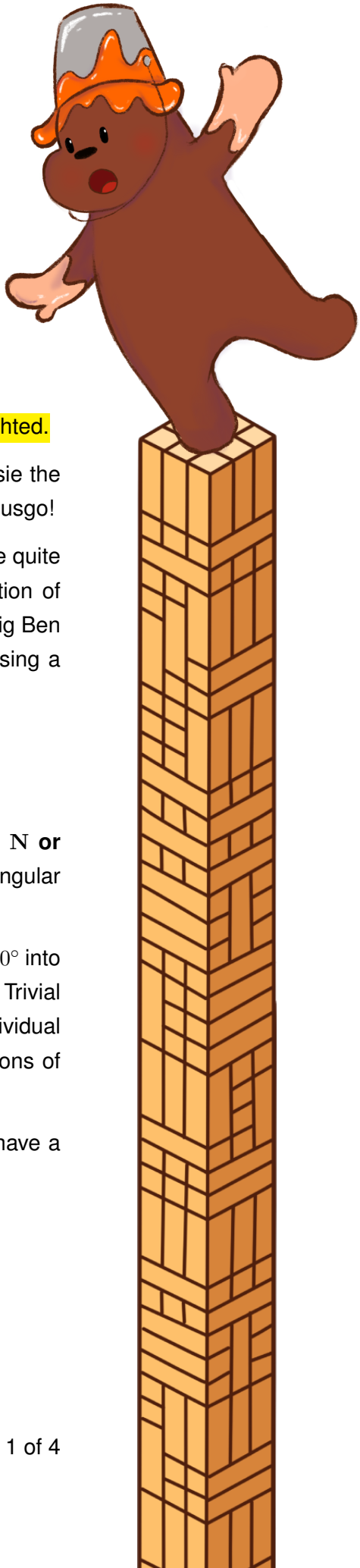
Problem 12: Big Ben's Benga Bricks 12+1=13 Points



Problem ID: `benga`
Rank: `4+4`

Introduction

This is a harder version of `jenga`! Key changes in `benga` are highlighted.

Big Ben loves Jenga! He is the undefeated champion, beating Bessie the Cow, P/NPenguin, and Ana, the Jenga National Champion from Mañusgo!

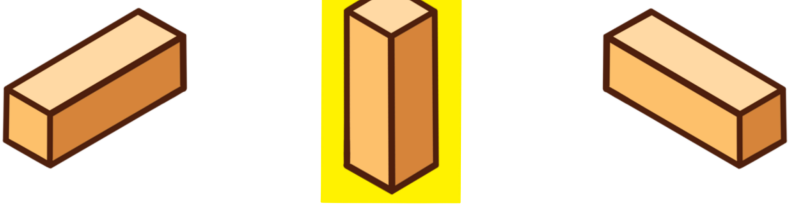
After beating all the computer science mascots, Big Ben has become quite bored of the game. One day, he decides to create his own variation of classic Jenga: Ben's Jenga (or as he likes to call it, Benga!). Now, Big Ben wants to know how many unique Benga towers he can engineer using a certain amount of bricks before getting bored again!

Problem Statement

Count the number of unique Benga towers that can be built using $\mathbf{N}$ **or fewer** bricks of size $1 \times 1 \times 3$. A Benga tower is a fully packed rectangular prism with a 3×3 base and a height of at least 1.

Bricks are indistinguishable from one another. Bricks can be rotated 90° into horizontal orientations and also vertical orientations as shown below. Trivial rotations (for example, rotating a brick along its major axis) of individual Jenga bricks should not be considered unique. However, 90° rotations of the entire tower along the vertical axis should be considered unique.

To celebrate CALICO's 2-year-old birthday and the fact that bricks have a length of 3, give your answer modulo $2^{3^2}3^{2^3} = 3359232$.



Input Format

The first line of the input contains a single integer $\mathbf{T}$ denoting the number of test cases that follow.

Each test case is described in a single line containing an integer $\mathbf{N}$ denoting the number of bricks.

Output Format

For each test case, output a single line containing an integer denoting the number of unique Benga towers modulo $2^{3^2}3^{2^3} = 3359232$.

Constraints

$1 \leq \mathbf{T} \leq 10$

Main Test Set

$1 \leq \mathbf{N} \leq 10^{18}$

Bonus Test Set

$1 \leq \mathbf{N} \leq 10^{10^5} = 10^{100000}$

Yep, you read that correctly. This is not just Nacho's computer exploding. Thanks to CALICO, Big Ben has an (almost) infinite supply of Benga Bricks.

