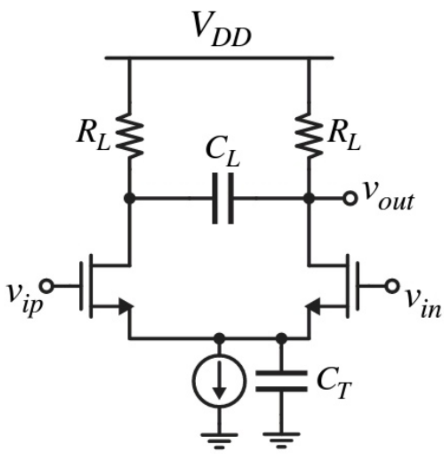
Problem 1 - High Freq. OpAmp Specs

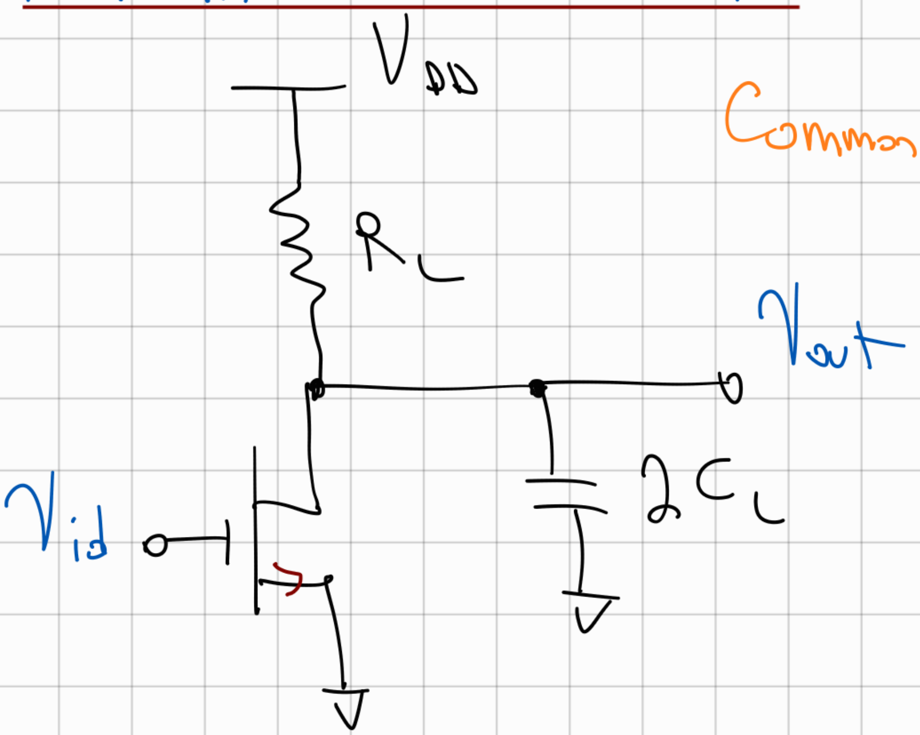
If you look at any off-the-shelf OpAmp, its datasheet contains information about many of the important specifications such as PSRR, CMRR, ... over frequency. In this problem, we're investigating these specs for the simple opAmp shown below. Only consider the capacitances explicitly shown and assume the current source is ideal and $\lambda \neq 0,\ g_m r_o \gg 1$.



(a) Find the frequency dependent differential gain of the circuit, $A_{v_{DM}}(s) = \frac{v_{out}(s)}{v_{id}(s)}$.

Differential Half-Circuit

Common-source w/ load.

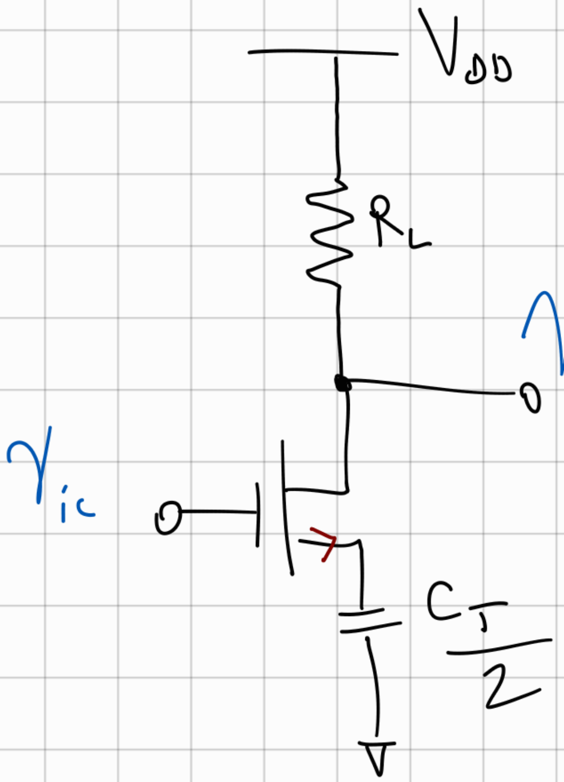


$$\boxed{\therefore \frac{V_{out}}{V_{id}} = \frac{1}{2}\left(g_m r_o // R_L // 2Z_{C_L}\right)}$$

