



Regression

Flervariabel & Polynom



Agenda

- Linjär regression med flera variabler
- Kodstruktur
- Kodexempel
- Polynomregression
- Kodexempel
- Statistik
 - p-value
 - Multivariable regression - variable selection
 - R_2
- Övningar



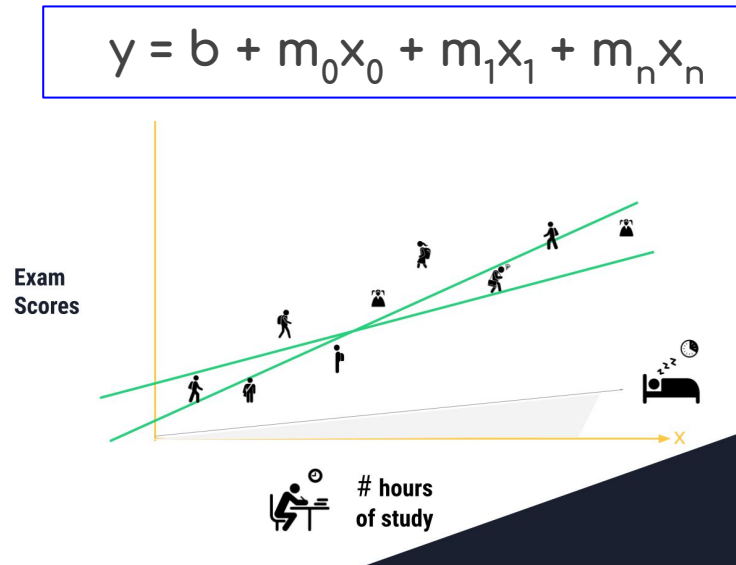
Linjär Regression med flera variabler

Flervariabel Linjär Regression

y är nu beroende av två, eller fler, variabler
(years of experience AND years of education)

Varje ny variabel är också oberoende.

Har sin egen riktningskoefficient.



<https://images.app.goo.gl/ZuAVS1F2JuiTXN1B8>

RESTRY...

Flervariabel LR

Simple
Linear
Regression

$$y = b_0 + b_1 * x_1$$

Multiple
Linear
Regression

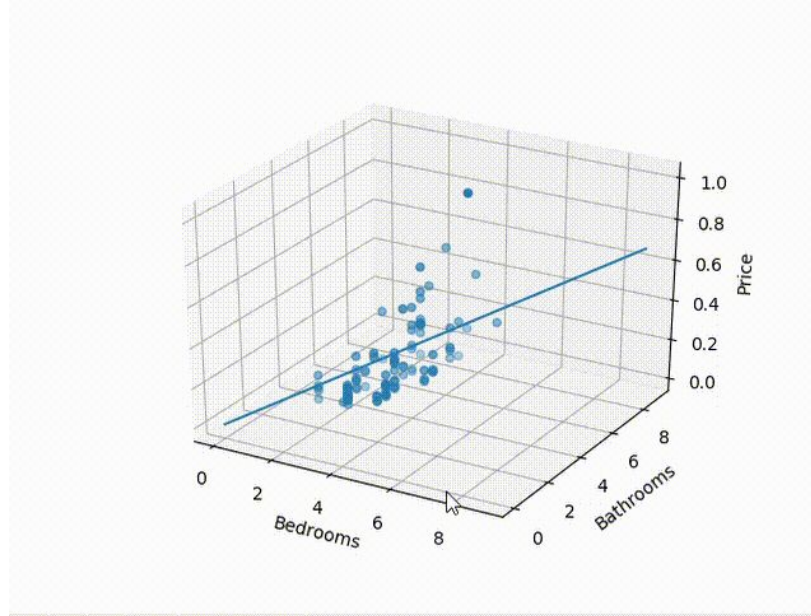
Dependent variable (DV) Independent variables (IVs)

The diagram shows the equation $y = b_0 + b_1 * x_1 + b_2 * x_2 + \dots + b_n * x_n$. Green arrows point from labels to parts of the equation: 'Dependent variable (DV)' points to 'y'; 'Independent variables (IVs)' points to 'x₁' and 'x₂'; 'Constant' points to 'b₀'; and 'Coefficients' points to 'b₁', 'b₂', and 'b_n'.

