ICS 271
Fall 2016
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Homework Assignment 6
Due Thursday, 11/17

1. (a) No two people have the same social security number.
   Not good. x and y should not be the same person and using $\Rightarrow$ with $\exists$ leads to a very weak statement.

$$\neg\exists x, y, n\ Person(x) \wedge Person(y) \wedge \neg(x = y) \wedge HasSS\#(x, n) \wedge HasSS\#(y, n)$$

   (b) John's social security number is the same as Mary's.
   Good. Do not need to modify.

$$\exists n\ HasSS\#(John, n) \wedge HasSS\#(Mary, n)$$

   (c) Everyone's social security number has nine digits.
   Not good. For each person, it should only have one SSN but not all.

$$\forall x\ Person(x) \Rightarrow [\exists n\ HasSS\#(x, n) \wedge Digits(n, 9)]$$

   (d) Rewrite as follows.

$$\neg\exists x, y\ Person(x) \wedge Person(y) \wedge [SS\#(x) = SS\#(y)]$$

$$SS\#(John) = SS\#(Mary)$$

$$\forall x\ Person(x) \Rightarrow Digits(SS\#(x), 9)$$

2. (a) Not.
   (b) $\{x/A, y/B, z/B\}$
   (c) $\{x/David\}$
   (d) $\{x/g(u), u/f(v)\}$
   (e) $\{x/B, y/B, z/B\}$

3. Define:
   $Alpine(x)$: x belong to Alpine Club.
   $Skier(x)$: x is a skier.
   $Climber(x)$: x is a mountain climber.
   $Like(x, w)$: x like weather w.

   Statement:
   A: $Alpine(Tony)$
   B: $Alpine(Mike)$
   C: $Alpine(John)$
   D: $\forall x\ Alpine(x) \Rightarrow Skier(x) \vee Climber(x)$
   E: $\neg\exists x\ [Climber(x) \wedge Like(x, Rain)]$
   F: $\forall x\ Skier(x) \Rightarrow Like(x, Snow)$
   G: $\forall x\ Like(John, x) \Rightarrow \neg Like(Mike, x)$
   H: $\forall x\ \neg Like(John, x) \Rightarrow Like(Mike, x)$
   I: $\neg Like(John, Rain)$
   J: $\neg Like(John, Snow)$

Question: $\textsc{AskVars}(KB, Alpine(x) \wedge Climber(x) \wedge \neg Skier(x))$

Convert them to CNF form and add negated goal to them:
A: $Alpine(Tony)$
B: $Alpine(Mike)$
C: $Alpine(John)$
D: $\neg Alpine(X_1) \vee Skier(X_1) \vee Climber(X_1)$
E: $\neg Climber(x_2) \vee \neg Like(x_2, Rain)$
F: $\neg Skier(X_3) \vee Like(X_3, Snow)$
G: $\neg Like(John, X_4) \vee \neg Like(Mike, X_4)$
H: $Like(John, X_5) \vee Like(Mike, X_5)$
I: $\neg Like(John, Rain)$
J: $\neg Like(John, Snow)$
K: $\neg Alpine(x_6) \vee \neg Climber(x_6) \vee Skier(x_6)$

Resolution refutation: 1: $\neg Skier(John) \vee Like(John, Snow)$ [F, $\{X_3/John\}$]
2: $\neg Skier(John)$ [1,J]
3: $\neg Alpine(John) \vee Skier(John) \vee Climber(John)$ [D, $\{X_1/John\}$]
4: $Climber(John)$ [3,C,2]
5: $\neg Alpine(John) \vee \neg Climber(John) \vee Skier(John)$ [K,$\{x_6/John\}$]
6: $\neg Climber(John)$ [5,C,2]
7: Conflict [4,6]
So John is the answer.

4. Only (a) is the result.

5. (a) $\forall x\ Horse(x) \Rightarrow Mammal(x)$
   $\forall x\ Cow(x) \Rightarrow Mammal(x)$
   $\forall x\ Sheep(x) \Rightarrow Mammal(x)$

   (b) $\forall x, y\ Offspring(x, y) \wedge Pig(y) \Rightarrow Pig(x)$

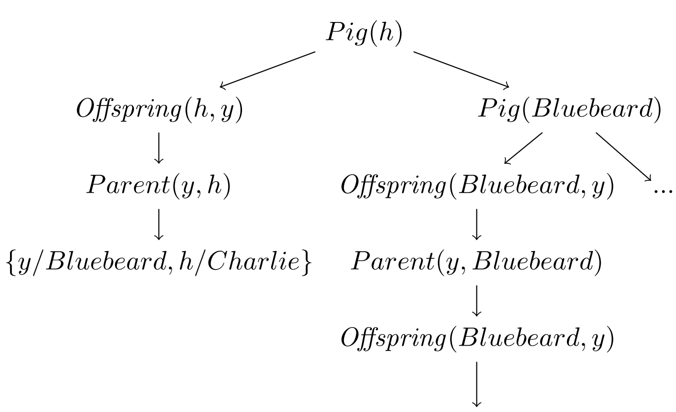
   (c) $Pig(Bluebeard)$

   (d) $Parent(Bluebeard, Charlie)$

   (e) $\forall x, y\ Offspring(x, y) \Rightarrow Parent(y, x)$
   $\forall x, y\ Parent(x, y) \Rightarrow Offspring(y, x)$