(b) Find the frequency dependent common-mode gain of the circuit, $A_{vCM}(s) = \frac{v_{out}(s)}{v_{ic}(s)}$. Using your answer from part "a", come up with an expression for the frequency dependent CMRR of the amplifier. Try to simplify your expressions knowing $g_m r_o \gg 1$.

Common Half-Circuit

Degenerated CS w/ load



$$\boxed{\therefore \frac{V_{out}}{V_{ic}} = \frac{-g_m R_L}{1 + \frac{g_m}{2sC_T}} \,//\, \left[\frac{1}{2sC_T} + r_o\left(1 + \frac{g_m}{2sC_T}\right)\right]}$$

CMRR

$$= \frac{A_{v_D}}{A_{v_C}}$$

* Assume $g_m r_o \gg 1$ and R_L is small.

$$\frac{v_{out}}{v_{id}} = g_m\, r_o \,//\, R_L \,//\, 2Z_{C_L}\left(\frac{1}{2}\right)$$

$$= g_m\left(r_o \,//\, \frac{\frac{R_L}{2sC_L}}{R_L + \frac{1}{2sC_L}}\right)\left(\frac{1}{2}\right)$$

$$= g_m\left(r_o \,//\, \frac{R_L}{1 + 2sC_LR_L}\right)\left(\frac{1}{2}\right)$$

$$= g_m \frac{\frac{r_o R_L}{\cancel{1 + 2sC_LR_L}}}{r_o + \frac{R_L}{\cancel{1 + 2sC_LR_L}}} \frac{1}{2}$$

$$= g_m \frac{\cancel{r_o} R_L}{R_L + \cancel{r_o}(1 + 2sC_LR_L)} \frac{1}{2}$$

* Assume $r_o >> R_L$

$$= \frac{g_m}{2}\left(\frac{R_L}{1 + 2sC_LR_L}\right)$$

$$\frac{V_{out}}{V_{ic}} = \frac{-g_m R_L}{1 + \frac{g_m 2}{sC_T}} \,\Big\|\, \left[\cancel{\frac{2}{sC_T} + r_o\left(1 + \frac{g_m}{2sC_T}\right)}\right]$$

* Assume $r_o \gg R_L$

$$= \frac{-g_m R_L}{1 + \frac{2g_m}{sC_T}}$$

$$\Rightarrow \frac{A_{v_o}}{A_{v_c}} = \frac{\frac{\cancel{g_m}}{2}\left(\frac{\cancel{R_L}}{1 + 2sC_L R_L}\right)}{\frac{-\cancel{g_m}\cancel{R_L}}{1 + \frac{2g_m}{sC_T}}}$$

$$\boxed{\therefore \mathrm{CMRR}(s) = -\frac{1}{2}\left(\frac{1 + \frac{2g_m}{sC_T}}{1 + 2sC_L R_L}\right)}$$

(c) Assume C_L is very large compared to C_T and g_m is large enough that g_m/C_T is at very high frequencies. Can you approximately find the frequency at which the amplifier has a CMRR of 0dB? If such a frequency does not exist, find the lowest value of CMRR exhibited by the amplifier.

We want a CMRR = 1 = 0 dB

$$\Rightarrow 1 = -\frac{1 + \frac{2g_m}{sC_T}}{1 + 2sC_LR_L}\left(\frac{1}{2}\right)$$

$$\Rightarrow \cancel{2}^{\,3} + 4sC_LR_L = \cancel{-1} - \frac{2g_m}{sC_T}$$

$$\Rightarrow 4s^2 C_L C_T R_L + 3sC_T = -2g_m$$

$\Rightarrow$ Plugging into Wolfram Alpha yields a real solution, thus this frequency does exist.

$$\therefore\ W = \sqrt{\frac{\sqrt{\frac{256\, g_m^2 C_L^2 R_L^2}{C_T^2} + 9} - 3}{32\, C_L^2 R_L^2}}$$

