ICS 271
Fall 2016
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Homework Assignment 4
Due Tuesday, 11/3

1. $X = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

   $D = \{D_1, D_2, D_3, D_4, D_5, D_6\}$
   - $D_1 = \{$desk, easy, dove, else, help, kind, soon, this$\}$
   - $D_2 = \{$eta, hat, her, him, one$\}$
   - $D_3 = \{$at, be, he, it, on$\}$
   - $D_4 = \{$dance, usage, first, loses, fuels, haste, given, sense, sound, think$\}$
   - $D_5 = \{$dance, usage, first, loses, fuels, haste, given, sense, sound, think$\}$
   - $D_6 = \{$desk, easy, dove, else, help, kind, soon, this$\}$

   $C_1 = \{v_2(1) = v_1(2), v_2(2) = v_3(2), v_2(3) = v_4(3), v_5(1) = v_4(4), v_5(3) = v_6(3)\}$, $v_i(j)$ means the j-th letter in word i.
   $C_2 = \{v_1 \neq v_6, v_4 \neq v_5\}$

   Cost function: the number of constraints unsatisified. (Range from 0 to 5)

   Start: v_1=this, v_2=eta, v_3=at, v_4=dance, v_5=haste, v_6=dove, cost=4
   Iter1: v_1=help, v_2=eta, v_3=at, v_4=dance, v_5=haste, v_6=dove, cost=3, update v_1
   Iter2: v_1=help, v_2=eta, v_3=at, v_4=usage, v_5=haste, v_6=dove, cost=2, update v_4
   Iter3: v_1=help, v_2=eta, v_3=at, v_4=usage, v_5=given, v_6=dove, cost=0, update v_5

2. Assume there's a classes(subjects) list S, the number of elements in S is NoS, s_i is i-th subject in S, $i = 1, 2, \ldots, NoS$. We have,

   $$S = \{s_i, i = 1, 2, \ldots, NoS\}$$

   Same as professors(P), classesrooms(C) and timeslots(T). That we have,

   $$P = \{p_\alpha, \alpha = 1, 2, \ldots, NoP\}$$
   $$C = \{c_\beta, \beta = 1, 2, \ldots, NoC\}$$
   $$T = \{t_\gamma, \gamma = 1, 2, \ldots, NoT\}$$

   $X = \{(s_i, p_\alpha, c_\beta, t_\gamma), i = 1, 2, \ldots, NoS\}$, it means that subject s_i is taught by professor p_α at classroom c_β on timeslot t_γ.

   $D_j = \{(s_i, p_\alpha, c_\beta, t_\gamma) | i, \alpha, \beta, \gamma$ could be any valid value$\}$

   C: - Each s_i should only appear once.
   - For any two subjects $(s_{i1}, p_{\alpha 1}, c_{\beta 1}, t_{\gamma 1})$ and $(s_{i2}, p_{\alpha 2}, c_{\beta 2}, t_{\gamma 2})$,
   if $p_{\alpha 1} = p_{\alpha 2}$, then $t_{\gamma 1} \neq t_{\gamma 2}$, it means a professor could not have two classes at same time;
   if $c_{\beta 1} = c_{\beta 2}$, then $t_{\gamma 1} \neq t_{\gamma 2}$, it means one classroom could no have two classes at same time.

3. Assume big rectangle locate in a 2-D coordinate system, range from $[0, X] \times [0, Y]$.

   $X = \{(x_i, y_i, dX_i, dY_i) | i = 1, 2, \ldots, n\}$, each X_i represents a small rectangle with it's position(bottom left corner) (x_i, y_i) and size (dX_i, dY_i).

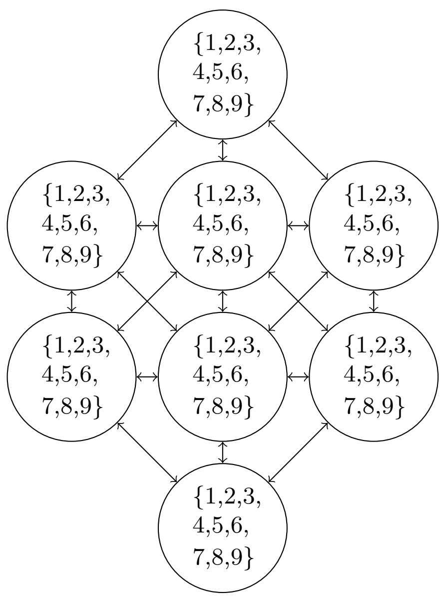
$D_i = \{(x, y, dX, dY) | dX \leqslant X, dY \leqslant Y\}$

C_1: For each small rectangle (x_i, y_i, dX_i, dY_i), we have,
$x_i \geqslant 0$ and $x_i + dX \leqslant X$ and $y_i \geqslant 0$ and $y_i + dY \leqslant Y$

C_2: For any of two small rectangles (x_i, y_i, dX_i, dY_i) and (x_j, y_j, dX_j, dY_j), we have,
$x_i + dX_i \leqslant x_j$ or $x_j + dX_j \leqslant x_i$ or $y_i + dY_i \leqslant y_j$ or $y_j + dY_j \leqslant y_i$

4. (a) For any neighboring pair of squares i and j such that $i \neq j$,

Relation	Domain
$\lvert X_i - X_j \rvert \geqslant 2$	(1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (2,4), (2,5), (2,6), (2,7), (2,8), (2,9), (3,1), (3,5), (3,6), (3,7), (3,8), (3,9), (4,1), (4,2), (4,6), (4,7), (4,8), (4,9), (5,1), (5,2), (5,3), (5,7), (5,8), (5,9), (6,1), (6,2), (6,3), (6,4), (6,8), (3,9), (7,1), (7,2), (7,3), (7,4), (7,5), (7,9), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (9,1), (9,2), (9,3), (9,4), (9,5), (9,6), (9,7)

