

## ## Basic Types of Algorithms

### ### \*\*Recursive Algorithms\*\*

#### #### Definition:

- An algorithm that calls itself in its definition.
  - **\*\*Recursive case\*\*** a conditional statement that is used to trigger the recursion.
  - **\*\*Base case\*\*** a conditional statement that is used to break the recursion.

#### #### What you need to know:

- **\*\*Stack level too deep\*\*** and **\*\*stack overflow\*\***.
  - If you've seen either of these from a recursive algorithm, you messed up.
  - It means that your base case was never triggered because it was faulty or the problem was so massive you ran out of allotted memory.
  - Knowing whether or not you will reach a base case is integral to correctly using recursion.
  - Often used in Depth First Search

### ### \*\*Iterative Algorithms\*\*

#### #### Definition:

- An algorithm that is called repeatedly but for a finite number of times, each time being a single iteration.
  - Often used to move incrementally through a data set.

#### #### What you need to know:

- Generally you will see iteration as loops, for, while, and until statements.
- Think of iteration as moving one at a time through a set.
- Often used to move through an array.

#### #### Recursion Vs. Iteration :

- The differences between recursion and iteration can be confusing to distinguish since both can be used to implement the other. But know that,
  - Recursion is, usually, more expressive and easier to implement.
  - Iteration uses less memory.
- **\*\*Functional languages\*\*** tend to use recursion. (i.e. Haskell)
- **\*\*Imperative languages\*\*** tend to use iteration. (i.e. Ruby)
- Check out this [Stack Overflow post] (<http://stackoverflow.com/questions/19794739/what-is-the-difference-between-iteration-and-recursion>) for more info.

#### #### Pseudo Code of Moving Through an Array (this is why iteration is used for this)

Recursion	Iteration
-----	-----
recursive method (array, n)	iterative method (array)
if array[n] is not nil	for n from 0 to size of array
print array[n]	print(array[n])
recursive method(array, n+1)	
else	
exit loop	
...	

### **### \*\*Greedy Algorithm\*\***

#### **#### Definition:**

- An algorithm that, while executing, selects only the information that meets a certain criteria.
- The general five components, taken from [Wikipedia] ([http://en.wikipedia.org/wiki/Greedy\\_algorithm#Specifics](http://en.wikipedia.org/wiki/Greedy_algorithm#Specifics)):
  - A candidate set, from which a solution is created.
  - A selection function, which chooses the best candidate to be added to the solution.
  - A feasibility function, that is used to determine if a candidate can be used to contribute to a solution.
  - An objective function, which assigns a value to a solution, or a partial solution.
  - A solution function, which will indicate when we have discovered a complete solution.

#### **#### What you need to know:**

- Used to find the expedient, though non-optimal, solution for a given problem.
- Generally used on sets of data where only a small proportion of the information evaluated meets the desired result.
- Often a greedy algorithm can help reduce the Big O of an algorithm.

#### **#### Pseudo Code of a Greedy Algorithm to Find Largest Difference of any Two Numbers in an Array.**

```
greedy algorithm (array)
  var largest difference = 0
  var new difference = find next difference (array[n], array[n+1])
  largest difference = new difference if new difference is > largest
  difference
  repeat above two steps until all differences have been found
  return largest difference
...
```

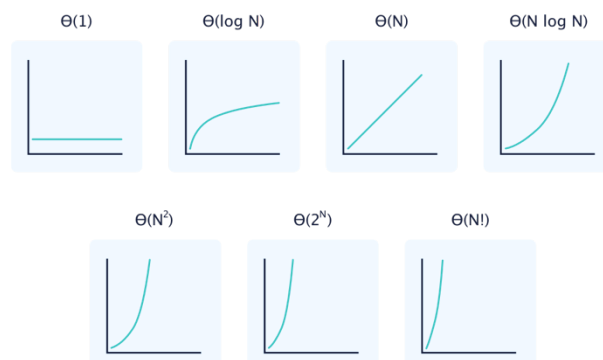
This algorithm never needed to compare all the differences to one another, saving it an entire iteration.

### **### Algorithmic Common Runtimes :**

The common algorithmic runtimes from fastest to slowest are:

- constant:  $\Theta(1)$
- logarithmic:  $\Theta(\log N)$
- linear:  $\Theta(N)$
- polynomial:  $\Theta(N^2)$
- exponential:  $\Theta(2^N)$
- factorial:  $\Theta(N!)$

Common Runtimes



## ## \*\*Hash Table or Hash Map\*\*

### #### Definition:

- Stores data with key value pairs.
- **Hash functions** accept a key and return an output unique only to that specific key.
  - This is known as **hashing**, which is the concept that an input and an output have a one-to-one correspondence to map information.
  - Hash functions return a unique address in memory for that data.
  - Our hash map implementation then takes that hash value mod the size of the array.

### #### What you need to know:

- Designed to optimize searching, insertion, and deletion.
- **Hash collisions** are when a hash function returns the same output for two distinct inputs.
  - All hash functions have this problem.
  - This is often accommodated for by having the hash tables be very large.
- Hashes are important for associative arrays and database indexing.

### #### Collision strategy:

- The first strategy we're going to learn about is called *separate chaining*. The separate chaining strategy avoids collisions by updating the underlying data structure. Instead of an array of values that are mapped to by hashes, it could be an array of linked lists! If a linked list already exists at the address, append the value to the linked list given. This is effective for hash functions that are particularly good at giving unique indices, so the linked lists never get very long. But in the worst-case scenario, where the hash function gives all keys the same index, lookup performance is only as good as it would be on a linked list. If we save both the key and the value, then we will be able to check against the saved key when we're accessing data in a hash map.

- Another popular hash collision strategy is called *open addressing*. In open addressing we stick to the array as our underlying data structure, but we continue looking for a new index to save our data if the first result of our hash function has a different key's data.

- A common open method of open addressing is called *probing*. Probing means continuing to find new array indices in a fixed sequence until an empty index is found.

- In a quadratic probing open addressing system, we add increasingly large numbers to the hash code. At the first collision we just add 1, but if the hash collides there too we add 4, and the third time we add 9. Having a probe sequence change over time like this avoids clustering.

- *Clustering* is what happens when a single hash collision causes additional hash collisions. Imagine a hash collision triggers a linear probing sequence to assign a value to the next hash bucket over. Any key that would hash to this "next bucket" will now collide with a key that, in a sense, doesn't belong to that bucket anyway.

