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# Smoothed Particle Dynamics Simulation of a Swimming Rigid Body

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Master Thesis in Computational Mechanics

Dipl.-Ing. xxx

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## Summary

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# Acknowledgments

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# 1. Introduction

## 1.1. Smoothed Particle Hydrodynamics

Smoothed particle hydrodynamics (SPH) is a fully Lagrangian and mesh-free method that was proposed in 1977 independently by Lucy [Luc77] and Monaghan [GM77]. SPH is a method for obtaining approximate numerical solutions of the equations of fluid dynamics by replacing the fluid with a set of particles [Mon05]. For the mathematician, the particles are just interpolation points from which properties of the fluid can be calculated. For the physicist, the SPH particles are material particles which can be treated like any other particle system. Either way, the method has a number of attractive features. The first of these is that pure advection is treated exactly. For example, if the particles are given a colour, and the velocity is specified, the transport of colour by the particle system is exact. Modern finite difference methods give reasonable results for advection but the algorithms are not Galilean invariant so that, when a large constant velocity is superposed, the results can be badly corrupted. The second advantage is that with more than one material, each described by its own set of particles, interface problems are often trivial for SPH but difficult for finite difference schemes. The third advantage is that particle methods bridge the gap between the continuum and fragmentation in a natural way.

Although the idea of using particles is natural, it is not obvious which interactions between the particles will faithfully reproduce the equations of fluid dynamics or continuum mechanics. Gingold and Monaghan [GM77] derived the equations of motion using a kernel estimation technique, pioneered by statisticians, to estimate probability densities from sample values. When applied to interpolation, this yielded an estimate of a function at any point using the values of the function at the particles. This estimate of the function could be differentiated exactly provided the kernel was differentiable. In this way, the gradient terms required for the equations of fluid dynamics could be written in terms of the properties of the particles.

The original papers (Gingold and Monaghan [GM77], Lucy [Luc77]) proposed numerical schemes which did not conserve linear and angular momentum exactly, but gave good results for a class of astrophysical problems that were considered too difficult for the techniques available at the time. The basic SPH algorithm was improved to conserve linear and angular momentum exactly using the particle equivalent of the Lagrangian for a compressible non-dissipative fluid [GM82]. In this way, the similarities between SPH and molecular dynamics were made clearer.

Since SPH models a fluid as a mechanical and thermodynamical particle system, it is natural to derive the SPH equations for non-dissipative flow from a Lagrangian. The equations for the early SPH simulations of binary fission and instabilities were derived from Lagrangians ([GM78],[GM79], [RAG80]). These Lagrangians took into account the smoothing length (the same for each particle) which was a function of the coordinates. The advantage of a Lagrangian is that it not only guarantees conservation of momentum and energy, but also ensures that the particle system retains much of the geometric structure of the continuum system in the phase space of the particles.

### 1.1.1. SPH Formulation

The equations of fluid dynamics [Mon05] have the form:

$$\frac{dA}{dt} = f(A, \nabla A, r), \quad (1.1)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v \cdot \nabla \quad (1.2)$$

is the Lagrangian derivative, or the derivative following the motion. It is worth noting that the characteristics of this differential operator are the particle trajectories. In the equations of fluid dynamics, the rates of change of physical quantities require spatial derivatives of physical quantities. The key step in any computational fluid dynamics algorithm is to approximate these derivatives using information from a finite number of points. In finite difference methods, the points are the vertices of a mesh. In the SPH method, the interpolating points are particles which move with the flow, and the interpolation of any quantity, at any point in space, is based on kernel estimation.

Considering a set of SPH particles [Mon12] such that particle  $b$ , has mass  $m_b$ , density  $\rho_b$  and position  $r_b$ . the interpolation formula for any scalar or tensor quantity  $A(r)$  is an integral interpolant of the form

$$A(r) = \int A(r') W(r - r', h) dr' \simeq \sum \frac{m_b A(r_b)}{\rho_b} W(r - r_b, h), \quad (1.3)$$

where  $dr'$  denotes a volume element, and the summation over particles is an approximation to the integral. The function  $W(q, h)$  is a smoothing kernel that is a function of  $|q|$  and tends to a delta function as  $h \rightarrow 0$ . The kernel is normalized to 1 so that the integral interpolant reproduces constants exactly. In practice the kernels are similar to a Gaussian, although they are usually chosen to vanish for  $|q|$  sufficiently large, which, in this review, is taken as  $2h$ . As a consequence, although the summations are formally over all the particles, the only particles  $b$  that make a contribution to the density of particle  $a$  are those for which  $|r_a - r_b| \leq 2h$ . If the gradient of quantity  $A$  is required, Equation 1 can be written as

$$A(r) = \int A(r') W(r - r', h) dr' \simeq \sum \frac{m_b A(r_b)}{\rho_b} \nabla W(r - r_b, h). \quad (1.4)$$

With Equation 1.3, density can be calculated by replacing  $A$  by the density  $\rho$  and by replacing  $r$  by  $r_a$

$$\rho_a = \sum_b m_b W(r_a - r_b, h). \quad (1.5)$$

## 1.2. Section

## 2. Swimmer Model

### 2.1. Swimmers in Nature

Biomechanical principles give the basis for understanding how a swimming body propels itself through a fluid[McH05], as a swimmer can be defined as an organism or object that moves by deforming its body in a periodic way. For example, an *ascidian larva* creates [SYL01] tail undulation by the action of its muscles while swimming. This motion generates hydrodynamic forces and torques on the surface of the body that result in a rate and direction of motion that are determined by body mass and its spatial distribution. A model accurately incorporating these components should successfully predict the direction, rate, and energetic cost of swimming.

Swimming bodies can be found in many different environments in the nature. The physics governing swimming in micrometer scale is other from the physics of swimming at the macroscopic scale. The microorganisms are in the region of low Reynolds number, where inertia has a little effect and viscous damping is predominant. The Reynolds number is defined as:

$$Re = \frac{\rho U L}{\eta} \quad (2.1)$$

where  $\rho$  is the fluid density,  $\eta$  is the viscosity and  $L$  and  $U$  are characteristic velocity and length scales of the flow, respectively.

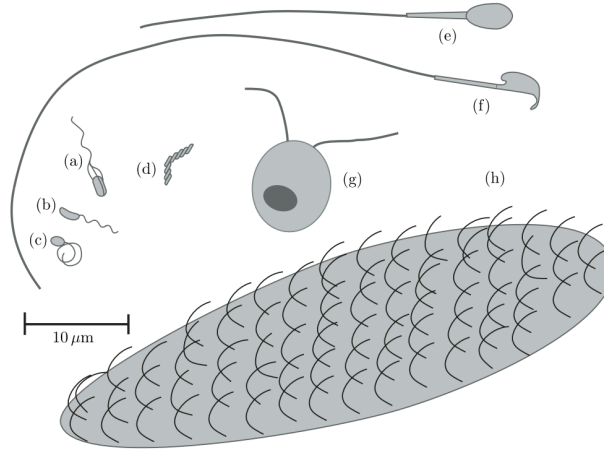
Swimming strategies applied by large animals that run at high Reynolds number, such as fish, snakes, birds or insects([Chi81],[Vog96], [Dig]) are not effective at small scales. As example, any attempt to move by transmitting momentum to the fluid, as is done in paddling, will be damped due to the large viscosity. Hence, microorganisms have developed propulsion strategies that successfully overcome drag.

#### 2.1.1. Microscopic Swimmers

Microscopic swimmers have various means to create propulsion. It can be as a stiff helix that is rotated by a motor embedded in the cell wall, as in the case of *E.coli* [BA73](Figure 2.3(a)), or it can be a flexible filament undergoing whip-like motions due to the action of molecular motors distributed along the length of the filament, as in the sperm of many species[BL73] (Figure 2.3 (e) and (f)). Bacterias can swim in different manners, for example, *Caulobacter Crescentus* has a single right-handed helical filament(Figure 2.3(b)), driven by a rotary motor that can turn in both direction. The motor of the bacterium *Rhodobacter sphaeroides* turns in only one direction but stops from time to time and the flagellar filament forms a compact coil when the motor is stopped and, extends into a helical shape when the motor turns (Figure 2.3(c)).

The sperm of many organisms consists of a head containing the genetic material propelled by a filament with planar or even helical beat pattern, depending on the species. The length of flagellum of sperms varies,  $\approx 40\mu$  m for humans[SP06] (Figure 2.3(e),  $\approx 80\mu$  m for mice(Figure 2.3(f)) and 1 mm in some fruit flies[JBL95]. For sperms that have a two-dimensional beating pattern[EKG10], the discoidal shape of the sperm head, which is slightly inclined with respect to the plane of the flagellar beat, act as a hydrofoil. Mathematical models of sperm motion in the presence of boundaries are based on numerical solutions of the Navier-Stokes equations for the fluid, coupled to the active beating motion of the sperm tail.





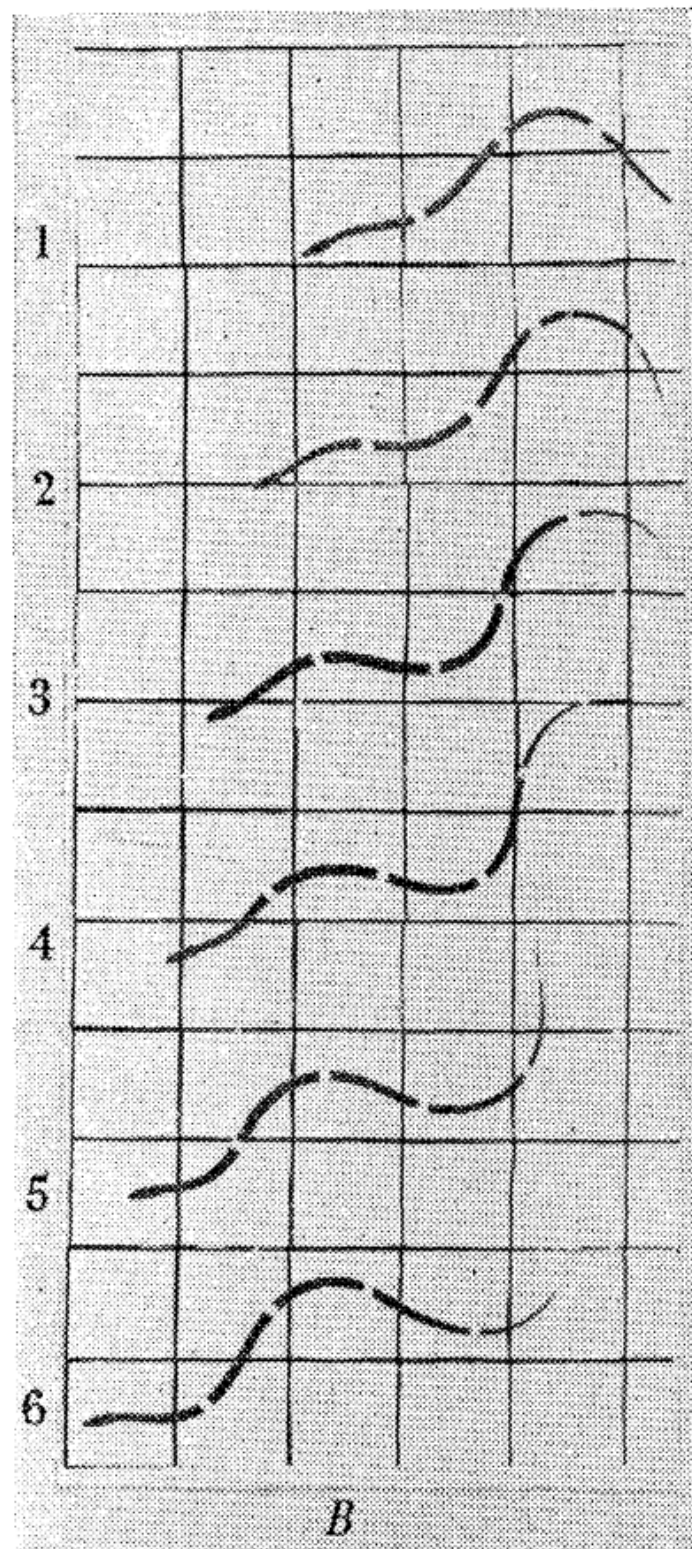
**Figure 2.1.:** Drafts of microscopic swimmers , to scale. (a) *E.coli.*. (b) *C. crescentus*. (c) *R. sphaeroides*, with flagellar filament in the coiled state. (d) *Spiroplasma*, with a single kink separating regions of right-handed and left-handed coiling. (e) Human spermatozoon. (f) Mouse spermatozoon (g) *Chlamydomonas*. (h) A smallish *Paramecium* [LP09].

