Non-linear systems

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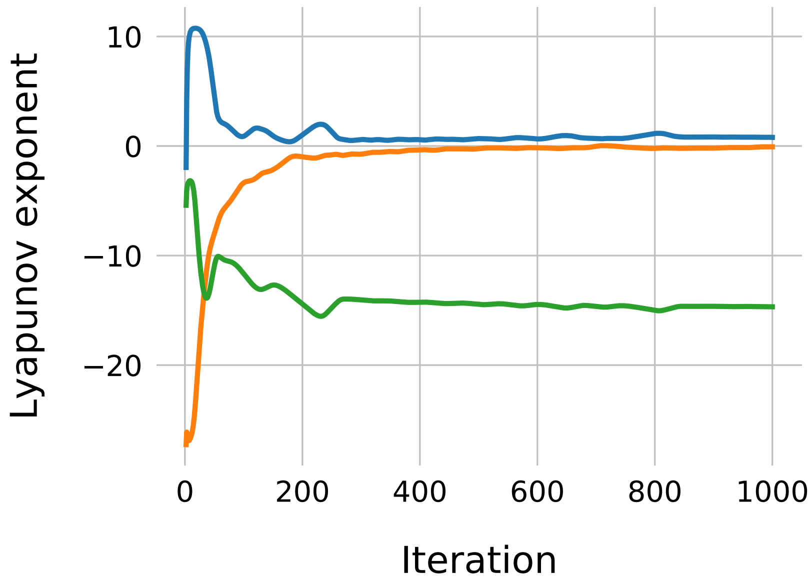


Figure 13: **Estimating Lyapunov exponents of the Lorenz system**. Initial phase point $(1, 1, 1)$. Final exponent estimates are 0.79, -0.07, and -14.7.

4 Chaos

4.1 Lyapunov exponents of the Lorenz equations

The algorithm integrates both an initial phase point and the spatial gradient of the phase point along the trajectory over fixed time steps `st` (via the simplest first order Euler method). Next, an orthogonal basis is sought for the new spatial gradient, via the Gram-Schmidt algorithm. The cumulative sum of logarithms of the Gram-Schmidt scaling factors divided by the total time elapsed is then the estimate of the Lyapunov exponent. This is integration-orthogonalisation loop is repeated for a given number of iterations `kkmax`.

When the time step is too large (e.g. `st = 0.1` here), the exponents diverge. When it is too small on the other hand (`st = 0.001` here), the convergence is extremely slow. Even for a balanced time step (like `st = 0.01` here), enough iterations need to be taken yield a decent result (e.g. `kkmax > 400` here; see fig. 13). The initial phase point also influences the results. A different phase point as in fig. 13 ($(6, 6, 6)$) yields better estimates for example (0.89, -0.04, and -14).

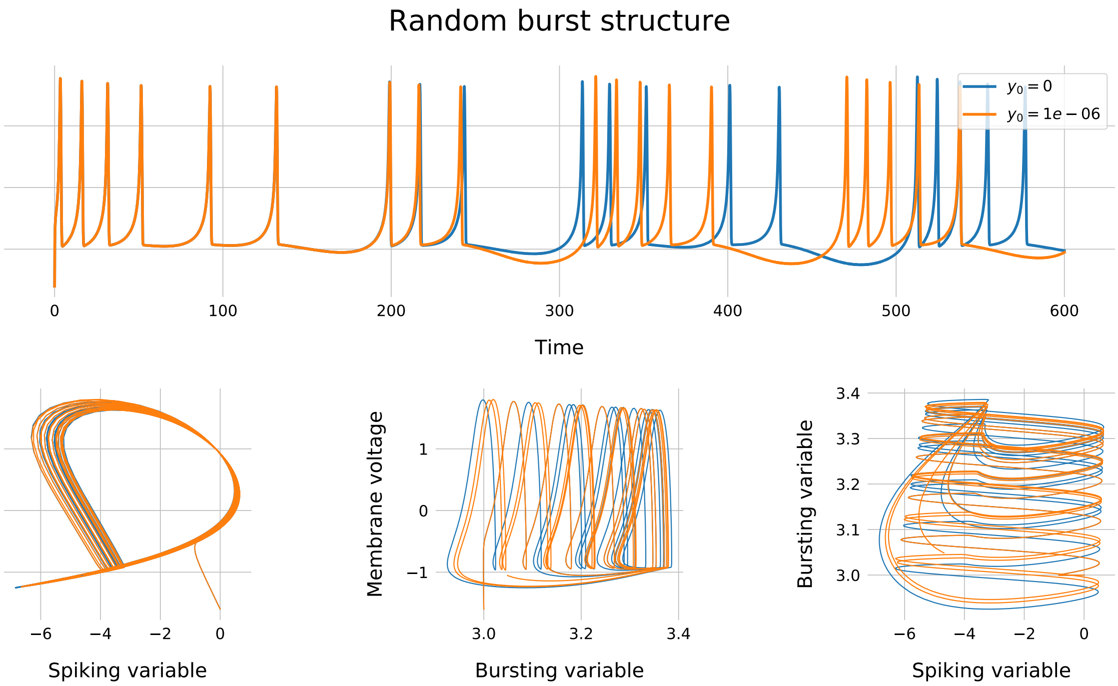
Interestingly, using a more advanced ODE solver (like a higher order Runge-Kutta method) or orthogonalisation algorithm (like a singular value decompostion via the QR-algorithm) yields markably worse results.

