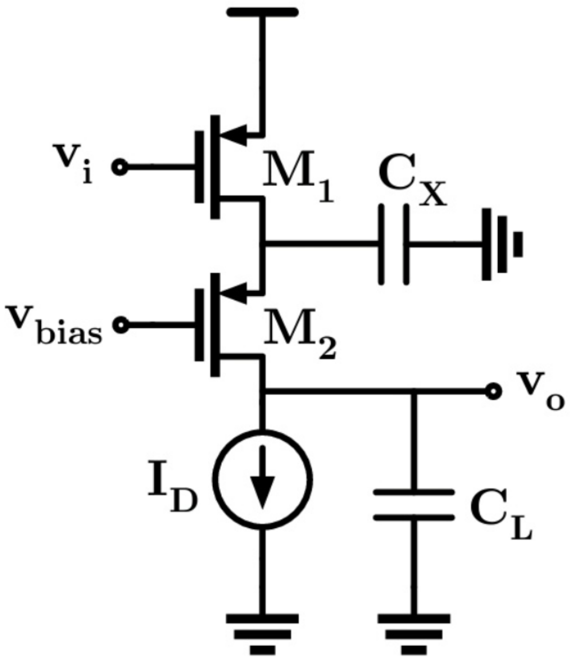
Problem 1 - Cascode Frequency Response

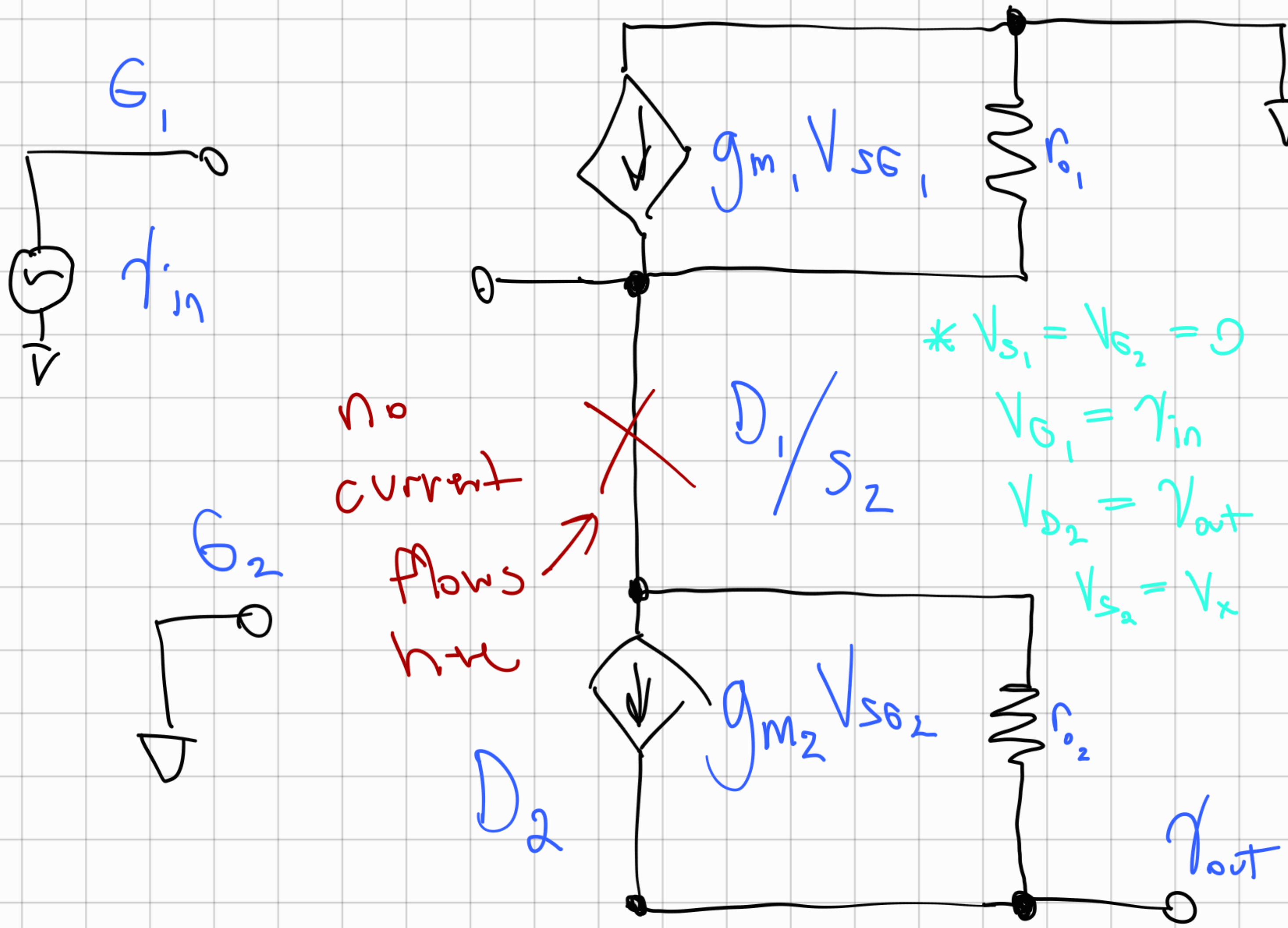
In this problem you will analyze the frequency response of the cascode amplifier below. Consider only the C_{gd} and C_{gs} of each transistor. Assume all transistors have the same small signal parameters and parasitic capacitances. Express your answers symbolically. You may assume that C_L, C_{gd}, and C_{gs} are all on the same order of magnitude, and $g_m r_o >> 1$.



(a) Compute the DC gain $A_v = \frac{v_o}{v_i}$ and DC output impedance R_{out}.

We will ignore the body effect for this part.

Small-signal Model



$* \; V_{S_1} = V_{G_2} = 0$

$V_{G_1} = v_{in}$

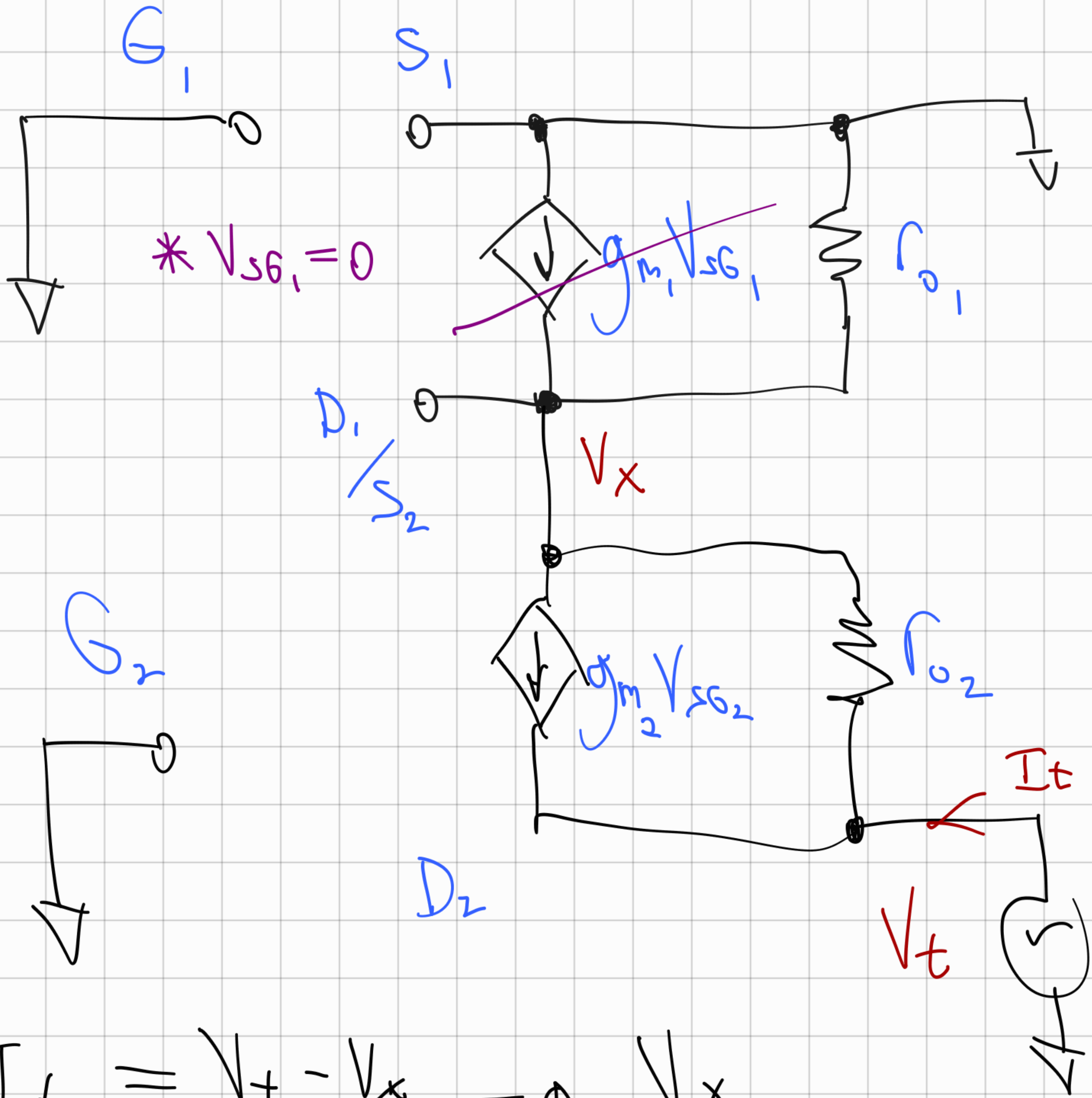
$V_{D_2} = v_{out}$

$V_{S_2} = V_x$

$$\Rightarrow \frac{v_{out}}{r_{o_1}} + g_{m_1} v_{in} = 0$$

$$\boxed{\therefore A_v = \frac{v_{out}}{v_{in}} = -g_{m_1} r_o}$$

Applying a Test Source to the Output



$$I_t = \frac{V_t - V_x}{r_{o_2}} - g_{m_2} V_x$$

$$= \frac{1}{r_{o_2}} V_t - \left(g_{m_2} + \frac{1}{r_{o_2}}\right) V_x \qquad (1)$$

$$\left[I_t = \frac{V_x}{r_{o_1}} \right] \cdot r_{o_1}\left(g_{m_2} + \frac{1}{r_{o_2}}\right) \qquad (2)$$

→ Add Eq. 1 to Eq. 2

$$1\,I_t = \left(\frac{1}{r_{o2}}\right)V_t - \cancel{\left(g_{m2}+\frac{1}{r_{o2}}\right)V_x}$$

$$+\left(r_{o1}g_{m2}+\frac{r_{o1}}{r_{o2}}\right)I_t = 0\,V_t + \cancel{\left(g_{m2}+\frac{1}{r_{o2}}\right)V_x}$$

$$= \left(1 + r_{o1}g_{m2} + \frac{r_{o1}}{r_{o2}}\right)I_t = \frac{1}{r_{o2}}V_t$$