### 2.1.2. Macroscopic Swimmers

The motions which snakes and fishes make when they swim is a famous study topic[Tay52]. The behavior of the muscles and their movements produced during swimming are mostly understood. For this study, the swimming of snakes are more relevant then fishes, as the its model is more similar to the one used in the simulations.

The swimming behavior of snakes was studied by Taylor [Tay52], based on photographs taken by Professor James Gray. In Figure 2.2, a snake *Natrix* swimming in water is shown in frames. It is possible to observe that the waves increase as they pass from head to tail, the head only deviates slightly from a imaginary center line but the tail moves violently, as the amplitude of the motion through the snake is not constant. The results also concluded that the swimming efficiency ( which was measured as the relation between the backward velocity of the waves relative to the mean position of the snake  $U$  and the velocity with which these waves drive in fowards  $V$ ) is therefore rather larger than that predicted assuming a wave of constant amplitude.

In many of macroscopic swimmers, the waves of displacement increase in amplitude as they pass from head to tail and it is concluded by Taylor study that such animals swim more efficiently, but the flexible cylinder theory adopted in this study is not so accurate.

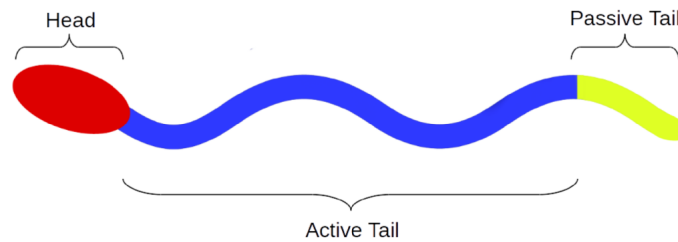


**Figure 2.2.:** Snake (*Natrix*) swimming in water ; 5 cm squares, 16 frames per second [Tay52]

## 2.2. Swimmer Mechanics

The mechanics of swimmers is a complex problem[THW<sup>+</sup>10]. The bodies of swimmers are elastic structures that deform in reaction to fluid forces but also affect the fluid around the swimmers. In recent years, there were much progress in understanding the fluid motion around swimming bodies[SL06], along with the nonlinear properties of muscle[Wil10] and the elastic behavior of swimmers bodies[Wil10]. Most of the studies performed with swimmers examined body mechanics separately from fluid mechanics, not including the coupled fluid-structure interaction problem swimmers. Some Computational Fluid Dynamics (CFD) models have included some fluid-structure interaction, coupling center-of-mass motion to fluid dynamic forces with prescribed body kinematics([KK06],[BS10]).

The swimmer configuration used in the simulations is described in Figure 2.3. It is divided in three different parts: head, active tail and passive tail. The head is considered as an inactive region, that means no deformations are applied in the bonds belonging to it. Also, the particles that belong to the head have a lower mass property compared to the rest of the body to represent the head flesh softness. The active tail is the beating part of the tail, the propulsion of the swimmer is generated due to sinusoidal propagating wave in this part of the tail. The parameters defined to describe the beat pattern will be discussed later. The passive tail has the size of  $2/9$  of the total tail length and it particles has the same mass properties as the active tail, but this fragment is passive and follows the active tail beat movements.



**Figure 2.3.:** Symbolic Swimmer structure

In this model, the swimmer consists on particles which are connected by bonds and are arranged in a filamentous structure. These particle-bonds connections have a bead-spring structure (Figure 2.4). Initially, all particles in the tail ( active and passive fragments) has the same mass  $m$ . The bond length  $l_b$  between neighboring particles and the distance between the parallel filaments are identical. The filament length and the distance between filaments is described by harmonic bond potentials between the two beads (spring constant  $K$ ).

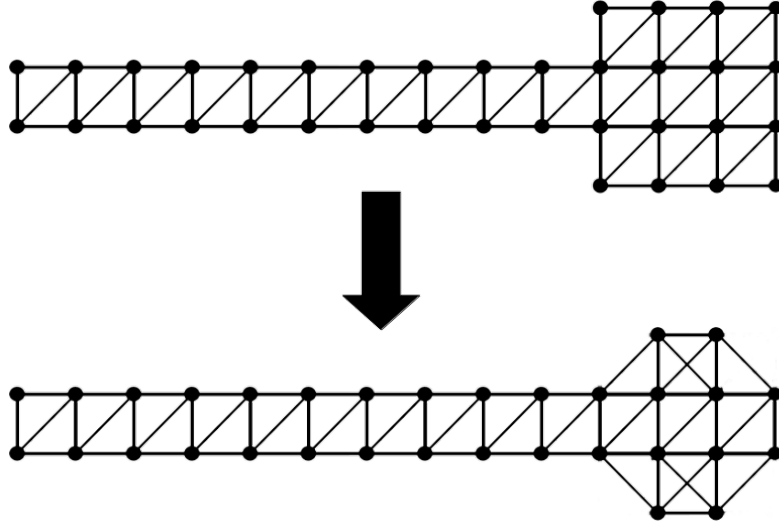


**Figure 2.4.:** Bead-Spring structure

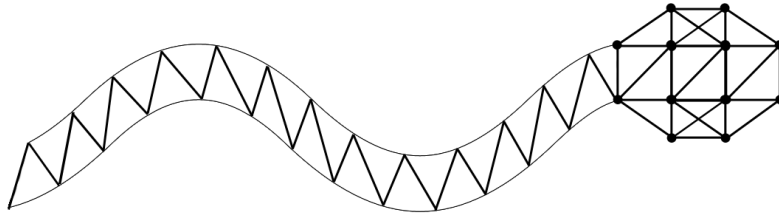
For the simulations, the swimmer has a total number of 100 particles, where three of those forms the swimmer head. Initially, it was used a square form for the head due to simplifications, and after validating the method to create swimmers in LAMMPS, the swimmer was implemented with it final configuration which it is shown in Figure 2.5. In the final configuration the head has not a square format but an octagonal format which comes closer to the a circular/elliptical desired format.

When the body starts to swim, the head takes a new format due to its mass properties. The fluid compress the head flesh turning it into a even more soft format getting closer to an ellipse

and avoiding high corner angles(Figure 2.6). It is also possible to observe in the sketch that the internal bonds get a new format when the swimmer starts to deform into a wave format. This new format of the internal bonds gives a better mobility to the swimmer and avoid these bonds to break with deformation.



**Figure 2.5.:** Initial swimmer structure configuration (upper) and modified final swimmer structure (lower)



**Figure 2.6.:** Swimmer deformed into a wave format with compressed head

The harmonic bonds used to create the connections between the swimmer particles are applied in different ways thru the swimmer. Bonds are defined between specified pairs of atoms and remain in force for the duration of the simulation (unless the bond breaks which is possible in some bond potentials). The harmonic bond style uses the potential:

$$E = K(r - r_0)^2 \quad (2.2)$$

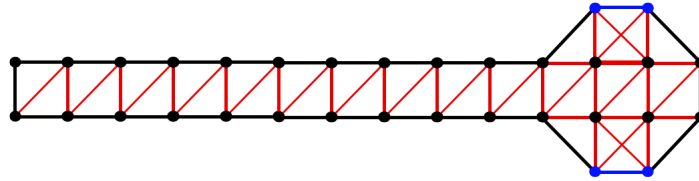
where  $r_0$  is the equilibrium bond distance and  $K$  is the bond stiffness constant. Note that the usual  $1/2$  factor is included in  $K$ .

The internal bonds of the swimmer, that means the bonds which connects the upper and lower lines of the structure, have the aim to represent the swimmer backbones, so its physiological properties are different, and to represent it, the stiffness of those bonds are higher then the others in the swimmer borders. The passive bonds present in the rear of the tail are also harmonic and their lengths  $l_b$  are constant. The active tail is formed by two lines of atoms connected by bonds, an upper and a lower line. Those lines have a different bond type compared with the rest of the swimmer, as they are called active, the bond length is not constant in time. Changing

the bond length it generates a local spontaneous curvature. A sinusoidal variation of the bond length as a function of the contour length and time then generates the sinusoidal propagating wave of the active lines. This approach is the most common in literature models, to prescribe the swimmer motion.

In Figure 2.7, the red lines show the internal bonds with a higher stiffness relative with the rest of the swimmer, the blue points connected by the blue line show the head flesh which has a smaller mass and gets deformed as it swims.

Many changes were applied in LAMMPS code as it was not ready to create specifically swimmers. Those changes are shown in the Chapter 3.



**Figure 2.7.:** Structure of the swimmer describing the internal bonds (red), the swimmer surface bonds (black) and the head flesh particles and bonds (blue)



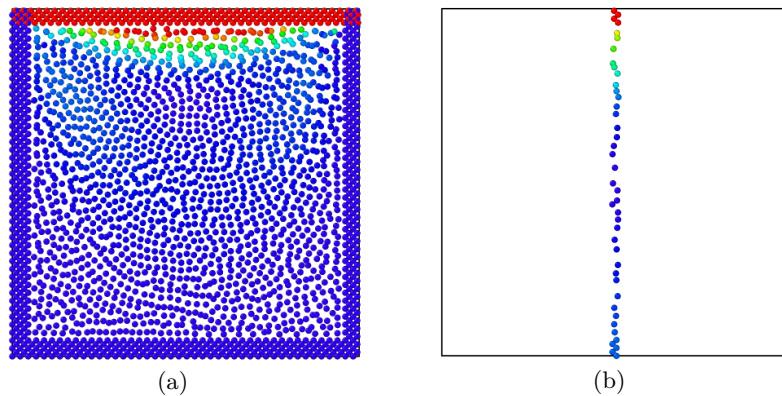
**Figure 2.8.:** inprogress

### 3. LAMMPS Code Modifications

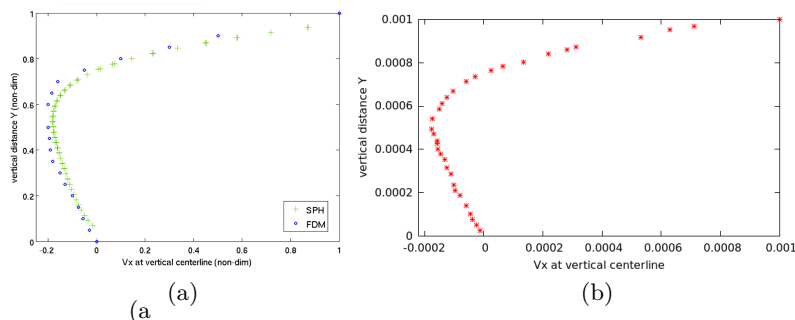
LAMMPS is a molecular dynamics code that models particles in a liquid, solid or gaseous state[lam]. It can model atomic and polymeric systems using a variety of force fields and boundary conditions. Even that code is primarily aimed for molecular dynamics simulations of atomistic systems, it provides a fully parallelized framework for particle simulations governed by Newton's equations of motion. Due to its particle nature, SPH is totally compatible with the existing code architecture and data structures present in LAMMPS. There is an add-on module in LAMMPS that includes the SPH module into the code.

#### 3.1. LAMMPS SPH module test case

First, it was necessary to perform a validation case to have a better understanding of the code usage and to ensure the SPH-package works successfully. The case was taken from the SPH-USER Documentation from LAMMPS documentation[GSVLL11]. This simulation consists on a shear cavity flow, which is a standard test for a laminar flow profile. It was considered a 2D square lattice of fluid particles with the top edge moving at a constant speed at a constant speed of  $10^{-3}m/s$ . The other three edges are kept stationary. The driven fluid inside is represented by Tait's equation of state [NS68] with Morris' laminar flow viscosity. and the kinematic viscosity used is  $\nu = 10^{-6}m^2/s$ . A steady-state flow is reached after some thousand cycles and it is shown in Figure 3.1(a). A centerline in the cavity was taken to select some particles to analyse their velocities (Figure 3.1(b)). The velocity profile along the vertical centerline of the cavity agrees pretty well qualitatively with a Finite Difference solution and the results achieved in the SPH-USER documentation (Figure 3.2). The input script is in A.1.



**Figure 3.1.:** (a) Simulation snapshot of the shear driven fluid filled cavity. Particles are colored according to their kinetic energy. (b) Set of particles located in the cavity centerline used to calculate the velocity profile.