The size of each input file will not exceed $2\,\mathrm{MB}$.

Sample Test Cases

Main Sample Input Download

```
7
2
3
6
11
16
705
333333333333333333
```

Main Sample Output Download

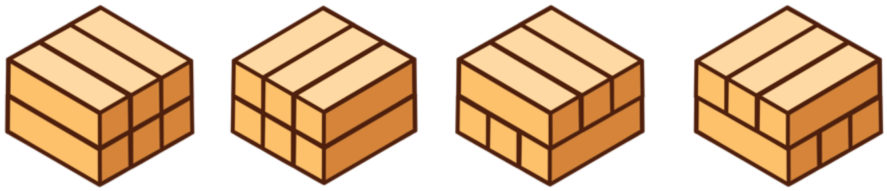
```
0
2
6
27
439
2941382
1635075
```

Main Sample Explanations

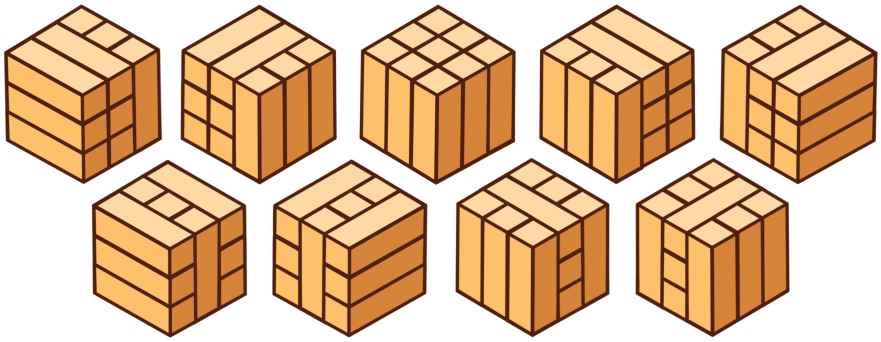
For test case #1, 2 bricks isn't enough to fill any layers, so our answer is 0.

For test case #2, 3 bricks lets us build only towers of height 1. The only 2 ways of building towers of height 1 are shown below.

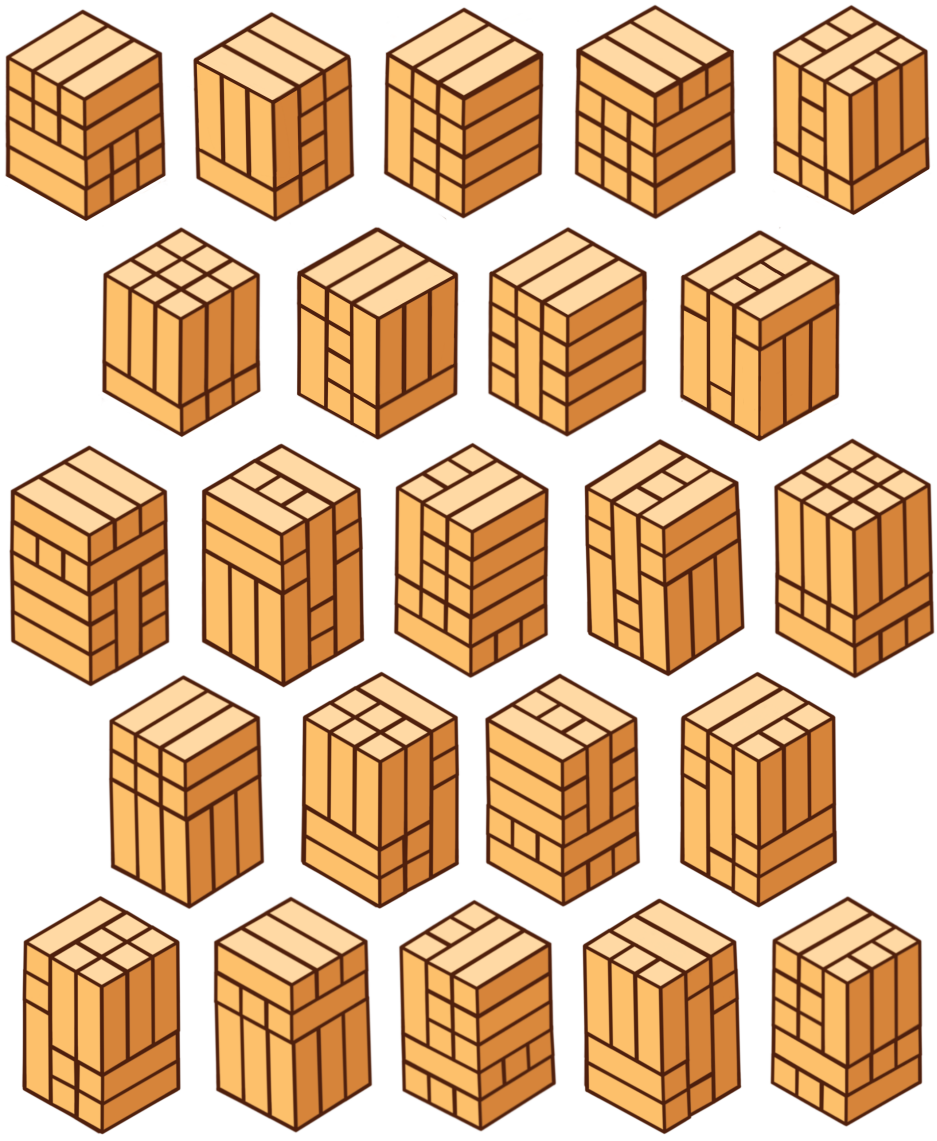
For test case #3, we can build towers of height 1 or 2. The 2 ways for height 1 are above and the 4 ways for height 2 are below, making a total of 6.