$$V_t = \left(r_{o2} + r_{o1}r_{o2}g_{m2} + r_{o1}\right)I_t$$

$$\therefore \boxed{R_{out} = r_{o1} + r_{o2}\left(1 + g_{m2}r_{o1}\right)}$$

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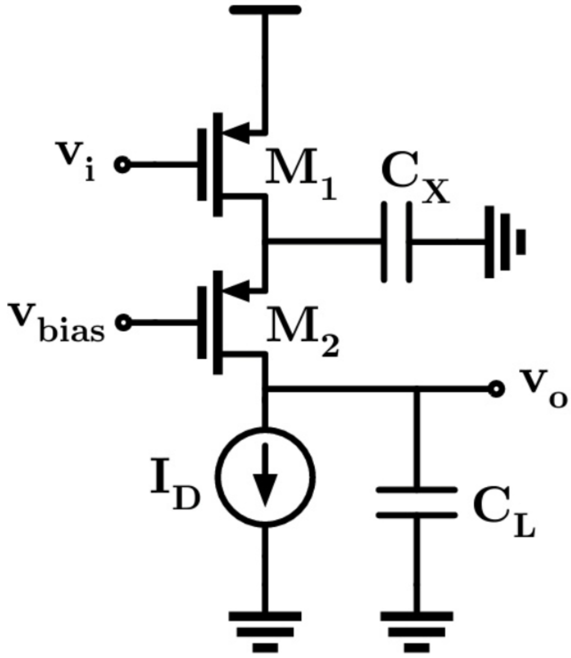
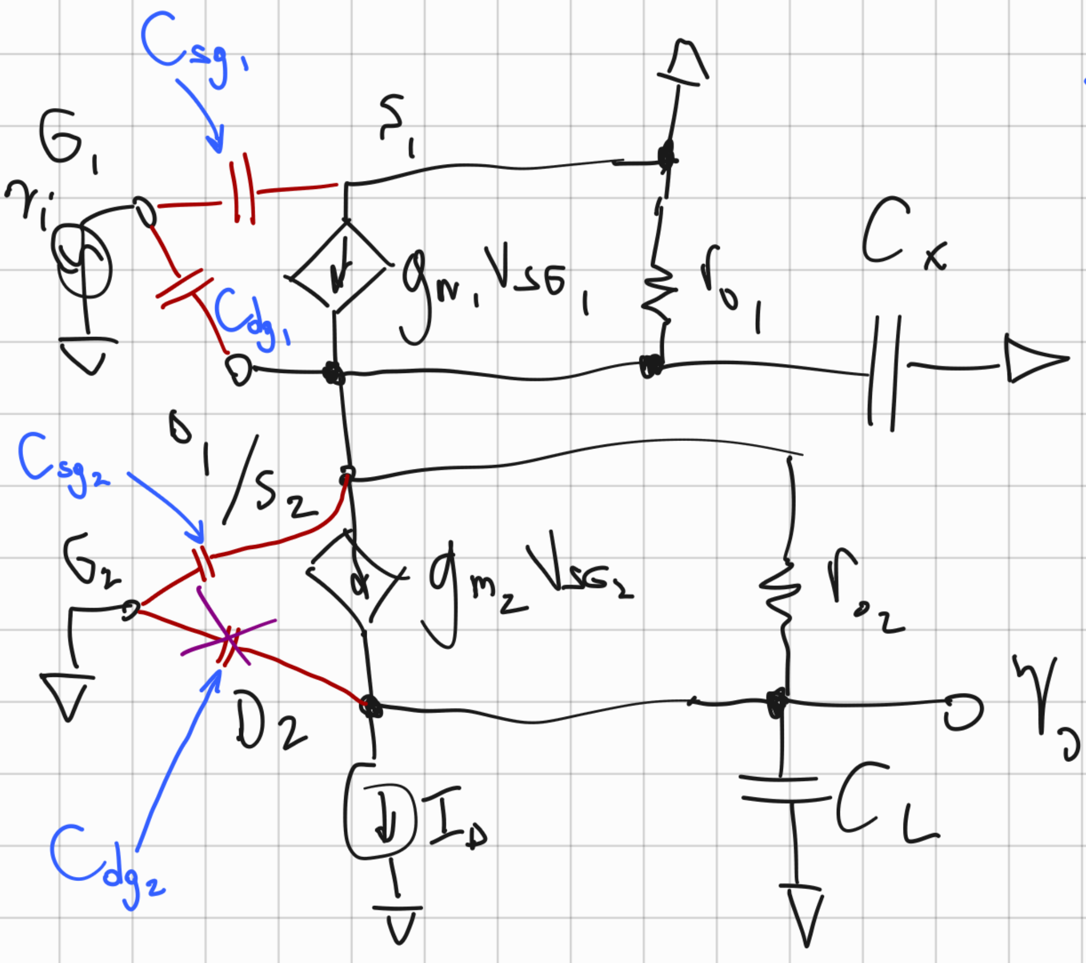
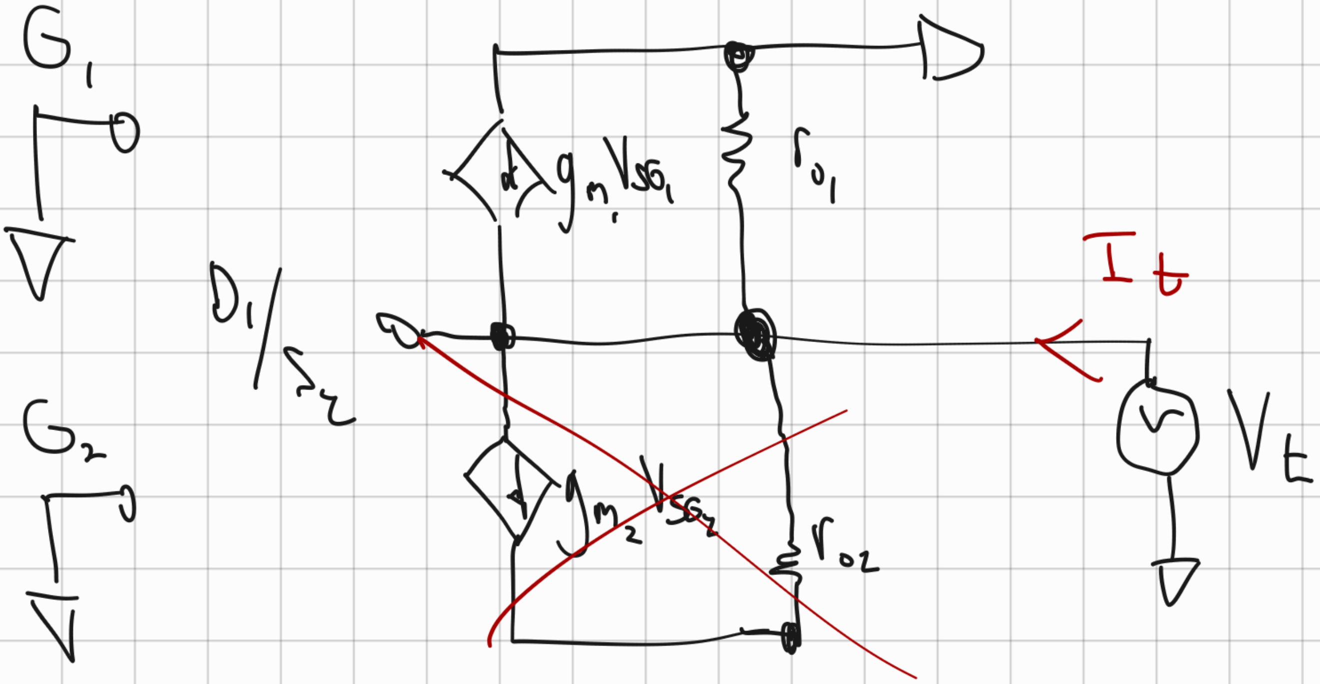


(b) Use OCTC to estimate the bandwidth of the amplifier in terms of circuit parameters. Under reasonable assumptions, which time constant would dominate the OCTC calculation?



* C_{dg_2} is grounded in OCTC analysis, so it will be ignored.

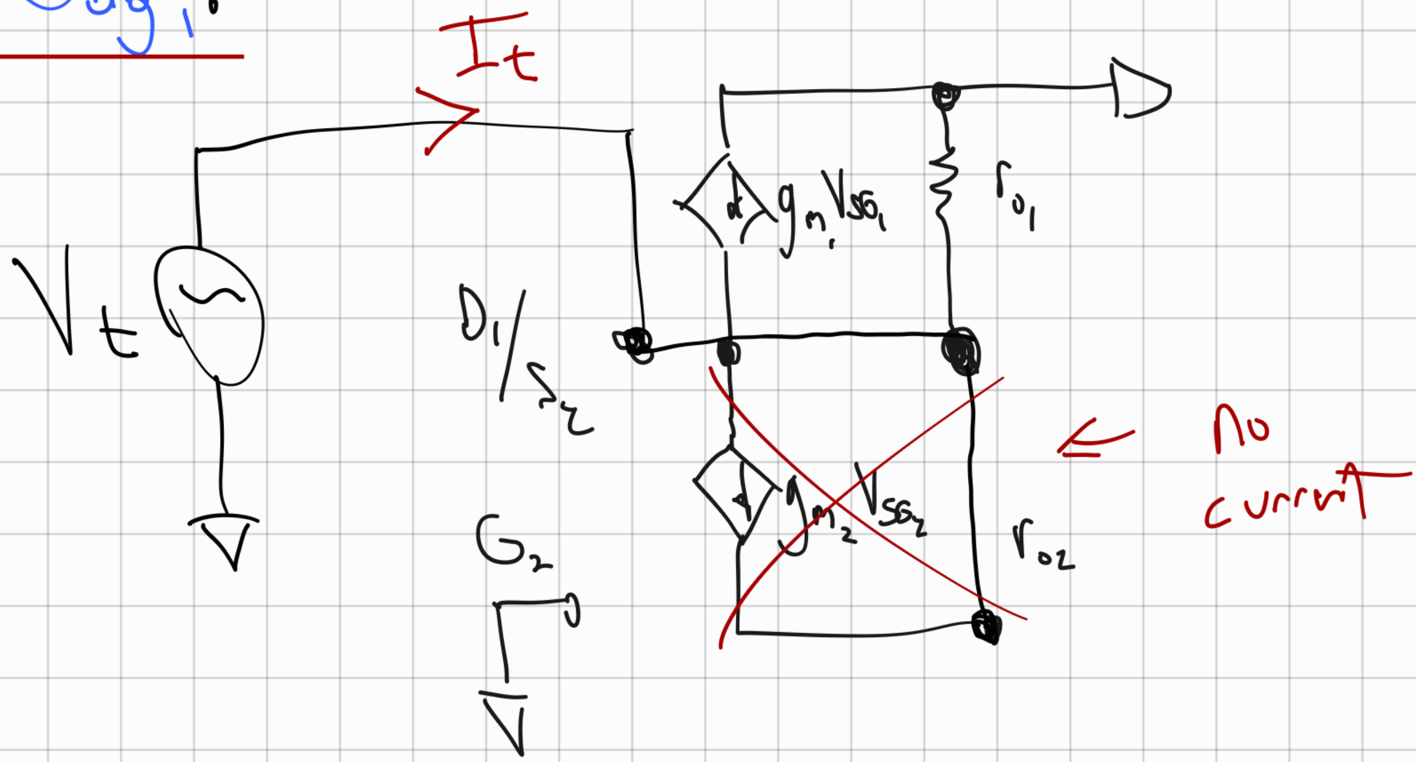
* By inspection, C_L should see the largest impedance, and will be the dominant pole. I will still proceed with mathematical analysis.



C_x : $I_t = \dfrac{V_t}{r_{o_1}} - \cancel{g_{m_1} V_{SG_1}}^{\to 0}$

$\Rightarrow \dfrac{V_t}{I_t} = r_{o_1} \quad \therefore \quad \boxed{R_{C_x} = r_{o_1}}$

C_{dg_1} :



$$\Rightarrow I_t = \frac{V_t}{r_{o_1}} - g_{m_1} V_{SG_1}$$

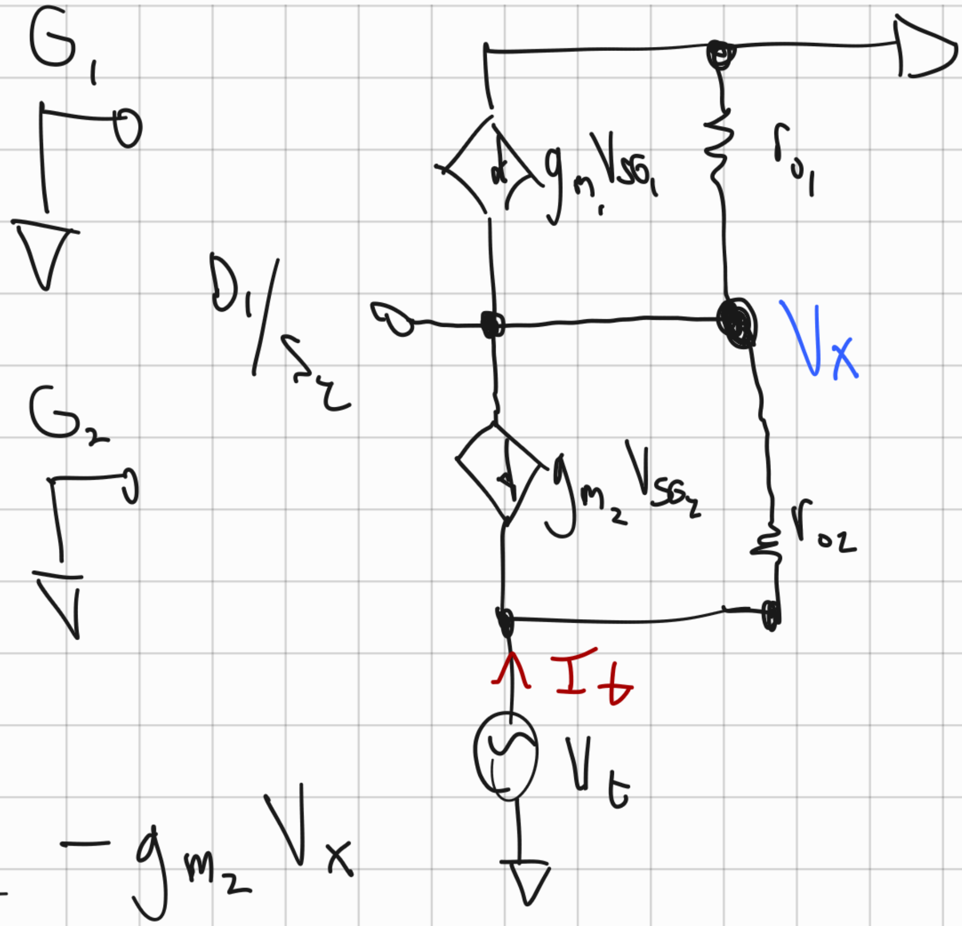
$$= \frac{V_t}{r_{o_1}} + g_{m_1} V_t$$

$$= V_t \left(\frac{1}{r_{o_1}} + g_{m_1} \right)$$

$$\Rightarrow \frac{V_t}{I_t} = \frac{r_{o_1}}{1 + g_m r_{o_1}} \simeq \frac{\cancel{r_{o_1}}}{\cancel{r_{o_1}}} \frac{1}{g_{m_1}}$$

$$\boxed{\therefore R_{Cdg_1} = \frac{1}{g_{m_1}}}$$

$C_L // C_{dg_2}$:



$$I_t = \frac{V_t - V_x}{r_{o2}} - g_{m_2} V_x$$

$$= \frac{V_t}{r_{o2}} - V_x\left(g_{m_2} + \frac{1}{r_{o2}}\right)$$

$$I_t = \frac{V_x}{r_{o_1}} \Rightarrow V_x = I_t \cdot r_{o_1}$$

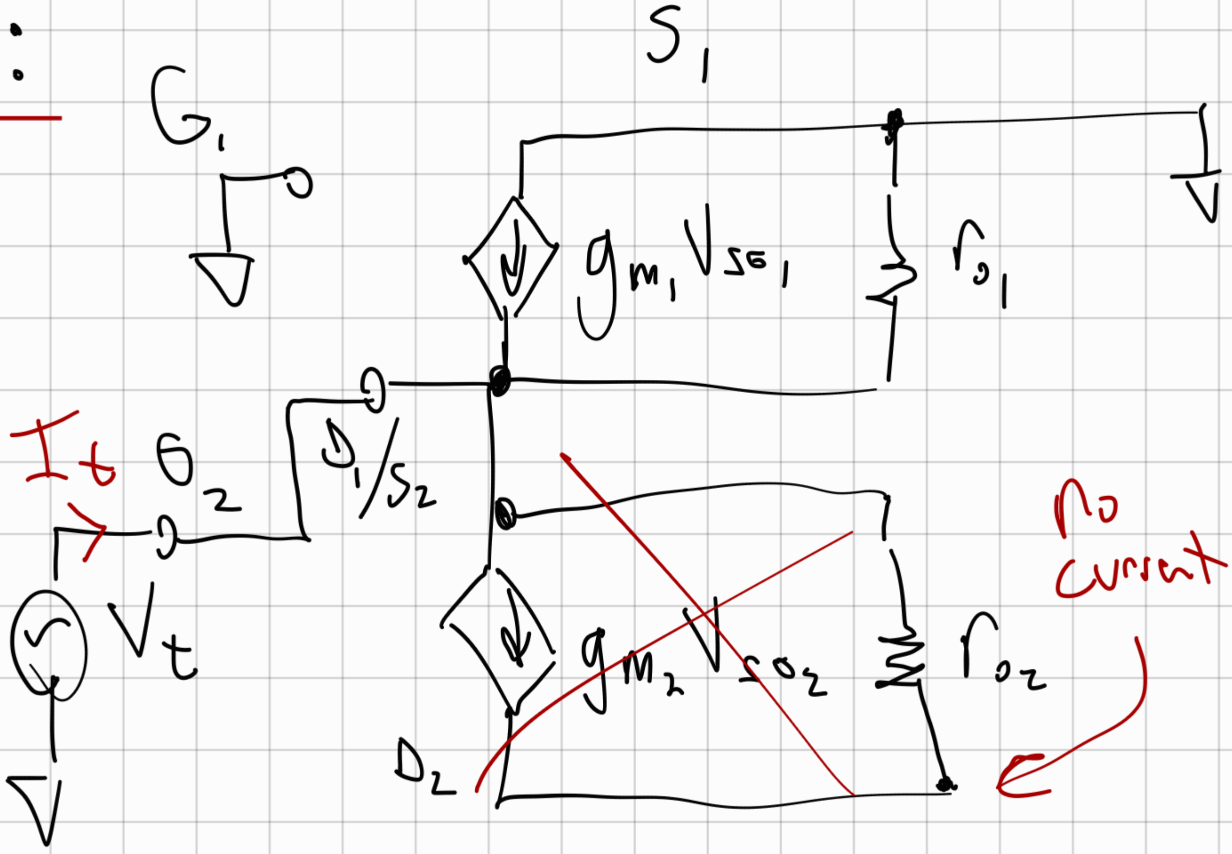
$$\Rightarrow I_t = \frac{V_t}{r_{o2}} - I_t\left(\frac{r_{o_1}}{g_{m2}} + \frac{r_{o_1}}{r_{o2}}\right)$$

$$\Rightarrow I_t\left(1 + \frac{r_{o_1}}{g_{m2}} + \frac{r_{o_1}}{r_{o2}}\right) = \frac{V_t}{r_{o2}}$$

$$\Rightarrow I_t\left(r_{o1} + r_{o2} + \frac{r_{o1}}{g_{m2}}\right) = V_t$$

$$\boxed{\Rightarrow R_{C_L \| C_{dg2}} = \frac{r_{o1} + g_{m2}(r_{o1} + r_{o2})}{g_{m2}}}$$

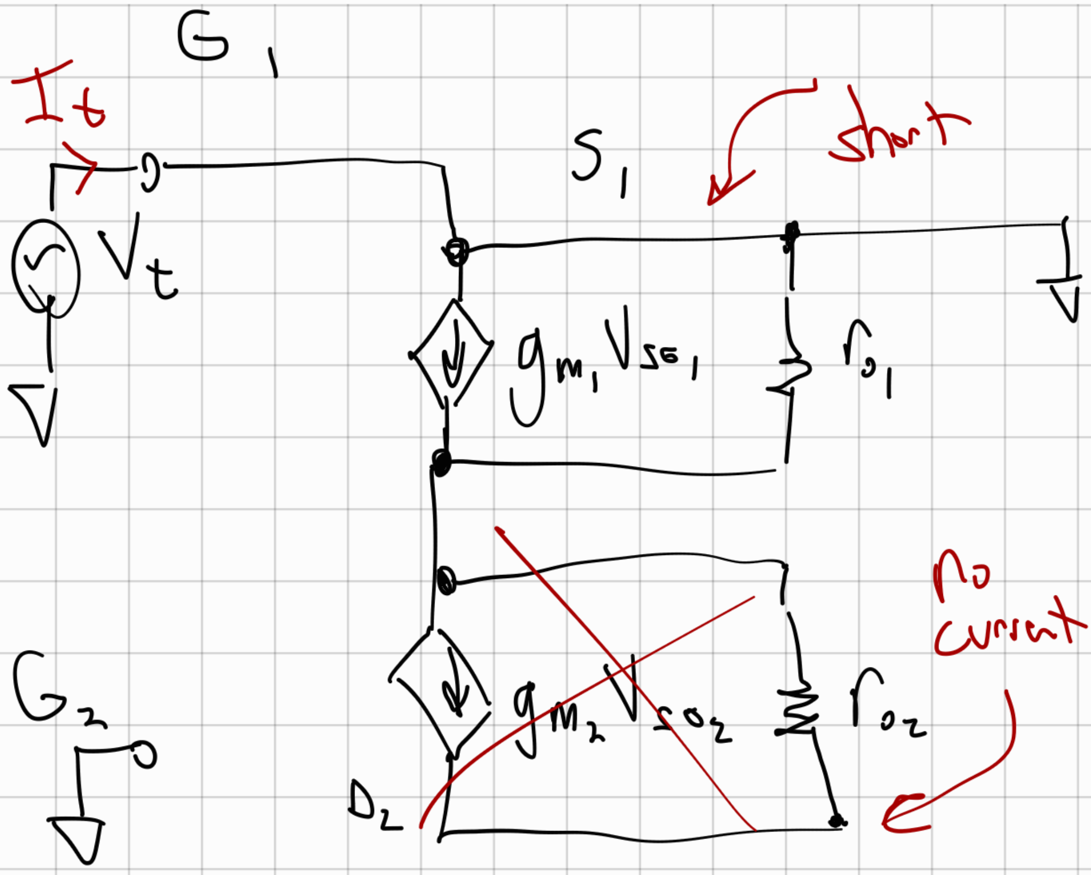
C_{sg1} :



$$\Rightarrow I_t = \frac{V_t}{r_{o1}} - \cancel{g_{m1}V_{SG1}}^{\,0}$$

$$\boxed{\therefore R_{C_{sg1}} = r_{o1}}$$

C_{sg_2}:



$$\Rightarrow I_t = \infty \qquad \boxed{\therefore\ R_{C_{sg_2}} = 0}$$

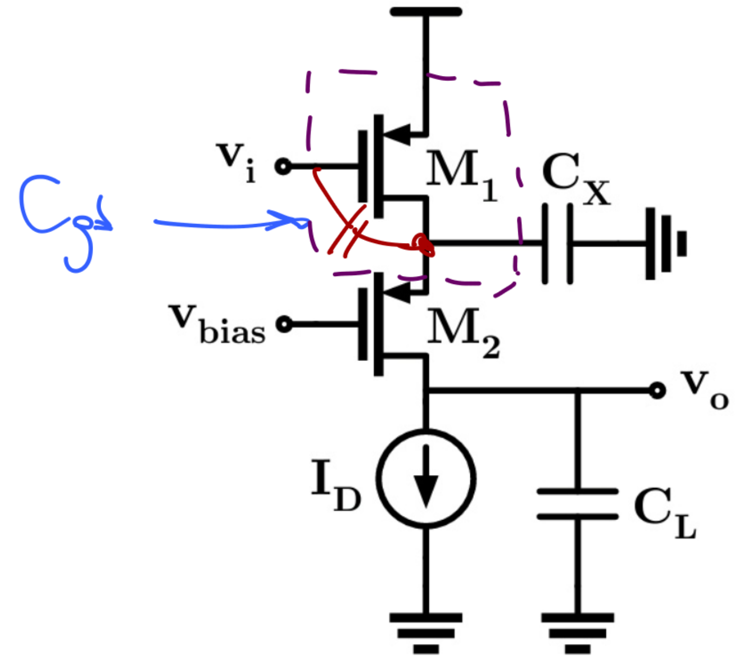
Overall 3dB Bandwidth

$$\boxed{\omega_o = \frac{1}{\left(R_{C_{sg_1}} \cdot C_{sg_1}\right) + \left(R_{C_L \parallel C_{dg_2}} \cdot \left(C_L + C_{dg_2}\right)\right) + \left(R_{C_x} \cdot C_x\right) + \left(R_{C_{dg_1}} \cdot C_{dg_1}\right)}}$$

$C_L \parallel C_{dg_2}$ is the dominant pole.

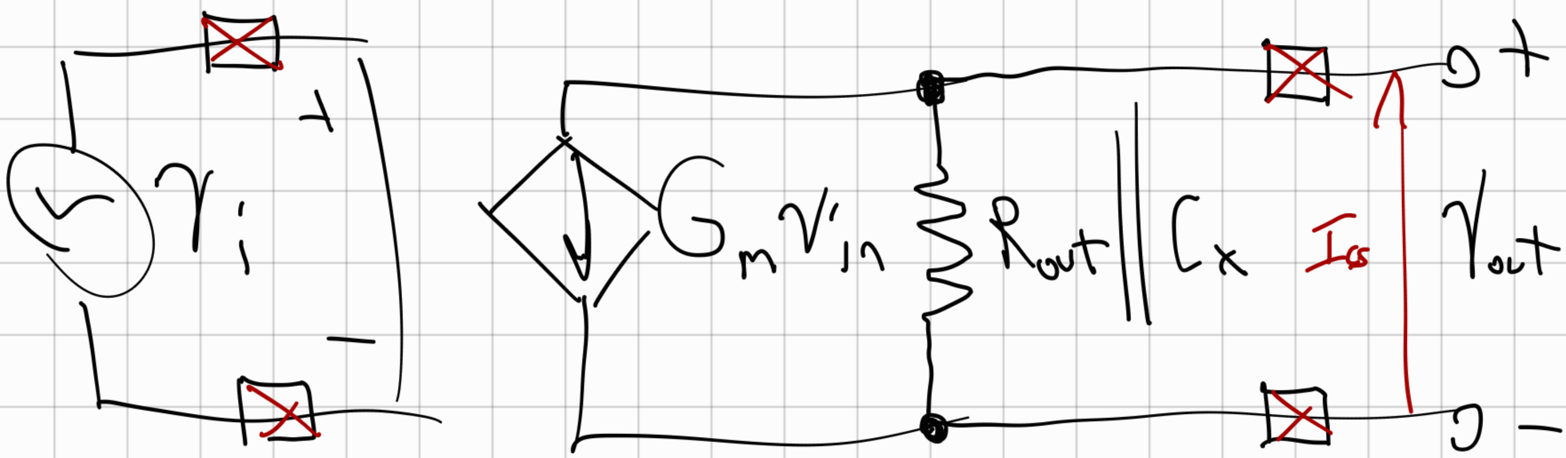
$$\Rightarrow \frac{g_{m_2}}{g_{m_2}} \left(\frac{r_{o_1}}{g_{m_2}} + r_{o_1} + r_{o_2} \right) \gg r_{o_1} \gg 1/g_{m_2}$$

