83-(ii) Sea $C \subseteq [0,1]$ el conjunto de cantor.

Probar $C+C=[0,2]$.

<u>Dem</u>: Recordar:

$$C=\left\{ x\in[0,1] \;\middle|\; x=\sum_{n=1}^{\infty}\frac{i_n}{3^n},\ i_n\in\{0,2\}\right\}$$

Definimos: $D=\left\{ \cancel{t}\in[0,1] \;\middle|\; \cancel{t}=\sum_{n=1}^{\infty}\frac{j_n}{3^n},\ j_n\in\{0,1\}\right\}$

Afi: $D+D=[0,1]$.

$\subseteq$: Si $t\in D$, $t\leq\sum_{n=1}^{\infty}\frac{1}{3^n}=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{3-1}=\frac{1}{2}$

$\therefore$ si $t\in D,\ s\in D \Rightarrow s+t\leq\frac{1}{2}+\frac{1}{2}=1$

$\supseteq$: Sea $y\in[0,1]$. En ternario:

$$y=\sum_{n=1}^{\infty}\frac{k_n}{3^n},\quad k_n\in\{0,1,2\}$$

definimos $j_n \not\exists\, i_n$ $\not+$ $i_n+j_n=k_n$ con:

$k_n=0$, $j_n=0=i_n$

$k_n=1 \Rightarrow j_n=1,\ i_n=0$

$k_n=2 \Rightarrow j_n=1=i_n$

Definimos $t=\sum_{n=1}^{\infty}\frac{j_n}{3^n}\in D$, $s=\sum_{n=1}^{\infty}\frac{i_n}{3^n}\in D$

y $t+s=y$.

P.D. $C+C=[0,2]$: $\subseteq$ claro.

$\supseteq$: $x\in[0,2] \Rightarrow \frac{x}{2}\in[0,1] \Rightarrow \frac{x}{2}=\underset{\in D}{t}+\underset{\in D}{s}$

$\Rightarrow x=2t+2s$

Notar $2t, 2s\in C$