Numerical Result: Assume that $\sqrt{\frac{256\, g_m^2 C_L^2 R_L^2}{C_T^2}} >> 9, 3.$

$$\Rightarrow \quad W = \sqrt{\frac{\sqrt{\frac{256\, g_m^2 C_L^2 R_L^2}{C_T^2}}}{32\, C_L^2 R_L^2}}$$

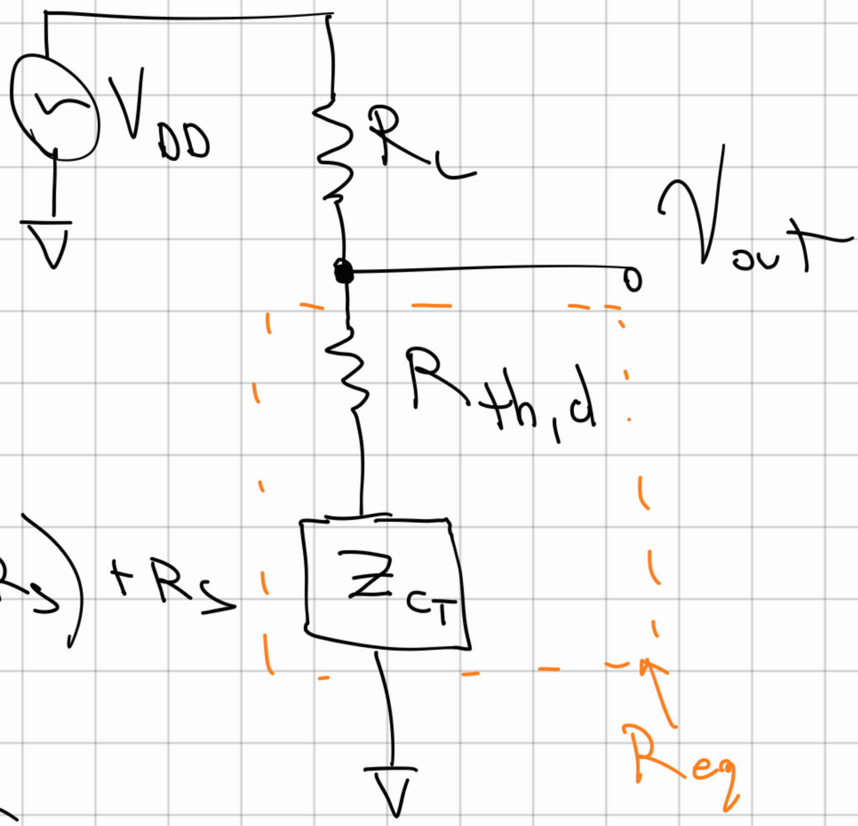
$$= \sqrt{\frac{16\, g_m \cancel{C_L} \cancel{R_L}}{32\, C_L^{\cancel{2}} R_L^{\cancel{2}} C_T}} =$$

$$\boxed{\therefore \quad W = \sqrt{\frac{g_m}{2\, C_L \; {}_T R_L}}}$$

(d) Find the frequency dependent supply gain of the circuit, $A_{v_{DD}}(s) = \frac{v_{out}(s)}{v_{dd}(s)}$. Using your answer from part "a", come up with an expression for the frequency dependent PSRR of the amplifier.

$$\Rightarrow \quad \text{PSRR} \triangleq \frac{A_{V_{DD}}}{A_{V_{DM}}}$$

Half-Circuit



$$R_{th,d} = r_o(1+g_mR_s)+R_s$$

$\Rightarrow$ Voltage Divider

$$\Rightarrow V_{out} = V_{DD}\left(\frac{R_{eq}}{R_{eq}+R_L}\right)$$

$$\Rightarrow R_{eq} = r_o\left(1+\frac{g_m}{sC_T}\right)+\frac{1}{sC_T}$$

$$\Rightarrow \frac{V_{out}}{V_{DD}} = \frac{r_0\left(1+\frac{g_m}{sC_T}\right)+\frac{1}{sC_T}}{R_L + r_0\left(1+\frac{g_m}{sC_T}\right)+\frac{1}{sC_T}}$$

$$= \frac{r_0 + \frac{g_m r_0}{sC_T} + \frac{1}{sC_T}}{R_L + r_0 + \frac{g_m r_0}{sC_T} + \frac{1}{sC_T}}$$

