ICS 271
Fall 2016
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Homework Assignment 5
Due Thursday, 11/10

1. (a) 4
   Only when both A and B are *True*, $B \wedge A$ would be *True*. C and D could be any value. So there should have $1 \times 2^2 = 4$ models.
   (b) 15
   $\neg A \vee \neg B \vee \neg C \vee \neg D = \neg(A \wedge B \wedge C \wedge D)$
   Only when A, B, C, D are all *True*, the sentences is *false*. So there should have $2^4 - 1 = 15$ models.
   (c) 0
   $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$
   $= (\neg A \vee B) \wedge A \wedge \neg B \wedge C \wedge D$
   $= \neg(A \wedge \neg B) \wedge (A \wedge \neg B) \wedge C \wedge D$
   $\neg(A \wedge \neg B)$ and $(A \wedge \neg B)$ could NOT be *True* at same time. So there's no models for this sentence.

2. (a) Define:
   A : The car is at John's house.
   B : The car is at Fred's house.
   (b) statement 1: $A \vee B$
   statement 2: $\neg B \Rightarrow A$
   (c) Statement 2 is equivalent with Statement 1, so we can not determine where the car is.

3. *unit resolution*:

$$\frac{\alpha \vee \beta,\ \neg\beta}{\alpha}$$

True table:

α	β	$\alpha \vee \beta$	$\neg\beta$	$(\alpha \vee \beta) \wedge \neg\beta$
T	T	T	F	F
<u>T</u>	F	T	T	<u>T</u>
F	T	T	F	F
F	F	F	T	F

From the True table, we can conclude that, when $(\alpha \vee \beta) \wedge \neg\beta$ is *True*, α is *True*. So the *unit resolution* is SOUND.

4.

$$\begin{aligned}
&\neg[((P \vee \neg Q) \rightarrow R) \rightarrow (P \wedge Q)]\\
=&\neg[(\neg(P \vee \neg Q) \vee R) \rightarrow (P \wedge Q)]\\
=&\neg[\neg(\neg(P \vee \neg Q) \vee R) \vee (P \wedge Q)]\\
=&(\neg(P \vee \neg Q) \vee R) \wedge \neg(P \wedge Q)\\
=&((\neg P \wedge Q) \vee R) \wedge (\neg P \vee \neg Q)\\
=&(\neg P \vee R) \wedge (Q \vee R) \wedge (\neg P \vee \neg Q)
\end{aligned}$$

5. N-Queen constraints: ($q_{k,l}$, (k,l) means the square on k-th row and l-th column)
   1. Any of two Queens should not be in the same column.
   $$\bigwedge_{k=1}^{N}\{\bigvee_{j=1}^{N}[q_{j,k} \wedge (\bigwedge_{\substack{i=1\\ i\neq j}}^{N} \neg q_{i,k})]\}$$
   2. Any of two Queens should not be in the same row.
   $$\bigwedge_{k=1}^{N}\{\bigvee_{j=1}^{N}[q_{k,j} \wedge (\bigwedge_{\substack{i=1\\ i\neq j}}^{N} \neg q_{k,i})]\}$$
   3. Any of two Queens should not be in the same diagnal line (NW to SE).
   $$\bigwedge_{k=1}^{N}\{\bigvee_{j=1}^{N}[q_{j,k} \wedge (\bigwedge_{\substack{i=1\\ i\neq j\\ 1\leqslant i-j+k\leqslant N}}^{N} \neg q_{i,i-j+k})]\}$$
   4. Any of two Queens should not be in the same diagnal line (NE to SW).
   $$\bigwedge_{k=1}^{N}\{\bigvee_{j=1}^{N}[q_{j,k} \wedge (\bigwedge_{\substack{i=1\\ i\neq j\\ 1\leqslant j-i+k\leqslant N}}^{N} \neg q_{i,j-i+k})]\}$$

6. (a) $P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$ (see Table 1: 6-(a))
   (b) $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ (see Table 2: 6-(b))
   (c) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ (see Table 3: 6-(c))
   (d) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ (see Table 4: 6-(d))

