1 Stability of equilibrium points & bifurcations

1.1 A simple population model

The population model has in general two solutions (and hence two fixed points) for $\dot{N} = 0$, namely

$$N_1 = 0 \quad \text{and} \quad N_2 = K\frac{\alpha - \beta}{\alpha}.$$

The stability of these fixed points in function of α and β can be summarised as follows:

Parameter region	Fixed points
$\alpha < \beta$	$N_1 = 0$: stable $N_2 < 0$: unstable
$\alpha = \beta$	$N_1 = N_2 = 0$: half-stable (unstable for $N < 0$, stable for $N > 0$)
$\alpha > \beta$	$N_1 = 0$: unstable $N_2 > 0$: stable

The system thus undergoes a transcritical bifurcation at $\alpha = \beta$.