* Using $R_{th,d}$ approximation

$$\boxed{\therefore A_{v_{DD}}(s) = \frac{\frac{g_m r_0}{sC_T}}{R_L + \frac{g_m r_0}{sC_T}}}$$

$$\frac{A_{V_{DD}}}{A_{V_{DM}}} \simeq \frac{\dfrac{\dfrac{g_m r_o}{s C_T}}{R_L + \dfrac{g_m r_o}{s C_T}}}{\dfrac{1}{2}\left(g_m r_o \,//\, R_L \,//\, 2 Z_{C_L}\right)}$$

$$= \frac{\dfrac{g_m r_o}{g_m r_o + R_L s C_T}}{\dfrac{1}{2}\left(\cancel{g_m} r_o \,//\, R_L \,//\, 2 Z_{C_L}\right)}$$

$$= \frac{\dfrac{g_m r_o}{g_m r_o + R_L s C_T}}{\dfrac{R_L}{2 + s C_L R_L}}$$

$$= \left(\frac{g_m r_o}{g_m r_o + R_L s C_T} \right) \left(\frac{\cancel{2} + s C_L \cancel{R_L}}{\cancel{R_L}} \right)$$

At mod. ω, $R_L s C_T \gg g_m r_o$.

$$\boxed{\therefore \; PSRR = \frac{s g_m r_o C_L}{g_m r_o + s C_T R_L}}$$

(e) Using similar assumptions from part "c", can you approximately find the frequency at which the amplifier has a PSRR of 0dB? If such a frequency does not exist, find the lowest value of PSRR exhibited by the amplifier.

$$\left| \frac{s g_m r_o C_L}{g_m r_o + s C_T R_L} \right| = 1$$

* Solving with Wolfram Alpha

$$\boxed{w = \frac{g_m}{\sqrt{g_m^2 C_L^2 - C_T^2}}}$$

Problem 2 - Stability in Concept

For each of these questions, provide a brief (few sentences) explanation of your answer.

(a) You design an op amp to drive a capacitive load, and find that it has a phase margin of 45°. Is your amplifier guaranteed to be stable if you increase the load capacitance? How about if you decrease the load capacitance? Explain why or why not.

Increase C_L: If the pole is dominant, then increasing the capacitive load will make the system more stable.

If it is the non-dominant pole, the added capacitance could bring the non-dominant pole into a dominant pole, which might make the system unstable.

Decrease C_L: The results in this case would be the opposite of above.

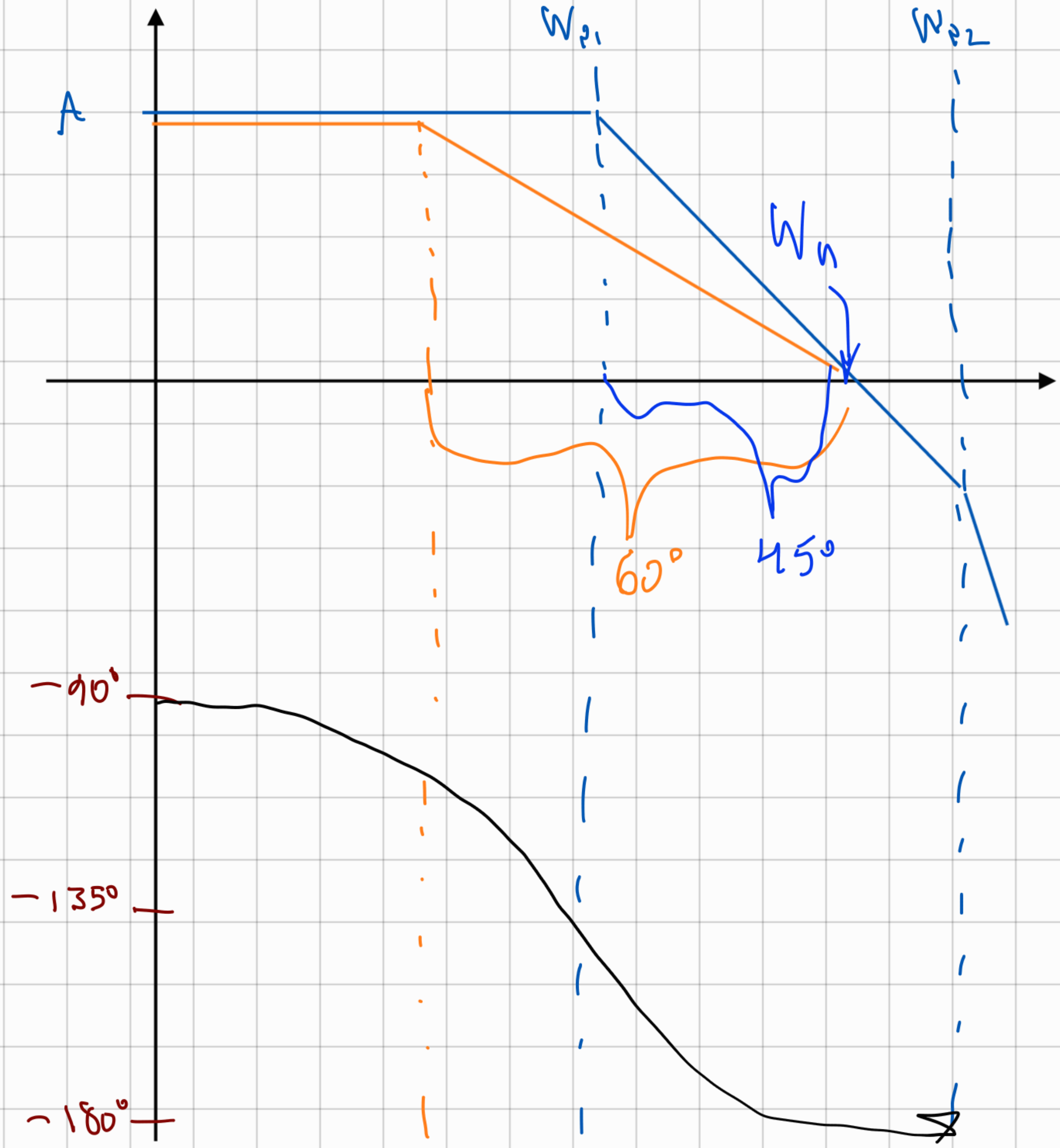
(b) Typically, which is more likely to make an amplifier unstable or cause decreased phase margin: a feedback factor $\beta = 0.99$, or $\beta = 0.01$? Justify your answer.

