P.D $q \neq 1$

$$\int_{(0,\infty)} f_1^q \, d\lambda = +\infty$$

Caso $q > 1$

P.D $\displaystyle\int_{(0,1)} f_1^q \, d\lambda = +\infty$

$$\boxed{s(1+\log(t)) \le t^s}$$

$x \in (0,1)$

$t = \frac{1}{x} \in (1,\infty)$

Lema $\Rightarrow$

$$s\left(1+\log\left(\tfrac{1}{x}\right)\right) \le \frac{1}{x^s}$$

$$\Rightarrow s(1-\log(x)) \le \frac{1}{x^s}$$

$$\Rightarrow s(\underbrace{1+|\log(x)|}) \le \frac{1}{x^s}$$

$$\Rightarrow (1+|\log(x)|)^2 \le \frac{1}{s^2 x^{2s}}$$

$$\Rightarrow \frac{1}{x(1+|\log(x)|)^2} \ge \frac{s^2 x^{2s}}{x}$$

$$\Rightarrow f_1(x) \ge \frac{s^2}{x^{1-2s}}$$

$$\Rightarrow f_1(x)^q \ge \frac{s^{2q}}{x^{q(1-2s)}}$$

s cercana a cero $\Rightarrow \boxed{q(1-2s)>1}$

$$\Rightarrow \int_{(0,1)} f_1^q \ge \int \frac{s^{2q}}{x^{\alpha}} \overset{\text{Lema}}{=} +\infty$$

$\alpha = q(1-2s) > 1$

Caso $0 < q < 1$

P.D $\displaystyle\int_{(1,\infty)} f_1^q \, d\lambda = +\infty$

$$s(1+\log(t)) \le t^s$$

$t = x \in (1,\infty)$

$$\Rightarrow 1+\log(x) \le \frac{x^s}{s}$$

$$\Rightarrow 1+|\log(x)| \le \frac{x^s}{s}$$

$$\Rightarrow \frac{1}{(1+|\log(x)|)^2} \ge \frac{s^2}{x^{2s}}$$

$$\Rightarrow \frac{1}{x(1+|\log(x)|)^2} \ge \frac{s^2}{x^{2s+1}}$$

$$\Rightarrow f_1(x) \ge \frac{s^2}{x^{2s+1}}$$

$x \in (1,\infty)$

$$\Rightarrow \int_{(1,\infty)} f_1^q \ge \int_{(1,\infty)} \frac{s^{2q}}{x^{q(2s+1)}}$$

$0 < q < 1$, tomar s cercana a 0 para que

$\alpha = q(2s+1) < 1$

Lema

$$\Rightarrow \int_{(1,\infty)} \frac{s^{2q}}{x^{\alpha}} \, d\lambda = +\infty$$

Fin caso $p=1$