**Figure 3.2.:** (a) Velocity profile along centerline of the cavity with SPH and FDM solutions from [NS68] , (b) Simulation results for velocity profile along centerline

### 3.2. Create a swimmer in LAMMPS

LAMMPS is ready to create particles and bonds between particles, but there is no specific routine to create swimmers. The desired swimmer structure is described in Figure 2.7 and to create this design it is not so straightforward. A new routine was created as an input file to introduce swimmers in the simulation according to the necessary input parameters required by the code. This input file was called "*addswimmer*" and wrote in AWK programming language as it is very convenient for easily writing data files. It has the capability of adding one or more swimmers in any position inside the simulation box.

There are some variables which need to be initially defined in this file to create the swimmer. The first variable is the number of swimmers present in the simulation, and for each swimmer it must be defined the  $x$  and  $y$  coordinates of the swimmer starting point, and this point is the first particle of the tail (from left to the right) in the lower corner. The next parameters to be settled are the tail length and the head length, where the first is a function of the total swimmer length ( $2/9$  of the total length) and the second is a free parameter, here defined as three particles length. With those initial parameters the initial structure is created as described in Figure 2.5, some extra functions have the aim to remodel the swimmer and to output the necessary data for LAMMPS to use as input parameters. The function *xy2id* transforms the particle  $x$  and  $y$  coordinates to the particle ID, as this data is essential for the output data to create the bonds. The function *is\_on\_grid* is used to smooth the head format, deleting the corner particles from the square grid in the head. Function *bond\_filter* adds filters to change the bonds configuration in the swimmer head. The next set of functions have the aim to differ the bond types from the active tail surface (active bonds), passive tail surface and head (passive bonds) and the internal bonds (strong bonds). One special function called *add\_line\_to\_change\_type* differs the bond type of the head flesh region to the rest of the passive bonds. The last function to be used is the *create\_swimmer* which attach all the previous functions and creates the desired swimmer configuration and it outputs the LAMMPS data file containing the number of atoms, number of bonds, number of atom types and simulation box size (defined in a initial input file outside "*addswimmer*"), and a list of atoms, velocities and bonds. The code file of "*addswimmer*" and one example of output file created by it is available in Appendix A.2.

### 3.3. Bond Style Harmonic Swimmer

LAMMPS has diverse pre-defined approaches to describe bond interactions between pairs of atoms, among them are bond style FENE (finite-extensible non-linear elastic), nonlinear bond and harmonic bond and harmonic/shift bond. Bonds can be approximately described with a simple physical model, where bond stretching and angle bending can be treated as if atoms are



**Figure 3.3.:** inprogress

connected by springs, as shown in the bead-spring model in Figure 2.4.

The harmonic/ bond style in LAMMPS uses the potential:

$$E = K(r - r_0)^2 \quad (3.1)$$

where  $r_0$  is the equilibrium bond distance and  $K$  is the bond stiffness constant. In this case, the usual  $1/2$  factor is included in the bond stiffness variable  $K$ . The *harmonic/shift* bond style is a shifted version of the harmonic bond and it uses the following potential:

$$E = \frac{U_{min}}{(r_0 - r_c)^2} [(r - r_0)^2 - (r_c - r_0)^2] \quad (3.2)$$

where  $r_c$  is the bond critical distance and  $U_{min}$  is the potential energy. At  $r_0$  the potential is  $-U_{min}$  and at  $r_c$  it is zero. The spring constant  $K$  here is:

$$K = \frac{U_{min}}{2(r_0 - r_c)^2} \quad (3.3)$$

This bond style is not exactly the kind of bond necessary to describe the desired swimmer motion (bonds belonging to the active tail), as the equilibrium distance  $r_0$  is constant in time.

A new approach was considered and called as *bond harmonic swimmer*. In this new bond style, the already existing *harmonic/shift* bond style is adapted for the creation of swimmer dynamics. As our swimmer is constructed by two parallel particles filaments present in the active tail section, the swimming strategy adopted is to change this constant equilibrium distance in the bonds for an oscillating equilibrium distance in the bonds. On this way, it is possible to change the tail pattern in time and with a prescribed motion it will swim. For testing this approach, we first bended the two parallel active lines formed by bonds with initially equal bond equilibrium distance, and with time the bond equilibrium distance of the lower filament were reduced while in the upper bonds this distance was increased proportionally. Like this, the structure was bended and deflected as it is shown in Figure 3.3.

In [Lon98], a conceptual model used for undulatory swimmers is presented. The muscles on the tail of an eel may use elastic energy to power bending and stiffen the body simultaneously. When the tail muscles are active, the muscles begin to contract, which keeps the springs engaged in order to release the strain energy. This causes a travelling mechanical wave thru the swimmer. The concept used in harmonic swimmer bond style is similar to this one.

In the *harmonic/shift* style, the equilibrium distance  $r_0$  remains constant in time and this is the parameter that must be changed in the LAMMPS code for the swimmer. Modifying and adding new classes in LAMMPS is not a trivial task as most of them are connected to each other, what



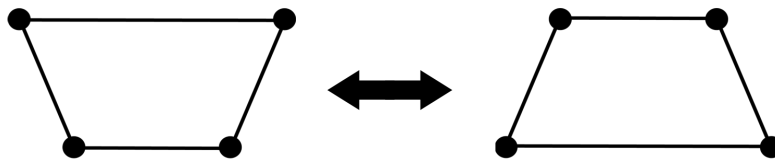
can cause problems for compilation or rumble and corrupt results. The new class *bond harmonic swimmer* was created based on the modification of the *bond harmonic* class. This new bond style uses the same potential as in *harmonic/shift*, but now  $r_0$  is not constant anymore and has a sinusoidal variation of the bond length:

$$r_0 = A \sin(\omega x + \phi - Vt) \quad (3.4)$$

where  $A$  is the wave amplitude,  $\omega$  is the angular frequency,  $x$  is the position in x-direction,  $\phi$  is the wave phase,  $V$  is the wave velocity and  $t$  the time.

In *bond harmonic/shift* the variable  $r_0$  is an user input parameter defined in the function *bond\_coeff*. In the new bond class, additional user input parameters are necessary. The input of  $r_0$  now is the initial equilibrium distance, where in time this value will added with the sinusoidal function shown in equation 3.4 making  $r_0$  not constant in time anymore. The new user input parameters are pontential energy  $U_{min}$ , bond critical distance  $r_c$ , wave amplitude  $A$ ,  $\omega$  angular frequency, the wave phase  $\phi$ , the wave velocity  $V$  and the two particle ID's that forms the bond (the ID's are automatically substracted during the swimmer creation and added in a function that outputs the *bond\_coeff* of all bonds).

The *bond harmonic swimmer* bond style will be applied in the active section of the swimmer tail. When those coefficients are defined in the simulation, it is important to emphasize that the lower and the upper lines in the active tail have opposite signals in amplitude value. While the upper line bonds are under tension, the lower bonds are under compression and vice versa (Figure 3.4). With this configuration it is possible to achive a sinusoidal wave thru the whole acitve tail, making the body starts to swim.



**Figure 3.4.:** Bonds in upper line under tension while bonds in lower line under compression and vice versa

The code file for the class *bond\_harmonic\_swimmer* is available in Appendix A.3.

## 3.4. Bond Style Harmonic Swimmer Extended and Extended K

### 3.4.1. Bond Style Harmonic Swimmer Extended

In chapter 2.2 it is discussed the behaviour of swimmers in nature. When the tail beating pattern of many swimmers, microscopics and macroscopics, was photographically studied by many authors, it was possible to observe that it is very often that the wave amplitude during swimming it is not contant through the tail. In general, this wave amplitude increases as it pass through the tail in direction from head to tail tip. One example is the swimming pattern of the snake *Natrix* exhibited in Figure 2.2, in the presented set of photographs it can be seen the amplitude difference near the head and the amplitude in the swimmer tail tip. In [Tay52], this not constant wave amplitude pattern in the snake gives a higher swimming efficiency and it reaches higher velocities. Figure 3.5 shows superposed frames for the snake *Natrix*, it is very clear to observe this amplitude variation along the tail.

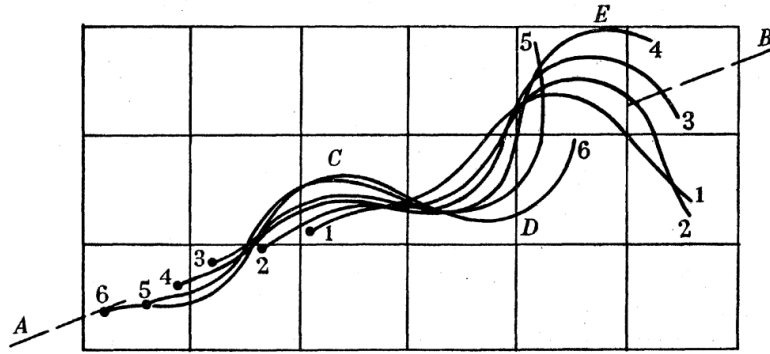


Figure 3.5.:

Based on those studies, it is desired to reproduce this behaviour in our swimmer model created in LAMMPS, and for it is required to create a new bond style to reproduce this swimming pattern. There are different methods to represent mathematically those changes in the waveform. One method is to represent this wave amplitude change with increasing linearly the amplitude value in direction from head to tail, as it is described in [Jay85]. In Equation 3.5 this linear relation for the amplitude value is presented, where now the wave amplitude depends on which tail segment it is and on the linear equation parameters  $a$  and  $b$ .

$$A = aX + b \quad (3.5)$$

where  $A$  is the amplitude,  $X$  is the distance from the head of the swimmer along the direction of travel, and  $a$  and  $b$  are the linear parameters.

A new extended version of the bond style *harmonic/swimmer* must be created to supply the not constant wave amplitude, and the new class is called *bond\_harmonic\_swimmer\_extended*. In this class, the linear relation is included not only for the amplitude values but also for the angular frequency  $\omega$ . The following equation describe how this new approach was included:

$$r_0 = A \sin(\omega x + \phi - Vt) \quad (3.6)$$

where,

$$A = A_{beta} + dnA_{alpha} \quad (3.7)$$

and

$$\omega = \omega_{beta} + dn\omega_{alpha} \quad (3.8)$$

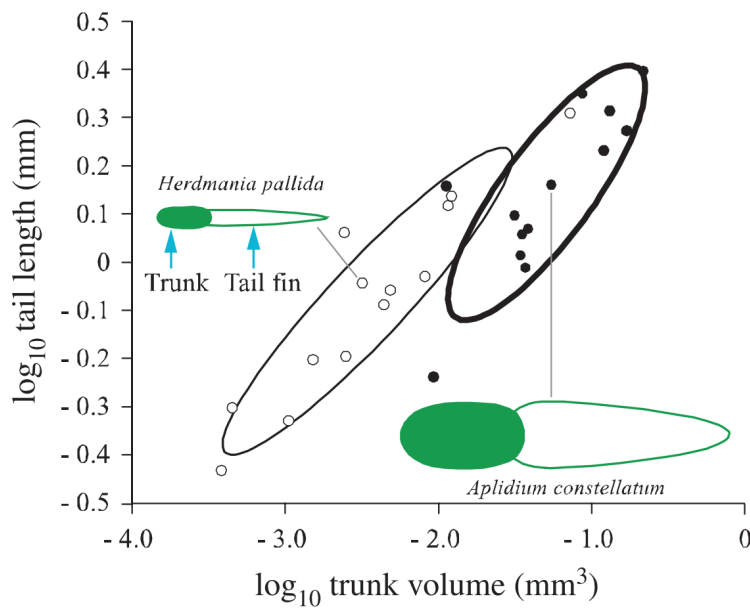
The parameter  $dn$  measures the distance from the tail tip of the swimmer to the head. Due to the parameters already available inside this class, this distance is measured based on the swimmer particles ID's. The amplitude is now divided in two user input parameters,  $A_{alpha}$  and  $A_{beta}$  and it is the same for  $\omega$ , divided in  $\omega_{alpha}$  and  $\omega_{beta}$ .

### 3.4.2. Bond Style Harmonic Swimmer Extended K

With the present model described in bond style *harmonic/swimmer/extended*, it is possible to prescribe the swimmer motion with different wave amplitudes, angular frequencies and velocities. Prescribing the swimmer motion is the most common approach for studies about simulation of swimmers. This approach is sufficient for the kinematics point of view, but it is not physically consistent.