$$y = b_0 + b_1 * x_1 + b_2 * x_2 + \dots + b_n * x_n$$

Constant Coefficients

Flervariabler - två oberoende variabler - 3D



bedrooms
bathrooms

price

The dummy variable trap

- Ordinal to numerical
- Third can be inferred by the absence of the other two (included in b_0).
- sklearn model is advanced and will figure that out

$$y = b_0 + b_1 * x_1 + b_2 * x_2 + b_3 * x_3 + b_4 * D_1$$

	R&D Spend	Administration	Marketing Spend	State	Profit
1	165349.2	136897.8	471784.1	New York	192261.83
2	162597.7	151377.59	443898.53	California	191792.06
3	153441.51	101145.55	407934.54	Florida	191050.39
4	144372.41	118671.85	383199.62	New York	182901.99
5	142107.34	91391.77	366168.42	Florida	166187.94
6	131876.9	99814.71	362861.36	New York	156991.12
7	134615.46	147198.87	127716.82	California	156122.51
8	130298.13	145530.06	323876.68	Florida	155752.6

NY = D_1

CA = D_2

$D_1 = 1 - D_2$

b_0 "includes" D_2

RESTORY...

Kodstruktur

1. Import libraries
2. Import dataset
3. Encode categorical variables
4. Split data from "answer" column
5. Split train/tests
 - a. Import "splitter"
6. Training
 - a. Import model?
7. Testing

Print options

- `np.set_printoptions(precision=2)`



Kod



RESTORY...

Övningar

- Ladda ned Iris data
 - Omvandla nominell data till numeriska kategorier
 - Gör en flervariabelregression för att förutspå Sepal längden.
-
- [Ladda ned Pima indians diabetes data](#)
 - Gör en flervariabelregression.



Polynom



RESTOR\...

Polynom Linjär Regression

Simple
Linear
Regression

$$y = b_0 + b_1x_1$$

Multiple
Linear
Regression

$$y = b_0 + b_1x_1 + b_2x_2 + \dots + b_nx_n$$

Polynomial
Linear
Regression

$$y = b_0 + b_1x_1 + b_2x_1^2 + \dots + b_nx_1^n$$

Polynom LR

$$y = b + m_0x_1 + m_1x_1^2 + \dots + m_nx_1^n$$

y är beroende av x

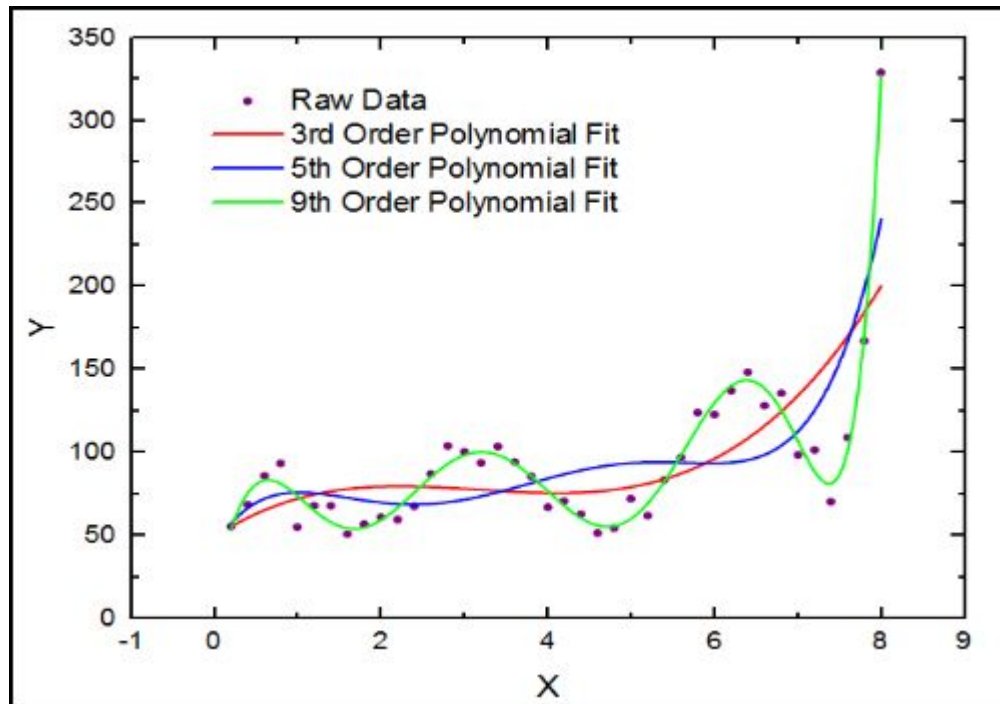
x är oberoende

förhållandet mellan dem som en
n-graders polynom av x
(relationship between them as a n-th
degree polynomial of x)

