Cost-effective Printing of 3D Objects with Skin-Frame Structures

[Weiming Wang.et.al SIG ASIA 2013]

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Outline

Motivation

- Material is expensive

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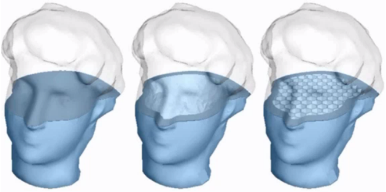
Background

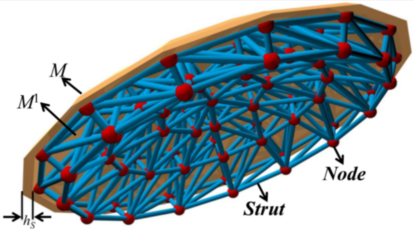
- Hollowing objects in 3D printing:



- Frame structure:

Key Idea

Mathematical Modeling



Goal: minimal Volume and minimal struts.

Solving

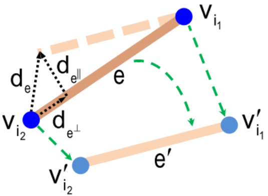
- For minimal struts: use L_0 optimization.
- For "goal" : use multi-objective optimization.

Problem Formulation

- Variants: radii r of struts and position V of nodes.
- Objective: $Vol(r,V,E)$ & $|E|$
- Constraints: Printability, Machanics, Geometry
  Printability: $r \in [\underline{\eta}, \bar{\eta}]$

Machanics constraints

- Stiffness: $K(V,r)D = F(r)$ where $K(V,r)$ is stiffness matrix, D is displacement, and F is inner and outer force on each node.
- Elastic:



$$\|d_{e\|}\|/\|e\|\gamma \le \sigma \quad (1)$$
$$\|d_{e^\perp}\|/\|e\|\mu \le \tau \quad (2)$$

- Buckling $r_j \ge l_j/\alpha$, where α is a given slenderness ratio.

Geometry constraints

- Geometry Approximation: $||d_i|| \leq \varepsilon_i$. Deformation of each strut should not larger than a given threshold ε_i
- Shape Barrier: $\forall e_i(v_{i_1} v_{i_2}) \in E_{int}$

$$v_{i_1}, v_{i_2} \in C \tag{3}$$

where C is a maximal convex region contains e_j and enclosed by M^1. Didn't say how to formulate this constraint.
- Balance: keep object stand on plane. [Prevost et.al 2013]

$$G_{proj} \in H \tag{4}$$

where G_{proj} is the projection of gravity center G, and H is a convex hull of its contactpoints on plane.

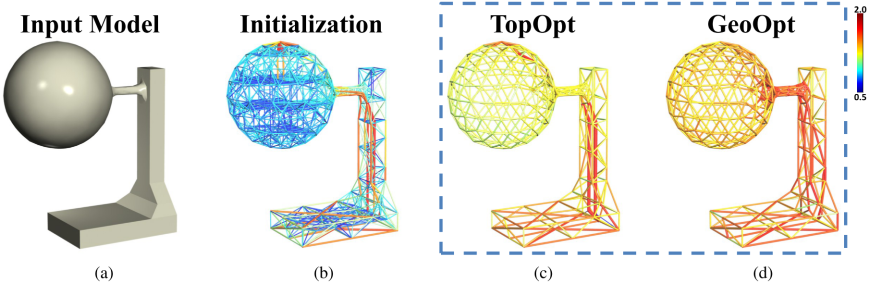
Solving

Since $\min Vol$ has higher priority than $\min |E|$, straightly formulate them into a single objective optimization will lead to a problem: How to choose an appropriate weight to trade off them?

The problem is splited into two steps:

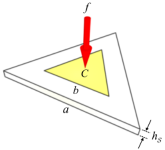
- Geometry Optimization: $\min Vol, \quad s.t. \quad cons$
- Topology Optimization: $\min |E|, \quad s.t. \quad cons \& Vol \leq \tilde{Vol}$

Solving Details



Initialization

- Determin node number and position
  - Node Number



$$
\begin{aligned}
d &\approx f(a-b)/(3\sqrt{3}\mu h_s b) && (5)\\
a &\leq 3\sqrt{3}\mu h_s b\varepsilon/f+b && (6)\\
\rightarrow |V_{skin}| &= 4Area(M^1)/(\sqrt{3}a^2) && (7)
\end{aligned}
$$

