Problem 1 - Cascode Current Mirror Biasing

Consider the wide-swing cascode current mirror in figure 1, with parameter values listed in the table. All devices are sized similarly (W) except for M_5 which has a different size (W_5).

I_B	$100\ \mu A$
L	$1\ \mu m$
$\lvert k' \rvert$	$225\ \mu A \cdot V^{-2}$
λ	$0.02\ V^{-1}$
$\lvert V_{TH} \rvert$	$0.6\ V$
γ	0

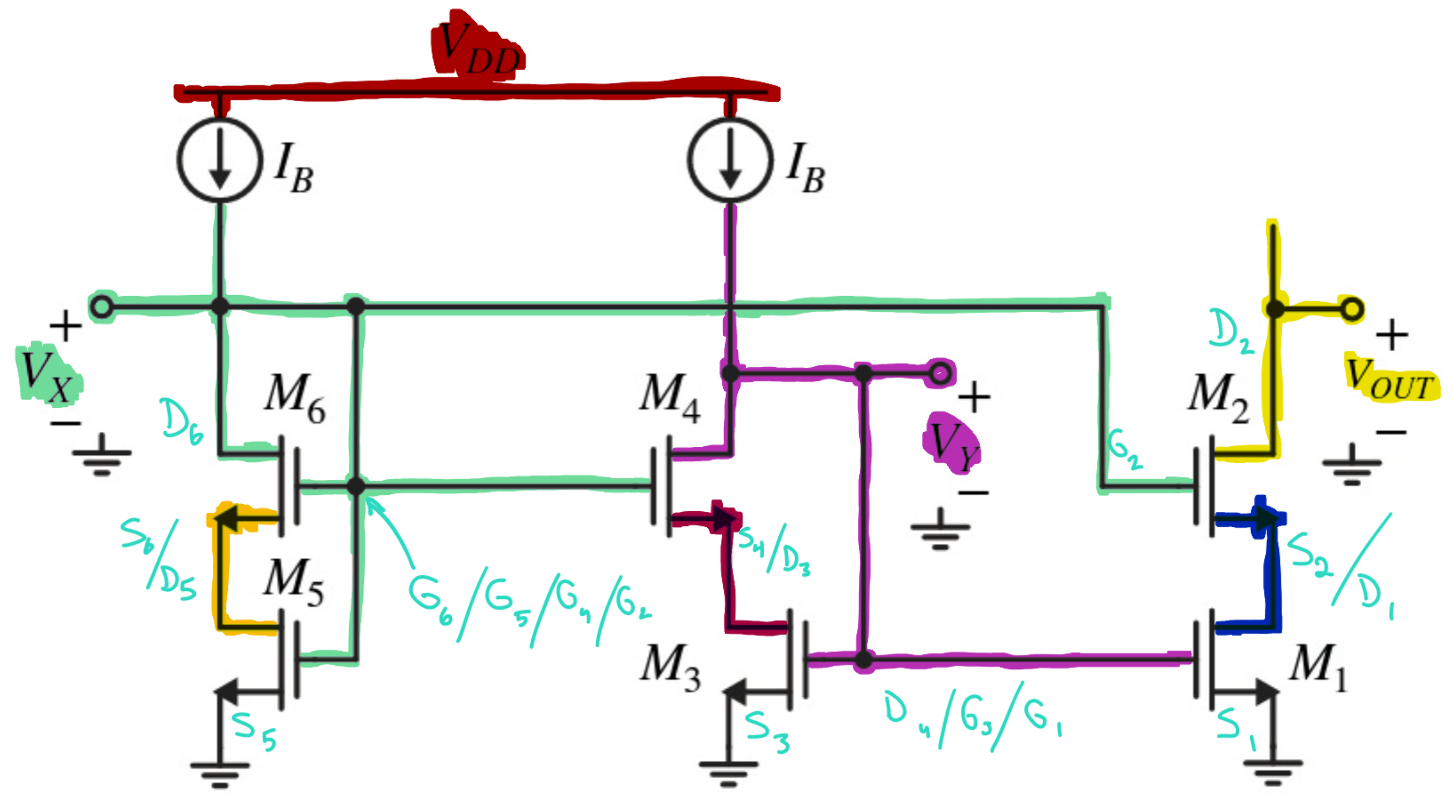


Figure 1: Circuit for problem 1

(i) Calculate W such that the minimum output voltage for which both M_1 and M_2 are in saturation is 0.5V. Assume that M_{3-5} can provide appropriate gate biases for M_1 and M_2. Neglect channel length modulation when performing large signal computations.

$$V_{DS_1} + V_{DS_2} = 0.5\ V = V_{out}$$

$$V_{D_1/S_2} = 0.25V \Rightarrow V_{DS_2} = 0.25V$$

$$\Rightarrow V_{DS_2} = V_{GS_2} - V_T$$

$$\Rightarrow 0.25V = V_{G_2} - 0.25 - V_T$$

$$\Rightarrow V_{G_2} = 0.25V + 0.25V + 0.6V$$

$$\Rightarrow V_{G_2} = 1.1V$$

$$\Rightarrow V_{GS_2} = 0.85\ V$$

$$\Rightarrow I_B = \left(\frac{W}{L}\right)_2 k' \left(V_{GS_2} - V_T\right)^2$$

$$\Rightarrow W = \frac{2 \cdot L \cdot I_B}{k' \left(V_{GS_2} - V_T\right)^2}$$

$$= \frac{1\,\mu m \cdot 100\,\mu A \cdot 2}{225\,\mu A/V^2 \left(0.85V - 0.6V\right)^2}$$

$$\boxed{\therefore\ W \cong 14.22\ \mu m}$$

(ii) Calculate W_5 such that the minimum output voltage in part "i" is achieved.

$\Rightarrow V_{GS_5} = 1.1V \quad \& \quad I_{D_5} = I_{D_6} = I_B$

$\Rightarrow I_{D_6} = \left(\frac{W}{2L}\right)_6 k' \left(V_{gs_6} - V_T\right)^2$

$* \; V_{gs_6} = V_x - V_{s_6} \quad * \; V_x = V_{g_2} = 1.1V$

$\Rightarrow V_x - V_{s_6} - V_T = \sqrt{\frac{2L \cdot I_{D_6}}{k' W}}$

$\Rightarrow V_{s_6} = V_x - \sqrt{\frown} - V_T$

$= 1.1V - \frac{2 \cdot 1 \cdot 100 \, \mu A}{225 \, \mu A/V^2 \cdot 14.22} - 0.6$

$1.1V - 0.25V - 0.6V$

$\cong 0.25V$

$\Rightarrow V_{gs_6} = 0.85V \quad * \; V_{g_2} = V_{g_6} = V_{g_5}$

$$\Rightarrow V_{ds_5} = V_{s_6} = 0.25V$$

$$< V_{gs_5} - V_T = 1.1V - 0.5V$$

$$= 0.6V$$

$\Rightarrow M_5$ is in triode.