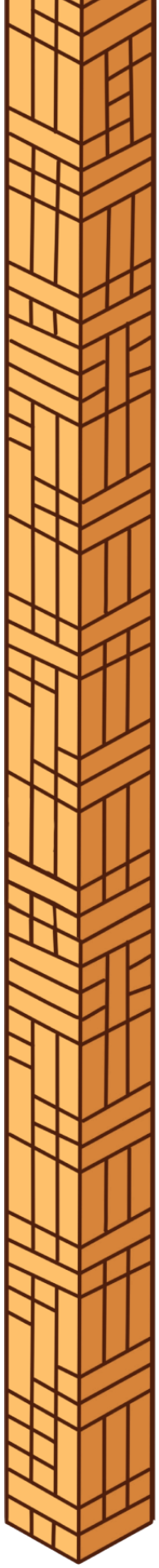
For test case #4, we can build towers of height up to 3. Some (but not all) towers with vertical pieces are shown below. This makes 27 in total.



For test case #5, there are 439 ways to build towers with height up to 5. Some (but not all) of these ways are shown below.



For test case #6, one of the towers that can be built using 705 bricks is shown to the right of this document.



Bonus Sample Input Download

```
1
314159265358979323846264338327950288419716939937510582
```

Bonus Sample Output Download

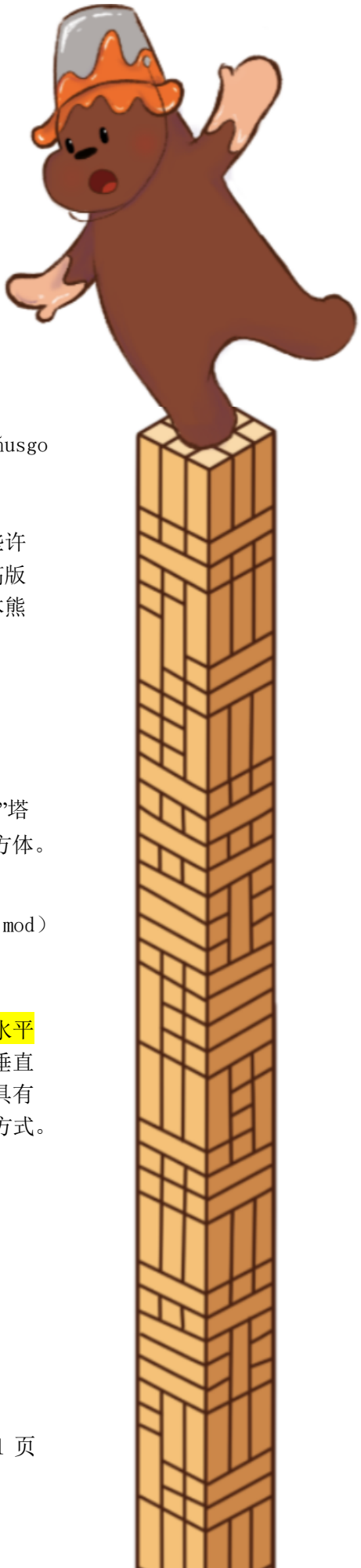
```
599783
```

第 12 题：大本熊的笨笨高塔楼

12+1=13 分

问题标识符：benga

难度等级：4+4



问题背景

这是一个更难的版本的“jenga”！关键变化已高亮显示。

大本熊超爱玩叠叠高！他战无不胜，击败了贝丝牛、P/NPenguin 和来自 Mañusgo 的叠叠高全国冠军 Don Matías！

在一一战胜了众多计算机领域的吉祥物之后，大本熊开始对原版游戏感到些许厌倦。于是有一天，他萌生了创新的想法，打造了一款以自己命名的叠叠高版本：“大本熊的叠叠高”（或者他喜欢称它为“笨笨高”！）。现在，大本熊想知道他使用一定数量的积木可以建造多少个独特的“笨笨高”塔楼！

