

Business Analytics (110-1)

Assignment 4

Due: 9:00 am, Tue 21-Dec-2021

1.

For the data in "Smoking & Lung Cancer" case, test whether the odds of lung cancer for smokers are equal to the odds of lung cancer for nonsmokers, using Fisher's Exact Test.

H0: $\phi = 1$

H1: $\phi > 1$

```
> fisher.test(case1803, alternative = 'greater')
```

Fisher's Exact Test for Count Data

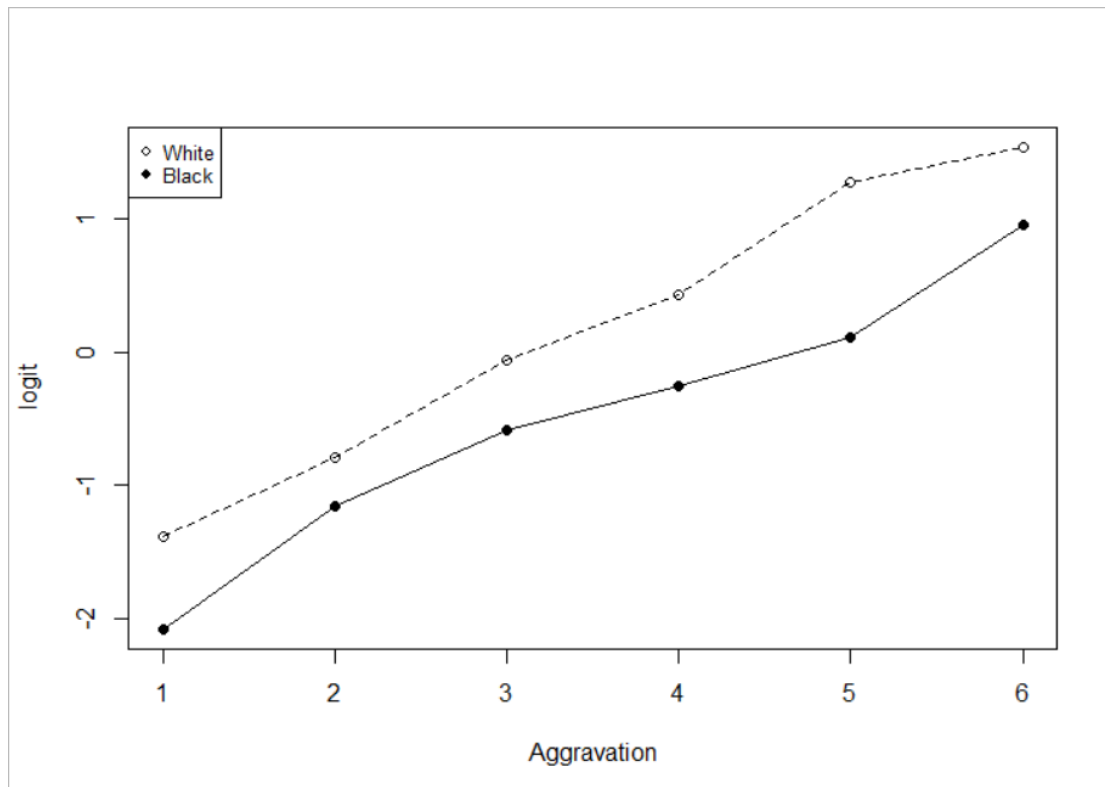
```
data: case1803
p-value = 0.004411
alternative hypothesis: true odds ratio is greater than 1
95 percent confidence interval:
 1.666128      Inf
sample estimates:
odds ratio
 5.333256
```

觀察圖中fisher's exact test結果可知 $p\text{-value} = 0.004411 < 0.05$ ，我們可以拒絕 H_0 ，得知 $\text{odds ratio} \neq 1$ ，兩組的odds並不相等。

2.

Reconsider the case Death Penalty and Race of Murder Victim discussed in lecture 10 (case1902). Reanalyze the data **using logistic regression**. The response variable is the number of convicted murderers in each category who receive the death sentence, out of the **m convicted murderers** in that category.