### #### Time Complexity:

- Indexing: Hash Tables:  $O(1)$
- Search: Hash Tables:  $O(1)$
- Insertion: Hash Tables:  $O(1)$

#### Code sample:

```
class HashMap:
    def __init__(self, array_size):
        self.array_size = array_size
        self.array = [None for item in range(array_size)]

    def hash(self, key, count_collisions=0):
        key_bytes = key.encode()
        hash_code = sum(key_bytes)
        return hash_code + count_collisions

    def compressor(self, hash_code):
        return hash_code % self.array_size

    def assign(self, key, value):
        array_index = self.compressor(self.hash(key))
        current_array_value = self.array[array_index]

        if current_array_value is None:
            self.array[array_index] = [key, value]
            return

        if current_array_value[0] == key:
            self.array[array_index] = [key, value]
            return

        # Collision!

        number_collisions = 1

        while(current_array_value[0] != key):
            new_hash_code = self.hash(key, number_collisions)
            new_array_index = self.compressor(new_hash_code)
            current_array_value = self.array[new_array_index]

            if current_array_value is None:
                self.array[new_array_index] = [key, value]
                return

            if current_array_value[0] == key:
                self.array[new_array_index] = [key, value]
                return

            number_collisions += 1

        return

    def retrieve(self, key):
        array_index = self.compressor(self.hash(key))
        possible_return_value = self.array[array_index]

        if possible_return_value is None:
            return None

        if possible_return_value[0] == key:
            return possible_return_value[1]

        retrieval_collisions = 1

        while (possible_return_value != key):
            new_hash_code = self.hash(key, retrieval_collisions)
```

```
retrieving_array_index = self.compressor(new_hash_code)
possible_return_value = self.array[retrieving_array_index]

if possible_return_value is None:
    return None

if possible_return_value[0] == key:
    return possible_return_value[1]

number_collisions += 1

return
```

## ## Linear Data Structure Basics

### ### \*\*Array\*\*

#### #### Definition:

- Stores data elements based on an sequential, most commonly 0 based, index.
- Based on [tuples](<http://en.wikipedia.org/wiki/Tuple>) from set theory.
- They are one of the oldest, most commonly used data structures.

#### #### What you need to know:

- Optimal for indexing; bad at searching, inserting, and deleting (except at the end).
- **Linear arrays**, or one dimensional arrays, are the most basic.
  - Are static in size, meaning that they are declared with a fixed size.
- **Dynamic arrays** are like one dimensional arrays, but have reserved space for additional elements.
  - If a dynamic array is full, it copies its contents to a larger array.
- **Multi dimensional arrays** nested arrays that allow for multiple dimensions such as an array of arrays providing a 2 dimensional spacial representation via x, y coordinates.

#### #### Time Complexity:

- Indexing:               Linear array:  $O(1)$ ,               Dynamic array:  $O(1)$
- Search:                Linear array:  $O(n)$ ,               Dynamic array:  $O(n)$
- Optimized Search:   Linear array:  $O(\log n)$ ,   Dynamic array:  $O(\log n)$
- Insertion:            Linear array:  $n/a$                Dynamic array:  $O(n)$

### ### \*\*Iterative Binary Search\*\*

```
def binary_search(sorted_list, target):
    left_pointer = 0
    right_pointer = len(sorted_list)

    # fill in the condition for the while loop
    while left_pointer < right_pointer:
        # calculate the middle index using the two pointers
        mid_idx = (left_pointer + right_pointer) // 2
        mid_val = sorted_list[mid_idx]
        if mid_val == target:
            return mid_idx
        if target < mid_val:
            # set the right_pointer to the appropriate value
            right_pointer = mid_idx
        if target > mid_val:
            # set the left_pointer to the appropriate value
            left_pointer = mid_idx + 1

    return "Value not in list"
}
```

### ### \*\*Linked List\*\*

#### #### Definition:

- Stores data with **nodes** that point to other nodes.
  - Nodes, at its most basic it has one datum and one reference (another node).
  - A linked list `_chains_` nodes together by pointing one node's reference towards another node.

#### #### What you need to know:

- Designed to optimize insertion and deletion, slow at indexing and searching.
- **Doubly linked list** has nodes that also reference the previous node.
- **Circularly linked list** is simple linked list whose **tail**, the last node, references the **head**, the first node.
- **Stack**, commonly implemented with linked lists but can be made from arrays too.
  - Stacks are **last in, first out** (LIFO) data structures.
  - Made with a linked list by having the head be the only place for insertion and removal.
- **Queues**, too can be implemented with a linked list or an array.
  - Queues are a **first in, first out** (FIFO) data structure.
  - Made with a doubly linked list that only removes from head and adds to tail.

#### #### Time Complexity:

- Indexing:               Linked Lists:  $O(n)$
- Search:                 Linked Lists:  $O(n)$
- Optimized Search: Linked Lists:  $O(n)$
- Insertion:             Linked Lists:  $O(1)$

#### #### Code Sample:

```
class Node:
    def __init__(self, value, next_node=None, prev_node=None):
        self.value = value
        self.next_node = next_node
        self.prev_node = prev_node

    def set_next_node(self, next_node):
        self.next_node = next_node

    def get_next_node(self):
        return self.next_node

    def set_prev_node(self, prev_node):
        self.prev_node = prev_node

    def get_prev_node(self):
        return self.prev_node

    def get_value(self):
        return self.value

class DoublyLinkedList:
    def __init__(self):
        self.head_node = None
        self.tail_node = None

    def add_to_head(self, new_value):
        new_head = Node(new_value)
        current_head = self.head_node

        if current_head != None:
            current_head.set_prev_node(new_head)
            new_head.set_next_node(current_head)

        self.head_node = new_head

        if self.tail_node == None:
            self.tail_node = new_head
```

```

def add_to_tail(self, new_value):
    new_tail = Node(new_value)
    current_tail = self.tail_node

    if current_tail != None:
        current_tail.set_next_node(new_tail)
        new_tail.set_prev_node(current_tail)

    self.tail_node = new_tail

    if self.head_node == None:
        self.head_node = new_tail

def remove_head(self):
    removed_head = self.head_node

    if removed_head == None:
        return None

    self.head_node = removed_head.get_next_node()

    if self.head_node != None:
        self.head_node.set_prev_node(None)

    if removed_head == self.tail_node:
        self.remove_tail()

    return removed_head.get_value()

def remove_tail(self):
    removed_tail = self.tail_node

    if removed_tail == None:
        return None

    self.tail_node = removed_tail.get_prev_node()

    if self.tail_node != None:
        self.tail_node.set_next_node(None)

    if removed_tail == self.head_node:
        self.remove_head()

    return removed_tail.get_value()

def remove_by_value(self, value_to_remove):
    node_to_remove = None
    current_node = self.head_node

    while current_node != None:
        if current_node.get_value() == value_to_remove:
            node_to_remove = current_node
            break

        current_node = current_node.get_next_node()

    if node_to_remove == None:
        return None

    if node_to_remove == self.head_node:

