Analysing the Tragedy of the Commons through Gradient Descent

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Abstract

This study simulates a multi-agent decision-making scenario based on the Tragedy of the Commons where multiple farmers share a limited resource — a pasture with a carrying capacity of a finite number of cows. Farmers can choose to adopt either cooperative or defecting strategies where cooperators limit their herd size while defectors attempt to graze more cows for greater short-term profit. Using gradient descent and adaptive learning rates, cooperating farmers optimize their grazing strategies to balance profit with pasture health while defectors are penalized for exceeding sustainable limits. The results reveal a gradual stabilization as cooperators adjust and defectors face penalties. Pasture health follows a pattern that shows the balance between resource degradation and recovery.

1. Problem Statement

1.1 The Tragedy of the Commons

> "That which is common to the greatest number gets the least amount of care. Men pay most attention to what is their own: they care less for what is common." – Aristotle

The Tragedy of the Commons is a game theory problem closely related to the Prisoner's Dilemma where individuals act in their self-interest, leading to the depletion of a shared resource even though this behaviour harms the group as a whole. This is a common-pool resource allocation problem. It highlights the concept of individuals neglecting the well-being of society in the pursuit of personal gain since individuals receive the greatest payoffs if they betray the group rather than cooperate.

1.2 Problem Formulation

A number of farmers can graze their cows on a common pasture that can only support a finite total number of cows. Initially, each farmer follows an agreed-upon sustainable strategy e.g. grazing a maximum of x number of cows each. As long as every farmer follows this rule, the pasture remains healthy and everyone benefits. However, each farmer has an incentive to increase the number of cows beyond the agreed limit to gain more profit. In the short term, if one farmer adds a few extra cows, he will benefit and the negative impact on the pasture will not be immediately noticeable. However, if all farmers act selfishly and exceed the agreed number of cows, the pasture becomes overgrazed leading to the depletion of the resource. Over time, the land is damaged and none of the farmers can graze their cows effectively resulting in a loss for everyone.

2. Literature Review

Recent development in the field

1. *Beyond the Tragedy of the Commons* by Xavier Basurto and Elinor Ostrom argued that one way in which we can go beyond Hardin's tragedy of the commons is by building a diagnostic theory of CPR management. It is necessary to avoid falling into "panacea" or "my-case-is-unique" analytical traps. They believe that a diagnostic theory needs to be able to help understand the complex interrelationship between social and biophysical factors at different levels of analysis.

2. *The tragedy of the digital commons* by Gian Maria Greco and Luciano Floridi shows how the Tragedy of the Commons can be applied to the Infosphere in order to uncover and model a new ethical dilemma, the Tragedy of the Digital Commons. It can also be applied to the Infosphere only if due attention is paid to the specific properties of the new environment, its differences with respect to the Biosphere and the nature of the agents that inhabit it.

3. In *The Tragedy of the Commons and Distributed AI Systems* by Roy M. Turner, the problem that the tragedy of the commons causes when agents in a distributed AI system share a resource is analyzed. This is done by designating one agent as planner to make resource allocation decisions for all the others and it serves as an arbiter.

4. In *A mathematical model for the TCP Tragedy of the Commons* by Luis López et al., they established a mathematical model for the TCP Tragedy of the Commons based on a simplifying assumption which consists in considering that the effect of the protocol is to render an ordered state to the network. In this state, an optimal collective strategy is obtained where the resources are shared fairly among all hosts participating in the game.

5. In *The tragedy of the AI commons* by Travis Lacroix and Aydin Mohseni, they use stochastic evolutionary game dynamics to model this social dilemma in the context of the ethical development of artificial intelligence. This enables the isolation of variables thus providing actionable suggestions for increased cooperation amongst numerous stakeholders in AI. The results show how stochastic effects can help make cooperation viable in such a scenario. They suggest that coordination for a common good should be attempted in smaller groups in which the cost for cooperation is low and the perceived risk of failure is high.

6. In *Modeling human-coupled common pool resource systems with techniques in evolutionary game theory and reinforcement learning* by Isaiah Farahbakhsh, an evolutionary common pool resource game is simulated on a social network with payoff functions that depend on the state of the resource. Model predictions under two types of learning: best response and imitation dynamics are compared and it is shown that best response dynamics lead to an increase in sustainability of the system and the persistence of cooperation while decreasing inequality and debt.

7. In *Resolving social dilemmas with minimal reward transfer* by Willis et al.,it is shown how social dilemmas are challenging in multi-agent cooperation because individuals are incentivised to behave in ways that undermine socially optimal outcomes. Consequently, self-interested agents often avoid collective behaviour. In response,

social dilemmas are formalised using the general self-interest level to quantify the disparity between individual and group rationality in such scenarios. This represents the maximum proportion of their individual rewards that agents can retain while ensuring that a social welfare optimum becomes a dominant strategy.

