Digital Image Processing

Image Formation

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April 27, 2021

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Cave paintings ≈40.000 years ago

Forced perspective

is a technique which employs optical illusion to make an object appear farther away, closer, larger or smaller than it actually is. It manipulates human visual perception through the use of scaled objects and the correlation between them and the vantage point of the spectator or camera. It has applications in photography, filmmaking and architecture.

Pietro della Francesca (1415-1492)

Trompe-l'œil (French for "deceive the eye")

is an art technique that uses realistic imagery to create the optical illusion that the depicted objects exist in three dimensions. Forced perspective is a comparable illusion in architecture.

Figure: *Escaping Criticism* by Pere Borrell del Caso, 1874

http://julianbeever.net/

http://julianbeever.net/

https://youtu.be/A4QcyW-qTUg?list=FLBJEFWqBtSxR67zLiyi6uAg

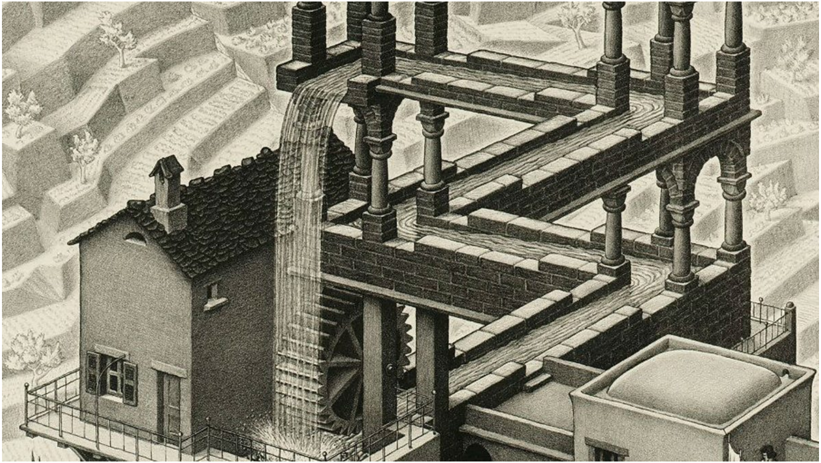
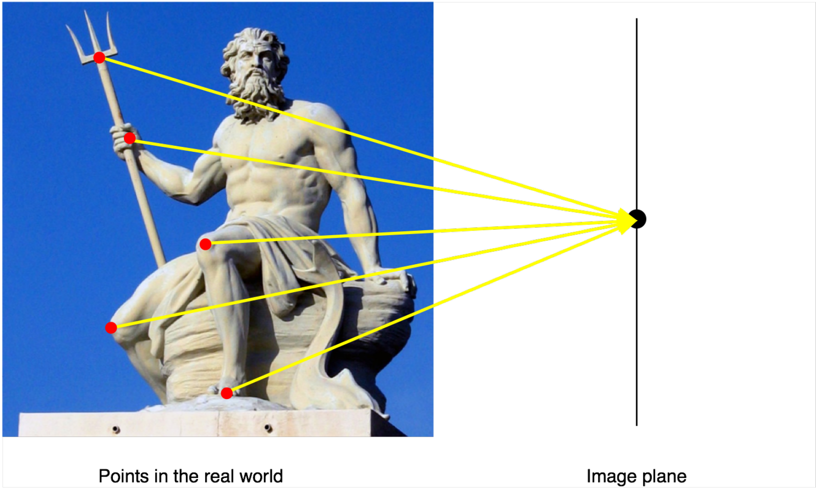
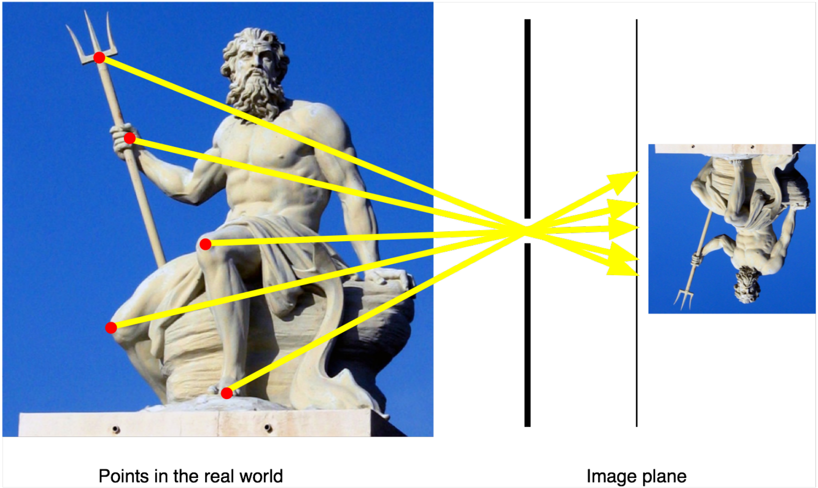