```



```

        self.remove_head()
    elif node_to_remove == self.tail_node:
        self.remove_tail()
    else:
        next_node = node_to_remove.get_next_node()
        prev_node = node_to_remove.get_prev_node()
        next_node.set_prev_node(prev_node)
        prev_node.set_next_node(next_node)

    return node_to_remove

def stringify_list(self):
    string_list = ""
    current_node = self.head_node
    while current_node:
        if current_node.get_value() != None:
            string_list += str(current_node.get_value()) + "\n"
        current_node = current_node.get_next_node()
    return string_list

```

### **### \*\*Queue\*\***

#### **#### Sample code:**

```

class Node:
    def __init__(self, value, next_node=None):
        self.value = value
        self.next_node = next_node

    def set_next_node(self, next_node):
        self.next_node = next_node

    def get_next_node(self):
        return self.next_node

    def get_value(self):
        return self.value

class Queue:
    def __init__(self, max_size=None):
        self.head = None
        self.tail = None
        self.max_size = max_size
        self.size = 0

    def enqueue(self, value):
        if self.has_space():
            item_to_add = Node(value)
            print("Adding " + str(item_to_add.get_value()) + " to the queue!")
            if self.is_empty():
                self.head = item_to_add
                self.tail = item_to_add
            else:
                self.tail.set_next_node(item_to_add)
                self.tail = item_to_add
            self.size += 1
        else:
            print("Sorry, no more room!")

    def dequeue(self):
        if self.get_size() > 0:
            item_to_remove = self.head

```

```

        print(str(item_to_remove.get_value()) + " is served!")
        if self.get_size() == 1:
            self.head = None
            self.tail = None
        else:
            self.head = self.head.get_next_node()
            self.size -= 1
        return item_to_remove.get_value()
    else:
        print("The queue is totally empty!")

def peek(self):
    if self.is_empty():
        print("Nothing to see here!")
    else:
        return self.head.get_value()

def get_size(self):
    return self.size

def has_space(self):
    if self.max_size == None:
        return True
    else:
        return self.max_size > self.get_size()

def is_empty(self):
    return self.size == 0

```

#### **#### \*\*Stack\*\***

##### #### Sample code:

```

class Node:
    def __init__(self, value, next_node=None):
        self.value = value
        self.next_node = next_node

    def set_next_node(self, next_node):
        self.next_node = next_node

    def get_next_node(self):
        return self.next_node

    def get_value(self):
        return self.value

class Stack:
    def __init__(self, limit=1000):
        self.top_item = None
        self.size = 0
        self.limit = limit

    def push(self, value):
        if self.has_space():
            item = Node(value)
            item.set_next_node(self.top_item)
            self.top_item = item
            self.size += 1
            print("Adding {} to the pizza stack!".format(value))
        else:
            print("No room for {}".format(value))

```

```
def pop(self):
    if not self.is_empty():
        item_to_remove = self.top_item
        self.top_item = item_to_remove.get_next_node()
        self.size -= 1
        print("Delivering " + item_to_remove.get_value())
        return item_to_remove.get_value()
    print("All out of pizza.")

def peek(self):
    if not self.is_empty():
        return self.top_item.get_value()
    print("Nothing to see here!")

def has_space(self):
    return self.limit > self.size

def is_empty(self):
    return self.size == 0
```

## ## Search Basics :

### ### \*\*Merge Sort\*\*

#### #### Definition:

- A comparison based sorting algorithm
  - Divides entire dataset into groups of at most two.
  - Compares each number one at a time, moving the smallest number to left of the pair.
  - Once all pairs sorted it then compares left most elements of the two leftmost pairs creating a sorted group of four with the smallest numbers on the left and the largest ones on the right.
  - This process is repeated until there is only one set.

#### #### What you need to know:

- This is one of the most basic sorting algorithms.
- Know that it divides all the data into as small possible sets then compares them.

#### #### Time Complexity:

- Best Case Sort: Merge Sort:  $O(n)$
- Average Case Sort: Merge Sort:  $O(n \log n)$
- Worst Case Sort: Merge Sort:  $O(n \log n)$

#### #### Code sample:

```
def merge_sort(items):
    if len(items) <= 1:
        return items

    middle_index = len(items) // 2
    left_split = items[:middle_index]
    right_split = items[middle_index:]

    left_sorted = merge_sort(left_split)
    right_sorted = merge_sort(right_split)

    return merge(left_sorted, right_sorted)

def merge(left, right):
    result = []

    while (left and right):
        if left[0] < right[0]:
            result.append(left[0])
            left.pop(0)
        else:
            result.append(right[0])
            right.pop(0)

    if left:
        result += left
    if right:
        result += right

    return result
```

### ### \*\*Quicksort\*\*

#### #### Definition:

- A comparison based sorting algorithm
  - Divides entire dataset in half by selecting the middle element and putting all smaller elements to the left of the element and larger ones to the right.
  - It repeats this process on the left side until it is comparing only two elements at which point the left side is sorted.
  - When the left side is finished sorting it performs the same operation on the right side.
- Computer architecture favors the quicksort process.

#### #### What you need to know:

- While it has the same Big O as (or worse in some cases) many other sorting algorithms it is often faster in practice than many other sorting algorithms, such as merge sort.
- Know that it halves the data set by the average continuously until all the information is sorted.

#### #### Time Complexity:

- Best Case Sort: Merge Sort:  $O(n)$
- Average Case Sort: Merge Sort:  $O(n \log n)$
- Worst Case Sort: Merge Sort:  $O(n^2)$

#### #### Merge Sort Vs. Quicksort

- Quicksort is likely faster in practice.
- Merge Sort divides the set into the smallest possible groups immediately then reconstructs the incrementally as it sorts the groupings.
- Quicksort continually divides the set by the average, until the set is recursively sorted.