högre grad -> mer komplex modell

Kan modellera mer komplexa
förhållanden.

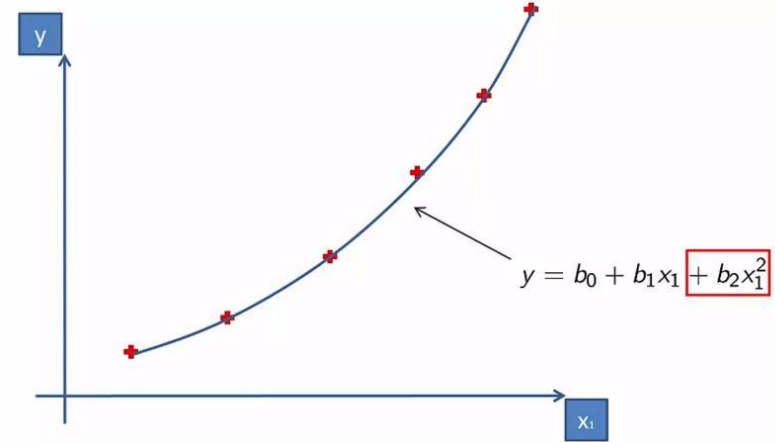
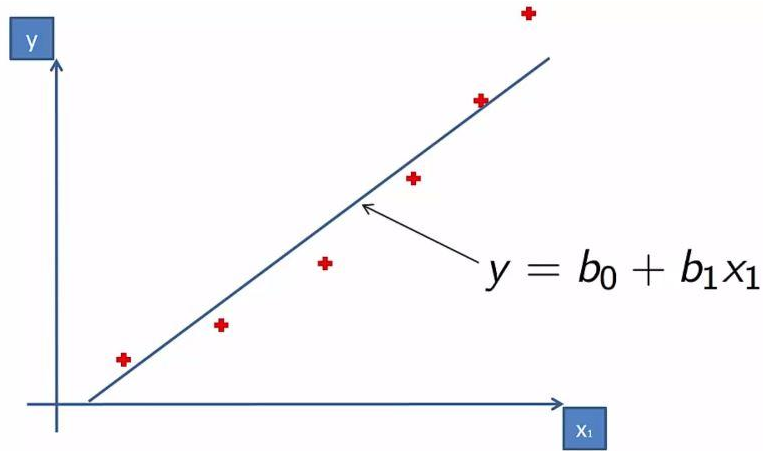
Linjär - coeffs



<https://images.app.goo.gl/qriDKw1nLNMr2fmf9>

RESTORY...

Polynom regression - när behövs det





Kod



RESTORY...

Övning

Titta på datan i följande Kaggle exemplet och försök göra en polynomregression utan att titta på koden.

[Pumpkin data](#)



Statistik-ish



RE STORY...

p-value

- Statistical significance
- Null hypothesis H_0 - assume true
- Is it correct? - experiment
 - 0.50
 - halveras ...
 - $\alpha = 0.05$
 - domain dependent

"A p-value is the probability that random chance generated the data, or something else that is equal or rarer"

p-value \neq probability

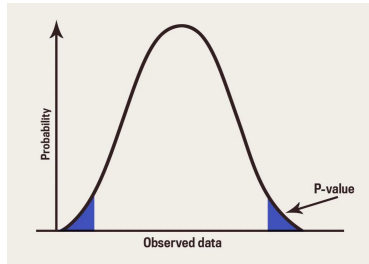
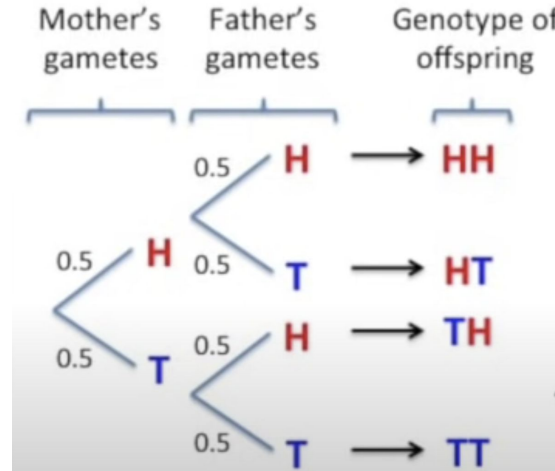