  - Adaptive sampling on M^1 and even sampling interior.
- Determin radis of each strut:
  Size optimization:

$$
\min_r Vol(r,V,E) \quad (8)
$$

$$
s.t. \quad cons. \quad (9)
$$

Topology Optimization

Formulation

$$\min_{r} |E_{int}| = \|r_{int}\|_0 \quad (10)$$

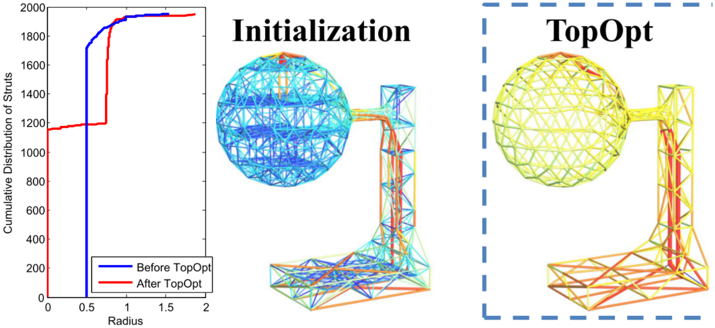
$$s.t. \quad cons. \, and \, Vol(r) \leq \tilde{Vol} \quad (11)$$

$\|r_{int}\|_0$ is approximated by reweighted L_1 minimization:

$$\|Wr\|_1 = W^T r \quad (12)$$

where $r > 0$, and W is a diagonal positive definite matrix, $w_i = \frac{1}{\varepsilon + (\bar{r}_j - \underline{\eta})}$

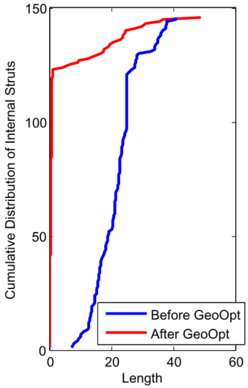
Topology Optimization Result



Q: Will there be hanging struts?

A: No proof.

Geometry Optimization



Formulation

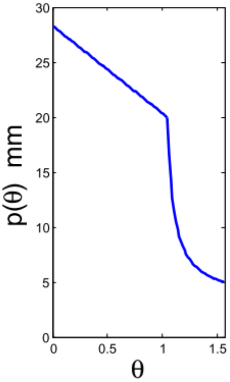
$$\min_{r,V} Vol(r,V,E) \tag{13}$$

$$s.t. \quad cons. \tag{14}$$

Short length struts will be collapsed.

Self supporting

For extrusion-type 3D printers, extra struts are added for self-supporting.



Algorithm 2 Self-supporting extension for extrusion-type printers

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Input: a frame $\mathcal{T}^* = (V^*, E^*)$ generated by Algorithm 1
Output: a self-supporting frame $\mathcal{T}^{(s)}$

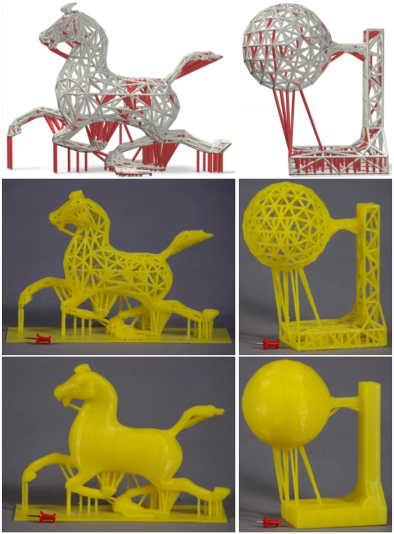
1: Let $V^{(s)} = V^*$ and $E^{(s)} = E^*$, and define a base plane B for the input frame.

2: For each $\mathbf{e}_j \in E^*$, compute its length l_j and vertical angle θ_j.
if $p(\theta_j) < l_j$, let $m(\mathbf{e}_j) = \lceil \frac{l_j}{p(\theta_j)} \rceil - 1$ and insert equidistant nodes $\mathbf{v}_k, k = 1, \cdots, m(\mathbf{e}_j)$, on the strut $\mathbf{e}_j$.
  for $k = 1, \cdots, m(\mathbf{e}_j)$
    Find a closest node $\mathbf{v}'_k \in V^{(s)} \cup B$, which lies below $\mathbf{v}_k$ such that the new strut $\mathbf{e}_k = \mathbf{v}'_k\mathbf{v}_k$ is self-supporting printable.
    Update the frame by setting $V^{(s)} = V^{(s)} \cup \{\mathbf{v}'_k\}$ and $E^{(s)} = E^{(s)} \cup \{\mathbf{e}_k\}$.
  endfor
endif

3: Call the size optimization with $d^z_{\mathbf{e}} \leqslant 0$ added for each $\mathbf{e} \in E^{(s)}$ to the constraints and obtain the optimized radius vector $\mathbf{r}^{(s)}$. Output a self-supporting frame $\mathcal{T}^{(s)}$ with its strut radii $\mathbf{r}^{(s)}$.

Results

Results



Conclusion

- First method considering cost-effective printing
- Nice addressing of approximation of L_0 norm.
- Nice addressing of multi-objective optimization. It's a good try to jump out of local minimal.
- Future Work:
  - Lack of considering orientation.
  - Only small size could be printed directly, large size model should be segmentated. How to design frame for those models should be considered.

Thank you!