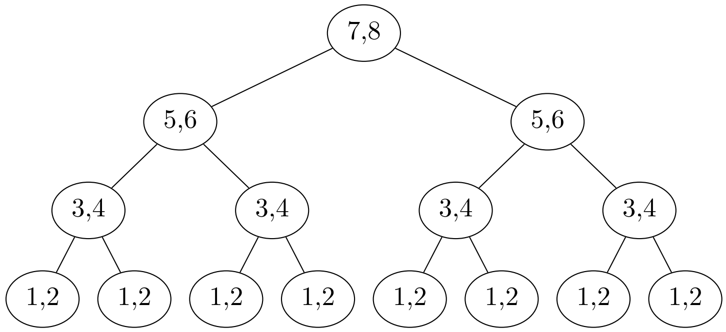


(b) Yes, it's arc-consistent.

(c) Yes, it's consistent. One solution is,

	2	
5	8	6
3	1	4
	7	

5. (a) Yes, it's arc-consistent.



(b) Yes, it's consistent. One solution is,

$$X_1 = 8 \quad X_2 = 6 \quad X_3 = 6$$

$$X_4 = 4 \quad X_5 = 4 \quad X_6 = 4 \quad X_7 = 4$$

$$X_8 = 2 \quad X_9 = 2 \quad X_{10} = 2 \quad X_{11} = 2$$

$$X_{12} = 2 \quad X_{13} = 2 \quad X_{14} = 2 \quad X_{15} = 2$$

(c) Do an in-order traversal: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

(d) Since this is a tree the complexity is $O(nd^2)$ when n is the number of variables and d is the domain size.

6. Solve problem:

$$\begin{array}{ccccc} & & \mathrm{T} & \mathrm{W} & \mathrm{O} \\ + & & \mathrm{T} & \mathrm{W} & \mathrm{O} \\ \hline & \mathrm{F} & \mathrm{O} & \mathrm{U} & \mathrm{R} \end{array}$$

Constraints:

$$O + O = R + 10X_1$$

$$W + W + C_1 = U + 10C_2$$

$$T + T + C_2 = O + 10C_3$$

$$C_3 = F$$

$$alldiff(T, W, O, F, U, R)$$

Domain:

$$D_F, D_{C_3} = \{1\}$$

$$D_{C_1}, D_{C_2} = \{0, 1\}$$

$$D_T = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$D_W, D_O, D_U, D_R = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

A solution:

$$\begin{array}{ccccc} & & 7 & 3 & 4 \\ + & & 7 & 3 & 4 \\ \hline & 1 & 4 & 6 & 8 \end{array}$$

A trace to a solution:

Node	State	Action
0	$D_F, D_{C_3} = \{1\}$, $D_{C_1}, D_{C_2} = \{0, 1\}$, $D_T = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $D_W, D_O, D_U, D_R = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$	Initial State
1	$F = 1, D_{C_3} = \{1\}$, $D_{C_1}, D_{C_2} = \{0, 1\}$, $D_T = \{2, 3, 4, 5, 6, 7, 8, 9\}$, $D_W, D_O, D_U, D_R = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$	Assign $F = 1$
2	$F = 1, C_3 = 1$, $D_{C_1}, D_{C_2} = \{0, 1\}$, $D_T = \{5, 6, 7, 8, 9\}$, $D_W, D_O, D_U, D_R = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$	Assign $C_3 = 1$
3	$F = 1, C_3 = 1, C_2 = 0$, $D_{C_1} = \{0, 1\}$, $D_T = \{5, 6, 7, 8, 9\}$, $D_W = \{0, 2, 3, 4\}$, $D_O = \{0, 2, 4, 6, 8\}$, $D_U, D_R = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$	Assign $C_2 = 0$
4	$F = 1, C_3 = 1, C_2 = 0, C_1 = 0$, $D_T = \{5, 6, 7, 8, 9\}$, $D_W = \{0, 2, 3, 4\}$, $D_O = \{0, 2, 4\}$, $D_U = \{0, 2, 4, 6, 8\}$ $D_R = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$	Assign $C_1 = 0$
5	$F = 1, C_3 = 1, C_2 = 0, C_1 = 0, T = 6, O = 2, R = 4$, $D_W = \{0, 3\}$, $D_U = \{0, 8\}$	Assign $O = 2$, then $T = 6, R = 4$
6	$F = 1, C_3 = 1, C_2 = 0, C_1 = 0, T = 7, O = 4, R = 8$, $D_W = \{0, 2, 3\}$, $D_U = \{0, 2, 6\}$	Assign $W = 3$ or $W = 0$, no solution in D_U; return to node 5, assign $O = 4$, then $T = 7, R = 8$
7	$F = 1, C_3 = 1, C_2 = 0, C_1 = 0, T = 7, O = 4, R = 8$, $D_W = \{3\}$, $D_U = \{6\}$	Assign $W = 0$ or $W = 2$, then no solution in D_U
8	$F = 1, C_3 = 1, C_2 = 0, C_1 = 0, T = 7, O = 4, R = 8, W = 3, U = 6$	Assign $W = 3$, then $U = 6$. Find Solution!

7. a_1: arc consistency with domain splitting
   a_2: variable elimination
   a_3: stochastic local search
   a_4: genetic algorithms

   (a) a_1 can determine that there is no solution, if the problem is inconsistent.

   (b) a_2, a_3 can find a solution if one exists.

   (c) a_4 can guarantee to find all solutions.