$$\Rightarrow I_B = \left(\frac{W}{L}\right)_5 k' \left(V_{gs_5} - V_{T_n} - \frac{V_{ds_5}}{2}\right) V_{ds_5}$$

$$\Rightarrow W_5 = \frac{I_B \cdot L_5}{k' \left(V_{gs_5} - V_{T_n} - \frac{V_{ds5}}{2}\right) V_{ds_5}}$$

$$= \frac{100\mu A \cdot 1\mu m}{225 \frac{\mu A}{V^2}\left(1.1V - 0.6V - \frac{0.25V}{2}\right) 0.25V}$$

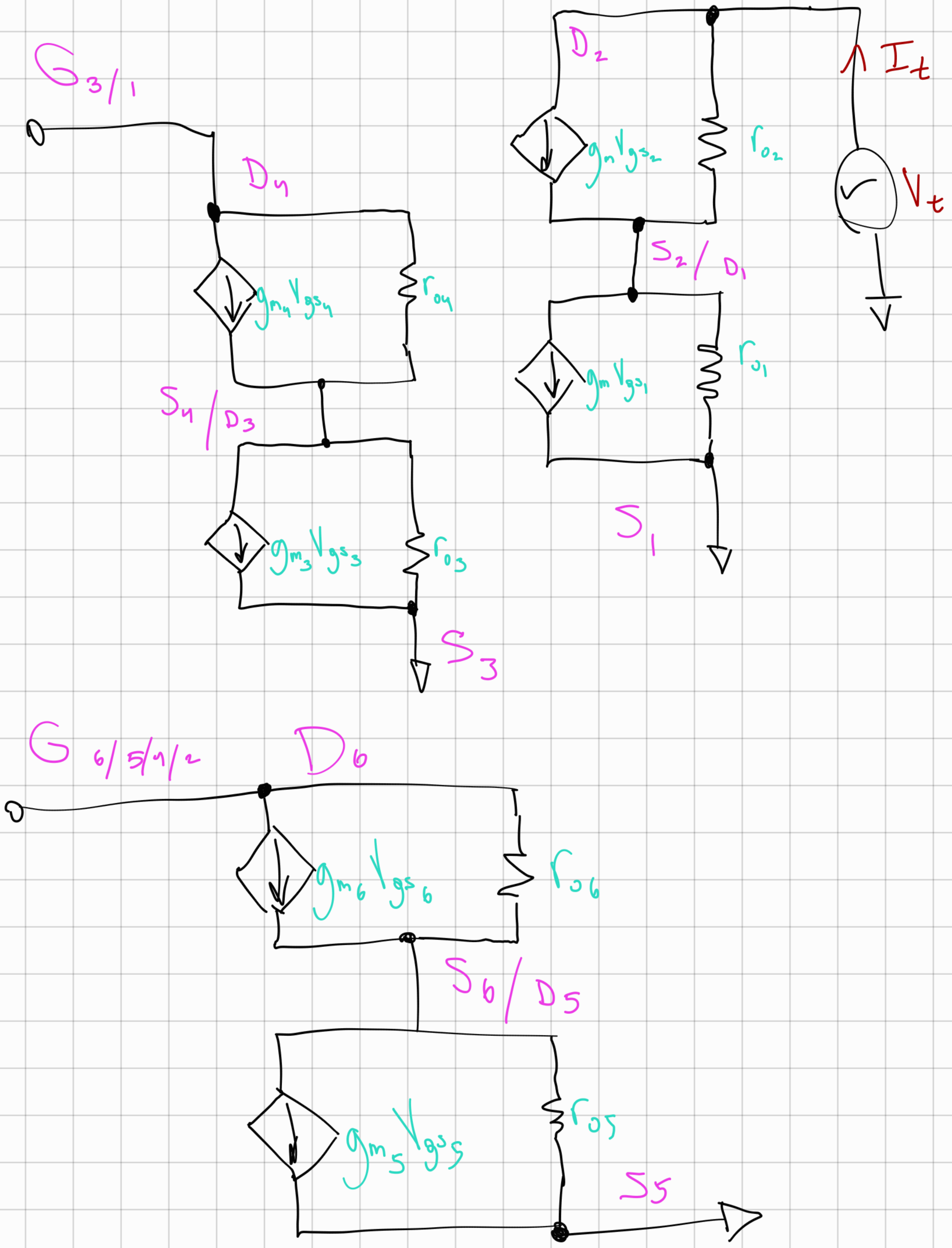
$$= 4.74074 \times 10^{-6}\ m$$

$$\boxed{\therefore W_5 \cong 4.74\mu m}$$

(iii) Briefly explain the function of M_4 in the circuit.

The main purpose of M_4 is to keep the V_{ds} across M_3 the same as where it is being mirrored from. This minimizes the mismatch and systematic gain error between M_1 & M_3.

(iv) What is the small signal output resistance of this current source? [R_{OUT} is the AC resistance looking into the output port]



$$I_t = \frac{V_t}{r_{o2}} - \frac{S_2}{r_{o2}} + g_{m_2} V_{gs_2} = \frac{V_{s_2}}{r_{o_1}}$$

$$\Rightarrow I_t = \frac{V_t}{r_{o2}} - \frac{1}{r_{o2}}\left(I_t r_{o1}\right) - g_{m_2}\left(I_t r_{o1}\right)$$

$$\Rightarrow I_t\left(1 + \frac{r_{o1}}{r_{o2}} + g_{m_2} r_{o1}\right) = \frac{V_t}{r_{o2}}$$

$$\Rightarrow \frac{V_t}{I_t} = \cancel{r_{o2}}\left(\frac{r_{o1} + r_{o2} + g_{m_2} r_{o1} r_{o2}}{\cancel{r_{o2}}}\right)$$

$$\Rightarrow \boxed{R_{out} = r_{o2} + r_{o1}\left(1 + g_{m_2} r_{o2}\right)}$$

$$\Rightarrow r_o = \frac{1}{\lambda I_D} = \frac{1}{0.02\,V^{-1} \cdot 100\,\mu A}$$

$$= 500\,k\Omega = r_{o1} = r_{o2}$$

$$\Rightarrow g_{m_2} = \frac{2 I_B}{V_{gs5} - V_T} = \frac{2 \cdot 100\,\mu A}{1.1V - 0.6V}$$