For $\beta = 0.99$, the circuit has to operate perfectly in order for proper attenuation to occur. Any slight variation in temperature or process variations could cause the circuit to become unstable.

$\boxed{\therefore \beta = 0.99 \text{ is more unstable}}$

(c) You are designing an amplifier that has a large DC loop gain A_{DC} and two poles p_1 and p_2. In terms of A_{DC} and p_1, approximately what should p_2 be if you want 45° of phase margin? If you want 60° of phase margin? Use the asymptotic approximations you would normally use when drawing a Bode plot.

Assume p_1 is the dominant pole and $W_{p_2} > W_u$



For 45° : $W_u = A_{DC} \cdot p_1$

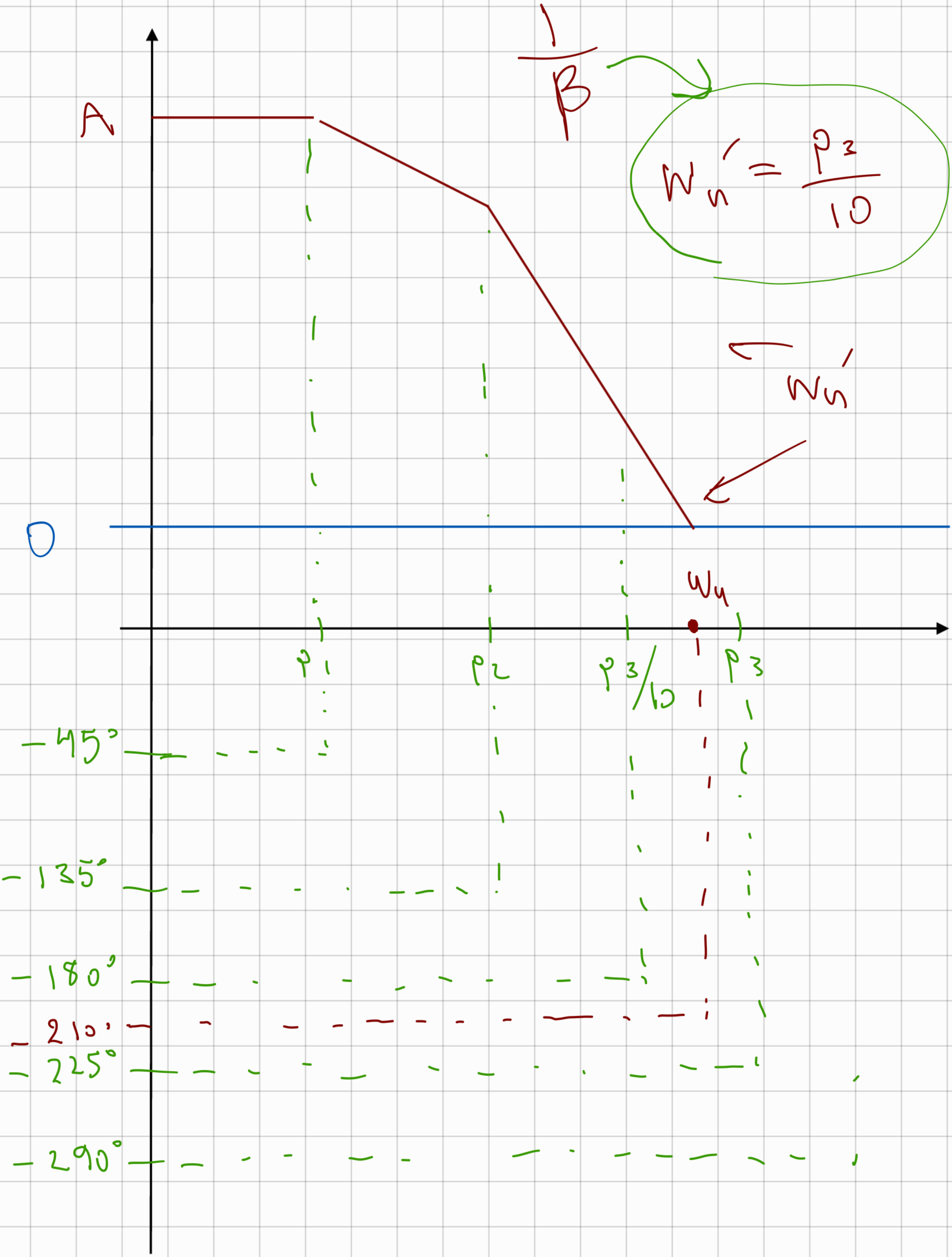
For 60° : $W_u = \frac{2}{3} A_{DC} \cdot p_1$