4.2 Hindmarsh-Rose neuron model

The system behaves chaotically in random burst mode (fig. 14, top): even the tiniest perturbation to the initial phase point renders $x(t)$eventually unpredictable. The burst generation mode is not chaotic: even relatively large perturbations do not qualitatively change the phase space trajectories (fig. 14, bottom).

4.3 Chua's circuit

A simulation of the Chua circuit displays chaotic behaviour (fig. 15). Estimating the Lyapunov exponents according to the method of section 4.1 confirms this: the largest exponent $\approx 0.16 > 0$ (fig. 16).



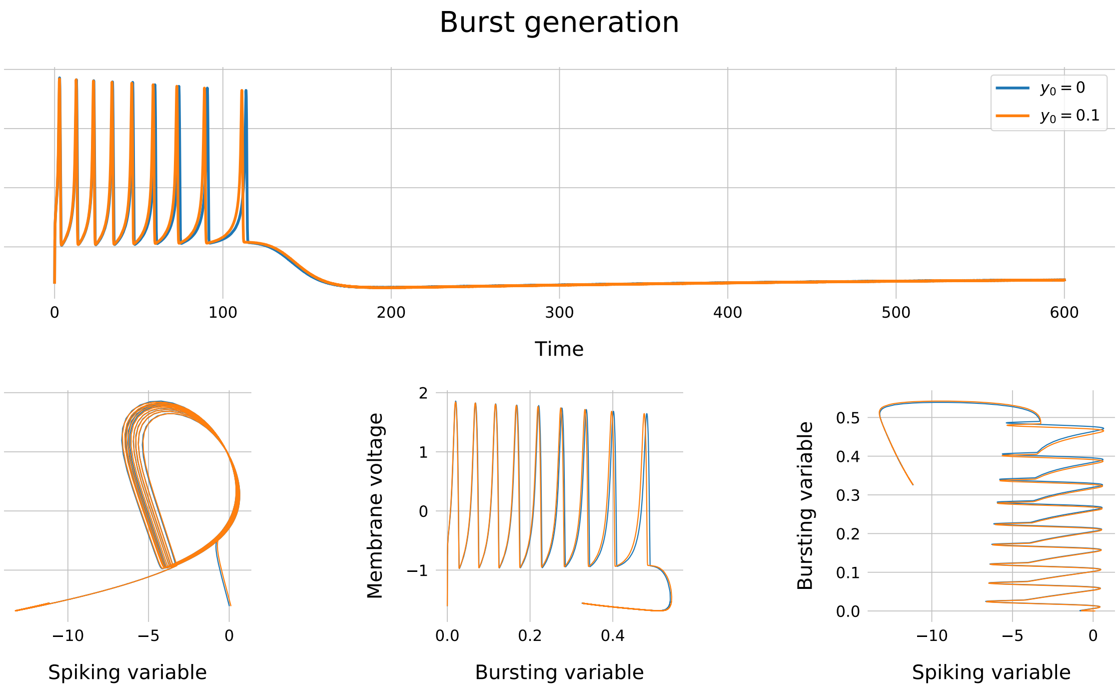


Figure 14: **Hindmarsh-Rose neuron model**. See text for interpretation. (In random burst mode, the initial slow current was $z(0) = 3$, while in burst generation mode, it was $z(0) = 0$. Other parameters as instructed).