#### #### Code sample:

```
def quicksort(list, start, end):
    # this portion of list has been sorted
    if start >= end:
        return
    print("Running quicksort on {}".format(list[start: end + 1]))
    # select random element to be pivot
    pivot_idx = randrange(start, end + 1)
    pivot_element = list[pivot_idx]
    print("Selected pivot {}".format(pivot_element))
    # swap random element with last element in sub-lists
    list[end], list[pivot_idx] = list[pivot_idx], list[end]

    # tracks all elements which should be to left (lesser than) pivot
    less_than_pointer = start

    for i in range(start, end):
        # we found an element out of place
        if list[i] < pivot_element:
            # swap element to the right-most portion of lesser elements
            print("Swapping {} with {}".format(list[i],
list[less_than_pointer]))
            list[i], list[less_than_pointer] = list[less_than_pointer], list[i]
            # tally that we have one more lesser element
            less_than_pointer += 1
    # move pivot element to the right-most portion of lesser elements
    list[end], list[less_than_pointer] = list[less_than_pointer], list[end]
    print("{} successfully partitioned".format(list[start: end + 1]))
```

```
# recursively sort left and right sub-lists
quicksort(list, start, less_than_pointer - 1)
quicksort(list, less_than_pointer + 1, end)
```

### **\*\*\* \*\*Bubble Sort\*\***

#### #### Definition:

- A comparison based sorting algorithm
  - It iterates left to right comparing every couplet, moving the smaller element to the left.
  - It repeats this process until it no longer moves an element to the left.

#### #### What you need to know:

- While it is very simple to implement, it is the least efficient of these three sorting methods.
- Know that it moves one space to the right comparing two elements at a time and moving the smaller one to left.

#### #### Time Complexity:

- Best Case Sort: Merge Sort:  $O(n)$
- Average Case Sort: Merge Sort:  $O(n^2)$
- Worst Case Sort: Merge Sort:  $O(n^2)$

#### #### Code sample:

```
def swap(arr, index_1, index_2):
    temp = arr[index_1]
    arr[index_1] = arr[index_2]
    arr[index_2] = temp

def bubble_sort_unoptimized(arr):
    iteration_count = 0
    for el in arr:
        for index in range(len(arr) - 1):
            iteration_count += 1
            if arr[index] > arr[index + 1]:
                swap(arr, index, index + 1)

    print("PRE-OPTIMIZED ITERATION COUNT: {}".format(iteration_count))

def bubble_sort(arr):
    iteration_count = 0
    for i in range(len(arr)):
        # iterate through unplaced elements
        for idx in range(len(arr) - i - 1):
            iteration_count += 1
            if arr[idx] > arr[idx + 1]:
                # replacement for swap function
                arr[idx], arr[idx + 1] = arr[idx + 1], arr[idx]

    print("POST-OPTIMIZED ITERATION COUNT: {}".format(iteration_count))
```

## ## Non-Linear Data Structure Basics

### ### \*\* Tree\*\*

#### #### Definition:

Tree data structures are built using tree nodes (a variation on the nodes you created earlier) and are another way of storing information. Specifically, trees are used for data that has a hierarchical structure, such as a family tree or a computer's file system. The tree data structure you are going to create is an excellent foundation for further variations on trees, including AVL trees, red-black trees, and binary search trees!

#### #### Code sample:

```
class TreeNode:
    def __init__(self, value):
        self.value = value # data
        self.children = [] # references to other nodes

    def add_child(self, child_node):
        # creates parent-child relationship
        print("Adding " + child_node.value)
        self.children.append(child_node)

    def remove_child(self, child_node):
        # removes parent-child relationship
        print("Removing " + child_node.value + " from " + self.value)
        self.children = [child for child in self.children
                          if child is not child_node]

    def traverse(self):
        # moves through each node referenced from self downwards
        nodes_to_visit = [self]
        while len(nodes_to_visit) > 0:
            current_node = nodes_to_visit.pop()
            print(current_node.value)
            nodes_to_visit += current_node.children
```

### ### \*\*Binary Tree\*\*

#### #### Definition:

- Is a tree like data structure where every node has at most two children.
- There is one left and right child node.

#### #### What you need to know:

- Designed to optimize searching and sorting.
- A **degenerate tree** is an unbalanced tree, which if entirely one-sided is essentially a linked list.
- They are comparably simple to implement than other data structures.
- Used to make **binary search trees**.
  - A binary tree that uses comparable keys to assign which direction a child is.
  - Left child has a key smaller than it's parent node.
  - Right child has a key greater than it's parent node.
  - There can be no duplicate node.
- Because of the above it is more likely to be used as a data structure than a binary tree.

#### #### Time Complexity:

- Indexing: Binary Search Tree:  $O(\log n)$
- Search: Binary Search Tree:  $O(\log n)$

- Insertion: Binary Search Tree:  $O(\log n)$

#### Code sample:

```
class BinarySearchTree:
    def __init__(self, value, depth=1):
        self.value = value
        self.depth = depth
        self.left = None
        self.right = None

    def insert(self, value):
        if (value < self.value):
            if (self.left is None):
                self.left = BinarySearchTree(value, self.depth + 1)
                print(f'Tree node {value} added to the left of {self.value} at
depth {self.depth + 1}')
            else:
                self.left.insert(value)
        else:
            if (self.right is None):
                self.right = BinarySearchTree(value, self.depth + 1)
                print(f'Tree node {value} added to the right of {self.value} at
depth {self.depth + 1}')
            else:
                self.right.insert(value)

    def get_node_by_value(self, value):
        if (self.value == value):
            return self
        elif ((self.left is not None) and (value < self.value)):
            return self.left.get_node_by_value(value)
        elif ((self.right is not None) and (value >= self.value)):
            return self.right.get_node_by_value(value)
        else:
            return None

    def depth_first_traversal(self):
        if (self.left is not None):
            self.left.depth_first_traversal()
        print(f'Depth={self.depth}, Value={self.value}')
        if (self.right is not None):
            self.right.depth_first_traversal()
```

### \*\*Heap\*\*

#### Introduction to Heaps :

Heaps are used to maintain a maximum or minimum value in a dataset. Heaps tracking the maximum or minimum value are *max-heaps* or *min-heaps*. We will focus on min-heaps, but the concepts for a max-heap are nearly identical.

Think of the min-heap as a binary tree with two qualities:

- The root is the minimum value of the dataset.
- Every child's value is greater than or equal to its parent.

These two properties are the defining characteristics of the min-heap. By maintaining these two properties, we can efficiently retrieve and update the minimum value.

Notice how by filling the tree from left to right; we're leaving no gaps in the array. The location of each child or parent derives from a formula using the index.

- left child:  $(\text{index} * 2) + 1$



- right child:  $(\text{index} * 2) + 2$
- parent:  $(\text{index} - 1) / 2$  – not used on the root!

