Topics

- Positional Number Systems
- Base conversion
- Special bases: 2,8,16
- Signed quantities
- Elementary arithmetic operations
- Binary Codes

Problems

1 Build a table with all the possible 3 binary digits (bits). For each combination determine the respective decimal, octal, and hexadecimal representation. Repeat the exercise with 4 bits.

2 Compute the decimal value of the following unsigned integer quantities:

a) 00001111_2 b) 1347_8 c) $DF5_{16}$

d) 10100011_2 e) 7751_8 f) $A7A2_{16}$

g) 11111111_2 h) 2013_8 i) $40FF_{16}$

3 Determine the octal, hexadecimal, decimal, and binary representations of the following non-negative integer quantities:

a) 1036_{10} b) 7354_8 c) $16B5_{16}$ d) 111100111_2

e) 7564_{10} f) 6102_8 g) $D3F9_{16}$ h) 110101011_2

4 Compute the decimal value of the following rational quantities. Do not exceed the precision of the original representation:

a) 110110.1101001_2 b) 127.444_8 c) $2D.8_{16}$

5 Determine the octal, hexadecimal and binary representations of the following rational non-negative quantities. Do not exceed the precision of the original representation:

a) 13.25_{10} b) 33.47_{10} c) 123.3_{10}

6 Compute the following additions and check the results with decimal representation:

a) $10101110_2 + 00011111_2$ b) $125_8 + 17_8$

c) $125_{16} + 1A7_{16}$ d) $00111011_2 + AD_{16}$

7 Compute the following subtractions and check the results with decimal representation:

a) 10101110_2 - 00011111_2 b) 125_8 - 17_8

c) 107_{16} - DC_{16} d) AD_{16} - 00111011_2

8 Compute the signed decimal value of the following quantities assuming a two's complement 8 bit encoding:

a) 11111110 b) 00000000 c) 11111111 d) 00110011

9 Assume a two's complement 8 bit encoding. Determine, whenever possible, the corresponding two's complement 4 bit encoding:

a) 11111110 b) 00000110 c) 11111111 d) 00110011

10 Assume a two's complement 4 bit encoding. Determine, the corresponding two's complement 8 bit encoding:

a) 1110 b) 0110 c) 1000 d) 0001

11 Consider a 12 bit quantity represented as 7650_8. Compute the corresponding signed decimal value assuming a two's complement 12 bit binary representation.

12 Show, whenever possible, the 8 bit binary representation of the following quantities assuming a two's complement encoding:

a) 45_{10} b) -13_8 c) $–F1_{16}$ d) 130_{10}

13 Compute the result of the following operations assuming an 8 bit two's complement representation. Verify the possible overflow cases.

a) $-1_{10} + 63_{10}$ b) $11111_2 + 10101_2$ c) $-11_{10} - 123_{10}$ d) $54_{16} + 2E_{16}$

14 Show in binary, octal, hexadecimal, and decimal the positive and negative limits of the representation of a 12 bit signed quantity

15 Determine *m*, the minimum number of bits necessary to code 6 different objects? Suggest an example. Compute the total number of different codes that can be produced in this case.

16 Represent the following numbers in BCD_{8421} code.

a) 111_{10} b) 125_8 c) ABC_{16}

17 Build the Gray tables with 3 and 4 bits. Build another table with the first 4 and last 4 Gray code words with 5 bits.

18 Determine the Gray code words corresponding to the following natural binary code words:

a) 00001111 b) 10011001 c) 11111111

19 Determine the natural binary code words corresponding to the following Gray code words:

a) 00001111 b) 10011001 c) 11111111

20 Compute the Hamming distance for the following code word pairs

a) 10101010 e 01010101 b) 11110000 e 11000011 c) 10101111 e 10101111

21 Verify that, for every Gray code, the Hamming distance for any pair of consecutive code words is always 1. Verify that the same happens for the first and the last code word pair.

ISD - P1-2

Problemas:

① binario com 3 digitos → decimal, octal, hexadecimal
... 4 digitos → ...