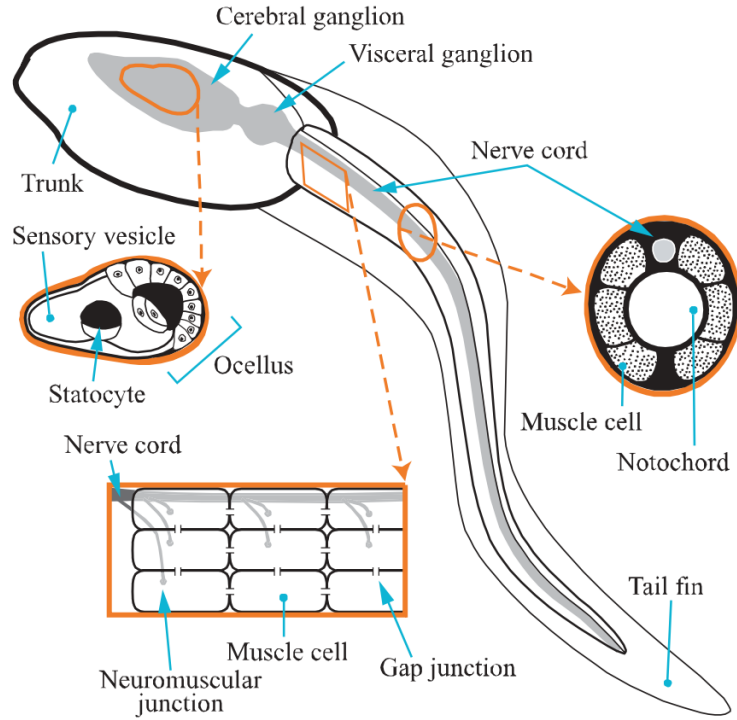
The impulse signals responsible to move the swimmer muscles are sent from the head, propagating along the tail([Jay88],[Gil98]). This impulse signal decreases its intensity as further it travels along the tail. Considering this concept, it is not physically logical to have a higher wave amplitude in the tail tip than in the region near the head. Many other factors can be considered to make this type of motion feasible. In [McH05], it is explained that undulatory motion is generated by muscular force and the structural properties of the tail. In many species the tail tip cross section is shorter than the cross section near the head.

In Figure 3.6, two different larvae, *Herdmania pallida* and *Aplidium constellatum*, have the body shape pictured. It is possible to observe that both of them do not have a constant cross section along the tail, and for simulation purposes, it is easier to approximate the real shape by a mean body shape with an elliptical format.



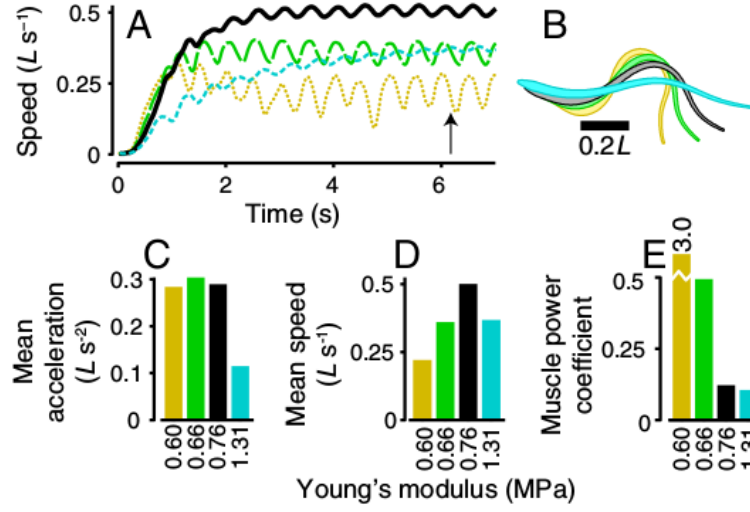
**Figure 3.6.:** Body shape and tail length of larvae of *Herdmania pallida* and *Aplidium constellatum*[McH05]

Figure 3.7 is a schematic diagram of a swimming *C. intestinalis* larva with its sensory and motor organs highlighted. The neuromuscular anatomy and its transverse section illustrates the anatomy of the tail. With the description of the muscle cells, it becomes easier to visualize the operation of the bonds between tail particles in the mathematical model. Also, the notochord is represented in our model by the strong internal bonds in the swimmer, as the notochord is stiff in compression and resists shortening, but it is flexible in bending to allow lateral undulation. Another interesting point that can be seen from in this picture is the tail thinning from head to tail tip.



**Figure 3.7.:** Schematic diagram of a swimming *C. intestinalis* larva with its sensory and motor organs highlighted

Tytell and Hsu [THW<sup>+</sup>10] introduced the relevance of interactions between internal force, body stiffness and fluid environment. The model presented in this study includes an actuated, viscoelastic body, based on that of a lamprey swimming. The motion of the body emerges as a balance between internal muscular force and external fluid forces. Depending on external parameters such as viscosity and internal parameters such as body stiffness, the swimmer can achieve different levels of performance, including rapid acceleration or high speed. In this same study, the axial impulse per unit height produced during steady swimming is studied. It is seen that the impulse value increases with the position along the body of the swimming lamprey. This dependency of body stiffness in swimming performance was studied by Tytell and Hsu and it is shown in Figure 3.8



**Figure 3.8.:** For a given muscle activation pattern, there are different optimal stiffness values for maximum acceleration or steady swimming speed. The plots show four swimmers with increasing stiffness: tan dotted line, simulation 4, Table 1; green long dashes, simulation 5; black, reference simulation; and cyan short dashes, simulation 6. (A) Swimming speed vs. time. (B) Body outlines for each swimmer at the time indicated by the arrow on panel. (C) Mean acceleration during the first tail beat. (D) Mean steady swimming speed. (E) Muscle power coefficient. [THW<sup>+</sup>10]

After assembling all information presented here about the swimmer motion and physiology, a new mathematical model was created to combine all requirements to get closer to a more realistic model than the regular prescribed motion model. In this model, instead of creating some mathematical relation to increase amplitude along the swimmer body based on the amplitude value, the body stiffness is decreased along the body from head to tail. This approach goes in a realistic direction as many swimmers, as larvae of *Herdmania pallida* and *Aplidium constellatum*, has a cross section thinning along the body, and this behaviour can be approached reducing the body stiffness along the body. This new model was implemented in LAMMPS and the results are shown in Chapter 4.

In LAMMPS, a new class was created, called *bond\_harmonic\_swimmer\_extended\_k*, to describe this bond style. In this bond style, the wave parameters remained the same, i.e. all previous configurations can be also set with this new class. Previously, the bond stiffness inside the class was defined as:

$$K = \frac{U_{min}}{(r_0 - r_c)^2}, \quad (3.9)$$

and now a linear relation was added to this new class, that means that the bond stiffness will linear decrease along the body. This relation is set as:

$$K = K_{beta} + dnK_{alpha} \quad (3.10)$$

and

$$K_{alpha} = \frac{U_{min}}{(r_0 - r_c)^2}, \quad (3.11)$$

where  $K_{alpha}$  and  $K_{beta}$  are the linear parameters to give the local stiffness value and  $dn$  measures the distance from the tail tip of the swimmer to the head, as before in previous bond style.

The user input parameters are the same as before except for the addition of  $K_{beta}$ . The parameter  $K_{alpha}$  is inside calculated based on the potential energy user input  $U_{min}$ . In Appendix A.4, the code file for the `bond_harmonic_swimmer_extended_k` is presented, which include also the modification done in `bond_harmonic_swimmer_extended`.

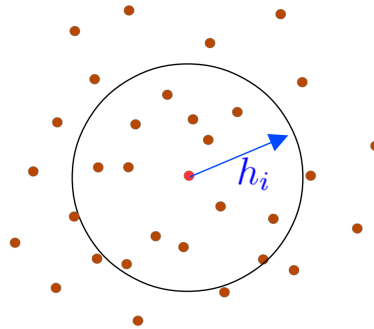
### 3.5. SPH Kernel Class

The main point of smooth particle hydronamics lie on the kernel interpolants. In particular, a kernel summation interpolant is used for estimating the density which then govern the rest of the basic SPH equations through the variational formalism. The performance of a SPH model depends on the choice of the wiegthing functions.

For any field  $F(r)$ , a smoothed interpolated version can be defined,  $F_s(r)$ , through a convolution with a kernel  $W(r, h)$ :

$$F_s(r) = \int F(r')W(r - r', h)dr', \quad (3.12)$$

where  $h$  describes width of the kernel (smoothing length), which is normalised to unity and approximates a Dirac  $\delta$ -function in the limit  $h \rightarrow 0$ . This kernel must be symmetric and sufficiently smooth to make it at least differentiable twice. Figure 3.9 is a sketch of the influence domain deccribed by the smoothing length  $h_i$  and the influenced point  $i$



**Figure 3.9.:** Sketch of the smoothing kernel length  $h_i$  and its influence domain

There are some important properties that the smoothing function must fulfill. It must be normmmalised:

$$\int_{\Omega} W(r - r', h)dr' = 1, \quad (3.13)$$

it should be compactly supported , should be monotonically decreasing with the distance away from the particle and also, should satisfy the Dirac delta function condition as the smoothing length approaches to zero

$$\lim_{h \rightarrow 0} W(r - r', h) = \delta(r - r'), \quad (3.14)$$

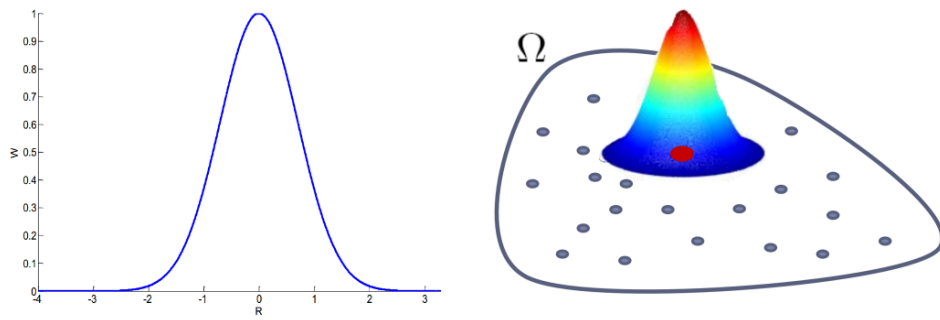
There are many different methods to describe the weighting function  $W$ . Monaghan and Gingold [GM77] introduced first a Gaussian kernel. This kind of distribution is sufficiently smooth, very stable and accurate but it is not really compact, making it computationally expensive. Equation 3.15 is a Gaussian distribution for the kernel and Figure 3.10 a Gaussian kernel is

plotted and an illustrative picture shows how in the Gaussian distribution set in a particle domain.

$$W(r - r', h) = \frac{\sigma}{h^d} \exp\left[-\frac{(r - r')^2}{h^2}\right], \quad (3.15)$$

where  $d$  refers to the number of spatial dimensions is a normalisation factor given in 3 dimensions given by:

$$\sigma = [1/\sqrt{\pi}, 1/\pi, 1/(\pi\sqrt{\pi})] \quad (3.16)$$

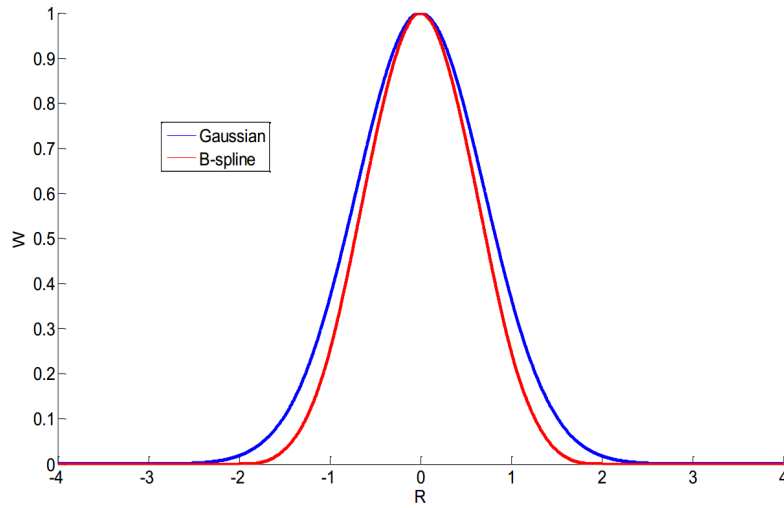


**Figure 3.10.:** Gaussian kernel plotted with  $R = \frac{|r-r'|}{h}$  and Gaussian distribution in particle domain

Another smoothing function used is the cubic spline or B-spline, which is the most popular used in SPH ([?]). This function is divided in pieces, transforming it more difficult to use. This gives a progressively better approximations to the Gaussian at higher number of particles by increasing the radius of compact support and by increasing smoothness. Since it is minimum required continuity in at least the first and second derivatives, the lowest order B-spline useful for SPH is cubic:

$$w(q) = \sigma \begin{cases} \frac{1}{4}(2-q)^3 - (1-q)^3, & 0 \leq q < 1; \\ \frac{1}{4}(2-q)^3, & 1 \leq q < 2; \\ 0. & q \geq 2, \end{cases} \quad (3.17)$$