- (a) Plot the logits of the observed proportions versus the level of aggravation. The logit, however, is undefined for the rows where the proportion is 0 or 1, so **compute the empirical logit = $\log[(y + 0.5) / (m - y + 0.5)]$ and plot this versus aggravation level**, using different plotting symbols to distinguish proportions based on white and black victims.



(b) Fit **the logistic regression of death sentence proportions** on aggravation level and an indicator variable for race of victim.

```
Call:
glm(formula = cbind(Death, Nodeath) ~ Victim + Aggravation, family = binomial,
    data = case1902_new)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-0.93570	-0.22548	0.05142	0.65620	1.01444

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-4.8653	0.6004	-8.103	5.37e-16	***
VictimBlack	-1.8106	0.5361	-3.377	0.000732	***
Aggravation	1.5397	0.1867	8.246	< 2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 212.2838 on 11 degrees of freedom

Residual deviance: 3.8816 on 9 degrees of freedom

AIC: 31.747

Number of Fisher Scoring iterations: 4

(c) Report the p-value from the deviance goodness-of-fit test for this fit.

H0: the chosen Logistic GLM holds

```
> # 2-c
> deviance(model)
[1] 3.881585
> model$df.residual
[1] 9
> 1 - pchisq(deviance(model), model$df.residual)
[1] 0.9190319
```

p-value = 0.919 > 0.05, there is no evidence that the model is inadequate

(d) Test whether the coefficient of the indicator variable for race is equal to 0, using Wald's test.

H0: Beta = 0

H1: Beta != 0

```
Call:
glm(formula = cbind(Death, Nodeath) ~ Victim + Aggravation, family = binomial,
    data = case1902_new)
```

```
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.93570 -0.22548  0.05142  0.65620  1.01444
```

```
Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept)  -4.8653      0.6004  -8.103 5.37e-16 ***
VictimBlack   -1.8106      0.5361  -3.377 0.000732 ***
Aggravation    1.5397      0.1867   8.246 < 2e-16 ***
---

```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)
```

```
Null deviance: 212.2838 on 11 degrees of freedom
Residual deviance: 3.8816 on 9 degrees of freedom
AIC: 31.747
```

```
Number of Fisher Scoring iterations: 4
```

VictimBlack的p-value為0.000732 < 0.05，因此可以推翻H0，coefficient of the indicator variable for race is not equal to 0.

(e) Construct a confidence interval for the same coefficient, and **interpret it in a sentence about the odds of death sentence for white-victim murderers relative to black-victim murderers**, accounting for aggravation level of crime.

```
              2.5 %      97.5 %
(Intercept) -6.171218 -3.7937756
VictimBlack  -2.929963 -0.8021809
Aggravation   1.207083  1.9464923
```

(f) Refit the model by treating the aggravation level as a factor. How would you interpret the results of this model?

```
Call:
glm(formula = cbind(Death, Nodeath) ~ Victim + as.factor(Aggravation),
     family = binomial, data = case1902_new)
```

Deviance Residuals:

1	2	3	4	5	6	7	8	9
0.02705	-0.03705	-0.27695	0.46062	-0.22255	0.33222	0.02846	-0.03695	1.21437
10	11	12						
-0.55797	0.00006	0.00007						

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-3.4207	0.6144	-5.567	2.59e-08	***
VictimBlack	-1.7409	0.5426	-3.208	0.00134	**
as.factor(Aggravation)2	1.6090	0.8506	1.892	0.05855	.
as.factor(Aggravation)3	3.3902	0.7474	4.536	5.74e-06	***
as.factor(Aggravation)4	4.5004	0.7858	5.727	1.02e-08	***
as.factor(Aggravation)5	5.8814	0.9128	6.443	1.17e-10	***
as.factor(Aggravation)6	26.2636	8772.8073	0.003	0.99761	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 212.2838 on 11 degrees of freedom
Residual deviance: 2.2391 on 5 degrees of freedom
AIC: 38.105

Number of Fisher Scoring iterations: 19