3-bit:

Binário	decimal	octal	Hexa decimal
000	0	0	0
001	1	1	1
010	2	2	2
011	3	3	3
100	4	4	4
101	5	5	5
110	6	6	6
111	7	7	7

4-bit:

Binario	decimal	octal	Hexadecimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	10	8
1001	9	11	9
1010	10	12	A
1011	11	13	B
1100	12	14	C
1101	13	15	D
1110	14	16	E
1111	15	17	F

② passar para decimal

a) $00001111_2 = 1\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3 + 0\times2^4 + 0\times2^5 \cdot 0\times2^6 + 0\times2^7 =$

$= 1 + 2 + 4 + 8 = 15_{10}$

b) $1347_8 = 7\times8^0 + 4\times8^1 + 3\times8^2 + 1\times8^3 =$

$= 7 + 32 + 192 + 512 = 743_{10}$

c) $DF5_{16} = 5\times16^0 + 15\times16^1 + 13\times16^2 =$

$= 5 + 240 + 3328 = 3573_{10}$

d) $10100011_2 = 1\times2^0 + 1\times2^1 + 0\times2^2 + 0\times2^3 + 0\times2^4 + 1\times2^5 + 0\times2^6 + 1\times2^7$

$= 1 + 2 + 32 + 128 = 163_{10}$

e) $7751_8 = 1\times8^0 + 5\times8^1 + 7\times8^2 + 7\times8^3 =$

$= 1 + 40 + 448 + 3584 = 4073_{10}$

f) $A7A2_{16} = 2\times16^0 + 10\times16^1 + 7\times16^2 + 10\times16^3 = ...$

$= 2 + 160 + 1792 + 40960 = 42914_{10}$

g) $11111111_2 = 1\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3 + 1\times2^4 + 1\times2^5 + 1\times2^6 + 1\times2^7 =$

$= 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255_{10}$

h) $2013_8 = 3\times8^0 + 1\times8^1 + 0\times8^2 + 2\times8^3 =$

$= 3 + 8 + 1024 = 1035_{10}$

i) $40FF_{16} = 15\times16^0 + 15\times16^1 + 0\times16^2 + 4\times16^3 =$

$= 15 + 240 + 16384 = 16639_{10}$

③ valores para binario, octal, decimal e hexadecimal

a) 1036_{10}

binário → 1036 |2

0 518 |2

0 259 |2

1 129 |2

1 64 |2

0 32 |2

0 16 |2

0 8 |2

0 4 |2

0 2 |2

0 1

$= 10000001100_2$

octal → 1036 ⌊8
4 129 ⌊8
1 16 ⌊8
0 2 ⌊8
2 0

$\rightarrow 02014_8 = 2014_8$

hexadecimal → 1036 ⌊16
12 64 ⌊16
0 4 ⌊16
4 0

$\rightarrow 040C_{16} = 40C_{16}$

b) 7354_8

binário → $4 \rightarrow 100$
$5 \rightarrow 101$
$3 \rightarrow 011$
$7 \rightarrow 111$

$= 111\ 011\ 101\ 100_2$

hexadecimal → com o binário: $\underbrace{1110}_{E}\ \overbrace{1110}^{E}\ \underbrace{1100}_{C}\ _2$

$= EEC_{16}$

decimal → $4 \times 8^0 + 5 \times 8^1 + 3 \times 8^2 + 7 \times 8^3 =$
$= 4 + 40 + 192 + 3584 = 3820_{10}$

c) $16B5_{16}$

binário: $5 \rightarrow 0101$
$B \rightarrow 1011$
$6 \rightarrow 0110$
$1 \rightarrow 0001$

$= 0001\ 0110\ 1011\ 0101_2 =$
$= 1011010110101_2$

acrescentar

octal: $\underbrace{001}_{1}\ \underbrace{011}_{3}\ \underbrace{010}_{2}\ \underbrace{110}_{6}\ \underbrace{101}_{5}$

$= 13265_8$

decimal: $5 \times 16^0 + 11 \times 16^1 + 6 \times 16^2 + 1 \times 16^3 =$
$= 5 + 176 + 1536 + 4096 = 5813_{10}$

d) 111100111_2

octal : $\underbrace{111}_{7}\,\underbrace{100}_{4}\,\underbrace{111}_{7}{}_2 = 747_8$

hexadecimal : $\underbrace{0001}_{1}\,\underbrace{1110}_{E}\,\underbrace{0111}_{7}{}_2 = 1E7_{16}$

decimal : $1\times2^0 + 1\times2^1 + 1\times2^2 + 0\times2^3 + 0\times2^4 + 1\times2^5 + 1\times2^6 + 1\times2^7 + 1\times2^8 =$

$= 1 + 2 + 4 + 32 + 64 + 128 + 256 = 487_{10}$

e) 7564_{10}

binário : 7564 |2
0 3782 |2
0 1891 |2
1 945 |2
1 472 |2
0 236 |2
0 118 |2
0 59 |2
1 29 |2
1 14 |2
0 7 |2
1 3 |2
1 1