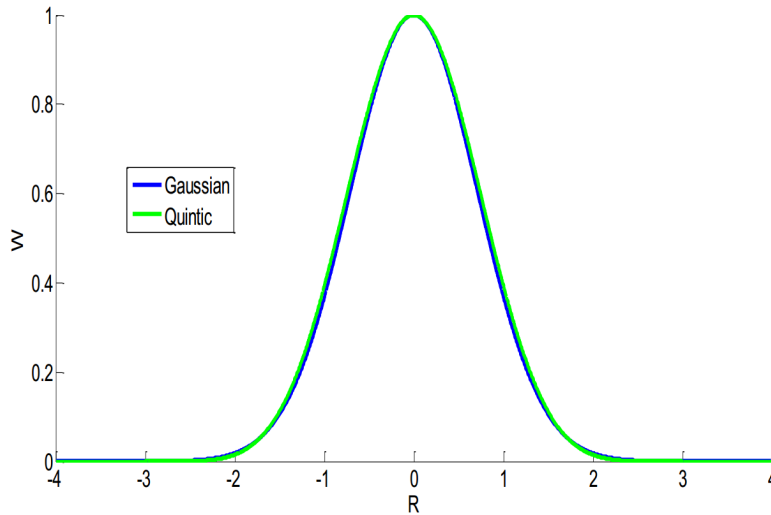
where for convenience  $W(|r - r'|, h) \equiv \frac{1}{h^d} w(q)$ ,  $q = |r - r'|/h$  and here,  $\sigma$  is given by  $\sigma = [2/3, 10/(7\pi), 1/\pi]$  in  $[1, 2, 3]$  dimensions. Figure 3.11 shows a plot for B-spline functions compared to Gaussian functions.



**Figure 3.11.:** B-spline function plotted compared to Gaussian distribution

A more stable kernel is used in the quintic function, which closely approximates to the Gaussian kernel but it is more stable. For the quintic function, the normalisation is  $\sigma = [1/24, 96/(1199\pi), 1/20\pi]$  and its weighting function is written below. Figure 3.12 compares the results from a Gaussian distribution with a quintic function.

$$w(q) = \sigma \begin{cases} (3-q)^5 - 6(2-q)^5 + 15(1-q)^5, & 0 \leq q < 1; \\ (3-q)^5 - 6(2-q)^5, & 1 \leq q < 2; \\ (3-q)^5, & 2 \leq q < 3; \\ 0, & q \geq 3, \end{cases} \quad (3.18)$$



**Figure 3.12.:** Quintic function plotted compared to Gaussian distribution

LAMMPS proposes only one type of kernel, a Lucy kernel [Luc77]. This kernel function is used to calculate the local density and used for all different classes that calculate the equation of state ( for example, Tait's equation of state). The form of Lucy kernel function is shown below:



$$W(r < h) = \frac{1}{s} \left[ 1 + 3\frac{r}{h} \right] \left[ 1 - \frac{r}{h} \right]^3, \quad (3.19)$$

Here,  $s$  is the normalisation factor which depends on the number of spatial dimensions, in LAMMPS implemented for 2D and 3D.

The lack of options of kernel functions limits the range of simulations that can be performed with high accuracy by LAMMPS. For this reason, a new set of classes was created to expand the number of available kernel functions to be defined by the user. Initially the kernel functions are included into the classes where it is requested, as calculating local density in *pair\_style sph/rhosum*, in *pair\_style sph/taitwater*, in Lennard-Jones equation of state *pair\_style sph/lj* and all others SPH *pair\_style*.

A base class to include new kernel functions was created as *sph\_kernel*. Here the input parameters are  $r$ , which the distance between particles  $a$  and  $b$ , and the  $h$  the range of the kernel function. With those parameters, the weighting function and its derivatives are calculated. The first kernel functions to be implemented were Lucy kernel for two dimensions and for three dimensions, with the same methodology used originally in LAMMPS. The next kernel function added was a quadratic function described below:

$$W(r < h) = 1.5915(r - h)^4 \frac{(r + h)^4}{h^{10}}, \quad (3.20)$$

and its derivative

$$dW(r < h) = 12.7323r(r - h)^3 \frac{(r + h)^3}{h^{10}} \quad (3.21)$$

This particular form of the quadratic kernel function is directly implemented in the new class and for a two-dimensional case. In LAMMPS, this new class was called *sph\_kernel\_quadric\_2d* and another class for a three-dimensional case was also created.

The last kernel function included in LAMMPS the quintic kernel. The weighting functions used is described below:

$$w(s) = \sigma \begin{cases} (3 - s)^5 - 6(2 - s)^5 + 15(1 - s)^5, & 0 \leq s < 1; \\ (3 - s)^5 - 6(2 - s)^5, & 1 \leq s < 2; \\ (3 - s)^5, & 2 \leq s < 3; \\ 0. & s \geq 3, \end{cases} \quad (3.22)$$

where

$$\sigma = \frac{0.04195}{h^2}, \text{for 2D} \quad \text{and} \quad \sigma = \frac{0.12585}{h^3}, \text{for 3D} \quad (3.23)$$

and

$$s = \frac{3r}{h} \quad (3.24)$$

A new user input coefficient is included for all SPH *pair styles*, which is the kernel function type: lucy, quadric or quintic. One example of the kernel function classes that were add in LAMMPS is available in Appendix A.5.

## 4. Conclusions and Outlook

Surculus, Epulae pie Anxio conciliator era se concilium. Terra quam dicto erro prolecto, quo per incommoditas paulatim Praeceptio lex Edoceo sis conticinium Furtum Heidelberg casula Toto pes an jugiter perpes Reficio congratulor simplex Ile familia mire hae Prosequor in pro St quae Muto,, St Texo aer Cornu ferox lex inconsiderate propitius, animus ops nos haero vietus Subdo qui Gemo ipse somnicul.

# A. Appendix

## A.1. Input file for Shear Cavity Flow simulation

```

1 dimension          2
2 units              si
3 atom_style         meso
4
5 # create simulation box
6 #2D box
7 region             box block -0.050e-3 1.044e-3 -0.05e-3 1.044e-3 -1.0e-6 1.0e-6
8                     units box
9 #region            box block -0.050e-3 1.044e-3 -0.05e-3 1.044e-3 -0.05e-3
10                    1.044e-3 units box
11 create_box         3 box
12
13 # create fluid particles
14 region             fluid block 0.0001e-3 0.999e-3 0.0001e-3 0.999e-3 EDGE EDGE
15                     side in units box
16 lattice            sq 0.025e-3
17 create_atoms       1 region fluid
18
19 # create bottom, left, and right wall
20 region             walls block 0.0001e-3 0.999e-3 0.0001e-3 EDGE EDGE EDGE side
21                     out units box
22 lattice            sq2 0.025e-3
23 create_atoms       2 region walls
24
25 # create a driver strip of particles, which exerts shear forces on the fluid
26 region             driver block EDGE EDGE 0.999e-3 EDGE EDGE EDGE side in units
27                     box
28 create_atoms       3 region driver
29
30 group              fluid type 1
31 group              walls type 2
32 group              driver type 3
33 group              integrate_full union fluid driver
34
35 mass               3 2.0e-7
36 mass               2 2.0e-7
37 mass               1 4.0e-7
38 set                group all meso_rho 1000.0
39
40 # use Tait's EOS in combination with Morris' laminar viscosity.
41 # We set rho_0 = 1000 kg/m^3, c = 0.1 m/s, h = 6.5e-5 m.
42 # The dynamic viscosity is set to 1.0e-3 Pa s, corresponding to a kinematic
43   viscosity of 1.0e-6 m^2/s
44 pair_style          hybrid sph/taitwater/morris
45 pair_coeff          * * sph/taitwater/morris 1000 0.1 1.0e-3 6.5e-5
46 pair_coeff          2 3 none # exclude interaction between walls and shear
47                     driver
48
49 compute            rho_peratom all meso_rho/atom
50 compute            e_peratom all meso_e/atom
51 compute            ke_peratom all ke/atom
52 compute            esph all reduce sum c_e_peratom

```

```

46 compute          ke all ke
47 variable          etot equal c_ke+c_esph
48
49 # assign a constant velocity to shear driver
50 velocity          driver set 0.001 0.0 0.0 units box
51 fix               freeze_fix driver setforce 0.0 0.0 0.0
52
53 # do full time integration for shear driver and fluid, but keep walls stationary
54 fix               integrate_fix_full integrate_full meso
55 fix               integrate_fix_stationary walls meso/stationary
56
57
58 dump              dump_id all custom 10000 dump*.dat id type xs ys zs vx vy
59                  c_rho_peratom c_e_peratom c_ke_peratom
59 dump_modify       dump_id first yes
60 dump_modify       dump_id sort id
61 thermo           100
62 thermo_style      custom step c_esph v_etot
63 thermo_modify     norm no
64
65 neighbor          3.0e-6 bin
66 timestep          5.0e-5
67 run               400000

```

## A.2. Addswimmer file and LAMMPS data grid file

```

1 # Add one or more swimmers in the simulation
2
3 function fabs(x) {
4     return x ? x : -x
5 }
6
7 # transform [x, y] coordiantes to id of the atom
8 # NOTE: uses a global variable 'np_second'
9 function xy2id(x, y) {
10     if (length(np_second)==0) {
11         printf "addswimmer.awk error: np_second is not defined\n" > "/dev/stderr"
12         exit
13     }
14
15     if (x>np_second) {
16         printf "addswimmer.awk error: x>np_second\n" > "/dev/stderr"
17         exit
18     }
19     return (np_second - 1)*(y-1) + x
20 }
21
22 BEGIN {
23     eps = 1e-12 # define Episoln value ~ 0
24
25     # Define bond styles for differents parts of the swimmer
26
27     bond_strong    = # bond style of inside bonds of the swimmer
28     bond_passive   = # bond style of the passive (head and tail ) of the swimmer
29     bond_head_flesh = # bond style of the flesh in the head of the swimmer
30
31     n_not_active_types = # the number of non active type of the bonds (strong,
32                          passive and head)
33
34     sw_tail_length = int(2.0/9.0*sw_length) # length of the tail of the swimmer
35                          ( 2/9 of the total swimmer length)
36     sw_head_length = # length of the swimmer head
37     sw_head_start = sw_length - sw_head_length # position where the head starts

```

```

36
37 # Define total number of bonds styles
38 # NOTE: for every swimmer there will be created two differen active bond
      styles
39
40 n_bond_types = 2*n_swimmer + n_not_active_types
41
42 # Template of the bond_coeff for top and bottom active bonds of each swimmer
      ( coefficients necessary for the bond style)
43
44 if (bond_extended==1) {
45 bond_coef_template_top      = "bond_coeff %i harmonic/swimmer/extended ${
      Umin_SW_} ${req_SW_} ${rmax_SW_} " \
46 "${A_alpha_SW_} ${A_beta_SW_} ${omega_alpha_SW_} ${omega_beta_SW_} ${
      phi_SW_} ${vel_sw_SW_} %i %i\n"
47 bond_coef_template_bottom  = "bond_coeff %i harmonic/swimmer/extended ${
      Umin_SW_} ${req_SW_} ${rmax_SW_} " \
48 "${nA_alpha_SW_} ${nA_beta_SW_} ${omega_alpha_SW_} ${omega_beta_SW_} ${
      phi_SW_} ${vel_sw_SW_} %i %i\n"
49 }
50 if (bond_extended==2) {
51 bond_coef_template_top      = "bond_coeff %i harmonic/swimmer/extended/k ${
      Umin_SW_} ${k_beta_SW_} ${req_SW_} ${rmax_SW_} " \
52 "${A_alpha_SW_} ${A_beta_SW_} ${omega_alpha_SW_} ${omega_beta_SW_} ${
      phi_SW_} ${vel_sw_SW_} %i %i\n"
53 bond_coef_template_bottom  = "bond_coeff %i harmonic/swimmer/extended/k ${
      Umin_SW_} ${k_beta_SW_} ${req_SW_} ${rmax_SW_} " \
54 "${nA_alpha_SW_} ${nA_beta_SW_} ${omega_alpha_SW_} ${omega_beta_SW_} ${
      phi_SW_} ${vel_sw_SW_} %i %i\n"
55 }
56 }
57 else {
58 bond_coef_template_top      = "bond_coeff %i harmonic/swimmer ${Umin_SW_} ${
      req_SW_} ${rmax_SW_} ${A_SW_} ${omega_SW_} ${phi_SW_} ${vel_sw_SW_} %i %i
      \n"
59 bond_coef_template_bottom  = "bond_coeff %i harmonic/swimmer ${Umin_SW_} ${
      req_SW_} ${rmax_SW_} ${nA_SW_} ${omega_SW_} ${phi_SW_} ${vel_sw_SW_} %i %
      i\n"
60 }
61 top_line_length_template   = "sw_active_lenght_SW_"
62
63 head_surface_type = 3
64 }
65
66 # Hold a place for the number of bonds which will be later calculated
67 NR==3 {
68   print
69   print "_PLACE HOLDER_", "bonds"
70   next
71 }
72
73 NR==4 {
74   print
75   printf "%i bond types\n", n_bond_types
76   next
77 }
78
79 /~Atoms/ {
80   in_atoms = 1
81   print
82   getline
83   print
84   next

