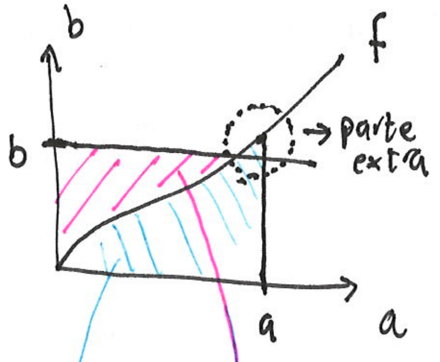
Lema (para Hölder) Dem 2.

Para $a, b > 0$ y $p, q > 1$ exp. conjugados:

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q} \quad \text{con igualdad} \iff a^p = b^q$$

Dem



$$f(t) = t^{p-1} = e^{(p-1)\log(t)}$$

f es estric. creciente

$f(0) = 0$

Obs: $f^{-1}(s) = s^{q-1}$

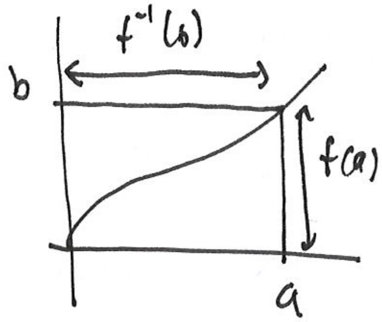
Razón: $f(f^{-1}(s)) = (s^{q-1})^{p-1} \overset{!}{=} s^{qp-p-q+1} = s$

$$\frac{1}{p} + \frac{1}{q} = 1 \Rightarrow p + q = pq$$

$$ab \leq \boxed{\int_0^a f(t)\,dt} + \boxed{\int_0^b f^{-1}(s)\,ds}$$

$$= \left[\frac{t^p}{p}\right]_0^a + \left[\frac{s^q}{q}\right]_0^b$$

Igualdad:



$\iff f^{-1}(b) = a, \; f(a) = b$

$\iff b^{q-1} = a, \quad a^{p-1} = b$

$\iff b^q = ab, \quad a^p = ab$

$\iff b^q = a^p.$