#### #### Adding an Element: Heapify Up :

Sometimes you will add an element to the heap that violates the heap's essential properties.

We need to restore the fundamental heap properties. This restoration is known as *heapify* or *heapifying*. We're adding an element to the bottom of the tree and moving upwards, so we're *heapifying up*.

As long as we've violated the heap properties, we'll swap the offending child with its parent until we restore the properties, or until there's no parent left. If there is no parent left, that element becomes the new root of the tree.

#### #### Removing an Element: Heapify Down :

Maintaining a minimum value is no good if we can never retrieve it, so let's explore how to remove the root node.

In the diagram, you can see removing the top node itself would be messy: there would be two children orphaned! Instead, we'll swap the root node, with the bottom rightmost child: The bottom rightmost child is simple to remove because it has no children. Unfortunately, we've violated the heap property. We'll *heapify down* to restore the heap property.

This process is similar to heapifying up, except we have two options where we can make a swap. We'll choose the **lesser of the two values** and swap. This is necessary for the heap property. Just like that, we've retrieved the minimum value, allocated a new minimum, and maintained the heap property!

#### #### Code sample:

```
class MinHeap:
    def __init__(self):
        self.heap_list = [None]
        self.count = 0

    # HEAP HELPER METHODS
    # DO NOT CHANGE!
    def parent_idx(self, idx):
        return idx // 2

    def left_child_idx(self, idx):
        return idx * 2

    def right_child_idx(self, idx):
        return idx * 2 + 1

    # NEW HELPER!
    def child_present(self, idx):
        return self.left_child_idx(idx) <= self.count

    # END OF HEAP HELPER METHODS

    def retrieve_min(self):
        if self.count == 0:
            print("No items in heap")
            return None

        min = self.heap_list[1]
```

```

    print("Removing: {0} from {1}".format(min, self.heap_list))
    self.heap_list[1] = self.heap_list[self.count]
    self.count -= 1
    self.heap_list.pop()
    print("Last element moved to first: {0}".format(self.heap_list))
    self.heapify_down()
    return min

def add(self, element):
    self.count += 1
    print("Adding: {0} to {1}".format(element, self.heap_list))
    self.heap_list.append(element)
    self.heapify_up()

def heapify_down(self):
    idx = 1
    while self.child_present(idx):
        print("Heapifying down!")
        smaller_child_idx = self.get_smaller_child_idx(idx)
        child = self.heap_list[smaller_child_idx]
        parent = self.heap_list[idx]
        if parent > child:
            self.heap_list[idx] = child
            self.heap_list[smaller_child_idx] = parent
            idx = smaller_child_idx
        print("HEAP RESTORED! {0}".format(self.heap_list))

def get_smaller_child_idx(self, idx):
    if self.right_child_idx(idx) > self.count:
        print("There is only a left child")
        return self.left_child_idx(idx)
    else:
        left_child = self.heap_list[self.left_child_idx(idx)]
        right_child = self.heap_list[self.right_child_idx(idx)]
        if left_child < right_child:
            print("Left child is smaller")
            return self.left_child_idx(idx)
        else:
            print("Right child is smaller")
            return self.right_child_idx(idx)

def heapify_up(self):
    idx = self.count
    while self.parent_idx(idx) > 0:
        if self.heap_list[self.parent_idx(idx)] > self.heap_list[idx]:
            tmp = self.heap_list[self.parent_idx(idx)]
            print("swapping {0} with {1}".format(tmp, self.heap_list[idx]))
            self.heap_list[self.parent_idx(idx)] = self.heap_list[idx]
            self.heap_list[idx] = tmp
        idx = self.parent_idx(idx)
    print("HEAP RESTORED! {0}".format(self.heap_list))
    print("")

```

## ## Graphs and Graphs traversal :

### ### Graph :

#### #### Code sample:

```
class Vertex:
    def __init__(self, value):
        self.value = value
        self.edges = {}

    def add_edge(self, vertex, weight = 0):
        self.edges[vertex] = weight

    def get_edges(self):
        return list(self.edges.keys())

class Graph:
    def __init__(self, directed = False):
        self.graph_dict = {}
        self.directed = directed

    def add_vertex(self, vertex):
        self.graph_dict[vertex.value] = vertex

    def add_edge(self, from_vertex, to_vertex, weight = 0):
        self.graph_dict[from_vertex.value].add_edge(to_vertex.value, weight)
        if not self.directed:
            self.graph_dict[to_vertex.value].add_edge(from_vertex.value, weight)

    def find_path(self, start_vertex, end_vertex):
        start = [start_vertex]
        seen = {}
        while len(start) > 0:
            current_vertex = start.pop(0)
            seen[current_vertex] = True
            print("Visiting " + current_vertex)
            if current_vertex == end_vertex:
                return True
            else:
                vertices_to_visit =
set(self.graph_dict[current_vertex].edges.keys())
                start += [vertex for vertex in vertices_to_visit if vertex not in
seen]
        return False
```

### ### \*\*Breadth First Search\*\*

#### #### Definition:

- An algorithm that searches a tree (or graph) by searching levels of the tree first, starting at the root.
- It finds every node on the same level, most often moving left to right.
- While doing this it tracks the children nodes of the nodes on the current level.
- When finished examining a level it moves to the left most node on the next level.
- The bottom-right most node is evaluated last (the node that is deepest and is farthest right of it's level).

#### #### What you need to know:

- Optimal for searching a tree that is wider than it is deep.
- Uses a queue to store information about the tree while it traverses a tree.
  - Because it uses a queue it is more memory intensive than **depth first search**.
  - The queue uses more memory because it needs to store pointers

#### #### Time Complexity:

- Search: Breadth First Search:  $O(V + E)$
- E is number of edges
- V is number of vertices

#### #### Code sample:

```
def bfs(graph, start_vertex, target_value):
    path = [start_vertex]
    vertex_and_path = [start_vertex, path]
    bfs_queue = [vertex_and_path]
    visited = set()
    while bfs_queue:
        current_vertex, path = bfs_queue.pop(0)
        visited.add(current_vertex)
        for neighbor in graph[current_vertex]:
            if neighbor not in visited:
                if neighbor is target_value:
                    return path + [neighbor]
                else:
                    bfs_queue.append([neighbor, path + [neighbor]])
```

### **### \*\*Depth First Search\*\* :**

#### #### Definition:

- An algorithm that searches a tree (or graph) by searching depth of the tree first, starting at the root.
  - It traverses left down a tree until it cannot go further.
  - Once it reaches the end of a branch it traverses back up trying the right child of nodes on that branch, and if possible left from the right children.
  - When finished examining a branch it moves to the node right of the root then tries to go left on all its children until it reaches the bottom.
  - The right most node is evaluated last (the node that is right of all its ancestors).