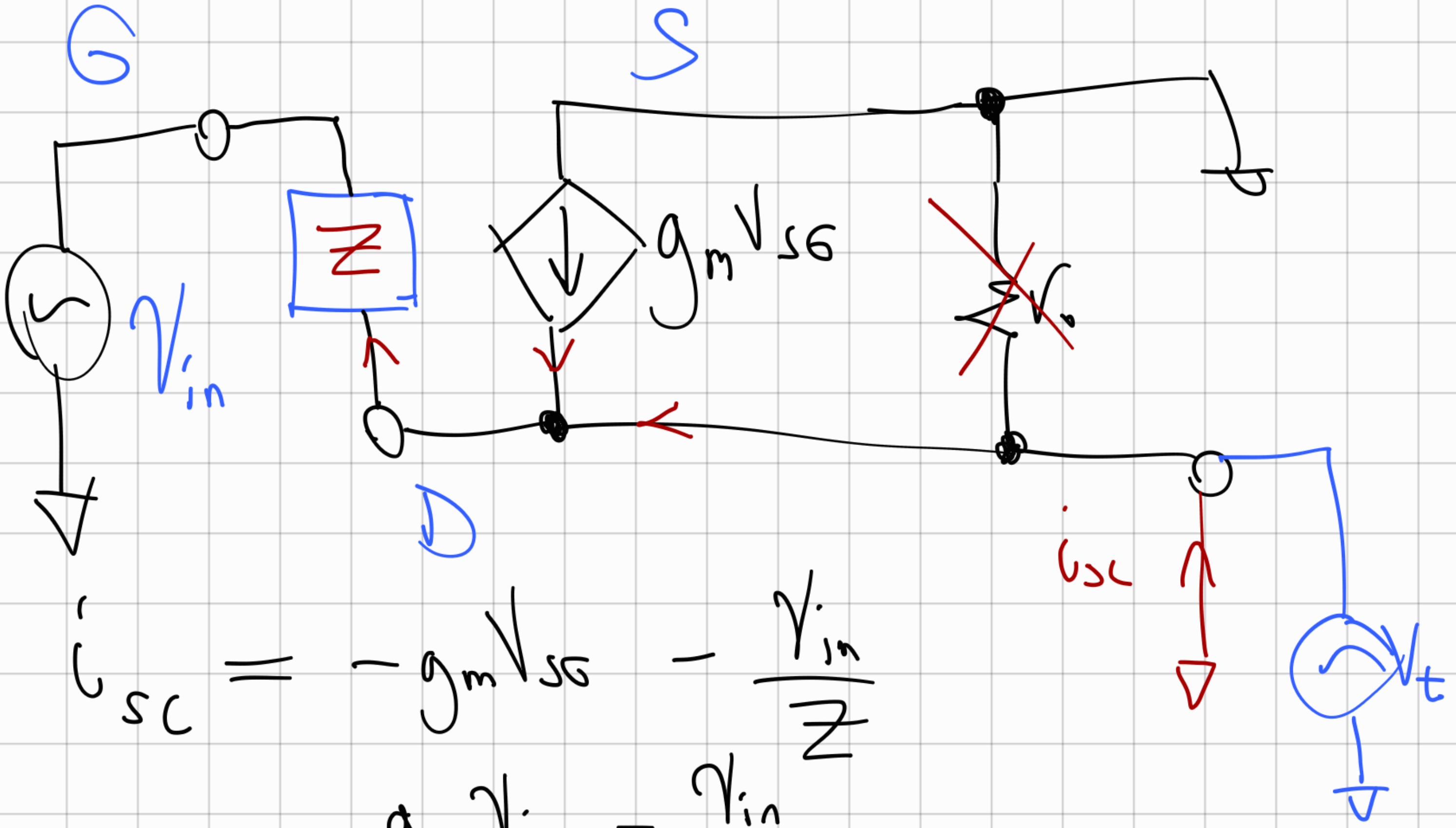
In this problem you will analyze the frequency response of the cascode amplifier below. Consider only the C_{gd} and C_{gs} of each transistor. Assume all transistors have the same small signal parameters and parasitic capacitances. Express your answers symbolically. You may assume that C_L, C_{gd}, and C_{gs} are all on the same order of magnitude, and $g_m r_o >> 1$.



(c) The transfer function $H(s) = v_o(s)/v_i(s)$ of this circuit has one zero. Estimate the zero frequency.
Hint: It is possible to find the zero without finding the entire $H(s)$. Calculate the equivalent Gm of the top transistor with Cgd included.

Two-port model





$$i_{sc} = -g_m V_{SG} - \frac{v_{in}}{Z}$$

$$= g_m v_{in} - \frac{v_{in}}{Z}$$

$$= v_{in}\left(g_m - sC_{gd}\right)$$

$$I_{sc} = G_m v_{in}$$

$$\Rightarrow \boxed{G_m = g_m - sC_{gd}}$$

$$i_t = \frac{V_t}{r_o} + \frac{V_t}{Z} = V_t\left(\frac{1}{r_o} + sC_{gd}\right)$$

$$\Rightarrow R_{out} = \frac{r_o}{1 + sC_{gd} r_o} \parallel C_x$$

$$
= \frac{\dfrac{\dfrac{r_o}{sC_x}}{1 + s r_o C_{gd}}}{\dfrac{r_o}{1 + s r_o C_{gd}} + \dfrac{1}{sC_x}}
$$

$$
= \left(\frac{r_o}{sC_x \cancel{\left(1 + s r_o C_{gd}\right)}} \right) \left(\frac{\cancel{sC_x \left(1 + s r_o C_{gd}\right)}}{s r_o C_x + 1 + s r_o C_{gd}} \right)
$$

$$
= \boxed{\frac{r_o}{1 + s r_o \left(C_x + C_{gd}\right)}} \quad R_{out}
$$

$$
G_m \cdot R_{out} = \frac{r_o \left(g_m - sC_{gd}\right)}{1 + s r_o \left(C_x + C_{gd}\right)}
$$

$$H(s) = \frac{g_m r_o \left(1 - s\frac{\cancel{r_o} C_{gd}}{g_m \cancel{r_o}}\right)}{1 + s r_o \left(C_x + C_{gd}\right)}$$

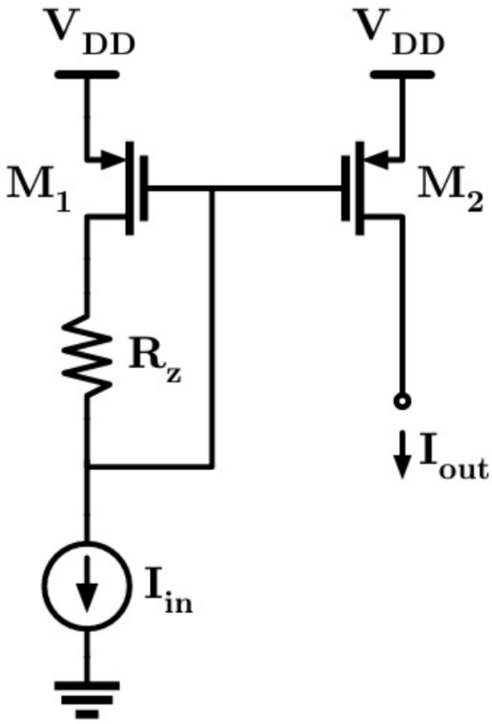
<u>Zero Frequency:</u> $\boxed{\dfrac{g_m}{C_{gd}}}$

Problem 2 - Current Amplifier

Figure below shows the schematic of a current input, current output amplifier. The input DC current is set such that M_1 is at the onset of saturation ($V_{DS} = V_{GS} - V_{TH}$). You may use the square-law model for the transistors with the following parameters.

$$\lambda = 0,\ \gamma = 0,\ |V_{TH}| = 0.5\text{V},\ V_{DD} = 2\text{V}$$

$$W_2 = A \times W_1,\ (\tfrac{W}{L})_1 = 10,\ k' = \mu_p C_{ox} = 100 \times 10^{-6}\ \text{A/V}^2$$



(a) We want to pick an R_z such that $R_z = \frac{1}{g_{m1}}$. What is this R_z value? What is the DC voltage at the gate of M_1?

We want to relate the following two equations:

$$\frac{V_{DS}}{R_z} = I_{in} \quad \& \quad g_m = \sqrt{2 \cdot k' \cdot \frac{W}{L} \cdot I_{in}}$$

We are given that: $V_{SD} = V_{SG} - V_T$

Rearranging $\Rightarrow$ $\cancel{V_S} - V_D = \cancel{V_S} - V_G - V_T$

$$\Rightarrow V_D = V_G + V_T$$

$$\Rightarrow \frac{\cancel{V_G} + V_T \cancel{-} V_G}{R_Z} = I_{in}$$

$$\Rightarrow \frac{V_T}{R_Z} = I_{in}$$

$$\Rightarrow \frac{V_T}{\frac{1}{g_m}} = I_{in}$$

$$\Rightarrow g_m V_T = I_n$$

$$\Rightarrow g_m = \frac{I_{in}}{V_T}$$

continued...

$$\Rightarrow \quad \frac{I_{in}}{V_T} = \sqrt{2 \cdot k' \cdot \frac{W}{L} \cdot I_{in}}$$

$$\Rightarrow \quad \left(\frac{I_{in}}{V_T}\right)^2 = 2k' \cdot \frac{W}{L} \cdot I_n$$

$$\Rightarrow \quad I_{in}^{\cancel{2}} = \underbrace{2k' \frac{W}{L} \cdot V_T^2}_{\phi} \cdot \cancel{I_n}$$

$$I_{in} = \phi = 0.5\,mA$$

$$\Rightarrow \quad R_z = \frac{V_T}{I_{in}} = \frac{0.5\,V}{0.5\,mA}$$

$$\boxed{\therefore \quad R_z = 1k\Omega}$$

DC Voltage at Gate of M_1

$$\Rightarrow g_m = \frac{1}{R_Z} = \frac{W}{L} k' \left(V_{SG} - V_T\right)$$

$$\Rightarrow \left(V_{SG} - V_T\right) = \frac{L}{R_Z W k'}$$

$$\Rightarrow V_{SG} = \frac{L}{R_Z W k'} + V_T$$

$$= \frac{1}{1k \cdot 10 \cdot 100 \times 10^{-6}} + 0{,}5$$

$$= 1.5$$

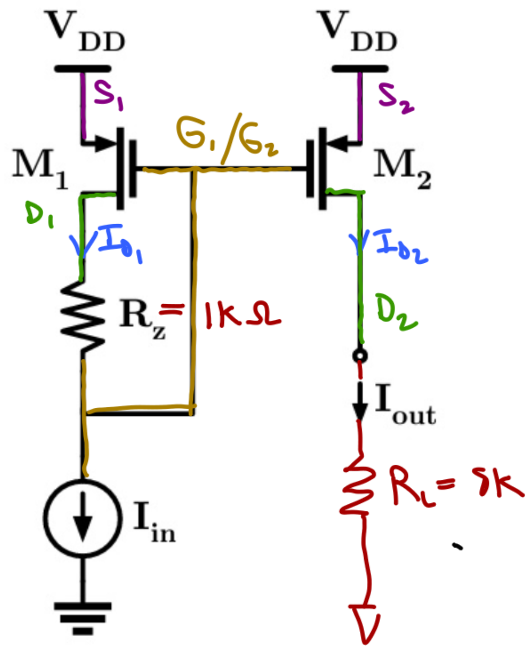
$$\Rightarrow V_{SG} = V_{DD} - V_G = 1.5$$

$$\Rightarrow V_G = V_{DD} - 1.5$$

$$\boxed{\therefore \; V_G = 0{,}5\,V}$$

$\lambda = 0$, $\gamma = 0$, $|V_{TH}| = 0.5\text{V}$, $V_{DD} = 2\text{V}$

$W_2 = A \times W_1$, $(\frac{W}{L})_1 = 10$, $k' = \mu_p C_{ox} = 100 \times 10^{-6}\ \text{A/V}^2$



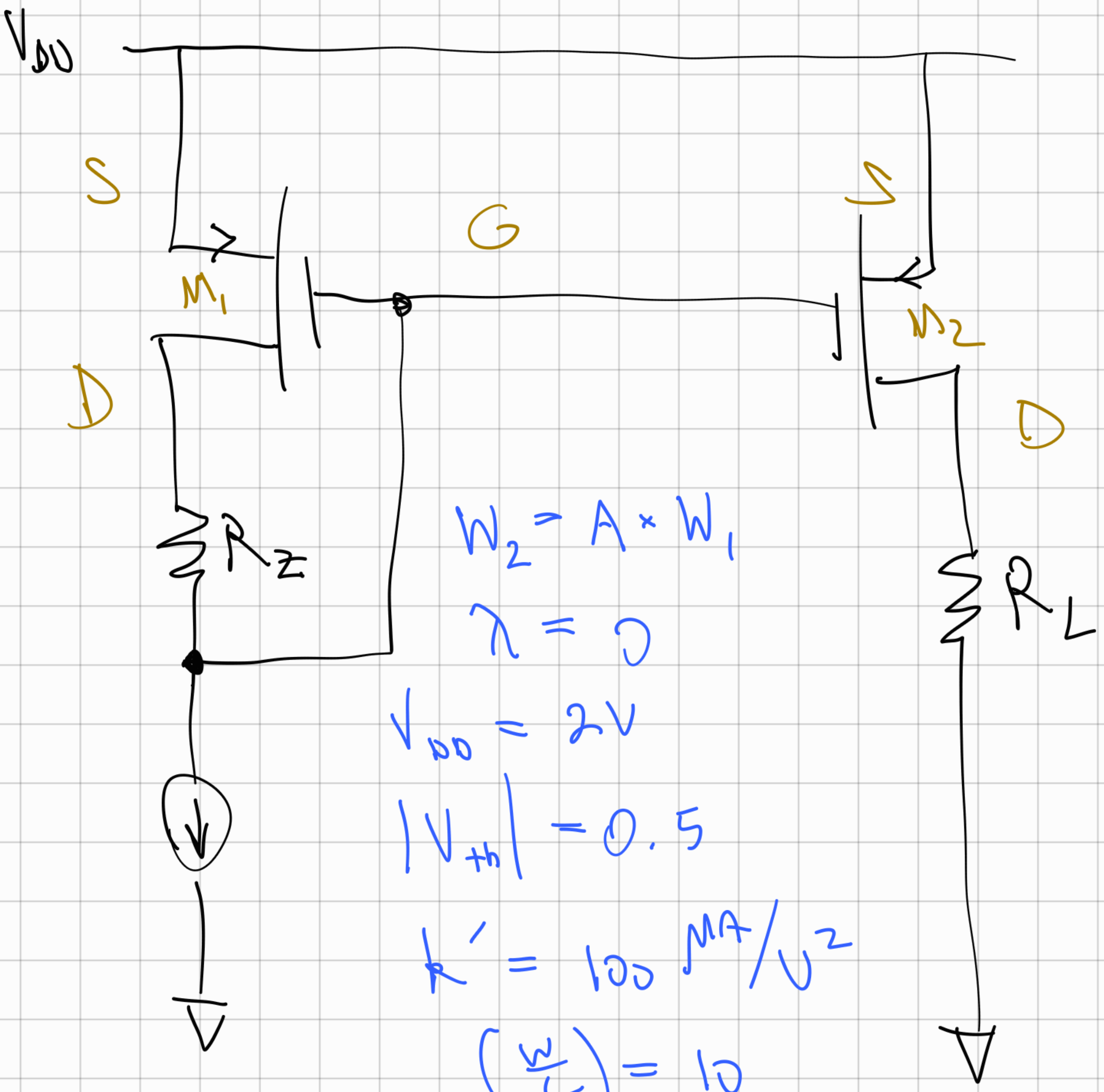
(b) If R_z is instead chosen to be 1 kΩ, and M_2 drives an 8 kΩ resistor to ground as its load, what is the maximum output current that this amplifier can supply without any devices entering triode? (You may assume the input current source itself works as long as the voltage across it is >0 V)

