2 = F_2 + H_1 \cdot F_1$$

On solving the above two equations simultaneously, we obtain:

$$F_1 = \frac{G_1 - H_2 \cdot G_2}{1 - H_1 \cdot H_2}$$

$$F_2 = \frac{G_2 - H_1 \cdot G_1}{1 - H_1 \cdot H_2}$$

The above formulae help us determine f_1 & f_2 as ifft(F_1) & ifft(F_2) respectively.

From the above derived formulae, it is evident that a problem would arise whenever $H_1 \cdot H_2 = 1$ [The denominators of F_1 & F_2 will blow up].

Since h_1 & h_2 are blur kernels, we have $\int_{-\infty}^{\infty} h_1(x)dx = 1$ & $\int_{-\infty}^{\infty} h_2(x)dx = 1$.
Now we observe that:

$$H_1(\mu) = \int_{-\infty}^{\infty} h_1(x) \cdot e^{-2\pi j \mu x} dx$$

$$H_2(\mu) = \int_{-\infty}^{\infty} h_2(x) \cdot e^{-2\pi j \mu x} dx$$

Thus, for $\mu = 0$, we have:

$$H_1(0) = \int_{-\infty}^{\infty} h_1(x)dx = 1$$

$$H_2(0) = \int_{-\infty}^{\infty} h_2(x)dx = 1$$

$$\implies H_1(0) \cdot H_2(0) = 1$$

Thus, $F_1(0)$ & $F_2(0)$ will be undefined, ie, we would not be able to recover the DC component of images f_1 & f_2.