ICS 271
Fall 2016
Student ID : 26642334
Student Name: Yu Guo
Instructor : Kalev Kask
Homework Assignment 6
Due Thursday, 11/17

1. (a) No two people have the same social security number.
   Not good. x and y should not be the same person and using $\Rightarrow$ with $\exists$ leads to a very weak statement.

$$\neg\exists x, y, n\ Person(x) \wedge Person(y) \wedge \neg(x = y) \wedge HasSS\#(x, n) \wedge HasSS\#(y, n)$$

   (b) John's social security number is the same as Mary's.
   Good. Do not need to modify.

$$\exists n\ HasSS\#(John, n) \wedge HasSS\#(Mary, n)$$

   (c) Everyone's social security number has nine digits.
   Not good. For each person, it should only have one SSN but not all.

$$\forall x\ Person(x) \Rightarrow [\exists n\ HasSS\#(x, n) \wedge Digits(n, 9)]$$

   (d) Rewrite as follows.

$$\neg\exists x, y\ Person(x) \wedge Person(y) \wedge [SS\#(x) = SS\#(y)]$$

$$SS\#(John) = SS\#(Mary)$$

$$\forall x\ Person(x) \Rightarrow Digits(SS\#(x), 9)$$

2. (a) Not.
   (b) $\{x/A, y/B, z/B\}$
   (c) $\{x/David\}$
   (d) $\{x/g(u), u/f(v)\}$
   (e) $\{x/B, y/B, z/B\}$

3. Define:
   $Alpine(x)$: x belong to Alpine Club.
   $Skier(x)$: x is a skier.
   $Climber(x)$: x is a mountain climber.
   $Like(x, w)$: x like weather w.

   Statement:

$$Alpine(Tony)$$

$$Alpine(Mike)$$

$$Alpine(John)$$

$$\forall x\ Alpine(x) \Rightarrow Skier(x) \vee Climber(x)$$

$$\neg\exists x\ Climber(x) \wedge Like(x, Rain)$$

$$\forall x\ Skier(x) \Rightarrow Like(x, Snow)$$

$$\forall x\ Like(John, x) \Rightarrow \neg Like(Mike, x)$$
$$\forall x\ \neg Like(John, x) \Rightarrow Like(Mike, x)$$
$$\neg Like(John, Rain)$$
$$\neg Like(John, Snow)$$

Question: $\textsc{AskVars}(KB, Alpine(x) \wedge Climber(x) \wedge \neg Skier(x))$

4. Only (a) is the result.

5. (a) $\forall x\ Horse(x) \Rightarrow Mammal(x)$
   $\forall x\ Cow(x) \Rightarrow Mammal(x)$
   $\forall x\ Sheep(x) \Rightarrow Mammal(x)$

   (b) $\forall x, y\ Offspring(x, y) \wedge Pig(y) \Rightarrow Pig(x)$

   (c) $Pig(Bluebeard)$
   (d) $Parent(Bluebeard, Charlie)$
   (e) $\forall x, y\ Offspring(x, y) \Rightarrow Parent(y, x)$
   $\forall x, y\ Parent(x, y) \Rightarrow Offspring(y, x)$
   (f) $\forall x\ Mammal(x) \Rightarrow \exists y\ Parent(y, x)$

6. Query $\exists h\ Pig(h)$

$$\begin{tikzcd}
& Pig(h) \arrow[dl] \arrow[dr] & & \\
Offspring(h, y) \arrow[d] & & Pig(Bluebeard) \arrow[d] \arrow[dr] & \\
Parent(y, h) \arrow[d] & & Offspring(Bluebeard, y) \arrow[d] & \ldots \\
\{y/Bluebeard, h/Charlie\} & & Parent(y, Bluebeard) \arrow[d] & \\
& & Offspring(Bluebeard, y) \arrow[d] & \\
& & \ldots &
\end{tikzcd}$$

7. (a)

$$\begin{aligned}
&((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \Rightarrow (\exists y)[P(y) \vee Q(y)] \\
=&\neg((\exists x)[P(x)] \vee (\exists x)[Q(x)]) \vee (\exists y)[P(y) \vee Q(y)] \\
=&((\neg\exists x)[P(x)] \wedge (\neg\exists x)[Q(x)]) \vee (\exists y)[P(y) \vee Q(y)] \\
=&((\forall x)[\neg P(x)] \wedge (\forall x)[\neg Q(x)]) \vee (\exists y)[P(y) \vee Q(y)] \\
=&((\forall x)[\neg P(x)] \wedge (\forall x)[\neg Q(x)]) \vee (P(Y) \vee Q(Y)) \\
=&(\neg P(x_1) \wedge \neg Q(x_2)) \vee (P(Y) \vee Q(Y)) \\
=&(\neg P(x_1) \vee P(Y) \vee Q(Y)) \wedge (\neg Q(x_2) \vee P(Y) \vee Q(Y))
\end{aligned}$$

   (b)

$$\begin{aligned}
&(\forall x)[P(x)] \Rightarrow (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\
=&(\neg\forall x)[P(x)] \vee (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\
=&(\exists x)[\neg P(x)] \vee (\exists z)[(\forall x)[Q(x, z)] \vee (\forall x)[R(x, y, z)]] \\
=&\neg P(X) \vee (\forall x)[Q(x, Z)] \vee (\forall x)[R(x, y, Z)] \\
=&\neg P(X) \vee Q(x_1, Z) \vee R(x_2, y, Z)
\end{aligned}$$

(c)

$$
\begin{aligned}
&(\forall x)[P(x) \Rightarrow Q(x,y)] \Rightarrow ((\exists y)[P(y)] \wedge (\exists v)[Q(y,v)]) \\
=&(\neg\forall x)[\neg P(x) \vee Q(x,y)] \vee ((\exists y)[P(y)] \wedge (\exists v)[Q(y,v)]) \\
=&(\exists x)[P(x) \wedge \neg Q(x,y)] \vee ((\exists y)[P(y)] \wedge (\exists v)[Q(y,v)]) \\
=&(P(X) \wedge \neg Q(X,y)) \vee (P(Y) \wedge Q(y,V)) \\
=&(P(X) \vee (P(Y)) \wedge (P(X) \vee Q(y,V)) \wedge (\neg Q(X,y) \vee (P(Y)) \wedge (\neg Q(X,y) \vee Q(y,V))
\end{aligned}
$$

8. (a) Statements.

$$(\exists x)[Push(x) \wedge Blue(x)] \Rightarrow (\forall y)[\neg Push(y) \Rightarrow Green(y)]$$

$$(\forall x)[(Blue(x) \wedge \neg Green(x)) \vee (\neg Blue(x) \wedge Green(x))]$$

$$(\exists x)[\neg Push(x)] \Rightarrow (\forall y)[Push(y) \Rightarrow Blue(y)]$$

$$Push(O_1)$$

$$\neg Push(O_2)$$

(b) Convert to clause form.

$$\neg Push(x_1) \vee \neg Blue(x_1) \vee Push(y_1) \vee Green(y_1)$$

$$(Blue(x_2) \vee Green(x_2)) \wedge (\neg Blue(x_2) \vee \neg Green(x_2))$$

$$Push(x_3) \vee \neg Push(y_2) \vee Blue(y_2)$$

$$Push(O_1)$$

$$\neg Push(O_2)$$

(c) Prove $\exists x\; Green(x)$