#### #### What you need to know:

- Optimal for searching a tree that is deeper than it is wide.
- Uses a stack to push nodes onto.
  - Because a stack is LIFO it does not need to keep track of the nodes pointers and is therefore less memory intensive than breadth first search.
  - Once it cannot go further left it begins evaluating the stack.

#### #### Time Complexity:

- Search: Depth First Search:  $O(|E| + |V|)$
- E is number of edges
- V is number of vertices

#### #### Breadth First Search Vs. Depth First Search

- The simple answer to this question is that it depends on the size and shape of the tree.
  - For wide, shallow trees use Breadth First Search
  - For deep, narrow trees use Depth First Search

#### #### Nuances:

- Because BFS uses queues to store information about the nodes and its children, it could use more memory than is available on your computer. (But you probably won't have to worry about this.)
- If using a DFS on a tree that is very deep you might go unnecessarily deep in the search. See [xkcd](http://xkcd.com/761/) for more information.
- Breadth First Search tends to be a looping algorithm.
- Depth First Search tends to be a recursive algorithm.

#### #### Code sample:

```
def dfs(graph, current_vertex, target_value, visited = None):
    if visited is None:
        visited = []
    visited.append(current_vertex)
    if current_vertex is target_value:
        return visited

    for neighbor in graph[current_vertex]:
        if neighbor not in visited:
            path = dfs(graph, neighbor, target_value, visited)
            if path:
                return path
```

#### **### \*\*Dijkstras\*\* :**

#### #### Definition:

- Dijkstra's algorithm is an algorithm to find all of the shortest distances between a start vertex and the rest of the vertices in a graph.
- The algorithm works by keeping track of all the distances and updating the distances as it conducts a breadth-first search.
- Dijkstra's algorithm runs in  $O((E+V)\log V)$ .

#### #### What you need to know:

Dijkstra's Algorithm works as following:

1. Instantiate a dictionary that will eventually map vertices to their distance from the start vertex
2. Assign the start vertex a distance of 0 in a min heap
3. Assign every other vertex a distance of infinity in a min heap
4. Remove the vertex with the smallest distance from the min heap and set that to the current vertex
5. For the current vertex, consider all of its adjacent vertices and calculate the distance to them as (distance to the current vertex) + (edge weight of current vertex to adjacent vertex).
6. If this new distance is less than the current distance, replace the current distance.
7. Repeat 4 and 5 until the heap is empty
8. After the heap is empty, return the distances

Within the while loop, create a variable called mid and set it to the average of first and last.

#### #### Code sample:

```
def dijkstras(graph, start):
```

```

distances = {}

for vertex in graph:
    distances[vertex] = inf

distances[start] = 0
vertices_to_explore = [(0, start)]

while vertices_to_explore:
    current_distance, current_vertex = heappop(vertices_to_explore)

    for neighbor, edge_weight in graph[current_vertex]:
        new_distance = current_distance + edge_weight

        if new_distance < distances[neighbor]:
            distances[neighbor] = new_distance
            heappush(vertices_to_explore, (new_distance, neighbor))

return distances

```

### **### \*\* A\* algorithm \*\* :**

#### **#### Definition:**

- The A\* algorithm is a greedy graph search algorithm that optimizes looking for a target vertex.
- A\* is a modification of Dijkstra's done by adding the estimated distance of each vertex to the goal vertex when searching.
- We can modify Dijkstra's and turn it into A\* by changing the following:
  - Adding a target for the search.
  - Gathering possible optimal paths and identify a single shortest path.
  - Implementing a heuristic that determines the likely distance remaining.
- The runtime of A\* is  $O(b^d)$  where  $b$  is the branching factor of the graph and  $d$  is the depth of the goal vertex from the start vertex.

#### **#### Code sample:**

```

# Manhattan Heuristic:
def heuristic(start, target):
    x_distance = abs(start.position[0] - target.position[0])
    y_distance = abs(start.position[1] - target.position[1])
    return x_distance + y_distance

# Euclidean Heuristic:
#def heuristic(start, target):
#    x_distance = abs(start.position[0] - target.position[0])
#    y_distance = abs(start.position[1] - target.position[1])
#    return sqrt(x_distance * x_distance + y_distance * y_distance)

def a_star(graph, start, target):
    print("Starting A* algorithm!")
    count = 0
    paths_and_distances = {}
    for vertex in graph:
        paths_and_distances[vertex] = [inf, [start.name]]

    paths_and_distances[start][0] = 0

```

```

vertices_to_explore = [(0, start)]
while vertices_to_explore and paths_and_distances[target][0] == inf:
    current_distance, current_vertex = heappop(vertices_to_explore)
    for neighbor, edge_weight in graph[current_vertex]:
        new_distance = current_distance + edge_weight + heuristic(neighbor,
target)
        new_path = paths_and_distances[current_vertex][1] + [neighbor.name]

        if new_distance < paths_and_distances[neighbor][0]:
            paths_and_distances[neighbor][0] = new_distance
            paths_and_distances[neighbor][1] = new_path
            heappush(vertices_to_explore, (new_distance, neighbor))
            count += 1
            print("\nAt " + vertices_to_explore[0][1].name)

    print("Found a path from {0} to {1} in {2} steps: ".format(start.name,
target.name, count), paths_and_distances[target][1])

return paths_and_distances[target][1]

```

## ## Practice:

### ### Memoization - Fibonacci :

```
memo = {}

def fibonacci(num):
    answer = None
    # Write your code here
    if num in memo:
        answer = memo[num]
    elif num == 0 or num == 1:
        answer = num
    else:
        answer = fibonacci(num - 1) + fibonacci(num - 2)
        memo[num] = answer
    return answer
```

### ### The Two Pointer Approach - Capturing Rainwater :

The previous solution had a quadratic runtime, but it's possible to solve this problem in  $O(n)$  time by using two pointers. The pointers will start at each end of the array and move towards each other. The two-pointer approach is a common approach for problems that require working with arrays, as it allows you to go through the array in a single loop and without needing to create copy arrays.