→ 1110110001100_2

octal : 7564 |8
4 945 |8
1 118 |8
6 14 |8
6 1

→ 16614_8

hexadecimal : 7564 |16
1D8C
12 (C) 472 |16
8 29 |16
13 (D) 1

→ $1D8C_{16}$

f) 6102_8

binário : $6 \to 110$ $\to 110\ 001\ 000\ 010_2$
$1 \to 001$
$0 \to 000$
$2 \to 010$

hexadecimal $\underbrace{1100}_{C}\underbrace{0100}_{4}\underbrace{0010}_{2}$

$\to C42_{16}$

Decimal : $0\times2^0 + 1\times2^1 + 0\times2^2 + 0\times2^3 + 0\times2^4 + 0\times2^5 + 1\times2^6 + 0\times2^7 + 0\times2^8 + 0\times2^9 + 1\times2^{10} + 1\times2^{11} =$

$= 2 + 64 + 1024 + 2048 = 3138_{10}$

g) $D3F9_{16}$

binario : $1101\ \ 0011\ \ 1111\ \ 1001_2$

octal : $\underbrace{1}_{1}\underbrace{101}_{5}\underbrace{001}_{1}\underbrace{111}_{7}\underbrace{111}_{7}\underbrace{001}_{1}$

$\to 151771_8$

decimal : $9\times16^0 + 15\times16^1 + 3\times16^2 + 13\times16^3 =$
$= 54\ 265_{10}$

h) 110101011_2

octal : $\underbrace{110}_{6}\underbrace{101}_{5}\underbrace{011}_{3}$ $\longrightarrow$ 653_8

hexadecimal : $\underbrace{1}_{1}\underbrace{1010}_{A}\underbrace{1011}_{B}$ $\to 1AB_{16}$

decimal $1\times2^0 + 1\times2^1 + 0 + 1\times2^3 + 0 + 1\times2^5 + 0 + 1\times2^7 + 1\times2^8 = 427_{10}$

④ valores decimal

a) 110 110 , 1101001$_2$

Decimal: 1º Parte inteira: $0 + 1\times2^1 + 1\times2^2 + 0 + 1\times2^4 + 1\times2^5 =$
$= 54_{10}$

2º Parte nº casas decimais: $n_1 = 7 \times \log_{10}(2) =$
$= 2$ casas

3º parte $1\times2^{-1} + 1\times2^{-2} + 0 + 1\times2^{-4} + 0 + 0 + 1\times2^{-7} =$
$= 82$

→ $54,82_{10}$

b) $127,444_8$

Decimal 1º parte inteira: $7\times8^0 + 2\times8^1 + 1\times8^2 = 87$
2º parte $n_2 = 3 \times \log_{10}(8) = 2$ casas
3º $= 4\times8^{-1} + 4\times8^{-2} + 4\times8^{-2} = 0,5703..$

→ $87,57_{10}$

c) $2D.8_{16}$

Decimal: $13\times16^0 + 2\times16 = 45_{10}$
$n_2 = 1\times\log_{10}(16) = 1$
$8\times16^{-1} = 0,5$

→ $45,5_{10}$

⑤ octal, hexadecimal e binário

a) $13,25_{10}$

Binário: $1101,010000_2$

$0,25 \times 2 = 0,5$
$0,5 \times 2 = 1,0$
$0 \times 2 = 0$

$n_2 = 2 \cdot \frac{\log 10}{\log 2} = 6$

octal: 13 | 8
5 1

$0,25 \times 8 = 2,00$ tirando
0×8
X → não existe > 0

→ $15,20_8$

$n_2 = 2 \cdot \frac{\log 10}{\log 8} = 2$

hexadecimal D,4 16   0,25 × 16 = 4,0 $n_2 = 2\frac{\log 10}{\log 16} = 1$

b) $33,47_{10}$

binário:

33 | 2
1 16 | 2
   0 8 | 2
      0 4 | 2
         0 2 | 2
            0 1

0,47 × 2 = 0,94
× 2 = 1,88
× 2 = 1,76
× 2 = 1,56
× 2 = 1,12
× 2 = 0,24

$n_2 = 2 \frac{\log 10}{\log 2} = 6$

→ $100001,011110_2$

octal:

33 | 8
1 4 | 8
   0

0,47 × 8 = 3,76
× 8 = 6,08

$n_2 = 2 \times \frac{\log 10}{\log 8} = 2$

→ 41,36

hexadecimal:

33 | 16
1 2

0,47 × 16 = 7,52

$n_2 = 2 \times \frac{\log 10}{\log 10} = 1$

→ $21,7_{16}$

c) $123,3_{10}$

binário:

123 | 2
1 61 | 2
   1 30 | 2
      0 15 | 2
         1 7 | 2
            1 3 | 2
               1 1