$$= 4 \times 10^{-4}\,S$$

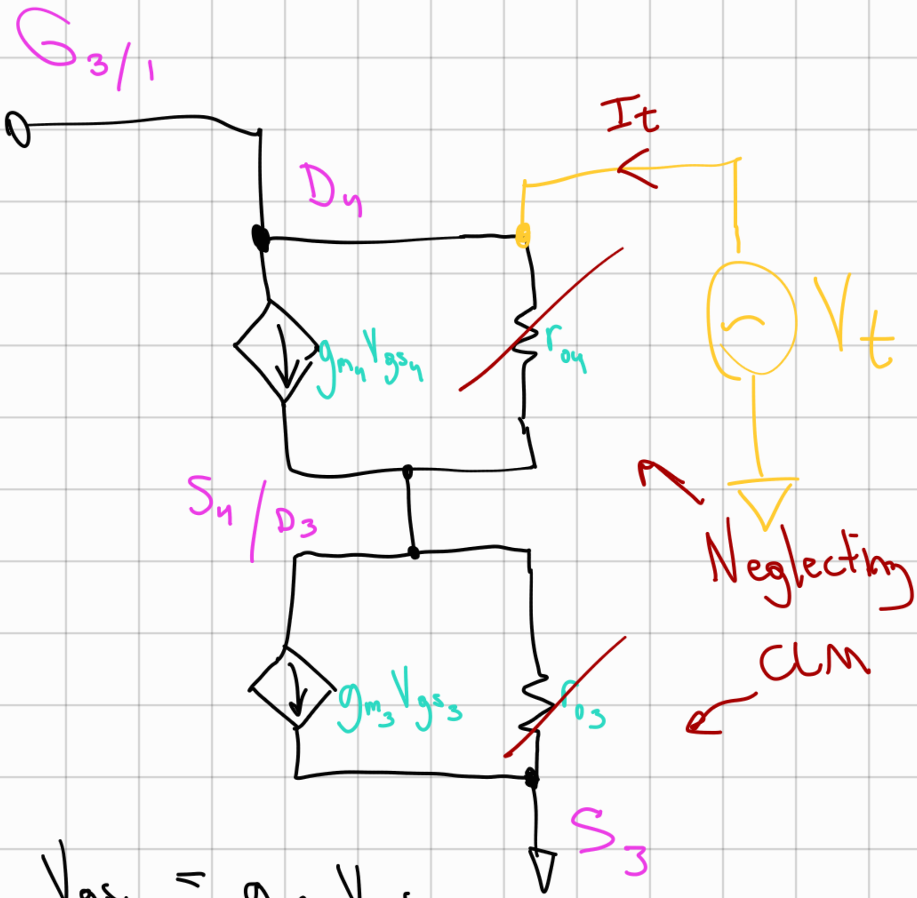
$$\Rightarrow R_{out} = 500k + 500k\left(1 + 4\times10^{-4}\,S \cdot 500k\right)$$

$$= 101000000\ \Omega$$

$$\boxed{\therefore\ R_{out} \cong 101\ M\Omega}$$

(v) What is the small signal resistance seen looking at the V_Y and V_X ports? In this part, you may neglect channel length modulation i.e. assume $\lambda = 0$.

V_Y



$$\Rightarrow I_t = g_{m_4} V_{gs_4} = g_{m_3} V_{gs_3}$$

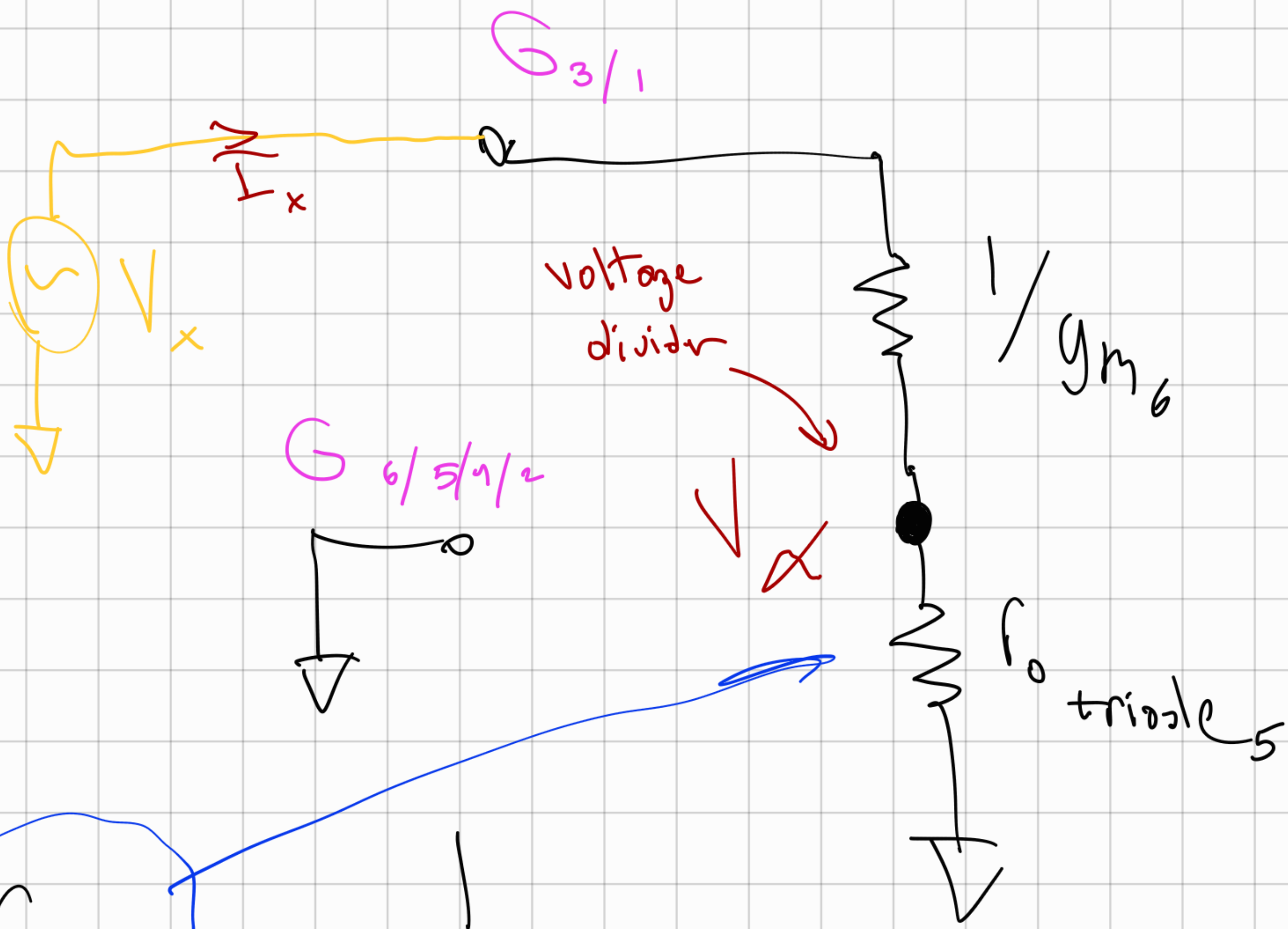
$$= -g_{m_4} V_{s_4} = g_{m_3} V_t$$

$$\Rightarrow I_t = g_{m_3} V_t \Rightarrow R_{at_Y} = 1/g_{m_3}$$

$$\therefore \boxed{R_{at_Y} = 1/g_{m_3}} = 1/4\times10^{-4}\,S$$

$$\therefore \boxed{R_{at_Y} \cong 2500\ \Omega}$$

$\underline{V_x}$



$$r_{o\,\text{triode}} = \frac{1}{\left(\frac{\partial I_D}{\partial V_{ds}}\right)} \} g_o$$