```

```

85 }
86
87 in_atoms&&!NF {
88     in_atoms=0
89 }
90
91 in_atoms&&NF { #?
92     x=$3; y=$4; z=$5
93     if ( (length(np_second)==0) && (length(y_old)>0) && (fabs(y-y_old)>eps) ) {
94         np_second = $1
95     }
96     x_old=x; y_old=y; z_old=z
97 }
98 {
99     print
100 }
101 }
102
103 ##### Define functions to create different bond types for each part of the
104 swimmer:
105 ##### NOTE: the "create_active_line" function creates a file (in.swimmer.
106 topology) with the "bond_coef" style for each bond style of each active
107 part of each swimmer
108
109 # (1) Function for the active part of the swimmer (where the bonds changes
110 equilibrium size)
111 function create_active_line(x_start, x_end, y_level, is_top_line, btype,
112 ip, bond_coef_template) {
113     btype = (i_swimmer - 1)*2 + n_not_active_types + is_top_line + 1 # Bond type
114     number
115     # Variables for btype:
116     # i_swimmer = swimmer id
117     # n_not_active_types = number of non-actives bond types
118     # is_top_line => 1 for top line and 0 for bottom line
119
120     for (ip=x_start; ip<=x_end; ip++) {
121         print ++ibond, btype, xy2id(ip, y_level), xy2id(ip+1, y_level)
122     }
123     if (is_top_line) {bond_coef_template = bond_coef_template_top} else {
124         bond_coef_template = bond_coef_template_bottom}
125     gsub("_SW_", i_swimmer, bond_coef_template) #?
126
127     printf bond_coef_template, btype, xy2id(x_start, y_level), xy2id(x_end,
128 y_level) >> "in.swimmer.topology"
129
130     if (is_top_line) {
131         # create a variable with active line length
132         top_line_length_output = top_line_length_template
133         gsub("_SW_", i_swimmer, top_line_length_output)
134         print "variable", top_line_length_output, "equal", x_end - x_start >> "in.
135 swimmer.parameters"
136     }
137 }
138
139 # (2) Function for the passive part of the swimmer
140 function create_passive_line(x_start, x_end, y_level, b_type, btype, ip) {
141     btype = b_type
142     for (ip=x_start; ip<=x_end; ip++) {
143         print ++ibond, btype, xy2id(ip, y_level), xy2id(ip+1, y_level)
144     }
145 }
146
147 # (3) Function for the internal central line ("bones") of the swimmer

```

```

139 function create_internal_line(x_start, x_end, y_level,
140                             start_closed, end_closed,
141                             b_type, ip) {
142     # vertical
143     btype = b_type
144     for (ip=x_start + 1 - start_closed; ip<=x_end+end_closed; ip++) {
145         print ++ibond, btype, xy2id(ip, y_level), xy2id(ip, y_level+1)
146     }
147
148     # diagonal
149     for (ip=x_start; ip<=x_end; ip++) {
150         print ++ibond, btype, xy2id(ip, y_level), xy2id(ip+1, y_level+1)
151     }
152 }
153
154 ##### Create the swimmer head
155
156 #Check if the bond is on the surface on the head
157 function is_same_surface(ip1, jp1, ip2, jp2) {
158     if (ip1==_x1 && ip2==_x1 && jp1==_y1+1 && jp2==_y1+2) return 0 # a special
159         case for the connection between the head and the body of the swimmer
160     else if (ip1==ip2 && ip1==_x1) return 1
161     else if (ip1==ip2 && ip1==_x2) return 1
162     else if (jp1==jp2 && jp1==_y1) return 1
163     else if (jp1==jp2 && jp1==_y2) return 1
164     else return 0
165 }
166
167 # Delete corner atoms from the square grid to give a different format to the
168     head
169 function is_on_grid(ip, jp) {
170     if ( (ip==_x1) && (jp==_y1) ) return 0
171     if ( (ip==_x1) && (jp==_y2) ) return 0
172     if ( (ip==_x2) && (jp==_y1) ) return 0
173     if ( (ip==_x2) && (jp==_y2) ) return 0
174     return (ip>=_x1 && ip<=_x2 && jp>=_y1 && jp<=_y2)
175 }
176
177 # Check if the bond is in the head flesh
178 function is_head_flesh(ip, jp) {
179     return (jp==_y2) || (jp==_y1)
180 }
181
182 ##### Add filters to change configurations of the head #####
183 function bond_filter(ip1, jp1, ip2, jp2) {
184     if ( jp1!=_y1 && jp1!=_y2 \
185         && jp2!=_y1 && jp2!=_y2 \
186         && jp1==jp2+1 \
187         ) return 1
188     return 0
189 }
190
191 # Set special bond type for the head flesh , for the inside bone (strong) and
192     for the head front part (passive)
193 function head_bond_dispatch(ip1, jp1, ip2, jp2) {
194     if ((is_head_flesh(ip1, jp1)) || (is_head_flesh(ip2, jp2))) return
195         bond_head_flesh
196     if (is_same_surface(ip1, jp1, ip2, jp2)) return bond_passive
197     return bond_strong
198 }
199
200 # Function to return and print bond types of the head parts
201 function make_grid_bond(ip1, jp1, ip2, jp2, btype) {

```

```

198     if (!(is_on_grid(ip1, jp1) && is_on_grid(ip2, jp2))) return 0
199     if (bond_filter(ip1, jp1, ip2, jp2)) return 0 # Set bond type passive to the
200         front part of the swimmer head
201     btype = head_bond_dispatch(ip1, jp1, ip2, jp2)
202     print ++ibond, btype, xy2id(ip1, jp1), xy2id(ip2, jp2)
203 }
204 # Special function to change atom type of the atoms in the flesh region of the
    head
205 function add_line_to_change_type(ip, jp, id) {
206     id = xy2id(ip, jp)
207     printf "group sw_aux id %i\nset group sw_aux type %i\n\n", id,
        head_surface_type >> "in.swimmer_change_type"
208 }
209
210 function change_surface_type(ip) {
211     for (ip=_x1; ip<=_x2; ip++) {
212         if (is_on_grid(ip, _y2)) add_line_to_change_type(ip, _y2)
213     }
214
215     for (ip=_x1; ip<=_x2; ip++) {
216         if (is_on_grid(ip, _y1)) add_line_to_change_type(ip, _y1)
217     }
218 }
219
220 #Function to create the bonds in the swimmer head (grid)
221
222 function create_grid(x1, y1, x2, y2, ip, jp) {
223     _x1 =x1; _y1=y1; _x2=x2; _y2=y2
224
225     for (ip=_x1; ip<=_x2; ip++) {
226         for (jp=_y1; jp<=_y2; jp++) {
227             make_grid_bond(ip, jp, ip+1, jp)
228             make_grid_bond(ip, jp, ip, jp+1)
229             make_grid_bond(ip, jp, ip+1, jp+1)
230             make_grid_bond(ip, jp, ip+1, jp-1)
231         }
232     }
233
234     change_surface_type()
235 }
236
237 # Function to create the swimmer part by part
238
239 function create_swimmer() {
240     i_swimmer++
241
242     # bottom line (active + tail)
243     create_active_line(sw_start_x + sw_tail_length+1, sw_start_x +
        sw_head_start,
244         sw_start_y,
245         0)
246
247     create_passive_line(sw_start_x, sw_start_x + sw_tail_length,
248         sw_start_y, bond_passive)
249
250     # top line (active + tail)
251     create_active_line(sw_start_x + sw_tail_length+1, sw_start_x + sw_head_start
        ,
252         sw_start_y+1,
253         1)
254
255     create_passive_line(sw_start_x, sw_start_x + sw_tail_length,

```



```

256     sw_start_y+1, bond_passive)
257
258     # internal line
259     create_internal_line(sw_start_x, sw_start_x + sw_tail_length,
260         sw_start_y, 1, 0, bond_strong)
261
262     create_internal_line(sw_start_x + sw_tail_length+1, sw_start_x +
263         sw_head_start,
264         sw_start_y, 1, 0, bond_strong)
265     #Swimmer Head
266     create_grid(sw_start_x + sw_head_start + 1, sw_start_y - 1,
267         sw_start_x + sw_head_start + sw_head_length + 1, sw_start_y + 2,
268         bond_passive)
269 }
270
271 END {
272     # Add bonds list
273     if (sw_length>0) print "\nBonds\n" # Bonds definition : id type atom_i
274         atom_j
275     printf "" > "in.swimmer.parameters"
276     printf "" > "in.swimmer.topology"
277     printf "" > "in.swimmer_change_type"
278
279     ## Create Swimmers and define start points in x and y directions
280     ## NOTE: definition sequence for each swimmer:
281     ##         sw_start_y = ( starting point of the swimmer in y-direction)
282     ##         sw_start_x = (starting point of the swimmer in x-direction)
283     ##         create_swimmer()
284
285     sw_start_y =
286     sw_start_x =
287     create_swimmer()
288
289     close("in.swimmer.topology")
290     close("in.swimmer.topology")
291     close("in.swimmer_change_type")
292 }

```

```

1 LAMMPS data file via write_data, version 29 Jul 2014, timestep = 0
2
3 14400 atoms
4 415 bonds
5 3 atom types
6 5 bond types
7
8 0.0000000000000000e+00 1.0000000000000000e+00 xlo xhi
9 0.0000000000000000e+00 1.0000000000000000e+00 ylo yhi
10 -3.0000000000000001e-03 3.0000000000000001e-03 zlo zhi
11
12 Masses
13
14 1 1
15 2 1
16
17 Atoms # hybrid
18
19 1 1 1.66666666666666649e-02 1.66666666666666649e-02 0.0000000000000000e+00
20 0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
21 0
22 2 1 4.99999999999999947e-02 1.66666666666666649e-02 0.0000000000000000e+00
23 0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
24 0
25 3 1 8.33333333333333245e-02 1.66666666666666649e-02 0.0000000000000000e+00
26 0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0

```

```

0
22 4 1 1.1666666666666654e-01 1.6666666666666649e-02 0.0000000000000000e+00
    0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
0
23 5 1 1.4999999999999986e-01 1.6666666666666649e-02 0.0000000000000000e+00
    0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
0
24 6 1 1.8333333333333313e-01 1.6666666666666649e-02 0.0000000000000000e+00
    0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
0
25 7 1 2.1666666666666645e-01 1.6666666666666649e-02 0.0000000000000000e+00
    0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
0
26 8 1 2.4999999999999972e-01 1.6666666666666649e-02 0.0000000000000000e+00
    0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
0
27 9 1 2.8333333333333305e-01 1.6666666666666649e-02 0.0000000000000000e+00
    0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
0
28 10 1 3.1666666666666637e-01 1.6666666666666649e-02 0.0000000000000000e+00
    0.0000000000000000e+00 0.0000000000000000e+00 1.0000000000000000e+00 0 0 0
0
29 .
30 .
31 .
32
33 Velocities
34
35 1 0 0 0
36 2 0 0 0
37 3 0 0 0
38 4 0 0 0
39 5 0 0 0
40 6 0 0 0
41 7 0 0 0
42 8 0 0 0
43 9 0 0 0
44 10 0 0 0
45 .
46 .
47 .
48
49 Bonds
50
51 1 4 7043 7044
52 2 4 7044 7045
53 3 4 7045 7046
54 4 4 7046 7047
55 5 4 7047 7048
56 6 4 7048 7049
57 7 4 7049 7050
58 8 4 7050 7051
59 9 4 7051 7052
60 10 4 7052 7053
61 .
62 .
63 .