(d) Briefly explain why an RHP zero can cause stability issues.

In general, a zero provides an alternate path for the input signal to reach the output of an amplifier.

RHP zero generally contribute to phase loss. With a loss in phase, we could bypass a stage that causes the phase margin to drop below a point where the circuit becomes unstable.

(e) Assume you have an amplifier with a phase margin of $-30°$ at unity gain feedback due to a third pole (second non-dominant pole). What is the lowest closed loop gain that can barely keep the amplifier stable?



Loop gain $= A(\omega)B$

$= A(s)_{dB} + \beta_{dB}$

$\Rightarrow \beta = 1 \rightarrow 0\,dB$

$\beta = \frac{1}{10} \rightarrow -20\,dB$

Unity gain FB $\Rightarrow \beta = 1$

(f) Consider the amplifier in "e". Can we make the amplifier unity gain stable by adding more capacitance to its output? Why?

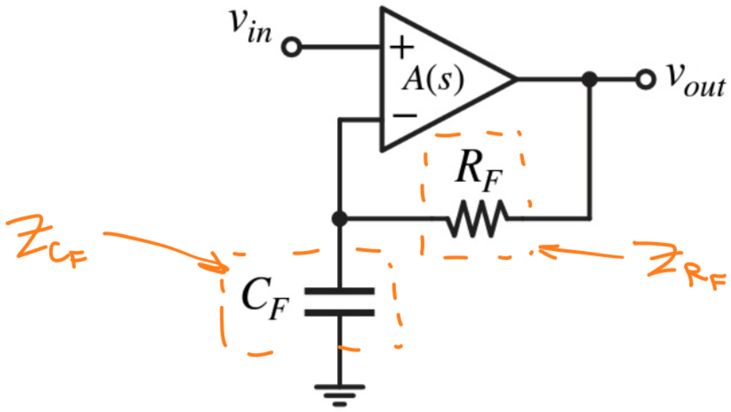
Yes. With a large enough increase in the load capacitance, the load capacitor can be made to be the dominant pole. Then we can hit the unity gain frequency before the other poles, thus making the circuit stable.

(g) Again think of the amplifier from part "e". If you set it up at unity gain feedback, what do you expect to see at the output?

The circuit from part (e) was not stable. With a unity gain feedback we have that $\beta = 1$. Thus, at the output we expect the circuit to blow up.

Problem 3 - Stabilize the Filter

Your designer friend is looking for a circuit that acts as a differentiator only for frequencies above a certain bandwidth, ω_c. You remember the circuit below from your last homework and suggest that it could do the job. Assume you only have access to a 10pF cap for C_F.



