3D Parameterization: PolyCube and CubeCover

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Outline

1. Introduction
2. CubeCover and PolyCube
3. Conclusion

What we want to do?

$f : V \to \Omega$

./img/cirque.png

What we need to do?

Analogous to 2D parameterization, a good 3D parameterization needs a suitable frame-field.
Which can be defined as follows:

$$\min \int_V \|\nabla f - X\|^2 dV$$
$$\text{s.t.} \quad C = 0$$

Where $C = 0$ is the boundary condition, and X is the frame-field.
Then the most important thing is to generate a good X:
smooth, topology well defined, controllable,....

Frame-Field Unknown

1. Smooth : It has been done.
2. Topology: *CubeCover* has found a few things, but it's still left unknown on how to place singularities.
3. Controllable: Depends on topology. Also unknown.

CubeCover

./img/meta-mesh-2.png

A good frame-field should statisfy the topology constaint: singularities should be placed well.

Frame-Field Construction

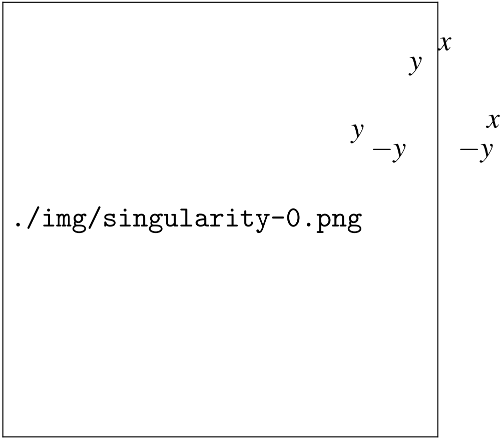
./img/frame-field-trilinear.png

By using the trilinear interpolation, it will get a frame on each point.

So that each tet has a frame at the barycenter.

It looks like a simple smoothing process.

Singularity



Frame-Field Construction

./img/singularity.png

The major character of frame field is to control the singularities.

./img/orientation.png

matching matrix $\prod_{st}$ defines the difference from orientation of s to orientation of t.

Inner-Singularity

./img/singularity-matching-matrix.png

The singularity type of edge e is defined as:

$type(e,t_0) := \prod_{t_k t_0} \circ \prod_{t_{k-1} t_k} \circ \ldots \circ \prod_{t_1 t_2} \circ \prod_{t_0 t_1}$

The tets $(t_0 \ldots t_k)$ is around the edge e.

Theorem: Let f be a parameterization where no tet is mapped to degenerated tet(a tet with vanishing volume).

Then the *type* of each edge is either the identity or a rotation of 90° around one of the coordinate axes.

An observation which may be usefull

Theorem:

Let p be an inner vertex of a non-degenerated hexahedral mesh and e_i its adjcaent edges. Then: $\sum_i (6 - valence(e_i)) = 12$

Where valence of e is the number of adjacent hexahedra.

./img/observation-0.png

./img/observation-1.png

An observation which may be usefull

The above theorem indicates that:

1. A singularity edge will not simply start or end interior. They usually meet at *node points* or hit the bounding surfaces.
2. Not all situations which can fulfill above equation exist in real life. Not complete.

./img/valence-6-2.png

./img/valence-6-3.png

PolyCube

./img/deformation.png

Deformation

./img/rotation.png

./img/position.png

Rotation

Align the model surface normal with the global axes: $\pm X, \pm Y, \pm Z$

Position

Align each PolyCube face with an appropriate axis.

Limitation

./img/non-planar.png

Enforcing the planarity constraint will result in large distortion.
That's because it's not a plane around the pink point.
It needs a chart cutting and inserting. But the chart insertion process is not guaranteed to produce a valid PolyCube.
No topology consistency guarantee

Conclusion

The two method are both semi-automatic parameterization.

- PolyCube:
  - All-hex mesh
  - Singularity-free interior
  - No theoretical guarantee on producing a valid PolyCube
- CubeCover:
  - Singularity controllable but need to manually generate meta-mesh
  - Lack of frame-field smoothing
  - There is no sufficient condition on finding a frame-field which adheres to given singularities.