3. Implementation

3.1 Parameters

The parameters defined are

- N: Number of farmers.
- K: Carrying capacity of the pasture.
- H: Initial health of the pasture.
- D: Degradation factor (when pasture is overgrazed).
- R: Recovery factor (when pasture is undergrazed).
- P: Profit per cow.
- λ: Weight for health in the objective function.
- η: Learning rate for gradient descent.
- T: Time step for the simulation rounds.
- $rounds$: Total number of rounds for the simulation.
- $cows_i$: Initial cow count for each farmer i, drawn from a uniform distribution.

3.2 Strategies

Strategy	Description
Cooperate	Farmers keep their cow numbers within limit to preserve the pasture.
Defect	One or more farmers exceed the limit to maximise personal gain.

Cooperation and Defection:

- $cooperation\_limit = \frac{K}{N}$: The cooperation limit is the maximum number of cows allowed for cooperating farmers.
- $defection\_factor = 1.5$: Defecting farmers can graze 1.5× the cows allowed for cooperating farmers, so the defecting limit is $defection\_limit = defection\_factor \times cooperation\_limit$.

3.3 Pasture Health

The health of the pasture is calculated based on the total number of cows grazing on it. It changes over time depending on whether the total number of cows exceeds the carrying capacity, K.
If the total number of cows exceeds the carrying capacity K, the health decreases proportionally to the overgrazing factor (which is raised to the power of 1.2 to smooth out the degradation curve). However, if the number of cows is below the carrying capacity, the health of the pasture increases due to recovery, proportional to the difference between K and C and multiplied by the recovery factor R. The health of the pasture is capped between 50 and 100 to ensure that it never falls below a sustainable level of 50 or exceeds the maximum health limit.

$$H(t+1) = \begin{cases} H(t) + R \cdot (K - C_{\text{total}}) & \text{if } C_{\text{total}} < K \\ H(t) - D \cdot (C_{\text{total}} - K)^{1.2} & \text{if } C_{\text{total}} > K \\ H(t) & \text{if } C_{\text{total}} = K \end{cases}$$

3.4 Loss Function

The loss function per farmer is calculated based on their number of cows, the health of the pasture and whether they are cooperating or defecting. Profit is earned by the farmer based on the number of cows.
The health of the pasture is weighted by λ. As a sustainability reward, farmers receive a reward for maintaining pasture health above a threshold (50). As penalty for defection, if a farmer is defecting and has more cows than the cooperation limit, they incur a penalty.

$$Loss = P \cdot cows_i - \lambda \cdot H + \text{sustainability reward} - \text{penalty}$$

where:

$$\text{sustainability reward} = 0.1 \cdot (H - 50)$$

and

$$\text{penalty} = \begin{cases} 0 & \text{if not defecting} \\ (cows_i - C_{\text{limit}})^{1.5} & \text{if defecting and } cows_i > C_{\text{limit}} \end{cases}$$

3.5 Update the number of cows

The number of cows for each farmer has is adjusted based on their strategy (either cooperating or defecting) and the current state of the pasture. For farmers who cooperate, the number of cows is decreased based on the gradient of the loss function. It has the profit and health penalties. For defecting farmers, the number of cows is increased based on a growth factor until they reach a limit defined by the defection factor times the cooperation limit to ensure they do not exceed this threshold.

$$cows_i^{\text{new}} = \begin{cases} \max(0, cows_i^{\text{old}} - \text{learning rate} \cdot (P - \lambda \cdot g)) & \text{if cooperating} \\ \min(F_{\text{defect}} \cdot C_{\text{limit}}, cows_i^{\text{old}} + G_{\text{rate}} \cdot (F_{\text{defect}} \cdot C_{\text{limit}} - cows_i^{\text{old}})) & \text{if defecting} \end{cases}$$

where

$$g = \begin{cases} -D & \text{if } C_{\text{total}} > K \\ R & \text{otherwise} \end{cases}$$

3.6 Gradient Descent

Gradient descent optimization algorithm is used to help farmers adjust their strategies regarding the number of cows they maintain based on the changing health of the pasture and their profits.

Each farmer tries to maximize their profits while minimizing the negative impact of their grazing on pasture health. Gradient descent adjusts the number of cows based on gradients derived from the loss function. For cooperating farmers, the adjustment to the number of cows is given by the gradient which represents the direction and rate of change of the profit relative to the number of cows. The gradient is influenced by the profit earned from grazing (P) which increases with more cows and the health penalty ($-\lambda \cdot g$) where g is either the recovery factor (R) or degradation factor (D) based on the health of the pasture. There is a negative impact on the overall profit when the pasture is unhealthy. In each iteration of the simulation, the number of cows for cooperating farmers is updated using the gradient calculated:

$$cows_i^{\text{new}} = cows_i^{\text{old}} - \text{learning rate} \cdot \text{grad}$$

Thus farmers can adjust their cow count in the direction that would lead to an increase in their overall profit as determined by the gradient.

The learning rate controls the size of each step taken, balancing the speed of convergence with the risk of going too much above the optimal number of cows. A small learning rate results in smaller updates to the cow count leading to more gradual adjustments while a larger learning rate leads to quicker adjustments to the cow count and hence accelerates the convergence process. The learning rate is important to find a balance in exploration and exploitation. The chosen learning rate of 0.002 provides a balance between stability and responsiveness.