The two-pointer approach is one that you can, and should, use in many interview questions. When you see a problem that requires you to iterate through an array (or string), take a moment and think if it would be possible to iterate through the array in sections at the same time instead of in separate loops. Common problems that can be solved using the two-pointer technique are the two sum problem (finding two numbers in an array that sum to a specified number) and reversing the characters in a string.

```
def efficient_solution(heights):
    total_water = 0
    left_pointer = 0
    right_pointer = len(heights) - 1
    left_bound = 0
    right_bound = 0

    # Write your code here
    while left_pointer < right_pointer:
        if heights[left_pointer] <= heights[right_pointer]:
            left_bound = max(heights[left_pointer], left_bound)
            total_water += left_bound - heights[left_pointer]
            left_pointer += 1
        else:
            right_bound = max(heights[right_pointer], right_bound)
            total_water += right_bound - heights[right_pointer]
            right_pointer -= 1

    return total_water
```



### ### The Knapsack Problem:

While this recursive solution works, it has a big O runtime of  $O(2^n)$ . In the worst case, each step would require us to evaluate two subproblems, sometimes repeatedly, as there's overlap between subproblems. We can drastically improve on this runtime by using dynamic programming.

```
def recursive_knapsack(weight_cap, weights, values, i):
    if weight_cap == 0 or i == 0:
        return 0
    elif weights[i - 1] > weight_cap:
        return recursive_knapsack(weight_cap, weights, values, i - 1)
    else:
        include_item = values[i - 1] + recursive_knapsack(weight_cap -
weights[i - 1], weights, values, i - 1);

        exclude_item = recursive_knapsack(weight_cap, weights, values, i - 1);

        return max(include_item, exclude_item)
```

This version has a big O runtime of  $O(n * \text{weight\_cap})$  compared to the recursive implementation's runtime of  $O(2^n)$ . While this optimized runtime might seem worse using small cases, it is much more efficient as the parameters grow.

```
def dynamic_knapsack(weight_cap, weights, values):
    rows = len(weights) + 1
    cols = weight_cap + 1
    # Set up 2D array
    matrix = [ [] for x in range(rows) ]

    # Iterate through every row
    for index in range(rows):
        # Initialize columns for this row
        matrix[index] = [ -1 for y in range(cols) ]

        # Iterate through every column
        for weight in range(cols):
            # Write your code here
            if index == 0 or weight == 0:
                matrix[index][weight] = 0
            # If weight at previous row is less than or equal to current weight
            elif weights[index - 1] <= weight:
                # Calculate item to include
                include_item = values[index - 1] + matrix[index - 1][weight -
weights[index - 1]]

                # Calculate item to exclude
                exclude_item = matrix[index - 1][weight]

                # Calculate the value of current cell
                matrix[index][weight] = max(include_item, exclude_item)
            else:
                # Calculate the value of current cell
                matrix[index][weight] = matrix[index - 1][weight]
    # Return the value of the bottom right of matrix
    return matrix[rows-1][weight_cap]
```

### ### Sieve of Eratosthenes:

#### Basic version :

```
def sieve_of_eratosthenes(limit):
    true_indices = []

    # handle edge cases
    if (limit <= 1):
        return true_indices

    # create the output list
    output = [True] * (limit+1)

    # mark 0 and 1 as non-prime
    output[0] = False
    output[1] = False

    # iterate up to the square root of the limit
    for i in range(2, limit+1):
        if (output[i] == True):
            j = i*2
            # mark all multiples of i as non-prime
            while j <= limit:
                output[j] = False
                j += i

    # remove non-prime numbers
    output_with_indices = list(enumerate(output))
    true_indices = [index for (index,value) in output_with_indices if value
== True]
    return true_indices
```

#### Optimized version:

The complexity of the Sieve of Eratosthenes with optimizations is  $O(n \log(\log n))$ . There are two operations to take into account: the creation of the array and the incrementing and multiple-marking loops. Creation happens in  $O(n)$  time, since it creates an element for each number from 2 to  $n$ . Multiple marking happens in  $O(n \log(\log n))$  time.

```
# import math library
import math

def sieve_of_eratosthenes (limit):
    # handle edge cases
    if (limit <= 1):
        return []

    # create the output list
    output = [True] * (limit+1)

    # mark 0 and 1 as non-prime
    output[0] = False
    output[1] = False

    # iterate up to the square root of the limit
    for i in range(2, math.floor(math.sqrt(limit))):
        if (output[i] == True):
            j = i ** 2    # initialize j to square of i
```

```
# mark all multiples of i as non-prime
while j <= limit:
    output[j] = False
    j += i

# remove non-prime numbers
output_with_indices = list(enumerate(output))
trues = [index for (index,value) in output_with_indices if value == True]
return trues
```

## ## TECHNICAL INTERVIEW PROBLEMS IN PYTHON: LISTS

### ### Palindrome:

```
# what if we wanted to use recursion?
def palindrome_1(str):
    if len(str) <= 1:
        return True
    if str[0] != str[-1]:
        return False
    return palindrome_1(str[1:-1])

# what if we didn't care about case?
def palindrome_2(str):
    lower = str.lower()
    for i in range(len(str) // 2):
        if lower[i] != lower[-i - 1]:
            return False
    return True

# what if we wanted to ignore punctuation?
def palindrome_3(str):
    punctuation = [',', '!', '?', '.']
    no_punc_str = str[:]
    for punc in punctuation:
        no_punc_str = no_punc_str.replace(punc, '')
    for i in range(len(no_punc_str) // 2):
        if no_punc_str[i] != no_punc_str[-i - 1]:
            return False
    return True

# what if we wanted to ignore space?
def palindrome_4(str):
    no_space_str = str.replace(' ', '')
    for i in range(len(no_space_str) // 2):
        if no_space_str[i] != no_space_str[-i - 1]:
            return False
    return True
```

### ### rotate list

```
# no time/space requirements
# return "rotated" version of input list

def rotate(l, k):
    for i in range(k):
        l.insert(0, l.pop())
    return l

def rotate_alternative(lst, degree):
    rotation = degree % len(lst)
    return lst[-rotation:] + lst[:-rotation]
```

### ### List Rotation: Indices

```
def rev(lst, low, high):
    while low < high:
        lst[low], lst[high] = lst[high], lst[low]
        high -= 1
        low += 1
    return lst

def rotate(my_list, num_rotations):
```

```

# first half
rev(my_list, 0, num_rotations - 1)

# second half
rev(my_list, num_rotations, len(my_list) - 1)

# whole list
rev(my_list, 0, len(my_list) - 1)
return my_list

```

### **### Rotation Point: Linear Search**

```

def rotation_point(rotated_list):
    rotation_idx = 0
    for i in range(len(rotated_list)):
        if rotated_list[i] < rotated_list[rotation_idx]:
            rotation_idx = i
    return rotation_idx