问题描述

请计算使用不超过 N 块 1 × 1 × 3 大小的积木可以建造多少个独特的“笨笨高”塔楼。一个“笨笨高”塔楼的定义为一个底面为 3 × 3 且高度至少为 1 的实心长方体。

为了庆祝 CALICO 的 2 岁生日，并且积木的长度为 3，请将答案进行取模（mod）$2^{3^2}3^{2^3} = 3359232$。

每块积木都是相同大小的。如下所示，积木可以沿任意方向旋转，不限于水平旋转。你可以忽略塔楼的对称性；也就是说，即使该塔楼在垂直轴旋转或垂直平面翻转后与其他塔楼相同，这也被视为不同的设计。然而，每块积木都具有旋转对称性的，因此沿其主轴旋转 90、180 或 270 度都被视为相同的放置方式。

输入格式

输入的第一行为一个整数 T，表示后面测试用例的数量。

每个测试用例为一行，包含一个整数 N，表示积木的数量。

输出格式

对于每个测试用例，请输出一个整数，表示独特的“笨笨高”塔楼数量在取模（mod）$2^{3^2}3^{2^3} = 3359232$ 后的结果。

数据范围

$1 \le T \le 10$

主测试集

$1 \le N \le 10^{18}$

附加测试集 2

$1 \le N \le 10^{10^5} = 10^{100000}$

是的，你没读错。不仅仅是 Nacho 的电脑会爆炸哦～
感谢 CALICO，大本熊拥有（几乎）无穷无尽的“笨笨高”积木～

每个输入文件的大小不会超过 2 MB。

测试样例

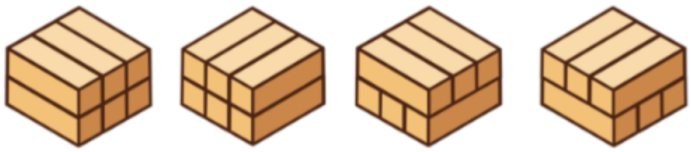
主样例输入 下载	主样例输出 下载
7	0
2	2
3	6
6	27
11	439
16	2941382
705	1635075
333333333333333333	