$$\frac{\partial I_D}{\partial V_{ds}} \left[\underbrace{\left(\frac{W}{L}\right)_5 k'}_{\phi} \left(V_{gs_5} - V_T - \frac{V_{ds_5}}{2} \right) V_{ds_5} \right]$$

$$= \frac{\partial I_D}{\partial V_{ds}} \left[\phi \left(V_{gs_5} - V_T \right) V_{ds_5} - \phi \frac{V_{ds_5}^2}{2} \right]$$

$$= \phi\left(V_{gs_5} - V_T\right) - \frac{\phi}{2} V_{ds_5}$$

$$= \phi\left(V_{gs_5} - V_T - \frac{1}{2} V_{ds_5}\right)$$

$$= \left(\frac{W}{L}\right)_5 k' \left(V_{gs_5} - V_T - \frac{1}{2} V_{ds_5}\right)$$

$$I_x = \frac{V_x}{\left(1/g_{m_6} + r_{tristate_5} \right)}$$

$$= \frac{V_x}{\frac{1 + g_{m_6} r_{tristate_5}}{g_{m_6}}}$$

$$\boxed{\therefore R_{out_x} = \frac{1 + g_{m_6} r_{tristate_5}}{g_{m_6}}}$$

$$\Rightarrow r_{o_{triode}} = \frac{1}{\left(\frac{W}{L}\right)_5 k' \left(V_{gs_5} - V_T - \frac{1}{2} V_{ds_5}\right)}$$

$$= \frac{1}{\left(\frac{4.74\,\mu m}{1\,\mu m}\right)\left(225\,\frac{\mu A}{V^2}\right)\left(1.1V - 0.6V - \frac{0.25V}{2}\right)}$$

$$\cong 2.5k\Omega$$

$$\Rightarrow g_{m_0} = g_{m_2} = 4 \times 10^{-4}\ S$$

$$\Rightarrow R_{out_x} = \frac{1 + (2.5k)(4 \times 10^{-4})}{4 \times 10^{-4}\ S}$$

$$\boxed{\therefore R_{out_x} \cong 5k\Omega}$$

Problem 2 - Advanced cascode current mirrors

Figure 2 shows two cascode current mirrors featuring different biasing techniques. Assume all devices are sized similarly except for M_9 in sub-figure (b). Also assume that $I_B = 10\mu A$, $\Delta V_{1-8} = 0.1V$, $V_{TH} = 0.5V$, $\lambda = 0.1V^{-1}$, and $\gamma = 0$.

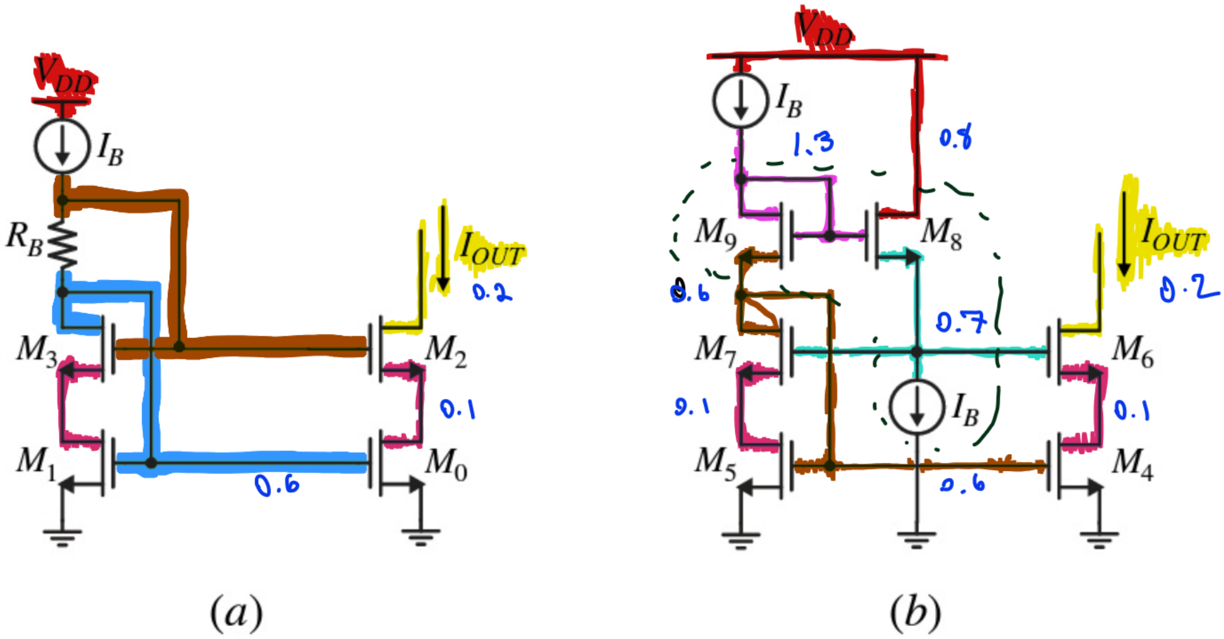


Figure 2: Circuits for problem 2

(a) Find the value of the output current, I_{OUT}, as well as the output resistance, R_{OUT}, in both circuits in figure 2.

All of I_B goes through M_3 and M_1. Since it is a current mirror and $\Delta V_1 = \Delta V_0$, the current on the mirrored side must be the same.

$$\boxed{\therefore\ I_{out} = I_B}$$

$$R_{out} = r_{o_2} + r_{o_3}\left(1 + g_{m_2} r_{o_2}\right)$$

$$r_{o_2} = r_{o_3} = \frac{1}{\lambda I_B} = \frac{1}{0.1\,V^{-1} \cdot 10\,\mu A}$$

$$\cong 1\,M\Omega$$

$$g_m = \frac{2 \cdot I_B}{\Delta V_1} = \frac{2 \cdot 10\,\mu A}{0.1\,V}$$

$$= 2 \times 10^{-4}\,S$$

$$\Rightarrow R_{out} = 1\,M\Omega + 1\,M\Omega\left(1 + 2 \times 10^{-4}\,S \cdot 1M\right)$$

$$\boxed{\cong 202\,M\Omega}$$

I_{out} and R_{out} will be the same in both circuits.

(b) For circuit in figure 2.a, determine the value of R_B that maximizes the output voltage swing.

$V_{out_{min}} = 2\Delta V$ & $V_{out_{max}} =$ undefined

$\Rightarrow$ We want a value of R_B that satisfies the minimum output swing, and assume V_{DD} has enough headroom for the maximum half of the swing.

$\Rightarrow V_{ds_0} = V_{gs_0} - V_T \Rightarrow 0.1 = V_{g_0} - 0.5 \Rightarrow V_{g_0} = 0.6\,V$

$\Rightarrow V_{ds_2} = V_{gs_2} - V_T \Rightarrow V_{g_2} = 0.1\,V + 0.1\,V + 0.5\,V = 0.7\,V$

$\Rightarrow V_{g_0} = V_{g_1} = V_{d_3} \Rightarrow V_{R_B} = V_{g_2} - V_{d_3}$

$\Rightarrow V_{R_B} = I_B \cdot R_B = V_{g_2} - V_{d_3} = 0.7V - 0.6V = I_B \cdot R_B$

$\Rightarrow R_B = \dfrac{0.1\,V}{10\,\mu A}$

$\boxed{\therefore R_B = 10\,K\Omega}$

(c) If I_B is reduced by four times ($I'_B = 1/4 \times I_B$), does the circuit in "a" still function? Explain.

