

$$\text{p.d.} \int_{(0, \infty)} f_1 d\lambda = 2$$

$$\int_{(0, \infty)} f_1 d\lambda = \underbrace{\int_{(0, 1)} f_1 d\lambda}_{1} + \underbrace{\int_{(1, \infty)} f_1 d\lambda}_{1}$$

$$\boxed{\text{P.D.} \int_{(0, 1)} \frac{d\lambda}{x(1+|\log(x)|)^2} = 1}$$

$$\int_{(0, 1)} \frac{d\lambda}{x(1+|\log(x)|)^2} \stackrel{\text{T.C.M.}}{=} \lim_n \int_{\frac{1}{n}}^1 \frac{d\lambda}{x(1+|\log(x)|)^2}$$

$$= \lim_n \int_{\frac{1}{n}}^1 \frac{dx}{x(1-\log(x))^2}$$

$$\downarrow \quad \begin{aligned} u &= 1 - \log(x) \\ du &= -\frac{dx}{x} \end{aligned}$$

$$= \lim_n \int_{1-\log(\frac{1}{n})}^1 \frac{-du}{u^2}$$

$$= \lim_n \int_1^{1+\log(n)} \frac{du}{u^2}$$

$$= 1$$