(a) Imagine the OpAmp is ideal with $A(s) = \infty$. Find the transfer function from the input to the output. Find the value of R_F that gives the expected response for $\omega > \omega_c = 10\ krad/s$.

$$\beta = \frac{Z_{C_F}}{Z_{C_F} + Z_{R_F}}$$

$$\Rightarrow H(s) = \frac{A(s)}{1 + A(s)\dfrac{Z_{C_F}}{Z_{C_F} + Z_{R_F}}}$$

$$A(s) \to \infty \Rightarrow A(s) \gg 1$$

$$\Rightarrow H(s) = \frac{\cancel{A(s)}}{\cancel{A(s)} \quad \frac{Z_{C_F}}{Z_{C_F} + Z_{R_F}}}$$

$$\Rightarrow H(s) = \frac{Z_{C_F} + Z_{R_F}}{Z_{C_F}}$$

$$= \left(\frac{\frac{1}{sC_F} + R_F}{\frac{1}{sC_F}} \right) \frac{sC_F}{sC_F}$$

$$= \frac{1 + sC_F R_F}{1}$$

$$\boxed{\therefore \quad H(s) = 1 + sC_F R_F}$$

$\Rightarrow$ Want $\dfrac{2\pi}{C_F R_F} = 10k\ rad/s$

$$\Rightarrow R_F = \frac{2\pi}{10\,pF \cdot 10k\ rad/s}$$

$$\boxed{\cong 62.8\ M\Omega}$$

(b) Now let's consider a more realistic OpAmp model where the amplifier has a frequency dependent gain, A(s), shown below.

$$A(s) = \frac{10000}{(1+s/10000)(1+s/100000)}$$

Determine the poles and zeros of the loop gain.

$$\beta = \frac{Z_{C_F}}{Z_{C_F} + Z_{R_F}} = \frac{\frac{1}{sC_F}}{\frac{1}{sC_F} + R_F}$$

$$= \frac{1}{1 + sC_F R_F}$$

$$\Rightarrow \text{Loop gain} = A(s) \cdot \beta$$

$$\left(\frac{1}{1 + s/1592}\right)\left(\frac{10k}{(1 + s/10k)(1 + s/100k)}\right)$$

$$\boxed{\Rightarrow \text{Loop Gain} = 10k\left[\frac{1}{(1 + s/1592)(1 + s/10k)(1 + s/100k)}\right]}$$

$$\boxed{\therefore f_{p_1} = 1592\ Hz \quad f_{p_2} = 10k\ Hz \quad f_{p_3} = 100k\ Hz}$$

(c) With R_F value you found in part "a", is the circuit stable? Explain your answer based on the stability criteria.

We can find the unity gain frequency of the loop gain to determine stability.

$$10k\left[\frac{1}{\left(1+\frac{s}{1592}\right)\left(1+\frac{s}{10k}\right)\left(1+\frac{s}{100k}\right)}\right] = 1$$

* Solving with Wolfram Alpha

$\Rightarrow \omega \hat{=} 245 \text{ kHz}$

$\Rightarrow$ This is too large of a frequency for stability, because all of the poles will have occurred before $\beta = 1$.

$\boxed{\therefore \text{ The circuit is unstable}}$

(d) Determine a feedback time constant, $\tau_F = R_F C_F$ that gives the circuit a phase margin of 60°.