Figure: *Waterfal* M. C. Escher 1961

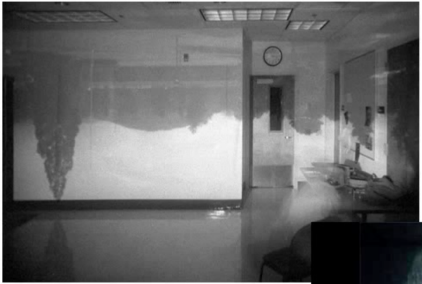
https://youtu.be/0v2xnl6LwJE

Specular Reflection. Lambertian Reflection. Retro-reflective reflection.

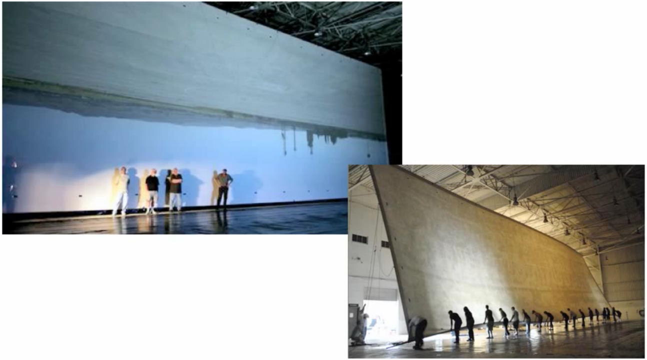

Points in the real world
Image plane


Points in the real world
Image plane

Pinhole images

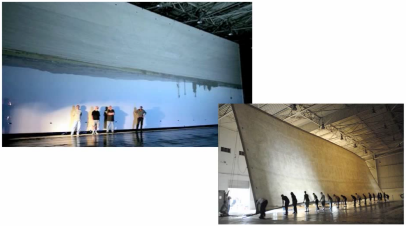


The world's largest pinhole camera



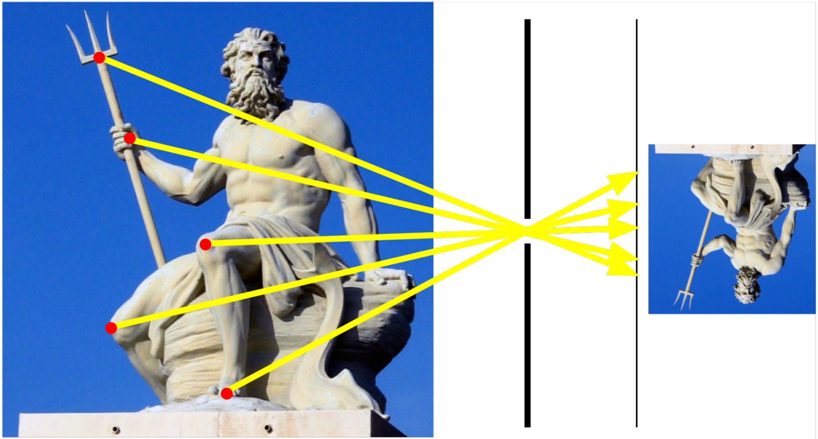
http://www.legacyphotoproject.com/

The world's largest pinhole camera



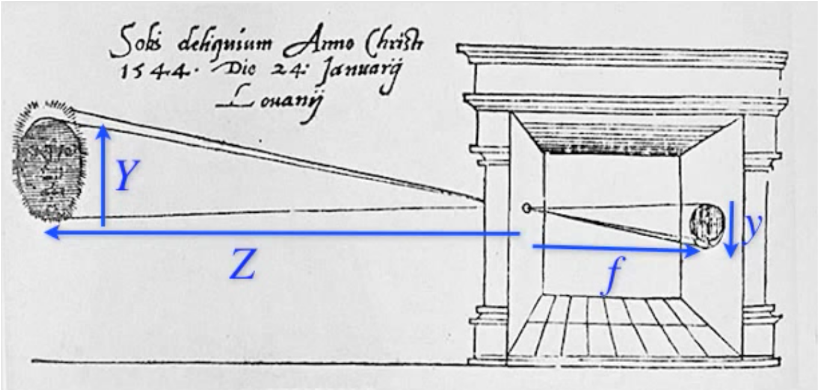
http://www.legacyphotoproject.com/

- No lenses.
- Long exposure.
- Cloth with light-sensitive chemicals.