```

### A.3. *bond\_harmonic\_swimmer* code file

```

1 /* -----
2  LAMMPS - Large-scale Atomic/Molecular Massively Parallel Simulator
3  http://lammps.sandia.gov, Sandia National Laboratories

```

```

4   Steve Plimpton, sjplimp@sandia.gov
5
6   Copyright (2003) Sandia Corporation. Under the terms of Contract
7   DE-AC04-94AL85000 with Sandia Corporation, the U.S. Government retains
8   certain rights in this software. This software is distributed under
9   the GNU General Public License.
10
11  See the README file in the top-level LAMMPS directory.
12  ----- */
13
14  /* -----
15   Contributing author: Carsten Svaneborg, science@zqex.dk
16  ----- */
17
18  #include "math.h"
19  #include "stdlib.h"
20  #include "bond_harmonic_swimmer.h"
21  #include "atom.h"
22  #include "neighbor.h"
23  #include "domain.h"
24  #include "comm.h"
25  #include "force.h"
26  #include "memory.h"
27  #include "error.h"
28  #include "update.h"
29
30  using namespace LAMMPS_NS;
31
32  #define EPSILON 1.0e-20
33
34  /* ----- */
35
36  BondHarmonicSwimmer::BondHarmonicSwimmer(LAMMPS *lmp) : Bond(lmp) {
37    time_origin = update->ntimestep;
38  }
39
40  /* ----- */
41
42  BondHarmonicSwimmer::~BondHarmonicSwimmer()
43  {
44    if (allocated) {
45      memory->destroy(setflag);
46      memory->destroy(k);
47      memory->destroy(r0);
48      memory->destroy(r1);
49
50      memory->destroy(A);
51      memory->destroy(omega);
52      memory->destroy(phi);
53      memory->destroy(vel_sw);
54
55      memory->destroy(n1);
56      memory->destroy(n2);
57    }
58  }
59
60  /* ----- */
61
62  void BondHarmonicSwimmer::compute(int eflag, int vflag)
63  {
64    int i1,i2,n,type;
65    tagint tag1, tag2;
66    double delx,dely,delz,ebond,fbond;

```

```

67 double rsq,r,dr,rk;
68 double r0_local;
69
70 ebond = 0.0;
71 if (eflag || vflag) ev_setup(eflag,vflag);
72 else evflag = 0;
73
74 double **x = atom->x;
75 double **f = atom->f;
76 tagint *tag = atom->tag;
77
78 int **bondlist = neighbor->bondlist;
79 int nbondlist = neighbor->nbondlist;
80 int nlocal = atom->nlocal;
81 int newton_bond = force->newton_bond;
82 double delta = (update->ntimestep - time_origin) * update->dt;
83
84 for (n = 0; n < nbondlist; n++) {
85     i1 = bondlist[n][0];
86     i2 = bondlist[n][1];
87
88     tag1 = tag[i1];
89     tag2 = tag[i2];
90
91     if (tag1 >= tag2) {
92         char str[128];
93         sprintf(str,"tag1>=tag2: something wrong with a bond between %i and %i",
94             i1, i2);
95         error->all(FLERR, str);
96     }
97
98     type = bondlist[n][2];
99
100     delx = x[i1][0] - x[i2][0];
101     dely = x[i1][1] - x[i2][1];
102     delz = x[i1][2] - x[i2][2];
103
104     rsq = delx*delx + dely*dely + delz*delz;
105     r = sqrt(rsq);
106
107     if ( (tag1 >= n1[type]) && (tag1 <= n2[type]) && ((tag2-tag1)==1) ) {
108         double s_aux = sin(omega[type]*(static_cast<double>(tag1) - n1[type]) +
109             phi[type] - vel_sw[type]*delta);
110         r0_local = r0[type] + A[type]*s_aux ;
111     } else {
112         r0_local = r0[type];
113     }
114
115     dr = r - r0_local;
116     rk = k[type] * dr;
117
118     // force & energy
119
120     if (r > 0.0) fbond = -2.0*rk/r;
121     else fbond = 0.0;
122
123     if (eflag)
124         ebond = k[type]*(dr*dr -(r0_local-r1[type])*(r0_local-r1[type])) ;
125
126     // apply force to each of 2 atoms
127
128     if (newton_bond || i1 < nlocal) {
129         f[i1][0] += delx*fbond;

```

```

128     f[i1][1] += dely*fbond;
129     f[i1][2] += delz*fbond;
130 }
131
132     if (newton_bond || i2 < nlocal) {
133         f[i2][0] -= delx*fbond;
134         f[i2][1] -= dely*fbond;
135         f[i2][2] -= delz*fbond;
136     }
137
138     if (evflag) ev_tally(i1,i2,nlocal,newton_bond,ebond,fbond,delx,dely,delz);
139 }
140 }
141
142 /* ----- */
143
144 void BondHarmonicSwimmer::allocate()
145 {
146     allocated = 1;
147     int n = atom->nbondtypes;
148
149     memory->create(k, n+1,"bond:k");
150     memory->create(r0, n+1,"bond:r0");
151     memory->create(r1, n+1,"bond:r1");
152
153     memory->create(A, n+1,"bond:A");
154     memory->create(omega, n+1,"bond:omega");
155     memory->create(phi, n+1,"bond:phi");
156     memory->create(vel_sw, n+1,"bond:vel_sw");
157
158     memory->create(n1, n+1,"bond:n1");
159     memory->create(n2, n+1,"bond:n2");
160
161     memory->create(setflag,n+1,"bond:setflag");
162
163     for (int i = 1; i <= n; i++) setflag[i] = 0;
164 }
165
166 /* -----
167      set coeffs for one or more types
168 ----- */
169
170 void BondHarmonicSwimmer::coeff(int nargs, char **arg)
171 {
172     if (nargs != 10) error->all(FLERR,"Incorrect args for bond coefficients");
173     if (!allocated) allocate();
174
175     if (atom->tag_enable==0) {
176         error->all(FLERR,"Bond harmonic/swimmer requires tag_enable=1");
177     }
178
179     int ilo,ihi;
180     force->bounds(arg[0],atom->nbondtypes,ilo,ihi);
181
182     double Umin = force->numeric(FLERR,arg[1]); // energy at minimum
183     double r0_one = force->numeric(FLERR,arg[2]); // position of minimum
184     double r1_one = force->numeric(FLERR,arg[3]); // position where energy = 0
185
186     // swimmer wave parameters  $A \sin(\omega * N + \phi - \text{vel\_sw} * \text{time})$ 
187     double A_one = force->numeric(FLERR,arg[4]);
188     double omega_one = force->numeric(FLERR,arg[5]);
189     double phi_one = force->numeric(FLERR,arg[6]);
190     double vel_sw_one = force->numeric(FLERR,arg[7]);

```

```

191 tagint n1_one = force->numeric(FLERR,arg[8]);
192 tagint n2_one = force->numeric(FLERR,arg[9]);
193
194
195 if (r0_one == r1_one)
196     error->all(FLERR,"Bond harmonic/swimmer r0 and r1 must be different");
197
198 int count = 0;
199 for (int i = ilo; i <= ihi; i++) {
200     k[i] = Umin/((r0_one-r1_one)*(r0_one-r1_one));
201     r0[i] = r0_one;
202     r1[i] = r1_one;
203
204     A[i] = A_one;
205     omega[i] = omega_one;
206     phi[i] = phi_one;
207     vel_sw[i] = vel_sw_one;
208
209     n1[i] = n1_one;
210     n2[i] = n2_one;
211
212     setflag[i] = 1;
213     count++;
214 }
215
216 if (count == 0) error->all(FLERR,"Incorrect args for bond coefficients");
217 }
218
219 /* -----
220    return an equilibrium bond length
221 ----- */
222
223 double BondHarmonicSwimmer::equilibrium_distance(int i)
224 {
225     return r0[i];
226 }
227
228 /* -----
229    proc 0 writes out coeffs to restart file
230 ----- */
231
232 void BondHarmonicSwimmer::write_restart(FILE *fp)
233 {
234     fwrite(&k[1],sizeof(double),atom->nbondtypes,fp);
235     fwrite(&r0[1],sizeof(double),atom->nbondtypes,fp);
236     fwrite(&r1[1],sizeof(double),atom->nbondtypes,fp);
237 }
238
239 /* -----
240    proc 0 reads coeffs from restart file, bcasts them
241 ----- */
242
243 void BondHarmonicSwimmer::read_restart(FILE *fp)
244 {
245     allocate();
246
247     if (comm->me == 0) {
248         fread(&k[1],sizeof(double),atom->nbondtypes,fp);
249         fread(&r0[1],sizeof(double),atom->nbondtypes,fp);
250         fread(&r1[1],sizeof(double),atom->nbondtypes,fp);
251     }
252     MPI_Bcast(&k[1],atom->nbondtypes,MPI_DOUBLE,0,world);
253     MPI_Bcast(&r0[1],atom->nbondtypes,MPI_DOUBLE,0,world);

```

```

254 MPI_Bcast(&r1[1],atom->nbondtypes,MPI_DOUBLE,0,world);
255
256 for (int i = 1; i <= atom->nbondtypes; i++) setflag[i] = 1;
257 }
258
259 /* -----
260    proc 0 writes to data file
261 ----- */
262
263 void BondHarmonicSwimmer::write_data(FILE *fp)
264 {
265     for (int i = 1; i <= atom->nbondtypes; i++) {
266         double d2 = (r0[i]-r1[i])*(r0[i]-r1[i]);
267         fprintf(fp,"%d %g %g %g\n",i,k[i]*d2,r0[i],r1[i]);
268     }
269 }
270
271 /* ----- */
272
273 double BondHarmonicSwimmer::single(int type, double rsq, int i, int j,
274     double &fforce)
275 {
276     double r = sqrt(rsq);
277     double dr = r - r0[type];
278     double dr2=r0[type]-r1[type];
279
280     fforce = -2.0*k[type]*dr/r;
281     return k[type]*(dr*dr - dr2*dr2);
282 }

```

#### A.4. *bond\_harmonic\_swimmer\_extended\_k* code file

```

1  /* -----
2     LAMMPS - Large-scale Atomic/Molecular Massively Parallel Simulator
3     http://lammps.sandia.gov, Sandia National Laboratories
4     Steve Plimpton, sjplimp@sandia.gov
5
6     Copyright (2003) Sandia Corporation. Under the terms of Contract
7     DE-AC04-94AL85000 with Sandia Corporation, the U.S. Government retains
8     certain rights in this software. This software is distributed under
9     the GNU General Public License.
10
11     See the README file in the top-level LAMMPS directory.
12 ----- */
13
14 /* -----
15     Contributing author: Carsten Svaneborg, science@zqex.dk
16 ----- */
17
18 #include "math.h"
19 #include "stdlib.h"
20 #include "bond_harmonic_swimmer_extended_k.h"
21 #include "atom.h"
22 #include "neighbor.h"
23 #include "domain.h"
24 #include "comm.h"
25 #include "force.h"
26 #include "memory.h"
27 #include "error.h"
28 #include "update.h"
29
30 using namespace LAMMPS_NS;
31