$$I_{D2_{sat}} = \left(\frac{AW_1}{2L_1}\right) k' \left(V_{SG_2} - V_T\right)^2 = \frac{V_{D_2}}{R_D} \quad (1)$$

$$I_{D_{1_{sat}}} = \left(\frac{W}{2L}\right)_1 k' \left(V_{SG_1} - V_T\right)^2 = \frac{V_{DG_1}}{R_Z} \quad (2)$$

$$I_{D_{2_{tri}}} = \left(\frac{AW_1}{L_1}\right) k' \left(V_{SG_2} - V_T - \frac{V_{SD_2}}{2}\right) V_{SD_2} = \frac{V_{D_2}}{R_D} \quad (3)$$

$$I_{D_{1_{tri}}} = \left(\frac{W}{L}\right)_1 k' \left(V_{SG_1} - V_T - \frac{V_{SD_1}}{2}\right) V_{SD_1} = \frac{V_{DG_1}}{R_Z} \quad (4)$$



$W_2 = A \times W_1$

$\lambda = 0$

$V_{DD} = 2V$

$|V_{th}| = 0.5$

$k' = 100\ \mu A / V^2$

$\left(\frac{W}{L}\right) = 10$

$\underline{M_1:}$ A_I $i_{in} \to 0 \to \uparrow$

$$\cancel{V_{SG_1}} - V_T < V_{SD_1}$$

$$= \cancel{V_{SG_1}} - R_Z \cdot I_{in}$$

$$\Rightarrow \quad I_{in} < \underbrace{\frac{V_T}{R_Z}}_{I_{in,MAX,}}$$ (upper bound for I_{in} #1)

M2: $V_{SD_2} > V_{SG_2} - V_T$

$$V_{DD} - I_{out} \cdot R_L > V_{SG_2} - V_T$$

$$\Rightarrow \quad I_{out} \cdot R_L < V_{DD} - V_{SG_2} + V_T$$

* Assuming both M_1 and M_2 in saturation, $I_{out} = A \cdot I_{in}$

$$\Rightarrow A \cdot I_{in} R_L < V_{DD} - V_{SG_2} + V_T$$

* $V_{SG_2} = V_{SG_1}$

* $I_{in} = \frac{1}{2} k' \left(\frac{W}{L}\right)_1 \left(V_{SG_1} - V_T\right)^2$

$$\Rightarrow V_{SG_1} = f(I_{in})$$

$\Downarrow$

$$I_{in} < \underbrace{\frac{V_{DD} - f(I_{in}) + V_T}{A \cdot R_L}}_{I_{in,\,MAX_2}} \qquad \left(\text{upper band \#2}\right)$$

$\Rightarrow \quad A \uparrow \;\rightarrow\; I_{in,\,MAX_2} \downarrow$

$\quad\;\; A \downarrow \;\rightarrow\; I_{in,\,MAX_1} \uparrow$

$\Rightarrow \quad A_0 \,;\; I_{in\,MAX_1} = I_{in,\,MAX_2}$

$$A > A_0 \Rightarrow \underbrace{I_{in\,MAX_2}}_{\text{limit}} < I_{in\,MAX_1}$$

$$A < A_0 \Rightarrow I_{in\,MAX_2} > \underbrace{I_{in\,MAX_1}}_{\text{limit}}$$

$$I'_{in,MAX} = \begin{cases} I'_{in\,MAX_2}, & A > A_0 \\ I'_{in\,MAX_1}, & A < A_0 \end{cases}$$

Solving for A_0

$$I'_{in} = \left(\frac{W}{L}\right)_1 k' \left(V_{SG_1} - V_T\right)^2$$

$$\Rightarrow \left(\Delta V_1\right)^2 = \frac{I'_{in}}{\left(\frac{W}{L}\right)_1 k'}$$

$$\Rightarrow V_{SG_1} = \sqrt{\frac{I'_{in}}{\left(\frac{W}{2L}\right)_1 k'}} + V_T$$

Substitute $I_{in,max,1}$

$$\Rightarrow V_{SG_1} = \sqrt{\frac{\frac{2V_T}{R_Z}}{\left(\frac{W}{L}\right)_1 k'}} + V_T$$

Relate $I_{in,max,1}$ to $I_{in,max,2}$

$$\Rightarrow \frac{V_T}{R_Z} = \frac{V_{DD} - \left(\sqrt{\frac{\frac{2V_T}{R_Z}}{\left(\frac{W}{L}\right)_1 k'}} + V_T\right) + V_T}{A \cdot R_L}$$

Rearranging

$$\Rightarrow A = \left[\frac{V_{DD} - \left(\sqrt{\frac{\frac{2V_T}{R_Z}}{\left(\frac{W}{L}\right)_1 k'}} + V_T\right) + V_T}{R_L \cdot V_T}\right] \cdot R_Z$$

Plugging in Values

$$A = \left[\frac{2 - \sqrt{\dfrac{\frac{2 \cdot 0.5}{1000}}{10 \cdot 100 \times 10^{-6}}} + 2 \cdot 0.5}{8K \cdot 0.5} \right] 1K$$

$$\boxed{\therefore \quad A = 0.25}$$

Solve for I_{in, max_2} in Terms of A

$$I_{in} = \frac{V_{DD} - \sqrt{\dfrac{I_{in}}{\left(\frac{W}{2L}\right)_1 k'}} + 2V_T}{A \cdot R_L}$$

$$\Downarrow$$

$$I_{in} = \frac{2 - \sqrt{\dfrac{I_{in}}{\frac{10}{2} \cdot 100 \times 10^{-6}}} + 2 \cdot 0.5}{0.25 \cdot 8K}$$

Plugging into Wolfram Alpha

$$\cancel{I'_{in_1} = \frac{-\frac{\sqrt{48A+1}}{A^2} + \frac{1}{A^2} + \frac{24}{A}}{64K}}$$

$$I'_{in_2} = \frac{+\frac{\sqrt{48A+1}}{A^2} + \frac{1}{A^2} + \frac{24}{A}}{64K}$$

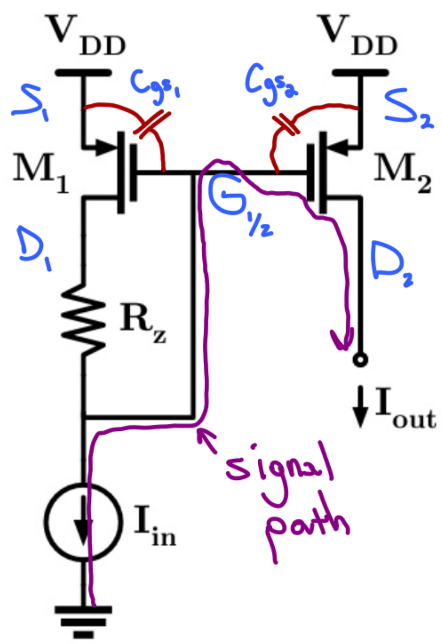
I'_{in_2} matches the value for $I'_{in,max}$

Final Piecewise Solution

$$I'_{in,MAX} = \begin{cases} \dfrac{0.5A}{1K}, & A < A_0 \\[2ex] \dfrac{\frac{\sqrt{48A+1}}{A^2} + \frac{1}{A^2} + \frac{24}{A}}{64K}, & A > A_0 \end{cases}$$

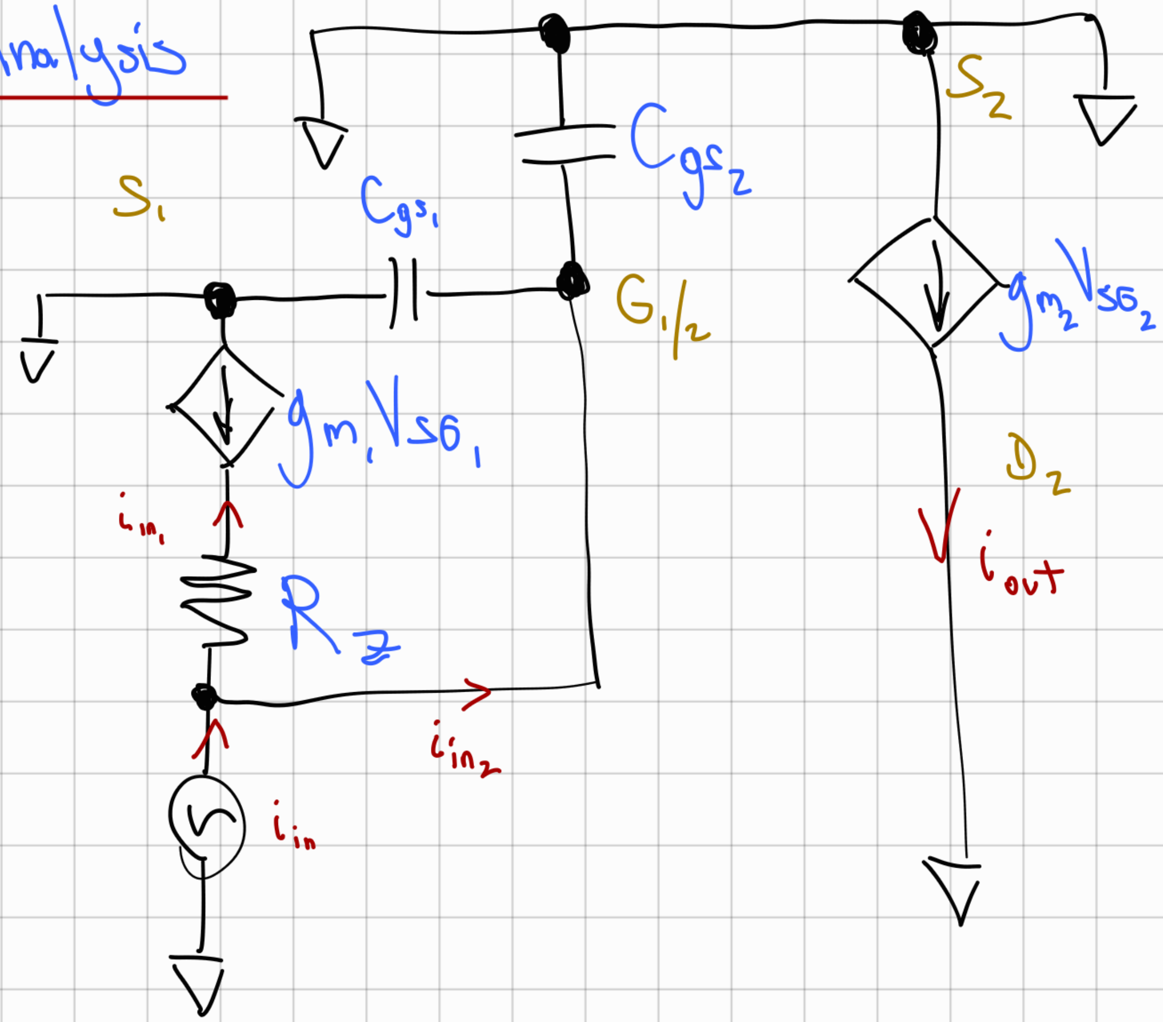
$\lambda = 0,\ \gamma = 0,\ |V_{TH}| = 0.5\text{V},\ V_{DD} = 2\text{V}$

$W_2 = A \times W_1,\ (\frac{W}{L})_1 = 10,\ k' = \mu_p C_{ox} = 100 \times 10^{-6}\ \text{A/V}^2$



