Caso general

$p \in (0, \infty)$

$$f_p(x) = \frac{1}{x^{\frac{1}{p}} \left(1 + |\log(|x|)|\right)^{2/p}}$$

$$\boxed{f_p(x) = \left(f_1(x)\right)^{\frac{1}{p}}}$$

$$f_p^p = f_1 \in L_1 \text{ ma } f_p \in L_p$$

$$f_p^q = (f_1)^{q/p} \in L_1 \iff \frac{q}{p} = 1$$