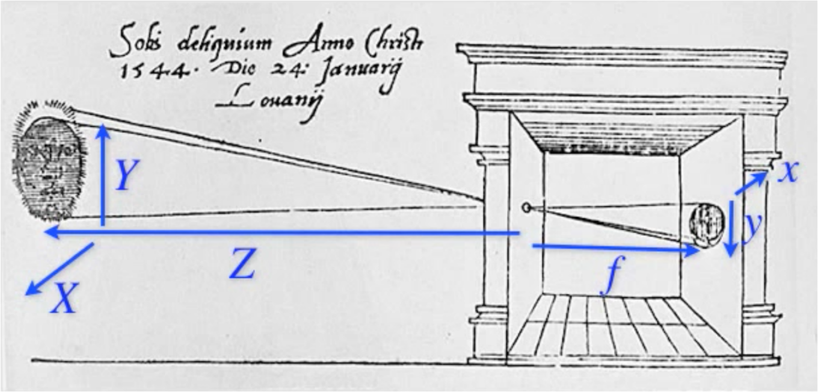


Points in the real world

Image plane

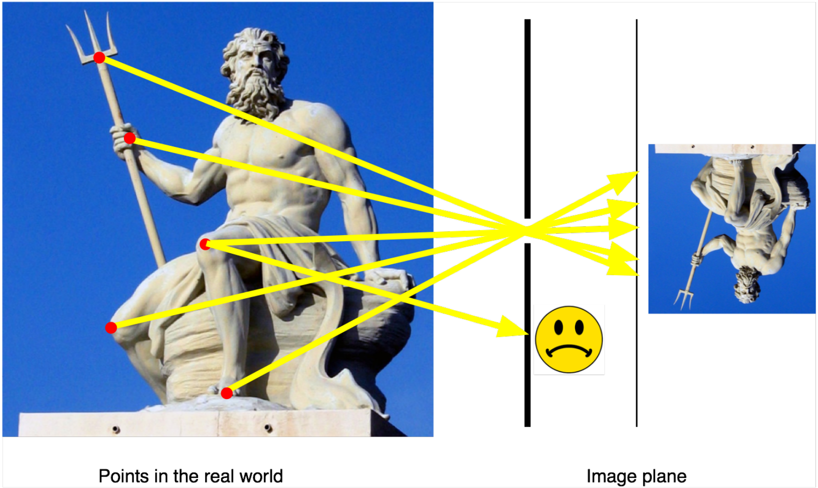


$$\frac{Y}{Z} = \frac{y}{f}$$

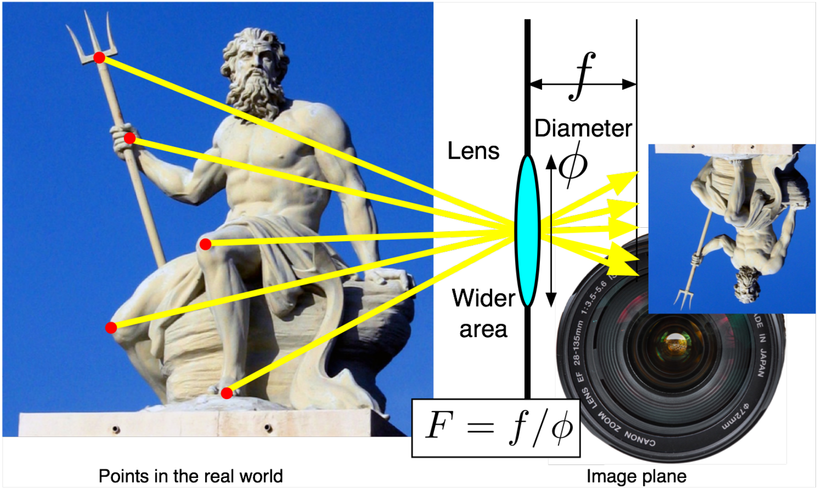


$$x = f\frac{X}{Z},\ y = f\frac{Y}{Z}$$

$$\mathbb{R}^3 \rightarrow \mathbb{R}^2$$


Points in the real world
Image plane

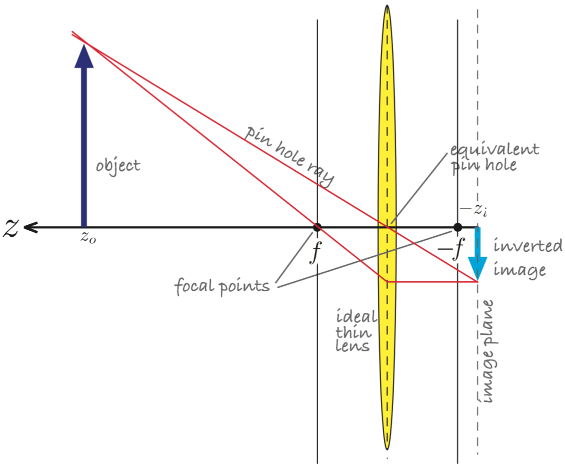
Use lens to capture more light