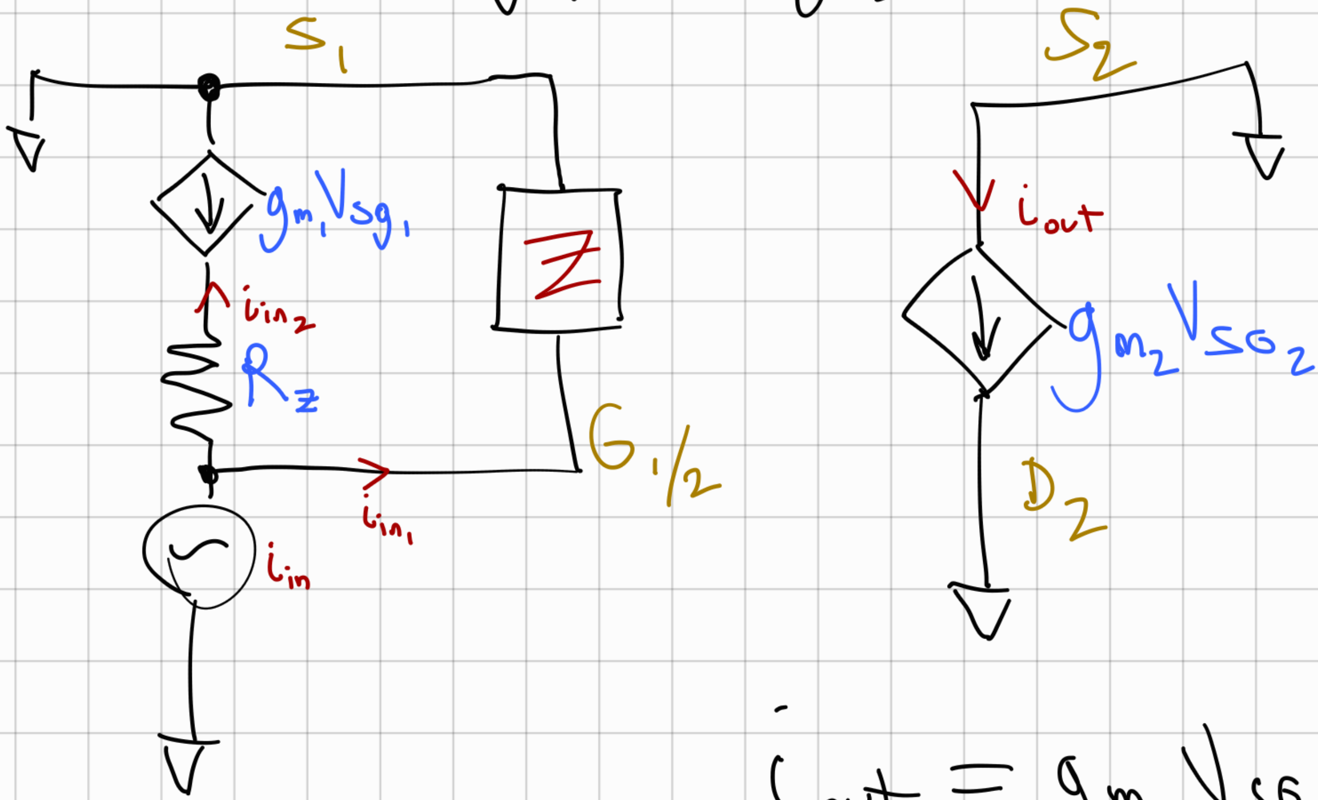
(c) Considering only the C_{gs} capacitors, derive the small signal transfer function for this amplifier $H(s) = \frac{i_{out}(s)}{i_{in}(s)}$ and draw the phase and amplitude Bode plots.

Nodal Analysis



Simplified Small-Signal Model

Let $Z = C_{gs_1} \parallel C_{gs_2}$



$$i_{out} = g_{m_2} V_{sg_2}$$

$$i_{in} = i_{in_1} + i_{in_2} = \frac{V_g}{Z} - g_{m_1} V_{sg_1}$$

$$= V_g\left(\frac{1}{Z} + g_{m_1}\right)$$

$$\frac{i_{out}}{i_{in}} = \frac{\cancel{V_g} - g_{m_2}}{\cancel{V_g}\left(\frac{1 + g_{m_1} Z}{Z}\right)}$$

$$= \frac{-g_{m_2} Z}{1 + g_{m_1} Z}$$

$$Z = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{\frac{1}{s^2 C_{gs_1} C_{gs_2}}}{\frac{1}{sC_{gs_1}} + \frac{1}{sC_{gs_2}}}$$

$$= \frac{\frac{1}{\cancel{s^2} \cancel{C_{gs_1} C_{gs_2}}}}{\frac{C_{gs_1} + C_{gs_2}}{\cancel{s C_{gs_1} C_{gs_2}}}} = \frac{1}{s\left(C_{gs_1} + C_{gs_2}\right)}$$

$$\Rightarrow \frac{i_{out}}{i_{in}} = \frac{\frac{-g_{m_2}}{s\left(C_{gs_1} + C_{gs_2}\right)}}{1 + \frac{g_{m_1}}{s\left(C_{gs_1} + C_{gs_2}\right)}}$$

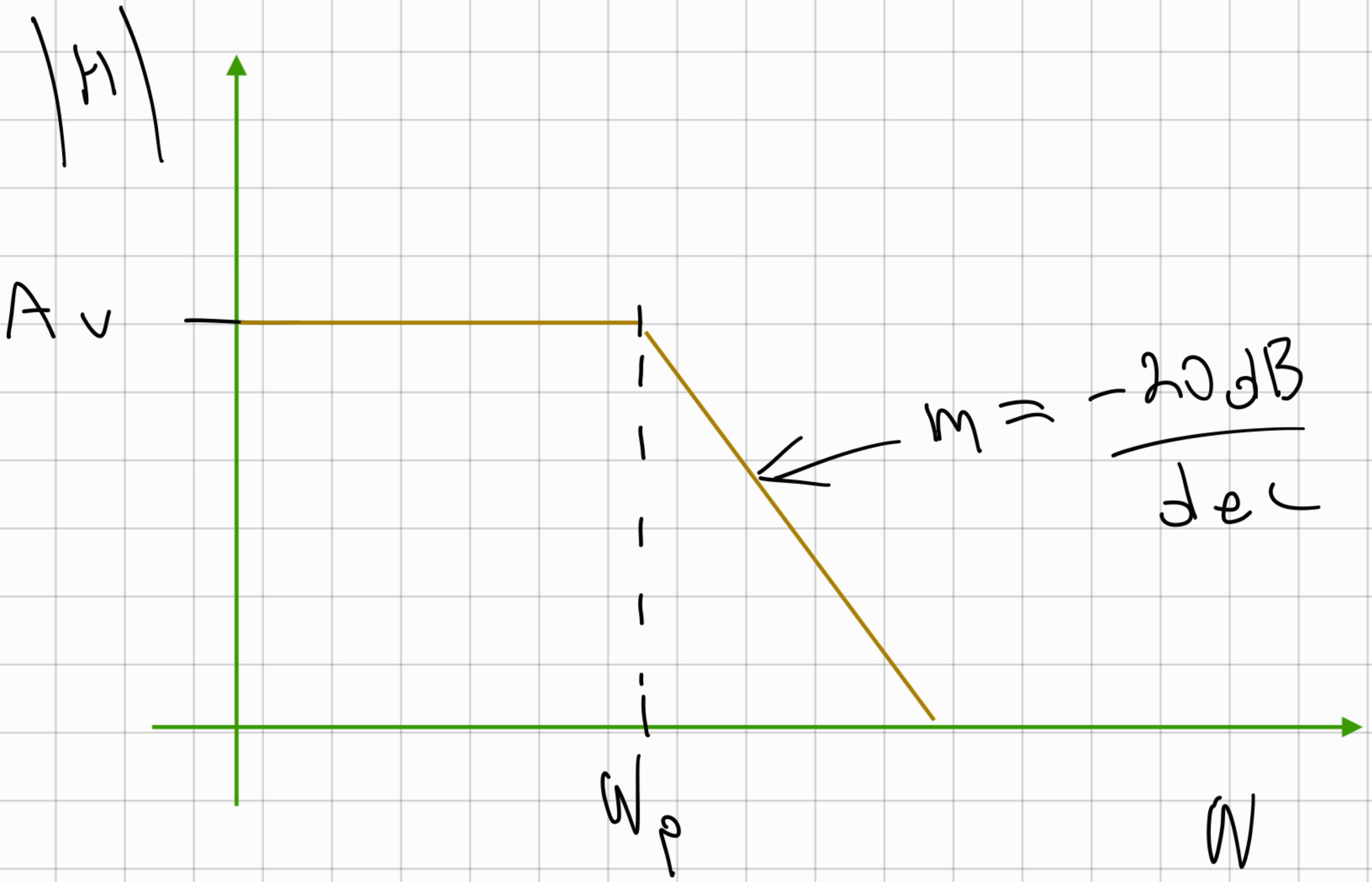
$$\Rightarrow \frac{i_{out}}{i_{in}} = \frac{\dfrac{-g_{m_2}}{\cancel{s\left(C_{gs_1}+C_{gs_2}\right)}}}{\dfrac{s\left(C_{gs_1}+C_{gs_2}\right)+g_{m_1}}{\cancel{s\left(C_{gs_1}+C_{gs_2}\right)}}}$$

$$= \frac{-g_{m_2}}{g_{m_1}}\left[\frac{1}{1+s\,\dfrac{C_{gs_1}+C_{gs_2}}{g_{m_1}}}\right]$$

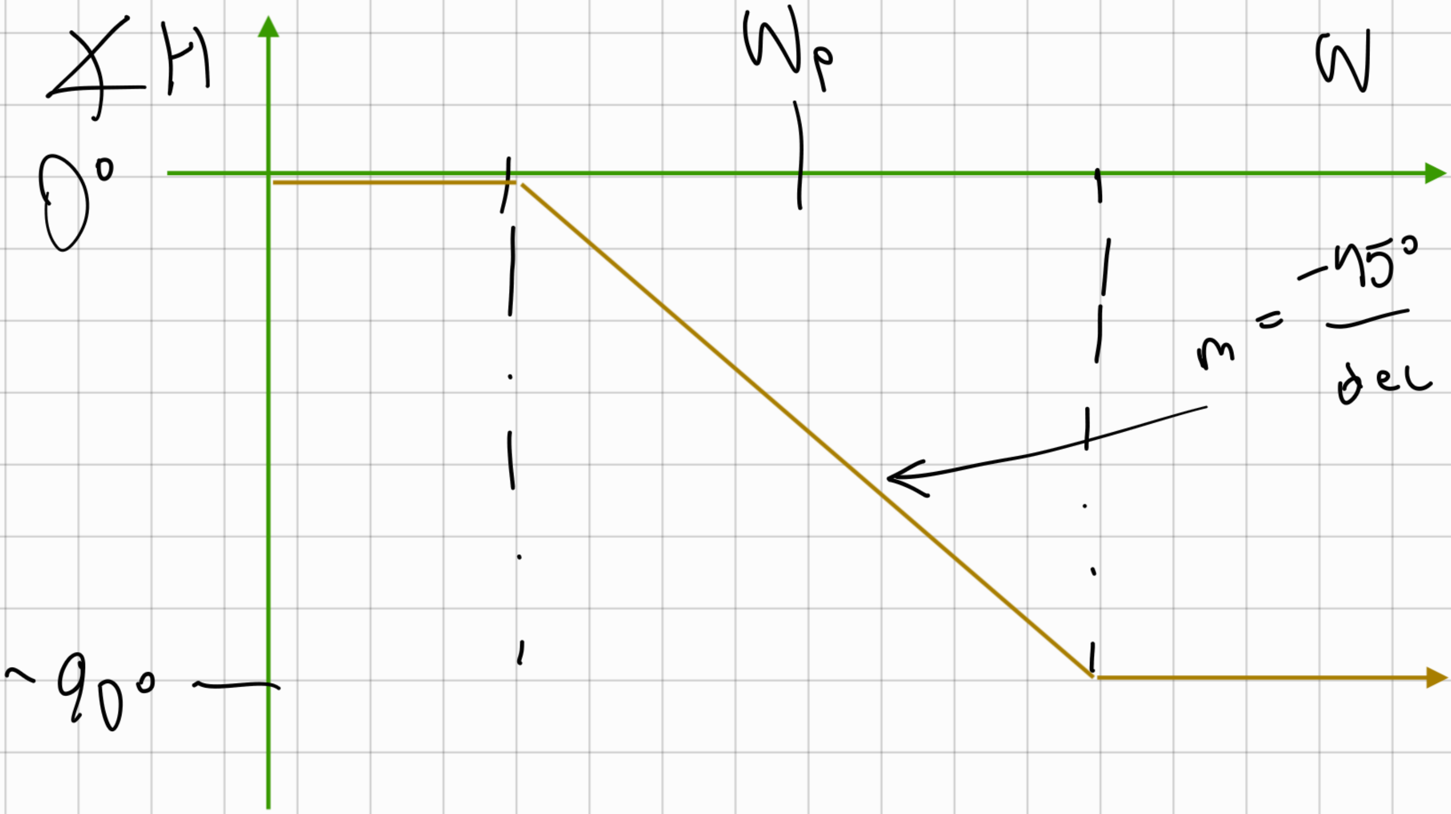
$\Rightarrow$ We have one LHP pole.

