Exercise 1

a) Without loss of generality assume the WIMP to travel in z-direction. The maximum momentum transfer is given, when the WIMP is scattered with an angle of 180°. We then get from energy and momentum conservation the following:

$$E_1 = E_2 + E_3 \qquad\qquad p_1 = p_2 + p_3$$

$$\frac{p_1^2}{2m_\chi} = \frac{p_2^2}{2m_\chi} + \frac{p_3^2}{2m_T} \qquad\qquad p_2 = p_1 - p_3$$

$$\Rightarrow \frac{p_1^2}{2m_\chi} = \frac{p_1^2 + p_3^2 + 2p_1p_3}{2m_\chi} + \frac{p_3^2}{2m_T}$$

$$E_1 = E_1 + E_3\frac{m_T}{m_\chi} - \frac{p_1p_3}{2m_\chi} + E_3$$

$$0 = E_3(\frac{m_T}{m_\chi} + 1) - 2\sqrt{E_1E_3\frac{m_T}{m_\chi}}$$

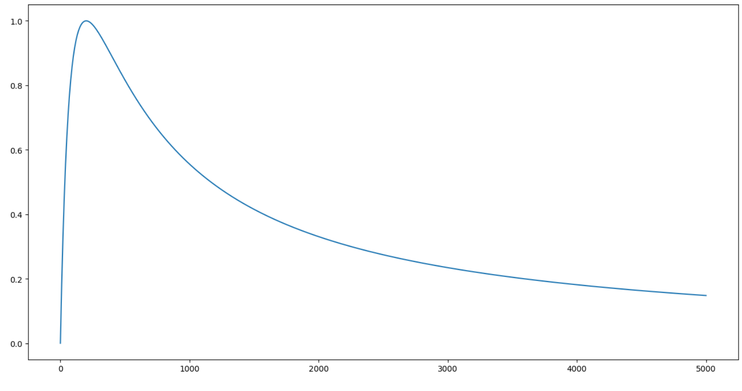
$$\Rightarrow E_3 = 4\frac{m_T}{m_\chi}(\frac{m_T}{m_\chi} + 1)^{-2} \cdot E_1$$

$$E_3 = r \cdot E_1$$

$$r = 4\frac{m_T m_\chi}{(m_T + m_\chi)^2}$$

Here 1 denotes the incoming χ, 2 the χ after scattering and 3 the target particle.

b)



The figure shows the behaviour of r for fixed $m_\chi = 100[randommassunit]$. One can see that for a target mass of zero r is zero as well. Going towards m_χ r rises fast to 1 ($m_\chi \approx m_T$). For $m_\chi << m_T$ r decreases towards zero again, but now the decrease is much weaker than the increase for $m_T << m_\chi$.

c) First case: $m_\chi = 10\frac{GeV}{c^2} \rightarrow E_{kin} = 5\ MeV$.

$$\begin{aligned} E_{r,e} &= 1.022\ keV \\ E_{r,p} &= 1.568\ MeV \\ E_{r,Si} &= 3.992\ MeV \\ E_{r,Xe} &= 1.390\ MeV \end{aligned}$$

Where for the mass of Si 14 proton and 14 neutron masses were added and for Xe 54 proton and 77 neutron masses.
Second case: $m_\chi = 100\frac{GeV}{c^2} \rightarrow E_{kin} = 50\ MeV$.

$$\begin{aligned} E_{r,e} &= 1.022\ keV \\ E_{r,p} &= 1.841\ MeV \\ E_{r,Si} &= 32.969\ MeV \\ E_{r,Xe} &= 49.467\ MeV \end{aligned}$$

d) Now one gets a scalar product at the point in a) where we have $2p_1p_3$ leading to $2p_1p_3cos\theta$. Following the same calculation one gets the additional factor $(1 - cos\theta)/2$, which is equal to one and maximal for $\theta = 180°$, confirming the assumption of maximal momentum transfer for that angle in a).