Use lens to capture more light

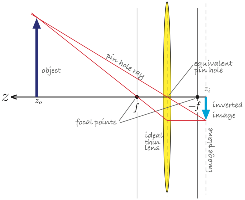
Figure: George Lawrence 1900

Thin lens model



- More light, but images are not always in focus.
-

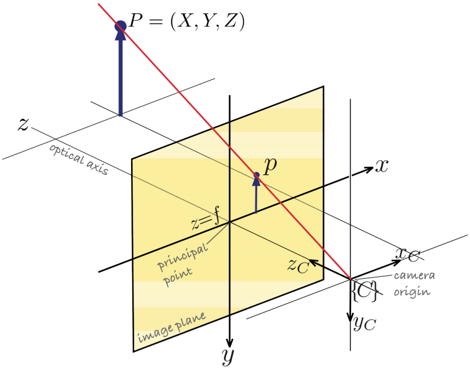
Thin lens model



- Thin-lens equation

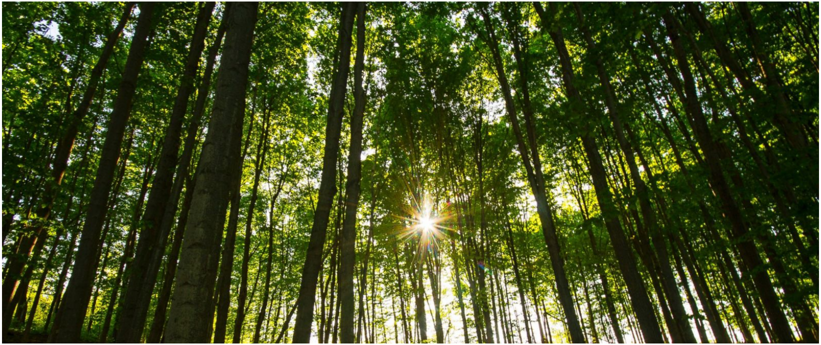
$$\frac{1}{z_0} + \frac{1}{z_i} = \frac{1}{f}$$

- If $z_o \mapsto \infty$ then $z_i \mapsto f$.