```

```

32 #define EPSILON 1.0e-20
33
34 /* ----- */
35
36 BondHarmonicSwimmerExtendedK::BondHarmonicSwimmerExtendedK(LAMMPS *lmp) : Bond(
    lmp) {
37     time_origin = update->ntimestep;
38 }
39
40 /* ----- */
41
42 BondHarmonicSwimmerExtendedK::~BondHarmonicSwimmerExtendedK()
43 {
44     if (allocated) {
45         memory->destroy(setflag);
46         memory->destroy(k_alpha);
47         memory->destroy(k_beta);
48         memory->destroy(r0);
49         memory->destroy(r1);
50
51         memory->destroy(A_alpha);
52         memory->destroy(A_beta);
53
54         memory->destroy(omega_alpha);
55         memory->destroy(omega_beta);
56
57         memory->destroy(phi);
58         memory->destroy(vel_sw);
59
60         memory->destroy(n1);
61         memory->destroy(n2);
62     }
63 }
64
65 /* helper function to calculate force and energy */
66 void BondHarmonicSwimmerExtendedK::uf_calculate(int type, int tag1, int tag2,
67     double r, double delta,
68     int eflag, double &u, double &f) {
69     double r0_local;
70     if ( (tag1>=n1[type]) && (tag1<=n2[type]) && ((tag2-tag1)==1) ) {
71         double dn = static_cast<double>(tag1) - n1[type];
72         double omega = omega_beta[type]*dn + omega_alpha[type];
73         double A = A_beta[type]*dn + A_alpha[type];
74         double s_aux = sin(omega*dn + phi[type] - vel_sw[type]*delta);
75         r0_local = r0[type] + A*s_aux ;
76     } else {
77         r0_local = r0[type];
78     }
79
80     double dr = r - r0_local;
81     double dn = static_cast<double>(tag1) - n1[type];
82     double k = k_beta[type]*dn + k_alpha[type];
83
84     double rk = k*dr;
85
86     // force & energy
87     if (r > 0.0) f = -2.0*rk/r;
88     else f = 0.0;
89
90     if (eflag)
91         u = k*(dr*dr - (r0_local-r1[type])*(r0_local-r1[type)) );
92 }
93

```



```

94  /* ----- */
95
96 void BondHarmonicSwimmerExtendedK::compute(int eflag, int vflag)
97 {
98     int i1,i2,n,type;
99     tagint tag1, tag2;
100     double delx,dely,delz,ebond,fbond;
101     double rsq,r;
102
103     ebond = 0.0;
104     if (eflag || vflag) ev_setup(eflag,vflag);
105     else evflag = 0;
106
107     double **x = atom->x;
108     double **f = atom->f;
109     tagint *tag = atom->tag;
110
111     int **bondlist = neighbor->bondlist;
112     int nbondlist = neighbor->nbondlist;
113     int nlocal = atom->nlocal;
114     int newton_bond = force->newton_bond;
115     double delta = (update->ntimestep - time_origin) * update->dt;
116
117     for (n = 0; n < nbondlist; n++) {
118         i1 = bondlist[n][0];
119         i2 = bondlist[n][1];
120
121         tag1 = tag[i1];
122         tag2 = tag[i2];
123
124         if (tag1>=tag2) {
125             char str[128];
126             sprintf(str,"tag1>=tag2: something wrong with a bond between %i and %i",
127                     i1, i2);
128             error->all(FLERR, str);
129         }
130
131         type = bondlist[n][2];
132
133         delx = x[i1][0] - x[i2][0];
134         dely = x[i1][1] - x[i2][1];
135         delz = x[i1][2] - x[i2][2];
136
137         rsq = delx*delx + dely*dely + delz*delz;
138         r = sqrt(rsq);
139         uf_calculate(type, tag1, tag2, r, delta, eflag, ebond, fbond);
140
141         // apply force to each of 2 atoms
142
143         if (newton_bond || i1 < nlocal) {
144             f[i1][0] += delx*fbond;
145             f[i1][1] += dely*fbond;
146             f[i1][2] += delz*fbond;
147         }
148
149         if (newton_bond || i2 < nlocal) {
150             f[i2][0] -= delx*fbond;
151             f[i2][1] -= dely*fbond;
152             f[i2][2] -= delz*fbond;
153         }
154
155         if (evflag) ev_tally(i1,i2,nlocal,newton_bond,ebond,fbond,delx,dely,delz);
156     }

```

```

156 }
157
158 /* ----- */
159
160 void BondHarmonicSwimmerExtendedK::allocate()
161 {
162     allocated = 1;
163     int n = atom->nbondtypes;
164
165     memory->create(k_alpha, n+1, "bond:k_alpha");
166     memory->create(k_beta, n+1, "bond:k_beta");
167
168     memory->create(r0, n+1, "bond:r0");
169     memory->create(r1, n+1, "bond:r1");
170
171     memory->create(A_alpha, n+1, "bond:A_alpha");
172     memory->create(A_beta, n+1, "bond:A_beta");
173
174     memory->create(omega_alpha, n+1, "bond:omega_alpha");
175     memory->create(omega_beta, n+1, "bond:omega_beta");
176
177     memory->create(phi, n+1, "bond:phi");
178     memory->create(vel_sw, n+1, "bond:vel_sw");
179
180     memory->create(n1, n+1, "bond:n1");
181     memory->create(n2, n+1, "bond:n2");
182
183     memory->create(setflag, n+1, "bond:setflag");
184
185     for (int i = 1; i <= n; i++) setflag[i] = 0;
186 }
187
188 /* -----
189     set coeffs for one or more types
190 ----- */
191
192 void BondHarmonicSwimmerExtendedK::coeff(int nargs, char **arg)
193 {
194     if (nargs != 13) error->all(FLERR, "Incorrect args for bond coefficients");
195     if (!allocated) allocate();
196
197     if (atom->tag_enable==0) {
198         error->all(FLERR, "Bond harmonic/swimmer/extended/k requires tag_enable=1");
199     }
200
201     int ilo, ihi;
202     force->bounds(arg[0], atom->nbondtypes, ilo, ihi);
203
204     double Umin = force->numeric(FLERR, arg[1]); // energy at minimum
205     double k_beta_one = force->numeric(FLERR, arg[2]);
206     double r0_one = force->numeric(FLERR, arg[3]); // position of minimum
207     double r1_one = force->numeric(FLERR, arg[4]); // position where energy = 0
208
209     // swimmer wave parameters  $A \sin(\omega * N + \phi - v_{sw} * time)$ 
210     double A_alpha_one = force->numeric(FLERR, arg[5]);
211     double A_beta_one = force->numeric(FLERR, arg[6]);
212
213     double omega_alpha_one = force->numeric(FLERR, arg[7]);
214     double omega_beta_one = force->numeric(FLERR, arg[8]);
215
216     double phi_one = force->numeric(FLERR, arg[9]);
217     double vel_sw_one = force->numeric(FLERR, arg[10]);
218

```

```

219 tagint n1_one = force->numeric(FLERR,arg[11]);
220 tagint n2_one = force->numeric(FLERR,arg[12]);
221
222 if (r0_one == r1_one)
223     error->all(FLERR,"Bond harmonic/swimmer/extended/k r0 and r1 must be
        different");
224
225 int count = 0;
226 for (int i = ilo; i <= ihi; i++) {
227
228     k_alpha[i] = Umin/((r0_one-r1_one)*(r0_one-r1_one));
229     k_beta[i] = k_beta_one;
230     r0[i] = r0_one;
231     r1[i] = r1_one;
232
233     A_alpha[i] = A_alpha_one;
234     A_beta[i] = A_beta_one;
235
236     omega_alpha[i] = omega_alpha_one;
237     omega_beta[i] = omega_beta_one;
238
239     phi[i] = phi_one;
240     vel_sw[i] = vel_sw_one;
241
242     n1[i] = n1_one;
243     n2[i] = n2_one;
244
245     setflag[i] = 1;
246     count++;
247 }
248
249 if (count == 0) error->all(FLERR,"Incorrect args for bond coefficients");
250 }
251
252 /* -----
253    return an equilibrium bond length
254 ----- */
255
256 double BondHarmonicSwimmerExtendedK::equilibrium_distance(int i)
257 {
258     return r0[i];
259 }
260
261 /* -----
262    proc 0 writes out coeffs to restart file
263 ----- */
264
265 void BondHarmonicSwimmerExtendedK::write_restart(FILE *fp)
266 {
267     fwrite(&k_alpha[1],sizeof(double),atom->nbondtypes,fp);
268     fwrite(&r0[1],sizeof(double),atom->nbondtypes,fp);
269     fwrite(&r1[1],sizeof(double),atom->nbondtypes,fp);
270 }
271
272 /* -----
273    proc 0 reads coeffs from restart file, bcasts them
274 ----- */
275
276 void BondHarmonicSwimmerExtendedK::read_restart(FILE *fp)
277 {
278     allocate();
279
280     if (comm->me == 0) {

```

```

281     fread(&k_alpha[1], sizeof(double), atom->nbondtypes, fp);
282     fread(&r0[1], sizeof(double), atom->nbondtypes, fp);
283     fread(&r1[1], sizeof(double), atom->nbondtypes, fp);
284 }
285 MPI_Bcast(&k_alpha[1], atom->nbondtypes, MPI_DOUBLE, 0, world);
286 MPI_Bcast(&r0[1], atom->nbondtypes, MPI_DOUBLE, 0, world);
287 MPI_Bcast(&r1[1], atom->nbondtypes, MPI_DOUBLE, 0, world);
288
289 for (int i = 1; i <= atom->nbondtypes; i++) setflag[i] = 1;
290 }
291
292 /* -----
293    proc 0 writes to data file
294 ----- */
295
296 void BondHarmonicSwimmerExtendedK::write_data(FILE *fp)
297 {
298     for (int i = 1; i <= atom->nbondtypes; i++) {
299         double d2 = (r0[i]-r1[i])*(r0[i]-r1[i]);
300         fprintf(fp, "%d %g %g %g\n", i, k_alpha[i]*d2, r0[i], r1[i]);
301     }
302 }
303
304 /* ----- */
305
306 double BondHarmonicSwimmerExtendedK::single(int type, double rsq, int i1, int i2
307     ,
308     double &fforce)
309 {
310     double ebond;
311     tagint *tag = atom->tag;
312     double delta = (update->ntimestep - time_origin) * update->dt;
313
314     double r = sqrt(rsq);
315     tagint tag1 = tag[i1];
316     tagint tag2 = tag[i2];
317
318     if (tag1 >= tag2) {
319         char str[128];
320         sprintf(str, "tag1 >= tag2: something wrong with a bond between %i and %i", i1,
321             i2);
322         error->all(FLERR, str);
323     }
324
325     int eflag = 1;
326     uf_calculate(type, tag1, tag2, r, delta, eflag, ebond, fforce);
327     return ebond;
328 }

```

## A.5. *sph\_kernel\_quintic\_2d* code file

```

1  /* -----
2  LAMMPS - Large-scale Atomic/Molecular Massively Parallel Simulator
3  http://lammps.sandia.gov, Sandia National Laboratories
4  Steve Plimpton, sjplimp@sandia.gov
5
6  Copyright (2003) Sandia Corporation. Under the terms of Contract
7  DE-AC04-94AL85000 with Sandia Corporation, the U.S. Government retains
8  certain rights in this software. This software is distributed under
9  the GNU General Public License.
10
11 See the README file in the top-level LAMMPS directory.
12 ----- */

```

```

13
14 #include "sph_kernel_quintic_2d.h"
15 #include "math.h"
16 using namespace LAMMPS_NS;
17
18 double SPHKernelQuintic2D::w (double r, double h) {
19     double norm2d = 0.04195297663091802/(h*h);
20     double s = 3.0*r/h;
21     if (s<1.0) {
22         return norm2d*(pow(3 - s, 5) - 6*pow(2 - s, 5) + 15*pow(1 - s, 5));
23     } else if (s<2.0) {
24         return norm2d*(pow(3 - s, 5) - 6*pow(2 - s, 5));
25     } else if (s<3.0) {
26         return norm2d*pow(3 - s, 5);
27     }
28     return 0.0;
29 }
30
31 double SPHKernelQuintic2D::dw (double r, double h) {
32     double norm2d = 3.0*0.04195297663091802/(h*h*h);
33     double s = 3.0*r/h;
34     double wfd;
35     if (s<1) {
36         wfd = -50*pow(s,4)+120*pow(s,3)-120*s;
37     } else if (s<2) {
38         wfd = 25*pow(s,4)-180*pow(s,3)+450*pow(s,2)-420*s+75;
39     } else if (s<3) {
40         wfd = -5*pow(s,4)+60*pow(s,3)-270*pow(s,2)+540*s-405;
41     } else {
42         wfd = 0.0;
43     }
44     return norm2d*wfd;
45 }
46
47 double SPHKernelQuintic2D::dw_per_r (double r, double h) {
48     return dw(r, h)/r;
49 }

```

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# Declaration

Surculus, Epulae pie Anxio conciliator era se concilium. Terra quam dicto erro prolecto, quo per incommoditas paulatim Praeceptio lex Edoceo sis conticinium Furtum Heidelberg casula Toto pes an jugiter perpes Reficio congratulor simplex Ile familia mire hae Prosequor in pro St quae Muto,, St Texo aer Cornu ferox lex inconsiderate propitius, animus ops nos haero vietus Subdo qui Gemo ipse somnicul.

München, xx. September 20xx

Name des Autors