No. Before reducing the current, we are precisely at one ΔV between G_0 and G_2, which is required for saturation. With a reduced current, the change in $V_{g2,3}$ will not scale the same as the change in $V_{g0,1}$, due to the fact that R_B^+ drops more than R_B^-. With a reduction of I_B by $1/4$, we have all $\Delta V \rightarrow \frac{1}{2}\Delta V$, but with different gate voltages, such that $M_{2,3}$ will not be in saturation.

(d) Size M_9 such that the output voltage swing is maximized. Determine the size of M_9, $(W/L)_9$, in terms of the size of other transistors.

M_8 & M_6 equally sized with the same current. $\Rightarrow \Delta V_8 = \Delta V_6$

$I_{D_9} = I_{D_6}$

$$\Rightarrow \cancel{k'} \left(\frac{W}{\cancel{L}}\right)_9 (\Delta V_9)^2 = \cancel{k'} \left(\frac{W}{\cancel{L}}\right)_6 (\Delta V_6)^2$$

$$\Rightarrow \frac{W_9}{W_6} = \left(\frac{\Delta V_6}{\Delta V_9}\right)^2 = \left(\frac{0.1}{0.2}\right)^2$$

$$= \frac{1}{4}$$

$$\boxed{\therefore W_9 = \frac{1}{4} W_4}$$

(e) If I_B is reduced by four times ($I'_B = 1/4 \times I_B$), does the circuit in "b" still function? Explain.

This circuit will still operate as it was intended to. The biasing I_B current is forced down M_9, M_7, and M_5. The mirror copies this current to force it down M_6 and M_4. In this circuit, the bias voltages are set by the current source, and not the voltage drop across a resistor. This means the circuit will self-adjust the bias voltages independent of I_B's value.

(f) If the ideal current sources need a minimum of 200mV voltage headroom to operate, determine the minimum required V_{DD} for the two circuits in figure 2.

We need to have a minimum V_{DD} of the gate voltage at $M_{8/9}$, which has already been calculated, and is indicated on the schematic, and the additional 200 mV.

$$\Rightarrow V_{g_{8/9}} + 200\,mV = 1.3V + 0.2V$$

$$\boxed{\therefore V_{DD_{min}} = 1.5V}$$

Problem 3 - Differential Amplifiers

Consider the amplifier shown in figure 3. Assume $\lambda \neq 0$.

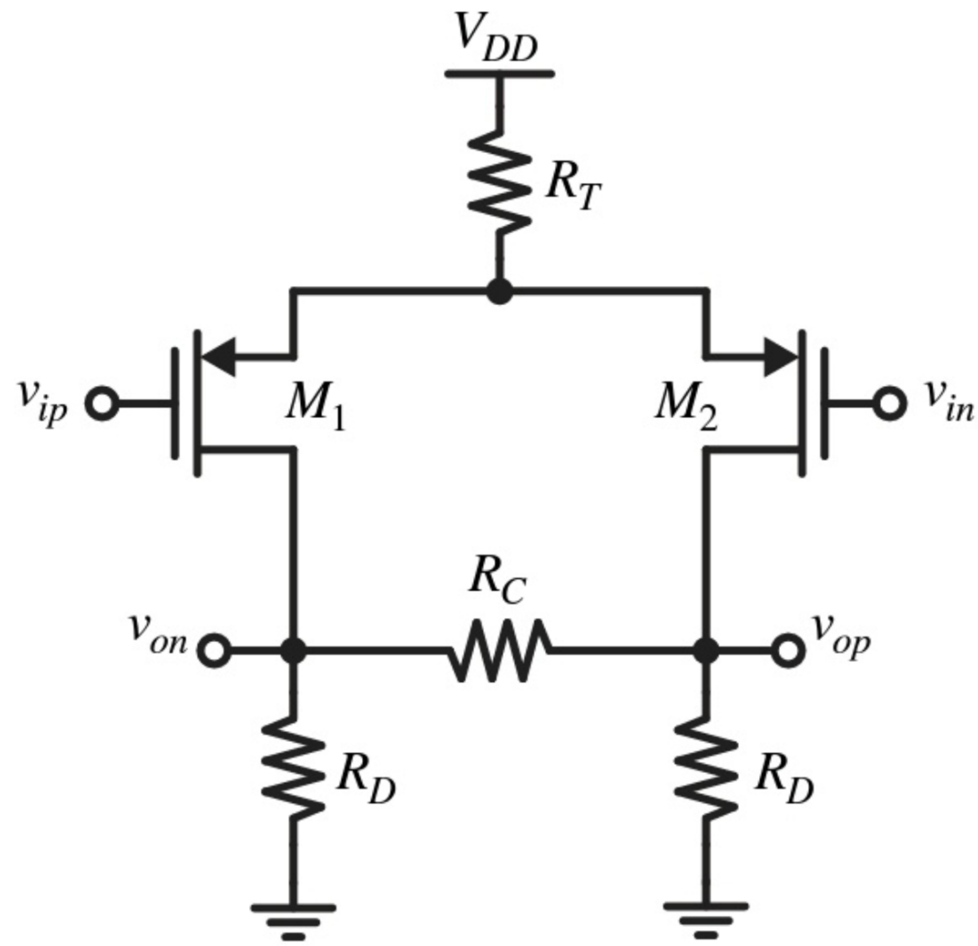
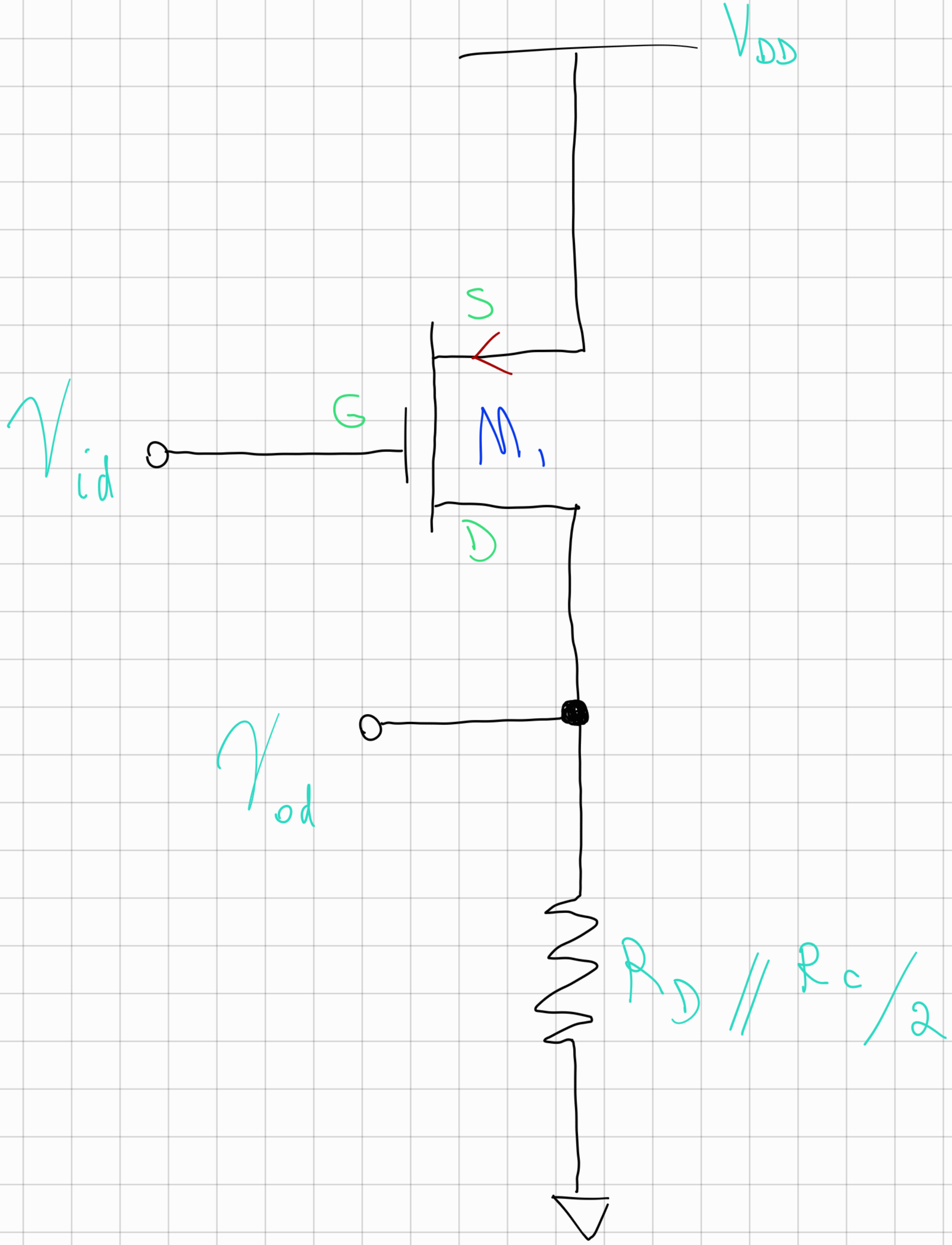