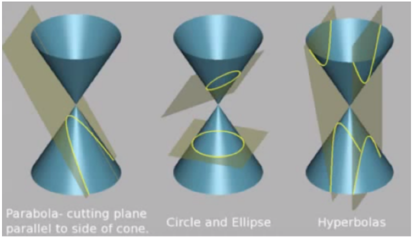
Perspective projection:

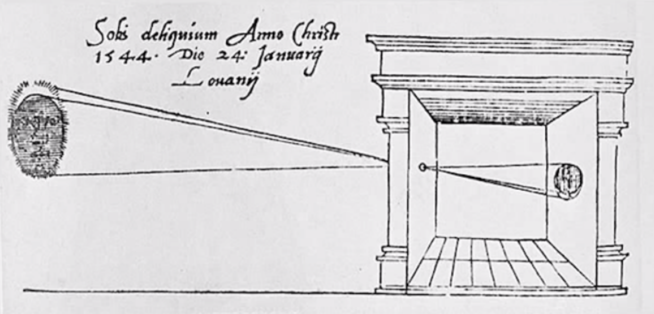
- $x = \dfrac{fX}{Z}$.
- $y = \dfrac{fY}{Z}$.
- 3D to 2D.
- $\mathbb{R}^3 \mapsto \mathbb{R}^2$.
- One dimension lost.



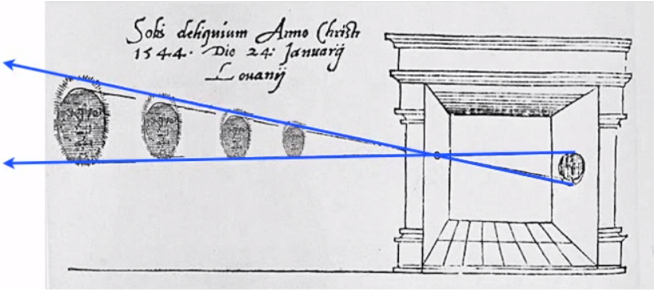
Perspective Projection

- Lines $\mapsto$ lines
  - Parallel lines no necessarily parallel.
  - Angles are not preserved.
- Conics $\mapsto$ conics.





Solis deliquium Anno Christi
1544. Die 24: Januarij
Lovanij


Solis deliquium Anno Christi
1544. Die 24. Januarij
Louanij

Homogeneous coordinates

- Cartesian $\mapsto$ Homogeneous:

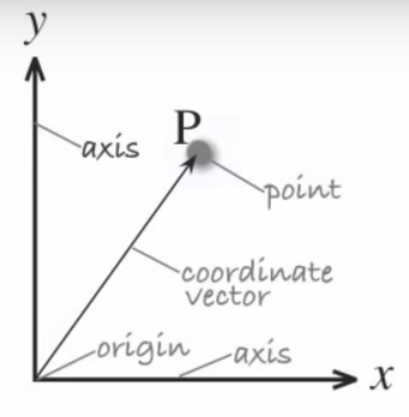
$$\mathbf{P} = (x, y) \quad \tilde{\mathbf{P}} = (x, y, 1)$$
$$\mathbf{P} \in \mathbb{R}^2 \qquad \tilde{\mathbf{P}} \in \mathbb{P}^2$$