7. (a) $\mathit{Smoke} \Rightarrow \mathit{Smoke}$ (see Table 5: 7-(a))
   Valid.
   (b) $\mathit{Smoke} \Rightarrow \mathit{Fire}$ (see Table 6: 7-(b))
   Unsatisfiable.
   (c) $(\mathit{Smoke} \Rightarrow \mathit{Fire}) \Rightarrow (\neg \mathit{Smoke} \Rightarrow \neg \mathit{Fire})$ (see Table 7: 7-(c))
   Unsatisfiable.
   (d) $\mathit{Smoke} \vee \mathit{Fire} \vee \neg \mathit{Fire}$ (see Table 8: 7-(d))
   Valid.
   (e) $((\mathit{Smoke} \wedge \mathit{Heat}) \Rightarrow \mathit{Fire}) \Leftrightarrow ((\mathit{Smoke} \Rightarrow \mathit{Fire}) \vee (\mathit{Heat} \Rightarrow \mathit{Fire}))$
   Left:
   $$\begin{aligned} &(S \wedge H) \Rightarrow F \\ =&\neg(S \wedge H) \vee F \\ =&\neg S \vee \neg H \vee F \end{aligned}$$
   Right:
   $$\begin{aligned} &(S \Rightarrow F) \vee (H \Rightarrow F) \\ =&(\neg S \vee F) \vee (\neg H \vee F) \\ =&\neg S \vee \neg H \vee F \end{aligned}$$
   Left and Right are equivalent. So it's valid.

(f) $Big \lor Dumb \lor (Dumb \Rightarrow Big)$

$$\begin{aligned} & B \lor D \lor (D \Rightarrow B) \\ = & B \lor D \lor (\neg D \lor B) \\ = & B \lor (D \lor \neg D) \end{aligned}$$

$D \lor \neg D$ is always *True*, so that $B \lor (D \lor \neg D)$ is always *True* too. So it's valid.

8. DPLL

$P \Rightarrow Q = \neg P \lor Q$
$L \land M \Rightarrow P = \neg L \lor \neg M \lor P$
$B \land L \Rightarrow M = \neg B \lor \neg L \lor M$
$A \land P \Rightarrow L = \neg A \lor \neg P \lor L$
$A \land B \Rightarrow L = \neg A \lor \neg B \lor L$
A
B

Step 1.
Clauses: $(\neg P \lor Q) \land (\neg L \lor \neg M \lor P) \land (\neg B \lor \neg L \lor M) \land (\neg A \lor \neg P \lor L) \land (\neg A \lor \neg B \lor L) \land A \land B$
Find pure symbols: Q
Model: $Q = True$

Step 2.
Clauses: $(\neg L \lor \neg M \lor P) \land (\neg B \lor \neg L \lor M) \land (\neg A \lor \neg P \lor L) \land (\neg A \lor \neg B \lor L) \land A \land B$
Find unit symbols: A, B
Model: $Q = True$, $A = True$, $B = True$

Step 3.
Clauses: $(\neg L \lor \neg M \lor P) \land (\neg L \lor M) \land (\neg P \lor L) \land L$
Find pure symbols: None
Find unit symbols: L
Model: $Q = True$, $A = True$, $B = True$, $L = True$

Step 4.
Clauses: $(\neg M \lor P) \land M$
Find pure symbols: P
Model: $Q = True$, $A = True$, $B = True$, $L = True$, $P = True$

Step 5.
Clauses: M
Find unit symbols: M
Model: $Q = True$, $A = True$, $B = True$, $L = True$, $P = True$, $M = True$

Step 6.
Done

Comparison between DPLL and FC algorithm:

They generate different traces based on KB. FC is data driven and DPLL has early termination. The complexity of DPLL would be much less than FC.

Table 1: 6-(a)

P	Q	R	$Q \wedge R$	$P \wedge Q$	$P \wedge (Q \wedge R)$	$(P \wedge Q) \wedge R$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Table 2: 6-(b)

P	Q	R	$Q \vee R$	$P \wedge Q$	$P \wedge R$	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Table 3: 6-(c)

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Table 4: 6-(d)

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg P \vee \neg Q$	$P \Leftrightarrow Q$	$(P \wedge Q) \vee (\neg P \vee \neg Q)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

Table 5: 7-(a)

S	$\neg S$	$S \Rightarrow S(\neg S \vee S)$
T	F	T
F	T	T

Table 6: 7-(b)

S	F	$\neg S$	$S \Rightarrow F(\neg S \vee F)$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Table 7: 7-(c)

S	F	$\neg S$	$\neg F$	$S \Rightarrow F$	$\neg S \Rightarrow \neg F$	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Table 8: 7-(d)

S	F	$\neg F$	$S \vee F \vee \neg F$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	T