Figure 3: Diff. Amp

(a) Draw the differential mode half circuit.



(b) Derive the symbolic differential gain of the circuit.

Gain of CS amp w/ no degeneration : $-g_m\left(r_o \,//\, R_D \,//\, R_C/2\right)$

$$V_{od} = V_{op} - V_{on}$$

$$= +g_m\left(r_o \,//\, R_D \,//\, R_C/2\right) \cdot \frac{V_{ip}}{2}$$

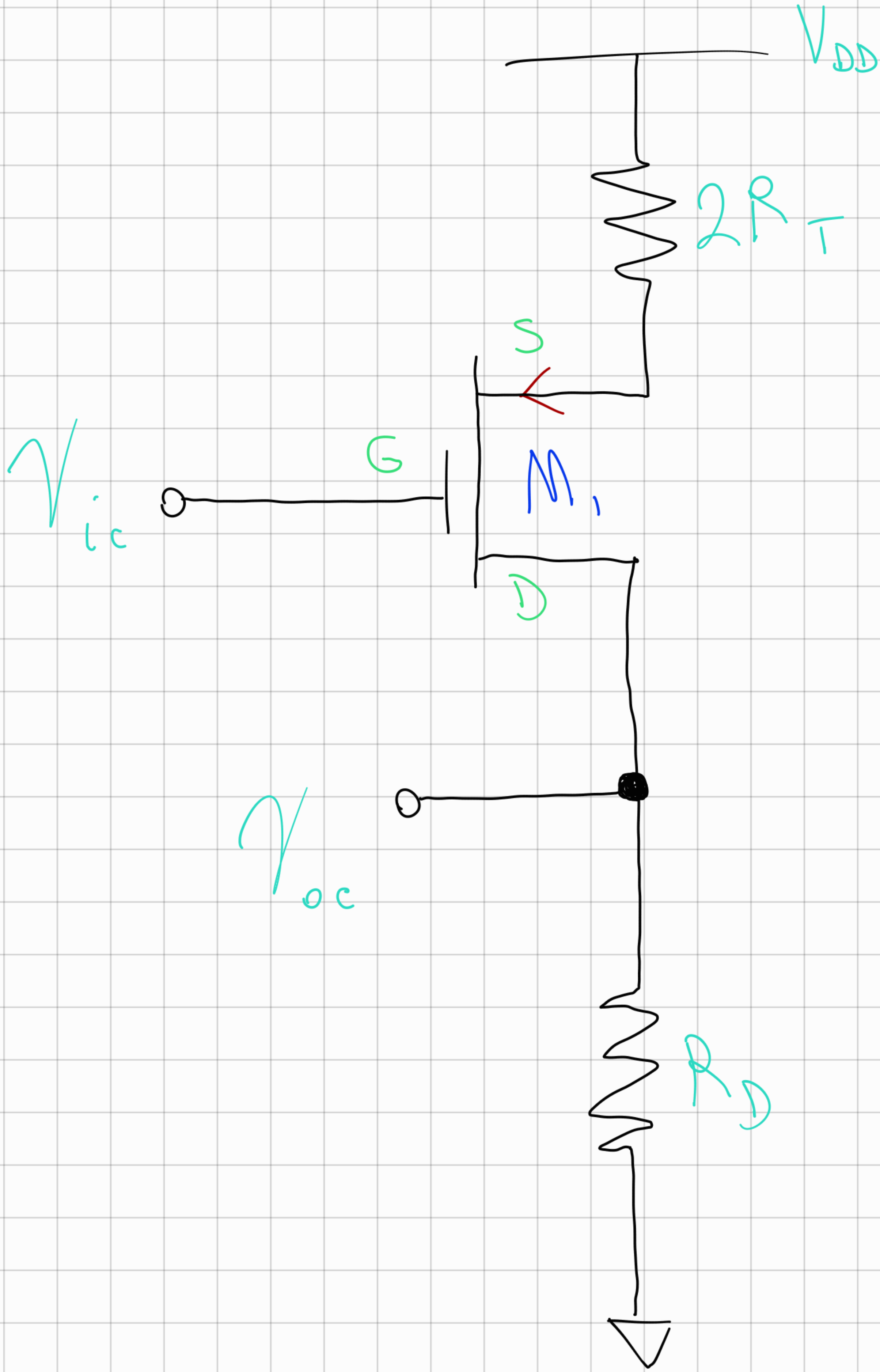
$$- \left[-g_m\left(r_o \,//\, R_D \,//\, R_C/2\right) \cdot \frac{V_{in}}{2}\right]$$

$$= g_m\left(r_o \,//\, R_D \,//\, R_C/2\right)\left[\frac{V_{ip}}{2} + \frac{V_{in}}{2}\right]$$

* Note that : $V_{id} = \frac{V_{ip}}{2} - \left(-\frac{V_{in}}{2}\right) = \frac{V_{ip}}{2} + \frac{V_{in}}{2}$

$$\boxed{\therefore \frac{V_{od}}{V_{id}} = g_m\left(r_o \,//\, R_D \,//\, R_C/2\right)}$$

(c) Draw the common mode half circuit.