$$\boxed{\therefore\ \omega_p = \frac{g_{m_1}}{C_{gs_1}+C_{gs_2}}}$$

Bode Plot of Magnitude

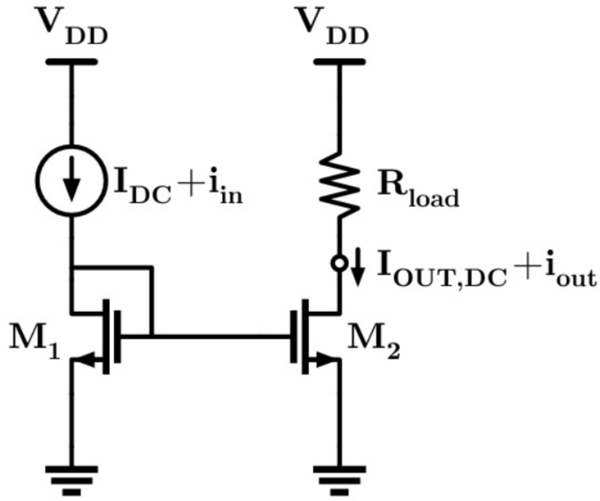


Bode Plot of Phase

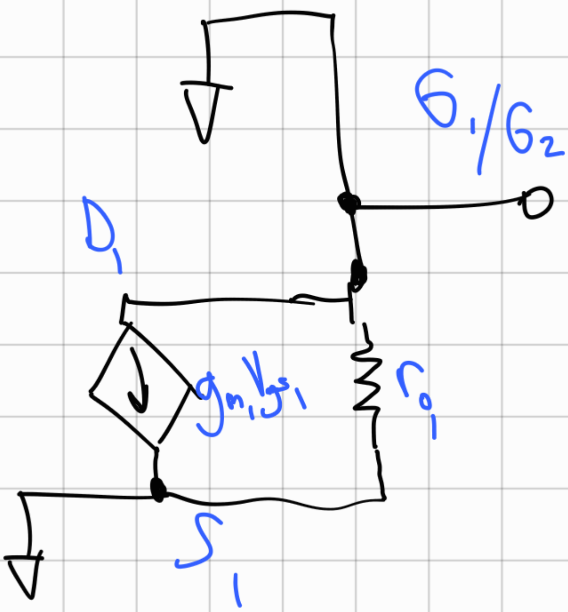


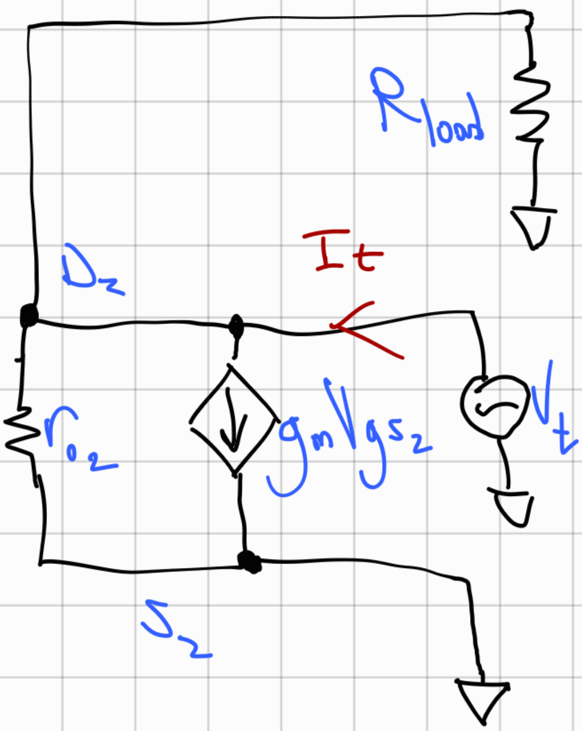
Problem 3 - Improving Current Mirrors

(a) For the below current mirror structure, with a small-signal input of i_{in} and an output of i_{out}, calculate the output impedance looking down from the output terminal. What would the small signal gain i_{out}/i_{in} be with a load resistor of $R_{load} = 10\ \text{k}\Omega$? $W_2 = 10 \times W_1$ and $L_2 = L_1$. Assume that I_{DC} puts M_1 in saturation, and M_1 has a g_m of $g_{m1} = 0.5$ mS and an r_o of $r_{o1} = 100\ \text{k}\Omega$.



Output Impedance





$$\frac{V_t}{R_{load}} + \frac{V_t}{r_{o2}} + \overset{0}{\cancel{g_{m_2} V_{gs_2}}} = I_t$$

$$\Rightarrow \frac{V_t}{I_t} = \boxed{R_{out} = r_{o_2} \,//\, R_{load}}$$

Down Up

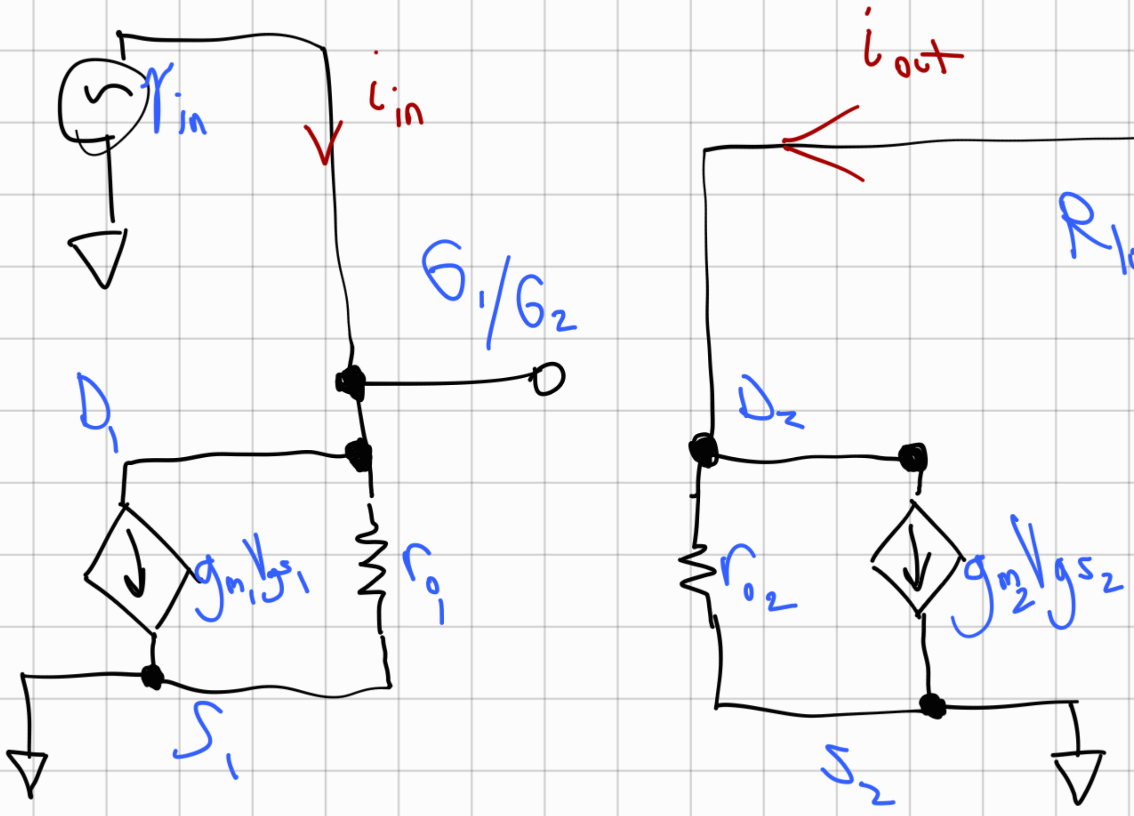
$$\Rightarrow r_{o_2} = \frac{r_{o_1}}{10} = \boxed{10K\,\Omega}$$

$$\Rightarrow R_{out} = \frac{r_{o_2} \cdot R_{LOAD}}{r_{o_2} + R_{LOAD}} = \frac{10K \cdot 10K}{10K + 10K}$$

$$\simeq \boxed{5k\,\Omega}$$ ← Total Rout

Small-signal gain

* Note that: We will assume a negligible body effect, $\lambda_1 = \lambda_2$, and $\mu C_{ox_1} = \mu C_{ox_2}$.



$$i_{out} = \frac{-V_{D_2}}{R_{load}} \quad (1)$$

$$i_{out} = \frac{V_{D_2}}{r_{o_2}} + g_{m_2} V_{g_2} \quad (2)$$

$$i_{in} = \frac{V_{g_2}}{r_{o_1}} + g_{m_1} V_{g_2} = V_{g_2}\left(g_{m_1} + \frac{1}{r_{o_1}}\right) \quad (3)$$

Rearrange (1) $\Rightarrow$ $$V_{D_2} = -i_{out} R_{load} \quad (4)$$

(4) into (2) $\Rightarrow$

$$i_{out} = -i_{out}\left(\frac{R_{load}}{r_{o2}}\right) + g_{m_2} V_{g_2} \quad (5)$$

Rearrange (2) $\Rightarrow$ $$V_{g_2} = i_{in}\left(\frac{r_{o_1}}{1 + g_{m_1} r_{o_1}}\right) \quad (6)$$

(6) into (5)

$$i_{out} = -i_{out}\left(\frac{R_{load}}{r_{o2}}\right) + g_{m2}\left[i_{in}\left(\frac{r_{o1}}{1+g_{m1}r_{o1}}\right)\right]$$

$$\Rightarrow \quad i_{out}\left(1+\frac{R_{load}}{r_{o2}}\right) = i_{in}\left(\frac{g_{m2}r_{o1}}{1+g_{m1}r_{o1}}\right)$$

$$\Rightarrow \quad \frac{i_{out}}{i_{in}} = \left(\frac{g_{m2}r_{o1}}{1+g_{m1}r_{o1}}\right)\left(\frac{r_{o2}}{r_{o2}+R_{load}}\right)$$

$$\boxed{\therefore \frac{i_{out}}{i_{in}} = \frac{g_{m2}r_{o1}r_{o2}}{(1+g_{m1}r_{o1})(r_{o2}+R_{load})}}$$

Find g_{m2}: Because of the scaling ratio of current mirrors, we know that $g_{m2} = 10 \cdot g_{m1}$

$$\Rightarrow \quad g_{m2} = 10 \cdot 0.5 \text{ mS}$$

$$\therefore \boxed{g_{m2} = 5 \text{ mS}}$$

$$\Rightarrow \frac{i_{out}}{i_{in}} = \frac{g_{m_2} r_{o_1} r_{i_2}}{(1+g_{m_1} r_{o_1})(r_{o_2}+R_{load})}$$

$$= \frac{(5\times10^{-3} S)(100k\Omega)(10k\Omega)}{(1+0.5\cdot\times10^{-3} S\cdot 100k\Omega)(20000)}$$

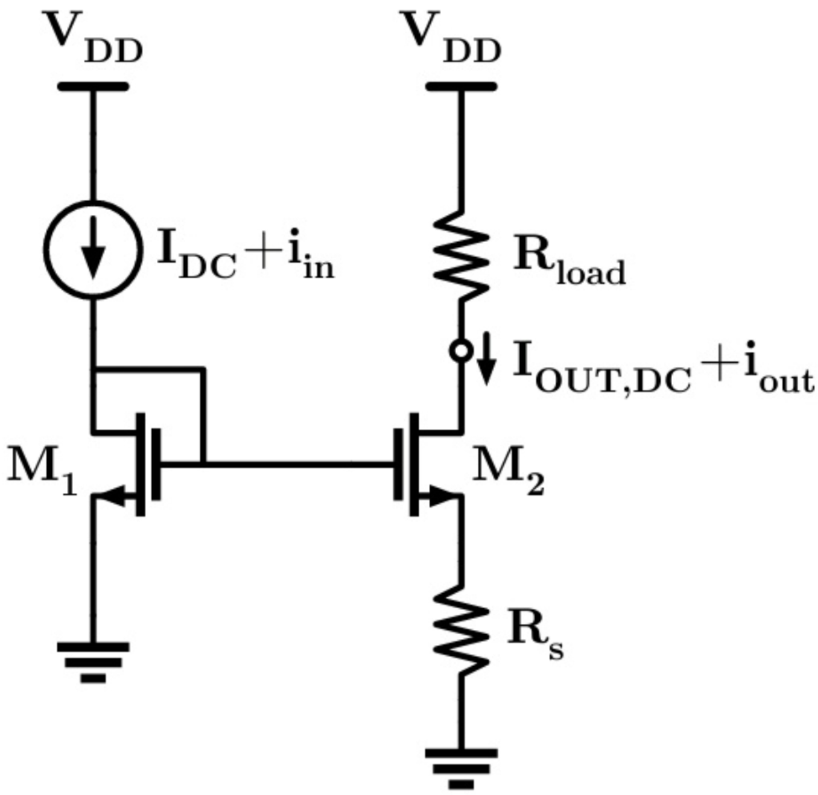
$$= \frac{5000000}{51\cdot 20k}$$

$$= \frac{50000000}{1020000}$$

$$= 4.901960784$$

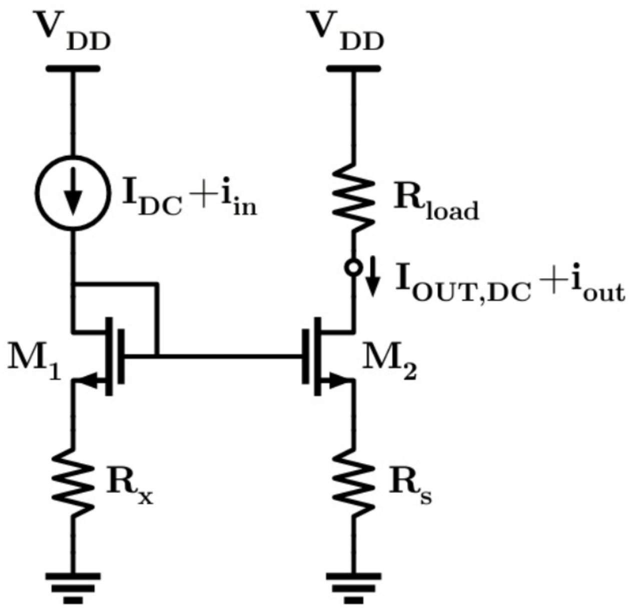
$$\boxed{\therefore A_i \cong 4.9}$$

(b) Your coworker has suggested that by adding a degeneration resistor $R_s = 1\ \text{k}\Omega$, you could improve the output impedance and hence the overall gain under high impedance loads. Would the new structure work as a current mirror? Why/why not?