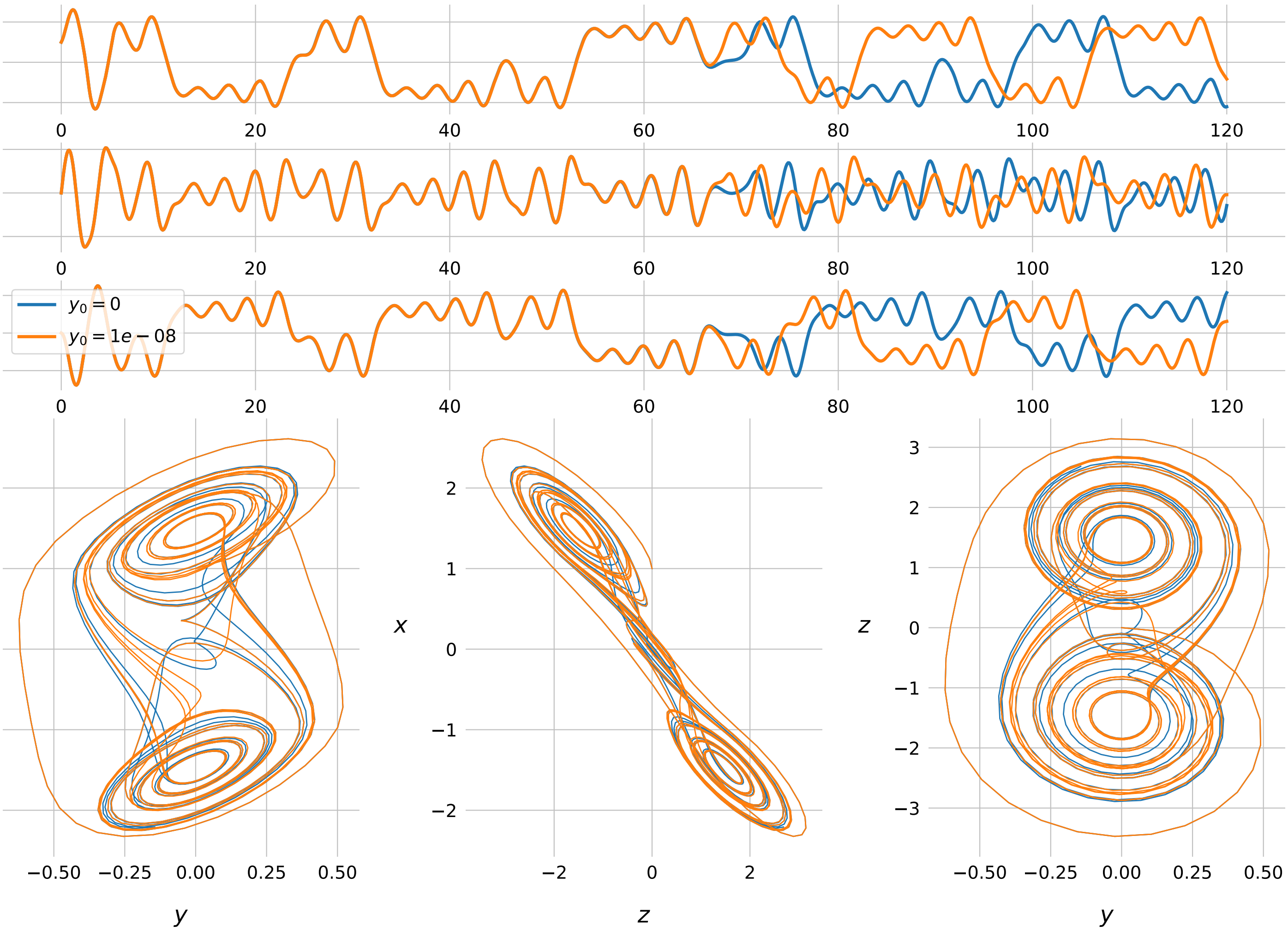


Figure 15: **Chua circuit model**. Time series (top) and phase space projections (bottom). Note the chaotic behaviour and the double-scroll dynamics. (Initical condition $(1, y_0, 0)$, with y_0 as indicated).

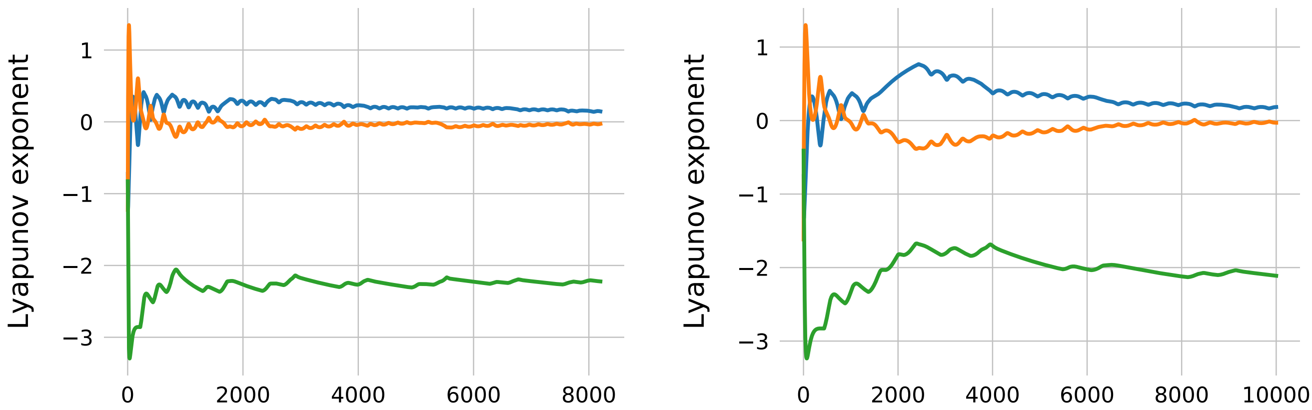


Figure 16: **Estimating Lyapunov exponents of the Chua circuit**. *Left*: `st = 0.01`, final largest exponent 0.146. *Right*: `st = 0.005`, final largest exponent 0.184.