0,3 × 2 = 0,6
× 2 = 1,2
× 2 = 0,4

$n = \frac{\log 10}{\log 2} = 3$

→ $1111011,010_2$

octal: $123 \,\underline{|8}$ $\quad 3 \quad 15 \,\underline{|8} \quad 7 \quad 1$

$0{,}2 \times 8$ $\quad$ $n = 1 \cdot \frac{\log 10}{\log 8} = 1$

$\rightarrow 173{,}2_8$

hexadecimal: $123 \,\underline{|16}$ $\quad 11 \quad 7 \,\underline{|16} \quad 0$

$n_1 = 1 \cdot \frac{\log 10}{\log 16} = 0$

$\rightarrow 7B_{16}$

6 Adição + confirmar com decimal

a) 10101110_2
$+\ 00011111_2$
11001101

verificação: $174 + 31 = 205$

b) 125_8
$+\ 17_8$
144_8

verificação: $85 + 15 = 100$

c) 125_{16}
$+\ 1A7_{16}$
$2CC_{16}$

verificação: $293 + 423 = 716$

d) $\underbrace{0011}_{3}\underbrace{1011}_{11\ B}{}_2 + AD_{16} = 3B_{16} + AD_{16}$

$3B_{16}$
$+\ AD_{16}$
$E8_{16}$

verificação $59 + 173 = 232$

7 Subtrações + confirmar com decimar

a) 10101110_2
$-\ 00011111_2$
10001111

verificação: $174 - 31 = 143$

b) 125_8
$- 17_8$
106_8

verificação 85
15
70

c) 107
$- DC$
$02B_{16}$

verificação: 263
-220
43

d) $AD_{16} - 00111011_2 = AD_{16} - 3B_{16}$

AD_{16}
$- 3B_{16}$
72_{16}

verificação 173
59
114

8) consideramos os negativos → para valor decimal

a) $11111110_2 =$

$= 0\times2^0 + 1\times2^1 + 1\times2^2 + 1\times2^3 + 1\times2^4 + 1\times2^5 + 1\times2^6 - 1\times2^7 =$
$= 0 + 2 + 4 + 8 + 16 + 32 + 64 - 128 =$
$= -2_{10}$

b) $00000000_2 =$
$= 0+0+0+0+0+0+0+0 =$
$= 0_{10}$

c) $11111111_2 =$
$= 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 - 2^7 =$
$= 1 + 2 + 4 + 8 + 16 + 32 + 64 - 128 =$
$= -1_{10}$

d) $00110011_2 =$
$= 2^0 + 2^1 + 0 + 0 + 2^4 + 2^5 + 0 + 0 =$
$= 1 + 2 + 0 + 16 + 32 =$
$= 51_{10}$

arc

9 Quando possível 8bit → 4 bit

a) $11111110_2 \rightarrow 1110_2$ (-2_{10})

b) $00000110_2 \rightarrow 0110_2$ (6_{10})

c) $11111111_2 \rightarrow 1111$ (-1_{10})

d) 00110011 → não é possível, apenas poderíamos ficar com 7 bit → 0110011

10 4bit → 8 bit

a) 1110 → 11111110

b) 0110 → 00000110

c) 1000 → 11111000

d) 0001 → 00000001

11 $76.50_8 \rightarrow$ 12 bit ~~[illegible]~~

7 → 111
6 → 110
5 → 101
0 → 000

$111110101000_2 \rightarrow -2^{11} + 2^{10} + 2^9 + 2^8 + 2^7 + 2^5 + 2^3 =$

$= -2048 + 1024 + 512 + 256 + 128 + 32 + 8 =$

$= -88_{10}$

(12) 8 bits binario quando possível

a) 45_{10}

45 |2
1 22 |2
0 11 |2
1 5 |2
1 2 |2
0 1

→ 101101

→ 8bit 2s → 00101101_2

b) -13_8 ⇒ binario: 001011 → ⊕ 00001011

Para 8 bits → negativo negar tudo e adicionar 1

- negado: 11110100
+ 00000001
11110101

* tambem temos a regra

c) $-F1_{16}$ = binário: 11110001_2

→ decimal: -15×16 + 1 = -241, não vai dar porque em decimal apenas temos o seguinte intervalo: [-128, 127], e o numero que temos

d) 130_{10}

130 |2
0 65 |2
1 32 |2
0 16 |2
0 8 |2
0 4 |2
0 2 |2
0 1

→ 10000010 não da para representar pela mesma razão, apenas temos do intervalo [-128, 127]