(d) Derive a symbolic expression for the common mode gain of the circuit.

Now we have a CS amplifier with source degeneration. The derivation for the gain will be the same process as the previous part, but with the appropriate gain expression

From lecture slides:

$$G_m \simeq \frac{g_m}{1 + g_m R_S}$$

$$(*R_S = 2R_T)$$

$$R_{out} = R_D // R_{th,d} = R_D // r_o + R_S + g_m R_S$$

$$= R_D // r_o + 2R_T + g_m 2R_T$$

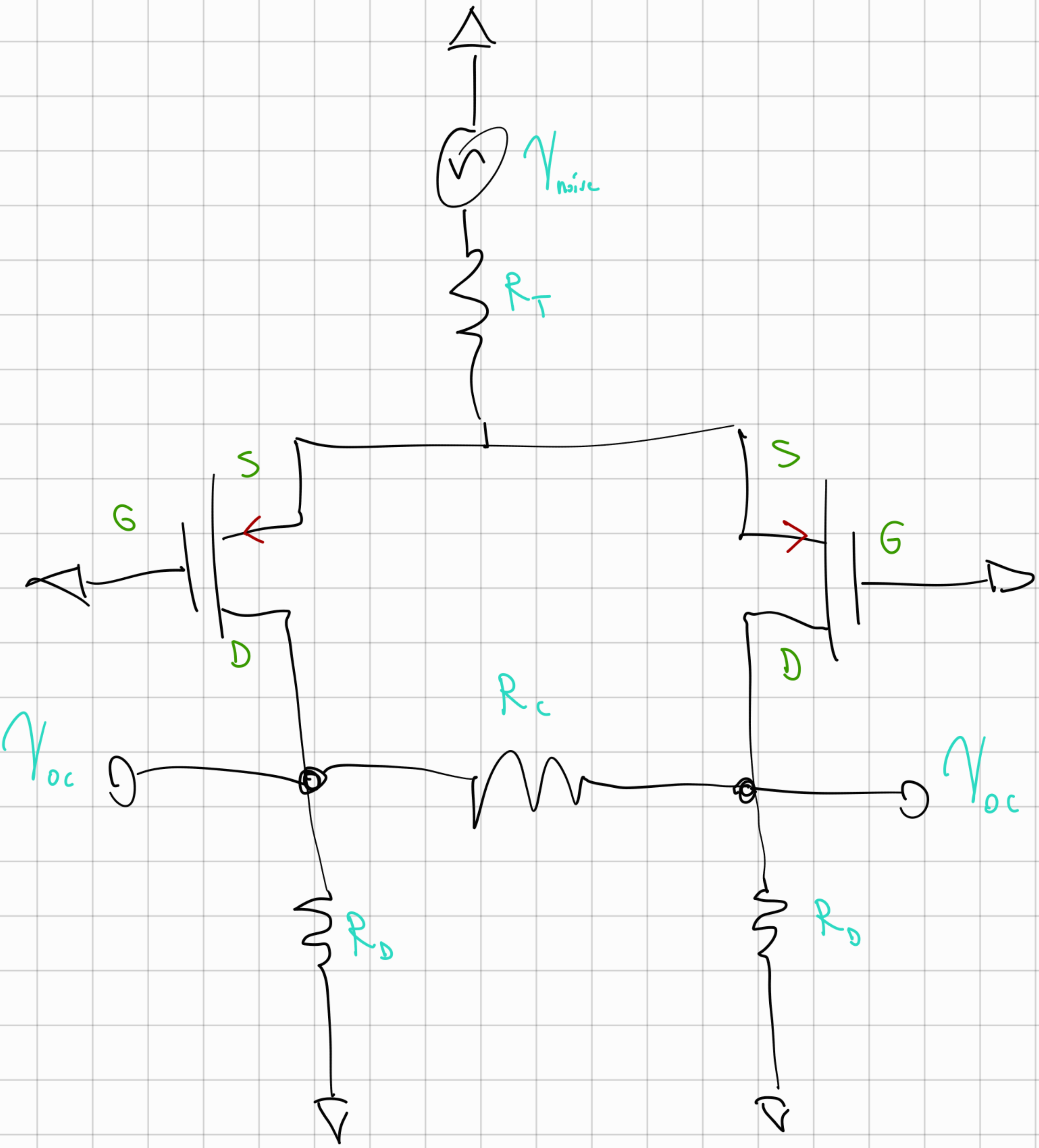
$$= \frac{R_D (r_o + 2R_T + g_m 2R_T)}{R_D + r_o + 2R_T + 2g_m R_T}$$

$$\Rightarrow G_m R_{out} = \frac{g_m R_D \left(r_o + 2R_T + 2 g_m R_T \right)}{\left(1 + g_m R_S \right) R_D + r_o + 2R_T + 2 g_m R_T}$$

