Synopsis of SIS 6116 Assignment II
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Analysing the Tragedy of the Commons through Gradient Descent

Topics: *Gradient Descent, Resource Allocation, Optimisation, Decision-making*

The Tragedy of the Commons is a game theory problem closely related to the Prisoner's Dilemma where individuals act in their self-interest, leading to the depletion of a shared resource even though this behaviour harms the group as a whole.

Problem: A number of farmers can graze their cows on a common pasture that can only support a finite total number of cows. Initially, each farmer follows an agreed-upon sustainable strategy e.g. grazing x cows each. As long as every farmer follows the rules, the pasture remains healthy and everyone benefits. However, each farmer has an incentive to increase the number of cows beyond the agreed limit to gain more profit. In the short term, if one farmer adds a few extra cows, he will benefit and the negative impact on the pasture will not be immediately noticeable. However, if all farmers act selfishly and exceed the agreed number of cows, the pasture becomes overgrazed leading to the depletion of the resource. Over time, the land is damaged and none of the farmers can graze their cows effectively resulting in a loss for everyone.

Agents: A number of farmers each having a number of cows
Shared Resource: Pasture i.e. common grounds for cows

Methods:

Cooperate	Farmers keep their cow numbers within limit to preserve the pasture.
Defect	One or more farmers exceed the limit to maximise personal gain.

Environment: There is a set initial number of cows in the pasture. The health of the pasture is initially 100%. The remaining capacity of the pasture before overgrazing occurs can be calculated. As the total number of cows on the pasture exceeds the pasture's capacity, the pasture degrades.

The simulation can be modelled as:

- N: Total number of farmers
- C_i: Number of cows grazed by a farmer (where $i = 1,2,\ldots,N$).
- K: Carrying capacity of the pasture
- $H(t)$: Health of the pasture at time t
- D: Degradation factor

- R: Recovery factor
- T: Time step for the simulation

The health of the pasture can be modelled as a function of total cow count relative to its carrying capacity:

$$H(t) = H0 - D \cdot \max(0, C\text{total} - K) + R \cdot \max(0, K - C\text{total})$$

- $H0$: the initial health of the pasture.
- $C\text{total} = \sum Ci$: the total number of cows grazed by all farmers.

The change in pasture health over time can be represented as:

$$H(t+T) = H(t) + R(K - C\text{total}) - D(C\text{total} - K)$$

- If $C\text{total} < K$, the pasture recovers according to the recovery factor R.
- If $C\text{total} > K$, the pasture degrades according to the degradation factor D.

To optimise each farmer's grazing strategy using Gradient Descent, the objective function combines profit and pasture health:

$$\text{Objective}(Ci) = P(Ci) - \lambda H(t)$$

- $P(Ci)$: Profit from grazing, where P is the profit per cow.
- $H(t)$: Current health of the pasture.
- λ: Weighting factor that balances profit and health of the pasture.

The gradient w.r.t each farmer's cow count can be calculated:

$$\partial Ci / \partial H = -D + R$$

This gradient indicates how changes in a farmer's grazing count affect overall pasture health.

Evaluation: Observing how gradient descent uses gradients to iteratively adjust cow counts for each farmer by monitoring loss function, plotting loss curves (to identify trends such as convergence towards a minimum loss) while experimenting with learning rates. Multiple runs of the simulation can be analysed with varying initial conditions.