13 casos de overflow (identificar)

a) $-1_{10} + 63_{10}$

$-1_{10} \rightarrow 11111111_2$

63 | 2
1 31 | 2
1 15 | 2
1 7 | 2
1 3 | 2
1 1

$63_{10} \rightarrow 00111111_2$

não existe overflow

verifica-se

$$
\begin{array}{rcccccccc}
\rightarrow & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
+ & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
\cancel{1} & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0
\end{array}
\qquad
\begin{array}{r}
(-1) \\
+\ (63) \\
\hline
(62)
\end{array}
$$

b) $11111_2 + 10101_2$

$$
\begin{array}{rcccccccc}
 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
+ & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
\hline
 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0
\end{array}
\qquad
\begin{array}{r}
(-1) \\
+\ (-11) \\
\hline
(-12)
\end{array}
$$

não existe overflow

c) $-11_{10} + (-123)_{10}$

Não é possivel porque com 8 bits temos o intervalo [-128, 127] e aqui passamos dele, dá overflow

d) $54_{16} + 2E_{16}$

não existe overflow

$$
\begin{array}{r}
54 \\
+\ 2E \\
\hline
82
\end{array}
\qquad
\begin{array}{r}
(84) \\
+\ (46) \\
\hline
(130)
\end{array}
$$

(14) os limites de 12 bit

[redacted]:

A representação em sinal e modulo tem 2 zeros 00...0 e 10...0
os limites em binario são:

$$\left[-(2^{N-1}-1) , (2^{N-1}-1) \right]$$

base 10 [-2047, ..., 0, ... 2047]
2 [11...11, ..., 10...0, 00...0, 01...1]
8 [7777_8, ..., 4000_8, 0000_8, ..., 3777_8]
16 [FFF_{16}, ..., 800_{16}, 000_{16}, ..., $7FF_{16}$]

→ zeros do lado positivo e do lado negativo

(15) Determinar m → nº minimo de bits para 6 objetos

$n \geq \log_2 (m) \geq \frac{\log 6}{\log 2} = 2,53 \Rightarrow 3$ bits   * 6 para base 2

nº total de codigos: $2^N = 2^3 = 8$

(16) Numeros para BCD_{8421} → temos cada digito representado por 4 bit

a) 111_{10} → 4 bits logo:

1 → 0001   1 → 0001   1 → 0001

→ 000100010001_{BCD}

b) 125_8 → binario: 001010101_2
→ decimal: $2^6 + 2^4 + 2^2 + 1 = 85_{10}$

→ 8 → 1000   5 → 0101

→ 10000101_{BCD}

e) ABC_{16} → binario: 1010 1011 1100

→ decimal: $= 2748_{10}$

→ 2 7 4 8 → $0010\,0111\,0100\,1000_{BCD}$

0010 0111 0100 1000

(17) Gray code com 3bits e 4 bit

3 bits	4 bits	Primeiros 4 numeros - 5 bits
000	0000	
001	0001	00000
011	0011	00001
010	0010	00011
110	0110	00010
111	0111	Ultimos 4 numeros - 5 bits
101	0101	
100	0100	10010
	1100	10011
	1101	10001
	1111	10000
	1110	
	1010	
	1011	
	1001	
	1000	

(18) binario → Gray code regra

a) $0000\,1111_2$

0 0 0 0 1 1 1 1

= = = = ≠ = = =

0 0 0 0 1 0 0 0

→ 00001000_{Gray}

b) 10011001_2

0 1 0 0 1 1 0 0 1

↘ ≠ ↓ ≠ ↓ = ↓ ≠ ↓ = ↓ ≠ ↓ = ↓ ≠ ↓

1 1 0 1 0 1 0 1

→ 11010101_{Gray}

c) 11111111_2

0 1 1 1 1 1 1 1 1

↘ ≠ ↓ = ↓ = ↓ = ↓ = ↓ = ↓ = ↓ = ↓

1 0 0 0 0 0 0 0

→ 10000000_{Gray}

(9) Gray → binario

a) 00001111_{Gray}

0 0 0 0 1 1 1 1

0 0 0 0 1 0 1 0

→ 00001010_2

b) 10011001

1 0 0 1 1 0 0 1

1 1 1 0 1 1 1 0

→ 11101110_2

c) 11111111

1 1 1 1 1 1 1 1

1 0 1 0 1 0 1 0

→ 10101010_2

20 distancia de Hamming → numeros diferentes

a) $\boxed{10101010}$ e $\boxed{01010101}$

→ distancia de Hamming: 8

b) 11110000 e 11000011

→ distancia de Hamming: 5

c) 10101111 e 10101111

→ distancia de Hamming: 0

21 A distancia de Hamming no codigo de Gray é de 1 em numeros consecutivos e também é de 1 entre o primeio e o último, tornando-se assim cíclico.

by Staples