- Homogeneous $\mapsto$ Cartesian:

$$\tilde{\mathbf{P}} = (\tilde{x}, \tilde{y}, \tilde{z}) \mapsto \mathbf{P} = (x, y)$$

where

$$x = \tilde{x}/\tilde{z}, \quad y = \tilde{y}/\tilde{z}.$$



Central Projection Model

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

I.e.,

$$\tilde{x} = fX, \quad \tilde{y} = fY, \quad \tilde{z} = Z.$$

As before:

$$x = \tilde{x}/\tilde{z}, \quad y = \tilde{y}/\tilde{z}.$$

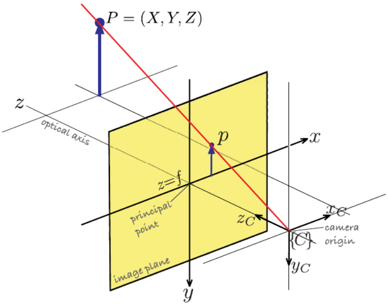
$$\mapsto x = \frac{fX}{Z}, \quad y = \frac{fY}{Z}$$

- Perspective transformation, with the divide by Z is **linear** in homogeneous coordinate form.

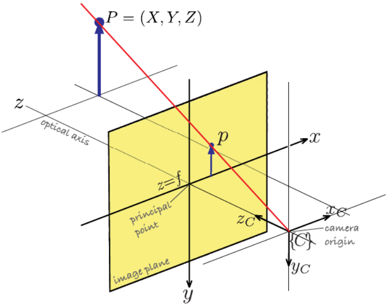
Central Projection Model

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

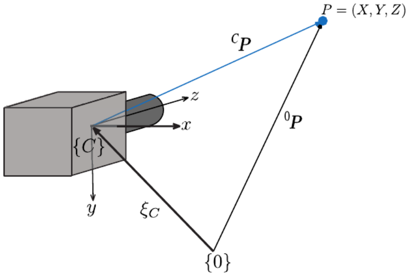
$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{3D \mapsto 2D} \underbrace{\begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{scaling/zooming}} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

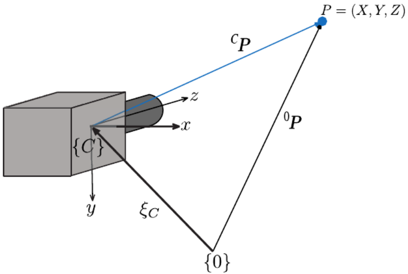


$$\begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

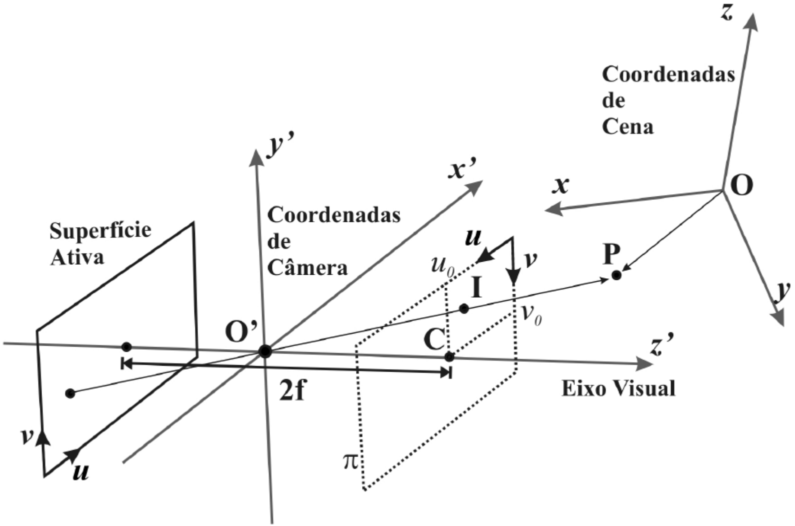


$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{R} & t \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix}^{-1} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{Intrinsic parameters}} \underbrace{\begin{pmatrix} \mathbf{R} & t \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix}^{-1}}_{\text{Extrinsic parameters}} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



z
Coordenadas
de
Cena
y'
x'
x
O
Coordenadas
de
Câmera
Superfície
Ativa
u
v
u_0
P
I
v_0
y
O'
C
z'
2f
Eixo Visual
v
π
u