This would not work as a current mirror, because with the addition of R_s the V_{GS} of both transistors will not be the same.

(c) You decide to improve upon the previous structure by adding an R_x. Now, for this structure to act as a current mirror, what should the value of R_x be if $R_s = 1\ \mathrm{k}\Omega$? Compute the new small signal output impedance R_{out} for the mirror. What would the new gain i_{out}/i_{in} be with a load resistor of $R_{load} = 10\ \mathrm{k}\Omega$?



Finding a Value for R_x

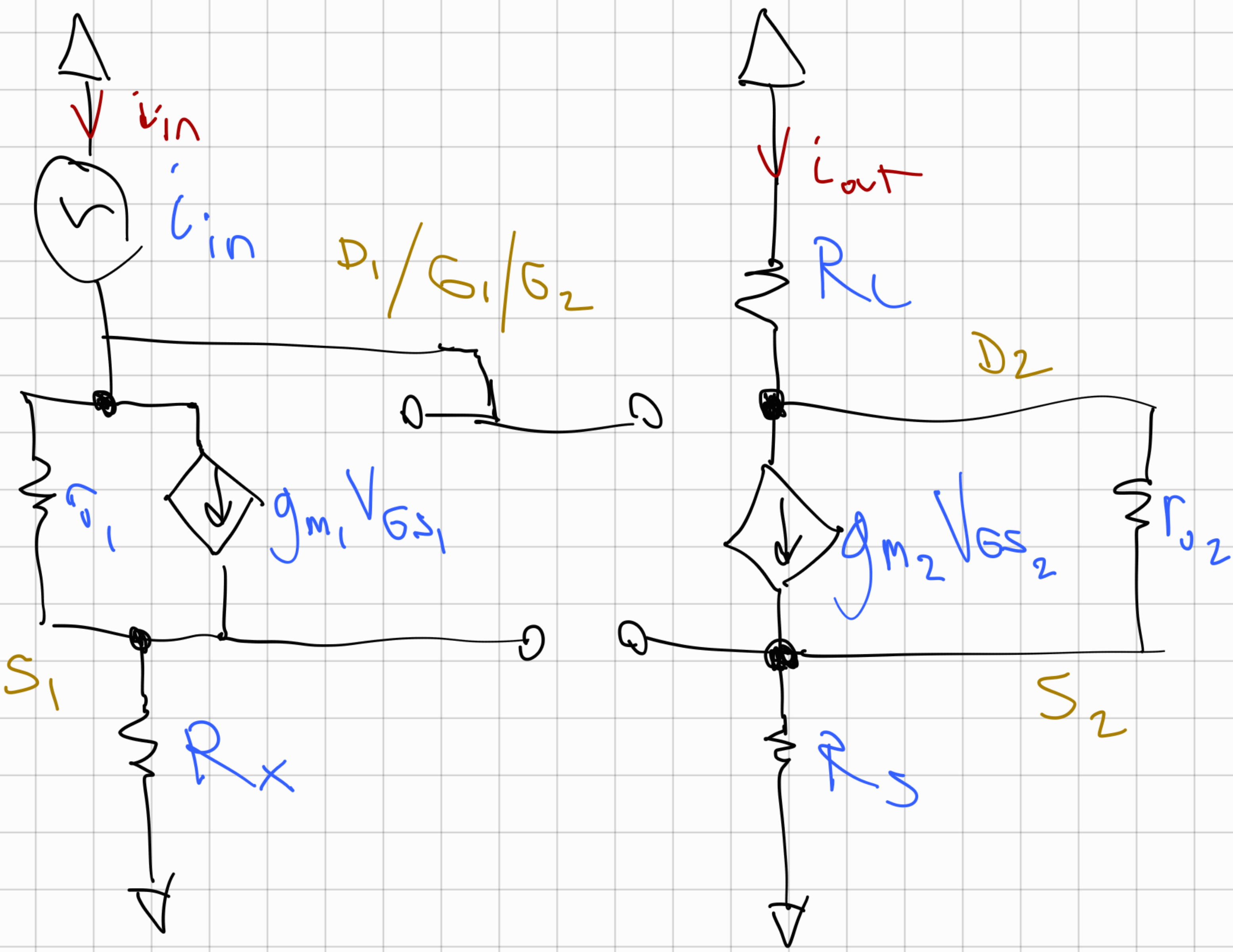
We want to match ΔV_1 to ΔV_2 for a proper current mirror. Since R_{LOAD} has no impact on ΔV_2, and M_2 scales the current in M_1 by a factor of 10, we need to reduce the value of R_x by a factor of 10 relative to the value of R_s.

$$\Rightarrow \quad R_x \cdot I_{DC} = 10 \cdot I_{DC} \cdot R_s$$

continued ...

$$\Rightarrow \boxed{R_x = 10 \cdot R_s = 10\,k\Omega}$$

Small-signal Model



$$i_{in} = \frac{V_g}{r_{o_1}} + g_{m_1} V_{gs_1} = V_g\left(g_{m_1} + \frac{1}{r_{o_1}}\right) - g_{m_1} V_{s_1} \quad (1)$$

$$i_{in} = \frac{V_{s_1}}{R_x} \Rightarrow V_{s_1} = i_{in} \cdot R_x \quad (2)$$

$$i_{out} = \frac{V_{d_2}}{r_{o_2}} + g_{m_2} V_{gs_2} \quad (3)$$

$$i_{out} = \frac{V_{s_2}}{R_S} \Rightarrow V_{S_2} = i_{out} \cdot R_S \quad (4)$$

$$i_{out} = -\frac{V_{d_2}}{R_L} \Rightarrow V_{d_2} = -i_{out} \cdot R_L \quad (5)$$

---

(2) into (1)

$$\Rightarrow i_{in} = V_g\left(g_{m_1} + \frac{1}{r_{o_1}}\right) - g_{m_1}\left(i_{in} \cdot R_x\right)$$

$$\Rightarrow i_{in}\left(1 + g_{m_1} R_x\right) = V_g\left(g_{m_1} + \frac{1}{r_{o_1}}\right)$$

$$\Rightarrow i_{in} = V_g \left(\frac{1 + g_{m1} r_{o1}}{r_{o1}} \right) \left(\frac{1}{1 + g_{m1} R_x} \right) \quad (6)$$

(4) and (5) into (3)

$$\Rightarrow i_{out} = -i_{out} \left(\frac{R_L}{r_{o2}} \right) + g_{m2} V_g - g_{m2} i_{out} \cdot R_S$$

$$\Rightarrow i_{out} \left(1 + \frac{R_L}{r_{o2}} + g_{m2} R_S \right) = g_{m2} V_g$$

$$\Rightarrow i_{out} = V_g \left(\frac{r_{o2}\, g_{m2}}{r_{o2} + R_L + g_{m2} r_{o2} R_S} \right)$$

$$\Rightarrow \frac{i_{out}}{i_{in}} = \frac{\cancel{V_g} \left(\frac{r_{o2}\, g_{m2}}{r_{o2} + R_L + g_{m2} r_{o2} R_S} \right)}{\cancel{V_g} \left(\frac{1 + g_{m1} r_{o1}}{r_{o1}} \right) \left(\frac{1}{1 + g_{m1} R_x} \right)}$$

$$\frac{i_{out}}{i_{in}} = \left(\frac{r_{o2}\, g_{m2}\, r_{o1} \left(1 + g_{m1} R_X\right)}{r_{o2} + R_L + g_{m2} r_{o2} R_S \left(1 + g_{m1} r_{o1}\right)} \right)$$

$g_{m1} = 0.5\,mS \qquad g_{m2} = 5\,mS$

$r_{o1} = 100K \qquad r_{o2} = 10K$

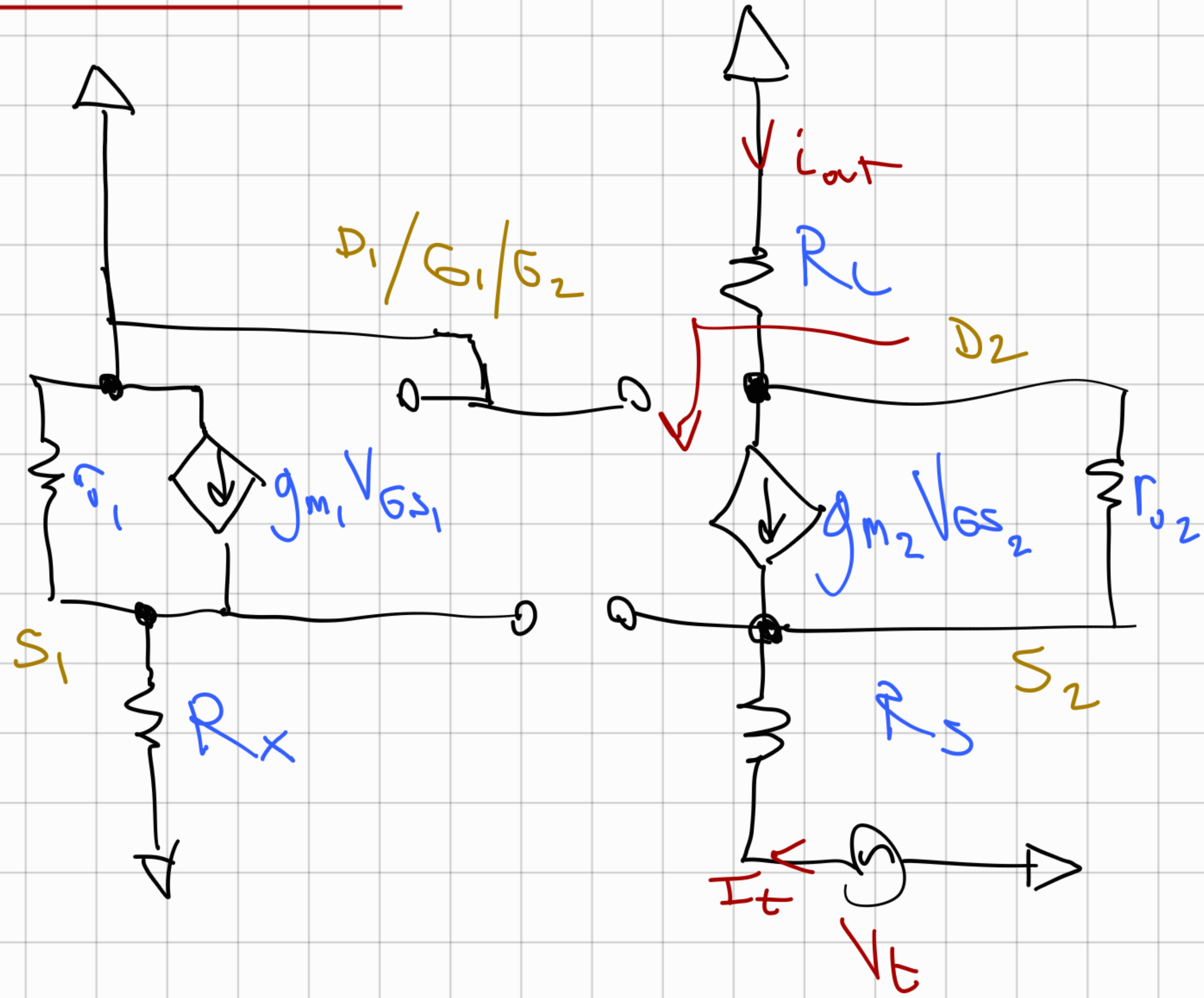
$R_X = 10K \qquad R_S = 1K$

$R_L = 10K$

$$\frac{i_{out}}{i_{in}} = \frac{\left(5 \times 10^{6}\right)\left(6\right)}{20k + 5000\left(51\right)}$$

$$\boxed{\therefore\ A_i \cong 11.67}$$

Output Resistance



Looking down

$$R_{th,d} = g_{m_2} r_{o_2} R_S + R_S + r_o$$

$$\therefore R_{out_{total}} = R_L \,//\, R_{th,d}$$

Numerical

$$R_{th,d} = 61\,k\Omega \quad R_{out} = 8.6\,k\Omega$$