```

### **### Rotation Point: Binary Search**

```

def rotation_point(rotated_list):
    low = 0
    high = len(rotated_list) - 1
    while low <= high:
        mid = (low + high) // 2
        mid_next = (mid + 1) % len(rotated_list)
        mid_previous = (mid - 1) % len(rotated_list)

        if (rotated_list[mid] < rotated_list[mid_previous]) and
(rotated_list[mid] < rotated_list[mid_next]):
            return mid
        elif rotated_list[mid] < rotated_list[high]:
            high = mid - 1
        else:
            low = mid + 1

```

### **### Remove Duplicates: Naive**

```

def remove_duplicates(dupe_list):
    unique_values = []
    for el in dupe_list:
        if el not in unique_values:
            unique_values.append(el)
    return unique_values

```

### **### Remove Duplicates: Optimized**

```

def move_duplicates(dupe_list):
    unique_idx = 0
    for i in range(len(dupe_list) - 1):
        if dupe_list[i] != dupe_list[i + 1]:
            dupe_list[i], dupe_list[unique_idx] = dupe_list[unique_idx],
dupe_list[i]
            unique_idx += 1
    dupe_list[unique_idx], dupe_list[len(dupe_list) - 1] =
dupe_list[len(dupe_list) - 1], dupe_list[unique_idx]
    return unique_idx

```

### **### Max list sub-sum: Naive**

```
def maximum_sub_sum(my_list):
    max_sum = my_list[0]
    for i in range(len(my_list)):
        for j in range(i, len(my_list)):
            sub_sum = sum(my_list[i:j + 1])
            if sub_sum > max_sum:
                max_sum = sub_sum
    return max_sum
```

### **### Max List Sub-Sum: Optimized**

```
def maximum_sub_sum(my_list):
    if max(my_list) < 0:
        return max(my_list)

    max_sum = 0
    max_sum_tracker = 0
    for i in range(len(my_list)):
        max_sum_tracker += my_list[i]
        if max_sum_tracker < 0:
            max_sum_tracker = 0
        if max_sum_tracker > max_sum:
            max_sum = max_sum_tracker

    return max_sum
```

### **### Pair Sum: Naive**

```
def pair_sum(nums, target):
    for i in range(len(nums)):
        for j in range(i, len(nums)):
            if nums[i] + nums[j] == target:
                return [i, j]
    return None
```

### **### Pair Sum: Optimized**

```
def pair_sum(nums, target):
    complements = {}
    indices = {}
    for i in range(len(nums)):
        x = complements.get(nums[i], None)
        if x is not None:
            return [indices[x], i]
        complements[target - nums[i]] = nums[i]
        indices[nums[i]] = i
```

## ## TECHNICAL INTERVIEW PROBLEMS IN PYTHON: LINKED LISTS

### ### Insert at Point

```
def insert(self, node_value, location):
    if not location:
        new_head = Node(node_value)
        new_head.next = self.head
        self.head = new_head
        return self

    prev = self.head
    node = Node(node_value)
    current_node = self.head.next

    while location > 1:
        prev = current_node
        current_node = current_node.next
        location -= 1

    prev.next = node
    node.next = current_node

    return self
```

### ### Nth From Last

```
def n_from_last(self, n):
    nodes_remaining = self.size() - 1 - n
    result = self.head

    while nodes_remaining:
        result = result.next
        nodes_remaining -= 1

    return result
```

### ### Remove Duplicates

```
def remove_duplicates(self):
    current_node = self.head

    while current_node:
        while current_node.next and current_node.next.val ==
current_node.val:
            current_node.next = current_node.next.next
            current_node = current_node.next
        return self
```

### ### Merge Sorted Linked Lists

```
def merge(linked_list_a, linked_list_b):

    current_node_a = linked_list_a.head
    current_node_b = linked_list_b.head

    if current_node_a.val < current_node_b.val:
        start_node = current_node_a
        current_node_a = current_node_a.next
    else:
        start_node = current_node_b
```

```

        current_node_b = current_node_b.next

    head = start_node

    while current_node_a or current_node_b:
        if not current_node_a:
            start_node.next = current_node_b
            current_node_b = current_node_b.next
        elif not current_node_b:
            start_node.next = current_node_a
            current_node_a = current_node_a.next
        elif current_node_a.val < current_node_b.val:
            start_node.next = current_node_a
            current_node_a = current_node_a.next
        else:
            start_node.next = current_node_b
            current_node_b = current_node_b.next
        start_node = start_node.next

    return LinkedList(head)

```

### **Find Merge Point**

```

def merge_point(linked_list_a, linked_list_b):
    size_of_a = linked_list_a.size()
    size_of_b = linked_list_b.size()

    diff = abs(size_of_a - size_of_b)

    if size_of_a > size_of_b:
        bigger = linked_list_a.head
        smaller = linked_list_b.head
    else:
        bigger = linked_list_b.head
        smaller = linked_list_a.head

    for i in range(diff):
        bigger = bigger.next

    while bigger and smaller:
        if bigger == smaller:
            return bigger
        bigger = bigger.next
        smaller = smaller.next

    return None

```

### **Reverse a Linked List**

```

def reverse(linked_list):
    prev = None
    current_node = linked_list.head
    while current_node:
        tmp = current_node.next
        current_node.next = prev
        prev = current_node
        current_node = tmp
    return LinkedList(prev)

```

### **Detect Cycle in a Linked List**



```

def has_cycle(linked_list):
    slow, fast = linked_list.head, linked_list.head
    while slow and fast:
        slow = slow.next
        fast = fast.next
        if fast:
            fast = fast.next
        else:
            return False
        if fast == slow:
            return True
    return False

```

### **### Add Two Numbers :**

```

def add_two(linked_list_a, linked_list_b):

    result = LinkedList()
    carry = 0

    a_node = linked_list_a.head
    b_node = linked_list_b.head

    while a_node or b_node:

        if b_node:
            b_val = b_node.val
            b_node = b_node.next
        else:
            b_val = 0

        if a_node:
            a_val = a_node.val
            a_node = a_node.next
        else:
            a_val = 0

        to_sum = a_val + b_val + carry

        if to_sum > 9:
            carry = 1
            to_sum %= 10
        else:
            carry = 0

        if not result.head:
            result.head = Node(to_sum)
            tmp = result.head
        else:
            tmp.next = Node(to_sum)
            tmp = tmp.next

    if carry:
        tmp.next = Node(carry)

    return result

```