Reebok CrossFit

Fish-eye lens

Imaging by reflection

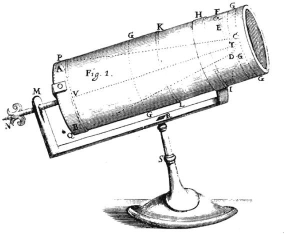
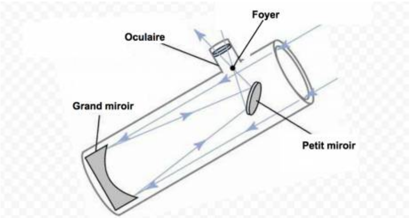
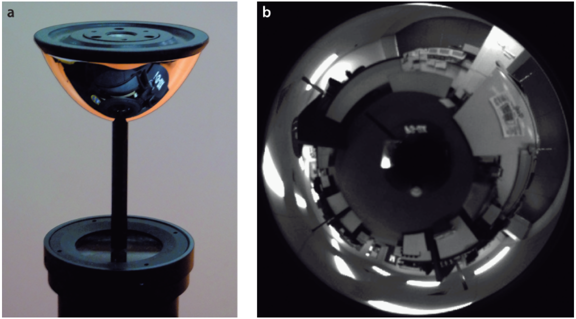


Figure: An accompt of a new catadioptrical telescope invented by Mr. Isaac Newton (≈1670)

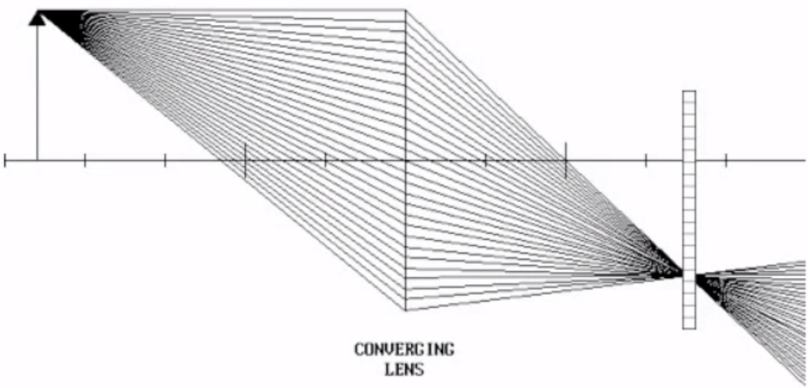


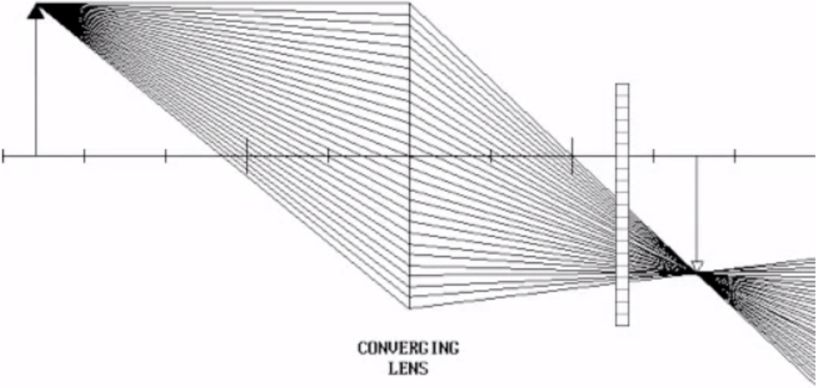
Spherical reflection

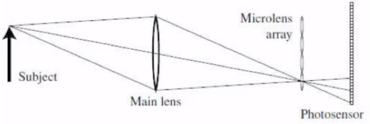


Light field

- A function that describes the amount of light travelling in **every direction** at **every point** in space.
- A light field sensor captures the color, intensity and vector **direction** of the rays of light.
  - a traditional camera simply adds up all the light rays and record a single amount of light
- Given the lightfield we can compute the image from any position.


CONVERGING
LENS


CONVERGING
LENS



- https://www.lytro.com/
- http://www.diyphotography.net/how-light-field-photography-works/
- https://vimeo.com/130983212

Thank you!

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