Exercise 1

a)

$$\begin{aligned}
E_{kin} &= \gamma m_0 c^2 - m_0 c^2 = m_0 c^2(\gamma - 1) \\
\frac{E_{kin}}{m_0 c^2} &= \frac{1}{\sqrt{1-\beta^2}} - 1 \\
\beta &= \sqrt{1 - \left(\frac{E_{kin}}{m_0 c^2} + 1\right)^{-2}}
\end{aligned}$$

Now rewriting the energy density as

$$\rho_E = \rho_{E_0} + \rho_{E_{kin}}$$

where ρ_{E_0} is the energy density occurring from the rest mass and $\rho_{E_{kin}}$ is the kinetic energy density. Analogous to the calculation in the script one gets:

$$\begin{aligned}
\rho_E &= \rho_{E_0} + \int_{E_{min}}^{\infty} dE_{kin}\ \frac{dn_{CR}}{dE_{kin}} E_{kin} \\
&= \rho_{E_0} + \int_{E_{min}}^{\infty} dE_{kin}\ \frac{4\pi}{\beta c}\frac{d\Phi}{dE_{kin}} E_{kin} \\
&= \rho_{E_0} + \frac{4\pi\Phi_0}{c}\int_{E_{min}}^{\infty} dE_{kin}\ \frac{E_{kin}^{-\gamma+1}}{\sqrt{1 - \left(\frac{E_{kin}}{m_0 c^2} + 1\right)^{-2}}}
\end{aligned}$$

In the last step the power-law for the kinetic energy flux is inserted and the expression for β from before.

b) From the graph one can read the points $P_1 = (10|2 \cdot 10^{-3})$ and $P_2 = (10^2|5 \cdot 10^{-6})$. This results in $\Phi_0 = 0.8\ (cm^2\ s\ sr\ GeV)^{-1}$ and $\gamma = 2.602$.

c) Inserting into the integral in b) gives a value

$$\int_{E_{min}}^{\infty} dE_{kin}\ \frac{E_{kin}^{-\gamma+1}}{\sqrt{1 - \left(\frac{E_{kin}}{m_0 c^2} + 1\right)^{-2}}} = 1.7323$$