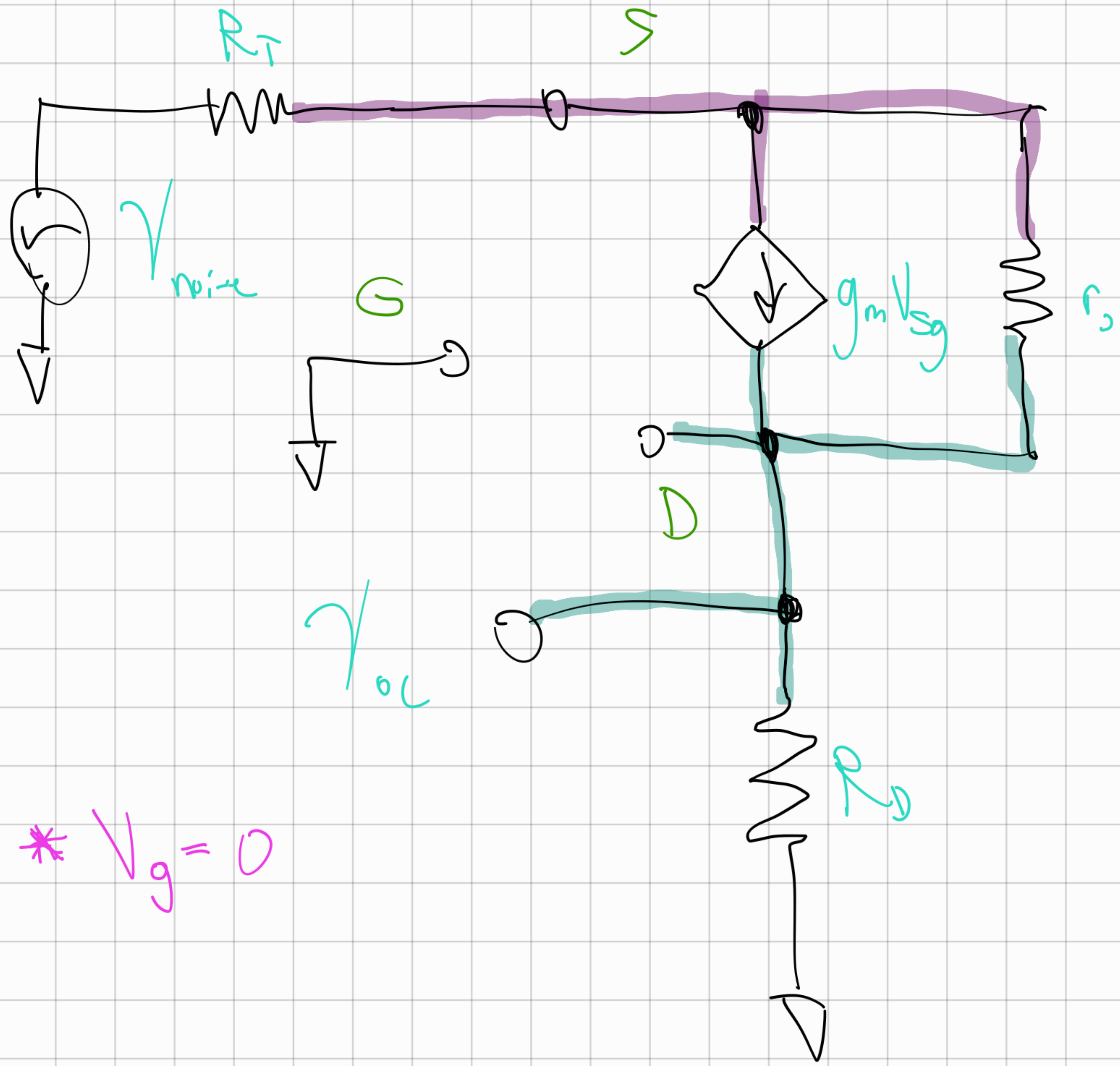
$$\Rightarrow V_{oc} = - V_{on} - V_{op}$$

$$\boxed{\therefore \frac{v_{oc}}{v_{ic}} = - \left[\frac{g_m R_D \left(r_o + 2R_T + 2 g_m R_T \right)}{\left(1 + 2 g_m R_T \right) R_D + r_o + 2R_T + 2 g_m R_T} \right]}$$

(e) Suppose the V_{DD} is taken from a noisy voltage source. Find the small signal gain for the supply noise to the common mode output. You may assume that the supply noise is a small signal AC perturbation at the V_{DD} terminal. You can refer to the figure below to see how the supply noise is modeled.



Small-signal Half-circuit



$* \; V_g = 0$

KCL Analysis:

$$\frac{V_{oc}}{R_D} + \frac{V_{oc}}{r_o} = g_m V_{sg}$$

$$\Rightarrow V_{oc}\left(\frac{1}{R_D} + \frac{1}{r_o}\right) = g_m V_s \quad (1)$$

$$\frac{V_s - V_{noise}}{R_T} + \frac{V_s - V_{oc}}{r_o} + g_m V_s = 0$$

$$\Rightarrow V_s \left(\frac{1}{R_T} + \frac{1}{r_o} + g_m \right) = \frac{V_{oc}}{r_o} + \frac{V_{noise}}{R_T}$$

$$\Rightarrow V_s = \left(r_o \,//\, R_T \,//\, {}^1\!/_{g_m} \right) \cdot r_o \cdot V_{oc} + \frac{V_{noise}}{R_T} \quad (2)$$

(2) into (1)

$$V_{oc} \left(\frac{1}{R_D} + \frac{1}{r_o} \right) = g_m \left[\left(r_o \,//\, R_T \,//\, {}^1\!/_{g_m} \right) \cdot r_o \cdot V_{oc} + \frac{V_{noise}}{R_T} \right]$$

$$V_{oc} = V_{oc} \left(r_o \,//\, R_D \right) \left(r_o \,//\, R_T \,//\, {}^1\!/_{g_m} \right) g_m r_o + V_{noise} \left(\frac{r_o \,//\, R_D}{R_T} \right)$$

$$\Downarrow$$

$$V_{oc} \left(1 - \left(r_o \,//\, R_D \right) \left(r_o \,//\, R_T \,//\, {}^1\!/_{g_m} \right) g_m r_o \right) = V_{noise} \left(\frac{r_o \,//\, R_D}{R_T} \right)$$

$\Downarrow$

$$\frac{V_{oc}}{V_{noise}} = \frac{\dfrac{r_o \parallel R_D}{R_T}}{1 - (r_o \parallel R_D)(r_o \parallel R_T \parallel {}^1/_{g_m}) g_m r_o}$$

$$= \frac{\dfrac{1}{R_T}}{\dfrac{1}{r_o \parallel R_D} - g_m r_o \left(r_o \parallel R_T \parallel {}^1/_{g_m}\right)}$$

$$= \frac{\dfrac{1}{R_T}}{\dfrac{r_o + R_D}{r_o R_D} - g_m r_o \left(\dfrac{1}{\dfrac{1}{r_o} + \dfrac{1}{R_T} + g_m}\right)}$$

$$= \frac{\dfrac{1}{R_T}}{\dfrac{r_o + R_D}{r_o R_D} - g_m r_o \left(\dfrac{1}{\dfrac{r_o + R_T + g_m r_o R_T}{r_o R_T}}\right)}$$

$$= \frac{\dfrac{1}{R_T}}{\dfrac{r_o + R_D}{r_o R_D} - \dfrac{g_m r_o^2 R_T}{r_o + R_T + g_m r_o R_T}}$$

$$= \frac{\dfrac{1}{R_T}}{\dfrac{(r_o + R_D)(r_o + R_T + g_m r_o R_T) - r_o R_D (g_m r_o^2 R_T)}{r_o R_D (r_o + R_T + g_m r_o R_T)}}$$

$$= \frac{r_o R_D (r_o + R_T + g_m r_o R_T)}{R_T \left[(r_o + R_D)(r_o + R_T + g_m r_o R_T) - r_o R_D (g_m r_o^2 R_T) \right]}$$

$$= \cancel{\left(\frac{R_T}{R_T}\right)}^{1} \frac{\frac{r_o R_D}{R_T}\left(r_o + R_T + g_m r_o R_T\right)}{\left(r_o + R_D\right)\left(r_o + R_T + g_m r_o R_T\right) - r_o R_D\left(g_m r_o^2 R_T\right)}$$

$$\boxed{\therefore \quad \frac{V_{oc}}{V_{noise}} = \left(\frac{r_o R_D}{R_T}\right) \frac{r_o + R_T + g_m r_o R_T}{\left(r_o + R_D\right)\left(r_o + R_T + g_m r_o R_T\right) - r_o R_D\left(g_m r_o^2 R_T\right)}}$$