Lema (Desigualdad de Young) Dem 3.

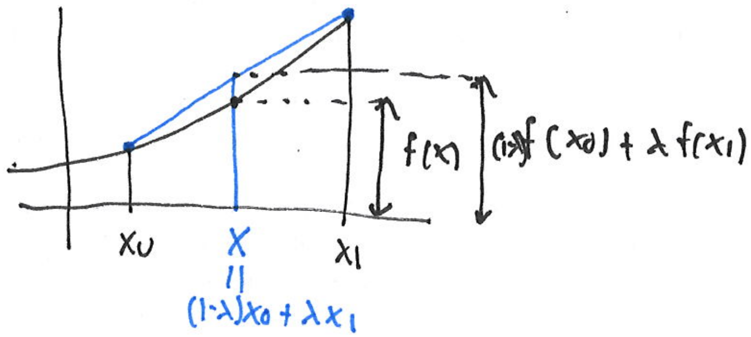
Para $a, b > 0$, $\frac{1}{p} + \frac{1}{q} = 1$ se cumple:

$$ab \le \frac{a^p}{p} + \frac{b^q}{q}$$

Dem Recordar: $f : \mathbb{R} \longrightarrow \mathbb{R}$ se llama estrictamente convexa si:
$\forall x_0, x_1 \in \mathbb{R}$, $0 < \lambda < 1$ se cumple

$$f((1-\lambda)x_0 + \lambda x_1) \le (1-\lambda) f(x_0) + \lambda f(x_1)$$

con igualdad $\iff x_1 = x_0$.



Ejemplo: $f(x) = e^x$.

Tomar $x_0 = \log(a^p)$, $x_1 = \log(b^q)$, $\lambda_0 = \frac{1}{p}$, $\lambda_1 = \frac{1}{q}$ $\therefore \lambda_0 + \lambda_1 = 1$

$$f(\lambda_0 x_0 + \lambda_1 x_1) \le \lambda_0 f(x_0) + \lambda_1 f(x_1)$$

$$e^{\frac{1}{p}\log(a^p) + \frac{1}{q}\log(b^q)} \qquad \frac{1}{p} e^{\log(a^p)} + \frac{1}{q} e^{\log(b^q)}$$

$$\Rightarrow \quad ab \le \frac{a^p}{p} + \frac{b^q}{q}$$

Nota: Se generaliza $a_1, \dots, a_n > 0$, $\frac{1}{p_1} + \dots + \frac{1}{p_n} = 1$

$$a_1 \cdots a_n \le \frac{a_1^{p_1}}{p_1} + \dots + \frac{a_n^{p_n}}{p_n}$$