Exercise 2

The Hillas criterion reads:

$$\frac{E_{max,obs}}{10^{20}\ eV} \lesssim \Gamma \cdot Z \cdot \frac{L_{source}}{pc} \cdot \frac{B}{G}$$

For the LHC B goes up to $7.7\ T = 7.7 \cdot 10^4\ G$. The circumference of the LHC is $\approx 27\ km$ resulting in a diameter $L_{source} = 2.7852 \cdot 10^{-13}\ pc$. For us the LHC itself is not moving relativistically ($\Gamma = 1$) and $Z = 1$ for protons.

$$\Rightarrow E_{max,obs} \lesssim 2.1446\ TeV$$

The maximum energy reached in the LHC is $6.5\ TeV$ for one proton. This can be explained by an additional factor in the Hillas criterion taking into account the acceleration efficiency. For shock acceleration a factor $\frac{\beta_S}{0.2}$ is added. A similar factor will be necessary for the acceleration efficiency of the LHC. Using the maximum for shock acceleration (factor 5) one gets

$$E_{max,obs} \lesssim 10.723\ TeV.$$

Exercise 4

a) For $T = 2.725\ K$ one gets for the integral $n_\gamma = 1.9334 \cdot 10^14\ m^{-3}$. The integral is taken from 0 to 10^{18}, since computational calculation above was not possible.

b) The mean free path is given by the inverse of the product of number density and cross section:

$$\lambda = \frac{1}{n_\gamma \sigma_T} = \frac{1}{1.2857 \cdot 10^{-14}}\ m = 7.7778 \cdot 10^{13}\ m = 3.2408 \cdot 10^{-3}\ pc$$

c) Since our galaxy is about 30 kpc across, it seems very unlikely that the electrons could come extra-galactic sources, since their mean free path is way too short.