   (f) $\forall x\ Mammal(x) \Rightarrow \exists y\ Parent(y, x)$

6. Query $\exists h\ Pig(h)$



7. (a)

$$
\begin{aligned}
&((\exists x)[P(x)] \lor (\exists x)[Q(x)]) \Rightarrow (\exists y)[P(y) \lor Q(y)] \\
=&\neg((\exists x)[P(x)] \lor (\exists x)[Q(x)]) \lor (\exists y)[P(y) \lor Q(y)] \\
=&((\neg\exists x)[P(x)] \land (\neg\exists x)[Q(x)]) \lor (\exists y)[P(y) \lor Q(y)] \\
=&((\forall x)[\neg P(x)] \land (\forall x)[\neg Q(x)]) \lor (\exists y)[P(y) \lor Q(y)] \\
=&((\forall x)[\neg P(x)] \land (\forall x)[\neg Q(x)]) \lor (P(Y) \lor Q(Y)) \\
=&(\neg P(x_1) \land \neg Q(x_2)) \lor (P(Y) \lor Q(Y)) \\
=&(\neg P(x_1) \lor P(Y) \lor Q(Y)) \land (\neg Q(x_2) \lor P(Y) \lor Q(Y))
\end{aligned}
$$

(b)

$$
\begin{aligned}
&(\forall x)[P(x)] \Rightarrow (\exists z)[(\forall x)[Q(x,z)] \lor (\forall x)[R(x,y,z)]] \\
=&(\neg\forall x)[P(x)] \lor (\exists z)[(\forall x)[Q(x,z)] \lor (\forall x)[R(x,y,z)]] \\
=&(\exists x)[\neg P(x)] \lor (\exists z)[(\forall x)[Q(x,z)] \lor (\forall x)[R(x,y,z)]] \\
=&\neg P(X) \lor (\forall x)[Q(x,Z)] \lor (\forall x)[R(x,y,Z)] \\
=&\neg P(X) \lor Q(x_1,Z) \lor R(x_2,y,Z)
\end{aligned}
$$

(c)

$$
\begin{aligned}
&(\forall x)[P(x) \Rightarrow Q(x,y)] \Rightarrow ((\exists y)[P(y)] \land (\exists v)[Q(y,v)]) \\
=&(\neg\forall x)[\neg P(x) \lor Q(x,y)] \lor ((\exists y)[P(y)] \land (\exists v)[Q(y,v)]) \\
=&(\exists x)[P(x) \land \neg Q(x,y)] \lor ((\exists y)[P(y)] \land (\exists v)[Q(y,v)]) \\
=&(P(X) \land \neg Q(X,y)) \lor (P(Y) \land Q(y,V)) \\
=&(P(X) \lor (P(Y)) \land (P(X) \lor Q(y,V)) \land (\neg Q(X,y) \lor (P(Y)) \land (\neg Q(X,y) \lor Q(y,V))
\end{aligned}
$$

8. (a) Statements:
   A: $(\exists x)[Push(x) \land Blue(x)] \Rightarrow (\forall y)[\neg Push(y) \Rightarrow Green(y)]$
   B: $(\forall x)[(Blue(x) \land \neg Green(x)) \lor (\neg Blue(x) \land Green(x))]$
   C: $(\exists x)[\neg Push(x)] \Rightarrow (\forall y)[Push(y) \Rightarrow Blue(y)]$
   D: $Push(O_1)$
   E: $\neg Push(O_2)$

   (b) Convert to clause form:
   A: $\neg Push(x_1) \lor \neg Blue(x_1) \lor Push(y_1) \lor Green(y_1)$
   B: $(Blue(x_2) \lor Green(x_2)) \land (\neg Blue(x_2) \lor \neg Green(x_2))$
   C: $Push(x_3) \lor \neg Push(y_2) \lor Blue(y_2)$
   D: $Push(O_1)$
   E: $\neg Push(O_2)$

   (c) Prove $\exists x\, Green(x)$:
   F: $\neg Green(x_4)$ [negate request in CNF form]

   1: $Push(O_2) \lor \neg Push(O_1) \lor Blue(O_1)$ [C, $\{x_3/O_2, y_2/O_1\}$]
   2: $Blue(O_1)$ [1,D,E]
   3: $\neg Push(O_1) \lor \neg Blue(O_1) \lor Push(O_2) \lor Green(O_2)$ [A,$\{x_1/O_1, y_1/O_2\}$]
   4: $Green(O_2)$ [3,D,E,2]
   5: $\neg Green(O_2)$ [F,$\{x_4/O_2\}$]
   6: Conflict [4,5]
   So Object 02 is green.