As this process is repeated over many simulation rounds, farmers can continuously refine their strategies based on the changing conditions of the pasture and their individual profits. Each round, the pasture's health is recalculated which then influences the gradients for the next update. The model is then expected to converge towards an equilibrium where the number of cows maintained by cooperating farmers is optimized based on the pasture health.

4. Simulation

The simulation runs and updates the pasture health and cow count for each farmer. For each round, the number of total cows is calculated, the pasture health $H(t+1)$ is updated and the loss function for each farmer is calculated. The number of cows is updated based on strategy - cooperating or defecting. The model follows an adaptive learning rate where the learning rate for cooperating farmers adjusts based on the health of the pasture for faster reaction to degrading conditions. Defectors are penalized more heavily if they overgraze which reduces their negative impact on the system. Then the objective loss is calculated for each farmer. The total number of cows and the health over time are recorded to visualize the effects of cooperation and defection strategies.

5. Evaluation

The simulation results show the evolution of pasture health $H(t)$ over time and the total profit earned by all farmers over time. By plotting these, it is to be expected that it will show how defection leads to short-term profit gains but long-term pasture degradation.

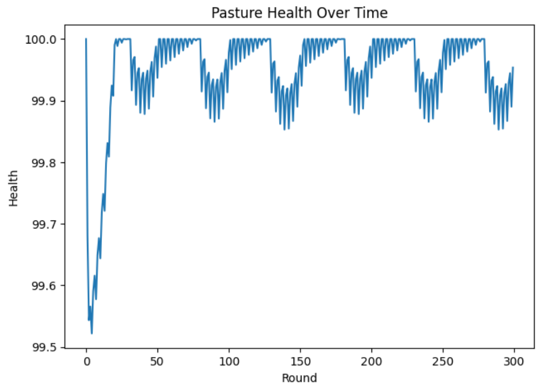

Figure 1: Pasture Health

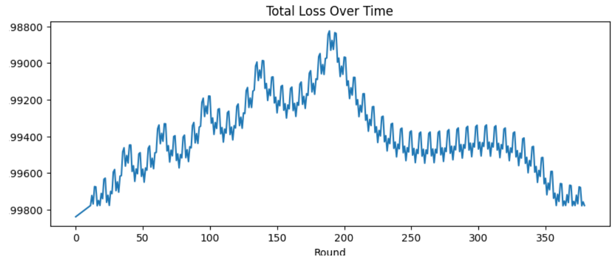

Figure 2: Objective Loss

Pasture Health Over Time

At first, there are fluctuations in pasture health: it drops below 100 but recovers quickly. This is because of a mismatch in the initial strategies of the farmers with defectors causing degradation and cooperators slowly adapting. After this, pasture health settles into a pattern, fluctuating between a little below 99.5 and 100. When the total number of cows goes beyond the carrying capacity, pasture health temporarily drops but the system's recovery mechanisms allow it to return to near-optimal health. While overgrazing still occurs periodically, the system is able to recover within a few rounds as there is a balance between degradation and recovery.

To facilitate recovery, the adaptive learning rate allows cooperative farmers to reduce the number of cows when the pasture health declines. The penalties for defecting farmers reduce the long-term overgrazing effects which stabilizes the system.

Total Loss Over Time

Initially, there is a steady rise in the total loss. This is because of overgrazing and mismanagement of resources as farmers are not yet adjusting their strategies to maintain balance. Defecting farmers have an aggressive grazing strategy leading to overgrazing and reduced pasture health. As cooperating farmers adjust their cow numbers through adaptive learning, the overall number of cows gradually decreases leading to a recovery in pasture health and a reduction in total loss. At later rounds, there is a downward trend and gradual stabilization in total loss which indicates that the system is learning and adjusting. Farmers have started to reduce their cows and adopt more sustainable strategies which results in decreasing losses over time. There is still a fluctuating pattern at later rounds because the system has not fully stabilized but it is trending towards a healthier balance.

The penalties introduced for defecting farmers seem to eventually take effect as overgrazing is reduced. Cooperating farmers also adjust more slowly through gradient descent which leads to a more sustainable outcome in the long run.

6. Conclusion

Through gradient descent, the model effectively models the Tragedy of the Commons scenario and demonstrates the impact of cooperative and defecting behaviors on shared resource sustainability. The results show that while defecting strategies lead to short-term profit gains, they cause a rise in total loss due to overgrazing which then degrades the pasture's health. Over time, as cooperating farmers adjust their strategies through gradient descent, total loss begins to stabilize which shows the importance of cooperation in managing the commons.

This model can be applied in various situations of the IT sector e.g. in cloud resource management where customers overuse shared resources like CPU, memory and storage, in data centers for shared data network bandwidth management, in IT teams to distribute resources across multiple projects, in large AI or machine learning models to share computational resources etc.

7. References

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