Bild ref.



$$\frac{HH}{HH, HT, TH, TT} = \frac{1}{4} = 0.25$$

+

$$\frac{TT}{HH, HT, TH, TT} = \frac{1}{4} = 0.25$$

StatQuest Youtube vid

RESTRY...

Multivariable regression - variable selection

Why select?

- GIGO
- Noise
- Explain

1. **All** Variables

- a. Domain knowledge - predictors
- b. Forced - by company
- c. Preparing for no. 2

2. **Backward** Elimination

- a. Set significance level (e.g. $sl = 0.05$)
- b. Fit full model
- c. Remove predictor with highest **p**-val if **$p > sl$** .
- d. Refit model
- e. Repeat c & d until no $p > sl$.

3. **Forward** Selection

- a. Select sl
- b. Fit all simple reg. models for y and X for all x separately.
- c. Select the one with the lowest p-val (\mathbf{x}_n).
- d. Fit \mathbf{x}_n with all other x variables ($X - \mathbf{x}_n$) separately for every model. (y and $[x_a, x_b]$)
- e. If the model with the lowest p-val has a $p < sl$, add it to the predictor, and repeat d.

Multivariable regression - variable selection

Stepwise regression: 2,3,4

4. ***Bidirectional Elimination***

- a. Select an entry and stay sl (can be different)
- b. Add on new variable using Forward Selection ($p < sl_enter$).
- c. Do all steps of Backward elimination. Keep only variables where $p < sl_stay$ is fulfilled.
- d. Repeat b and c until no new variables can enter or exit.

5. Score Comparison

- a. Set a fit score minimum
- b. Create a model for all possible variable combinations ($2^n - 1$)
- c. Select the one with the best score

Scikit-learn does this automatically!



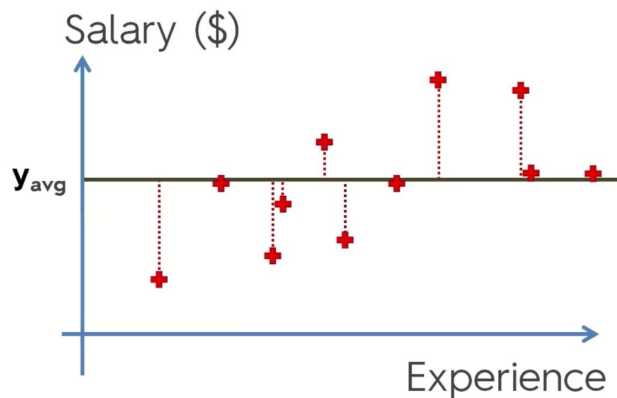
R^2

RESTORY...

R^2

How much better than average?

Simple Linear Regression:



$$SS_{\text{res}} = \text{SUM } (y_i - \hat{y}_i)^2$$

$$SS_{\text{tot}} = \text{SUM } (y_i - y_{\text{avg}})^2$$

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

R^2 och flera variabler

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

R^2 – Goodness of fit
(greater is better)

$$y = b_0 + b_1 * x_1$$

$$y = b_0 + b_1 * x_1 + b_2 * x_2$$

Problem:

$$+ b_3 * x_3$$

$$SS_{\text{res}} \rightarrow \text{Min}$$

R^2 will never decrease

Justerad R^2

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

$$\text{Adj } R^2 = 1 - (1 - R^2) \frac{n - 1}{n - p - 1}$$

p - number of regressors

n - sample size

Sammanfattning

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- Polynomregression
- Statistik
 - p-value
 - Multivariable regression - variable selection
 - R_2

Länkar

- [StatQuest](#)
- [Linear regression](#)