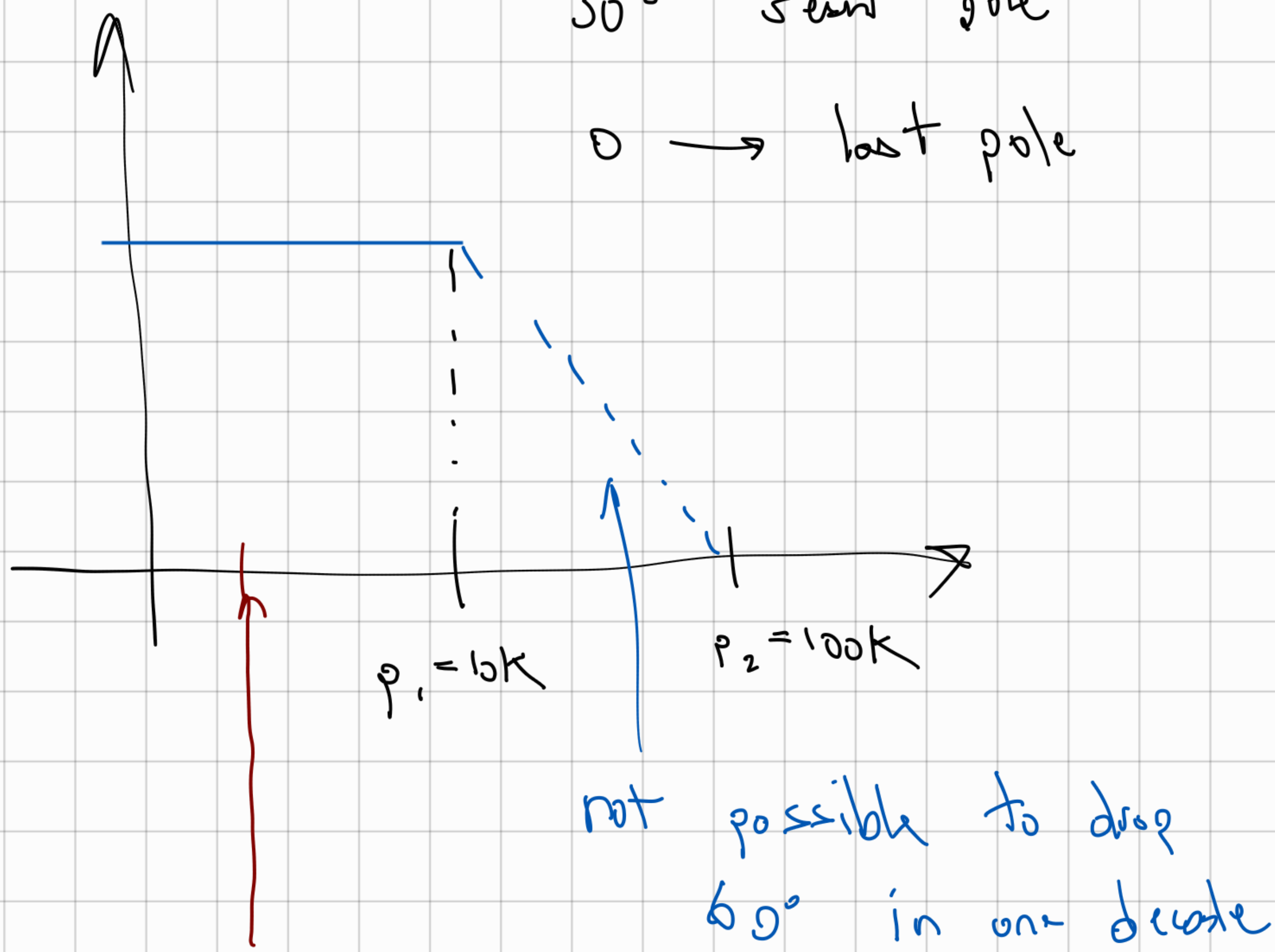
Phase margin of $60° \Rightarrow 120°$ phase drop

$\Downarrow$

90° first pole

30° second pole

0 $\rightarrow$ last pole



$\Rightarrow \tau_F$ comes before p_1.

$$\Rightarrow W_n = 10^4 \cdot \frac{1}{R_F C_F}$$

$$\Rightarrow \quad H(j\omega) = \frac{10^4}{(1 + j\omega R_F C_F)\left(1 + \frac{j\omega}{10^4}\right)}$$

$$\angle H(j\omega) = \angle 10^4 - \Big[\underbrace{\tan^{-1}(\omega R_F C_F)}_{\text{assume} \geq 90^\circ} + \tan^{-1}\left(\frac{\omega}{10^4}\right)\Big]$$

$$= 0^\circ - 90^\circ - \tan^{-1}\left(\frac{\omega_u}{10^4}\right) = -120^\circ$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{10^4}{\omega_u}\right) = 30^\circ$$

$$\Rightarrow \quad \tan(30^\circ) = 0.577$$

$$\Rightarrow \quad \omega_u = \frac{10^4}{0.577}$$

$$\cong 17320 \text{ Hz}$$

$$\Rightarrow \left| \frac{10^4}{(1 + j\omega R_F C_F)\left(1 + \frac{j\omega}{10^4}\right)} \right| = 1$$

* Solving with Wolfram Alpha

$$\Rightarrow R_F C_F = \frac{\sqrt{\frac{6249999750011}{249989}}}{17320}$$

$$\Rightarrow \boxed{\tau_F \approx 0.29\,s}$$

(e) Consider $v_{in}(t) = 100\text{mV} \cdot \text{Sin}(\omega_c \text{t}) + 100\text{mV}$ applied to the circuit with the settings of part "d". What is the time domain output of the circuit, $v_{out}(t)$?

$$W_{c_{new}} = \frac{1}{\tau_F} = \frac{1}{0.29}$$

$$\cong 3.46$$

$$\Rightarrow V_{in}(w) = 100\,mV \angle 0^\circ + 100\,mV$$

$$= 200\,mV \angle 0^\circ$$

$$\boxed{\therefore V_{out}(t) = 200\,mV \cdot \sin(3.46t)}$$