主样例解释

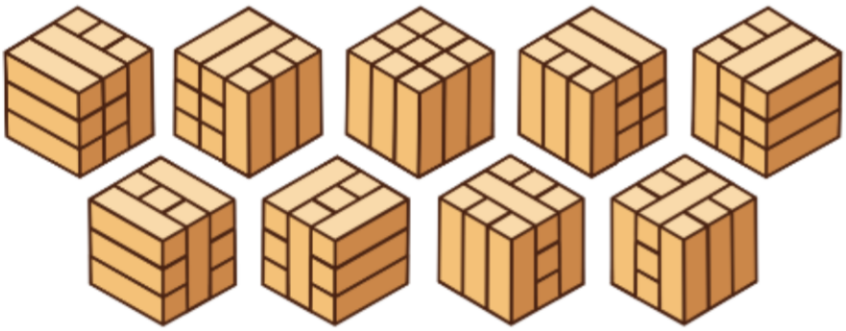
测试用例 #1：2 块积木不够建造任何一层，所以答案是 0。

测试用例 #2：3 块积木只能建造高度为 1 的塔楼。2 种方式如下所示。

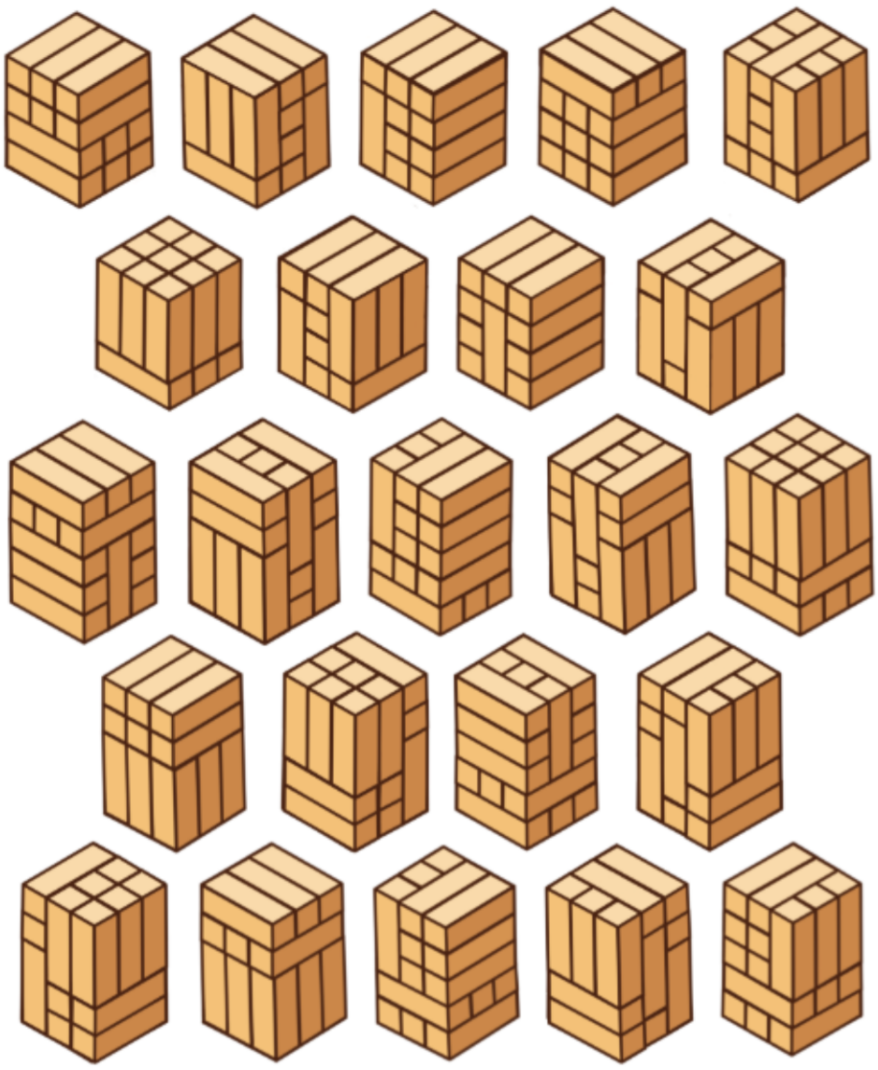
测试用例 #3：可以建造高度为 1 或 2 的塔楼。建造高度为 1 的塔楼的 2 种方式如上所示，建造高度为 2 的塔楼的 4 种方式如下所示，一共有 6 种方式。



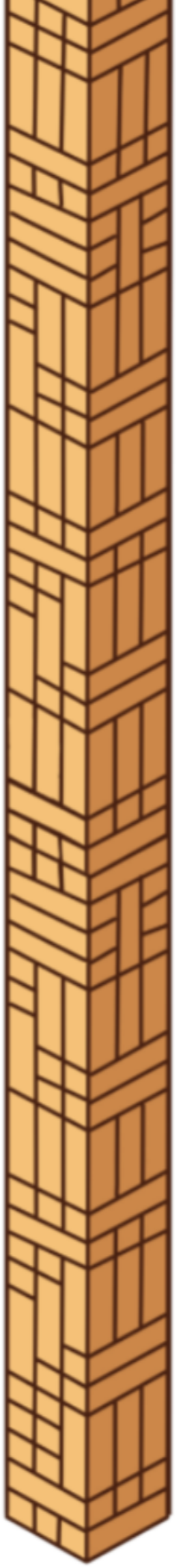
测试用例 #4：可以建造高度不超过 3 层的塔楼。一共有 27 种方式。下图展示了部分竖直搭建的塔楼。



测试用例 #5：有 439 种方式建造高度不超过 5 层的塔楼。下图展示了部分搭建方式。



测试用例 #6：使用 705 块积木可搭建的一个塔楼如本文件右侧所示。



附加样例输入 下载

```
1
31415926535897932384626433832795028841971693993751058 2
```

附加样例输入 下载

```
599783
```