3 Study of a predator-prey model

We study the system

$$\dot{x} = x(x-a)(1-x) - bxy$$
$$\dot{y} = xy - cy - d,$$

with $a = 0.1$ and $b = 1.5$. The following could be an ecological interpretation of this system as a predator-prey model:

The $-d$ term represents a constant decline of species y; This would correspond to a linear decrease in y over time, with slope d, if this was the only term present. Maybe a hunter shoots a fixed number d of y-type animals every time period.

The $-cy$ term represents a proportional pressure on y, corresponding to an exponential decline of y over time with time constant $1/c$ (i.e. faster decline for larger c). There might be a fixed amount of resources available for the y species. As a result, a larger number of y animals results in a proportionally smaller amount of the pie per animal.

The xy term represents a growth of y that is both proportional to the other species and to itself. For constant x, this would correspond to exponential growth of y with time constant $1/x$ (i.e. faster growth for more x). y could be a multiplying parasite, and x could be its host.

The $-bxy$ term represents a decline of the x species proportional to both itself and to the other species y. For constant y, this would correspond to an exponential decline of x with time constnat $1/(by)$. The y parasite might be pathological for x. Both more parasites y and more hosts x yield a higher probability of transmitting the parasite between hosts.

Finally, the $x(1-x)$ factors of the first $x(1-x)(x-a)$ term represent logistic growth (i.e. exponential growth, which switches to exponential slowing down when the carrying capacity of 1 is nearly reached). This is a common model for species growth. The $(x-a)$ multiplier has the effect that the growth does not start until x reaches a: for $x < a$, the species will decay instead of grow. This could model the fact that more than a few individuals are necessary for succesful long-term reproduction.

3.1 A qualitative study for $d = 0$