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Design of fractional order modeling based extended non-minimal state space MPC for temperature in an industrial electric heating furnace

Ridong Zhang*, Qin Zou, Zhixing Cao, Furong Gao

Department of Chemical and Biomolecular Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

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ABSTRACT

In this paper, an improved approach of extended non-minimal state space (ENMSS) fractional order model predictive control (FMPC) is presented and tested on the temperature model of an industrial heating furnace. In the fractional order model predictive control algorithm, fractional order single-input single-output (SISO) system is discretized via fractional order Grünwald-Letnikov (GL) definition. The ENMSS fractional order model that contains the state variable and the fractional order output tracking error is formulated by choosing appropriate state variables. Meanwhile, the fractional order integral is introduced into the cost function and the GL definition is used to obtain the discrete form of the continuous cost function. Then the control signals are derived by minimizing the fractional order cost function. Lastly, the temperature process control of a heating furnace is illustrated to reflect the performance of the proposed FMPC method. Simulation results show the effectiveness of the proposed FMPC method.

1. Introduction

In practice, the temperature control has a great influence on the production efficiency, quality and yields in many industries [1]. Therefore, the precision of temperature control has become one of the decisive factors for high product quality, operation security, etc. [2].

The widely used strategies for process control were divided as classical control and advanced control. Among the existing methods, proportional-integral-derivative (PID) control is the most common classical control that is widespread used and has developed drastically [3]. With the deep study on mathematical fractional order calculus, some scholars attempted to develop the fractional order PID that involves more tuning parameters than the integer order controller to improve the control performance [4–6]. Also, the genetic algorithm was introduced to improve the accuracy of fractional order PID controllers [7]. In [8], auto-tuning explicit PID based on fitting the optimal tuning parameters and minimizing the integral of the time weighted absolute error (ITAE) was presented, where the new optimal tuning rule was derived for fractional order plus time delay models. Fractional order PD$^{\mu}$ based on frequency and phase margin [9] and PI$^{\lambda}$D$^{\mu}$ controllers based on sensitivity constraint [10] were also applied prosperously. Synthesis of several methods for fractional order controllers has also yielded better performance in [11,12], which is still a hot topic of research for recent years. The study on the fractional order PID controllers has achieved the favorable dynamic performance and shown the feasibility and usefulness of the combination of the fractional order calculus and classical control.

As indicated in [13–18], model predictive control (MPC) can optimize process performance in terms of certain performance indexes and has been widely applied in industrial processes. As we all know, MPC depends on the precise process models to predict future outputs and the precision of the process models has a great influence on the control performance. In addition, more tuning degrees of freedom may also facilitate the controller to cope with complex requirements of process control. As formulated in [2,19], the non-minimal state space (NMSS) MPC offers the advantages of state space approach, such as ease of analysis, simple design structure, and so on. In view of this fact, a new extended non-minimal state space (ENMSS) model has been derived recently, where the output errors, measured outputs and inputs have been considered for MPC controller designs and the extended NMSS MPC maintains the good merits of the state space model and prevent the control performance deterioration when the controller is under plant/model mismatched cases [20]. The brief summary of

* Corresponding author at: Department of Chemical and Biomolecular Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong.

E-mail address: kerzhang@ust.hk (R. Zhang).

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the remaining representative methods can be seen as follows. Based on the ENMSS models, model predictive controllers were developed for industrial oxygen content process control [21] and coke fractionation tower [22]. A new model predictive fault-tolerant control algorithm based on ENMSS model was presented for batch processes with partial actuator failure [23]. For multivariable processes, the decoupling and output weighting were provided for ENMSS model based predictive controller to yield improved performance [24]. Also, the idea has been extended to optimize an improved PID controller and minmax control [25,26].

It is worth pointing out that many complex processes may not be described adequately by integer order models, and the extra fractional order integral and differential may lead to more flexible and accurate process models for representing the dynamic characteristics of practical processes [27]. As for the illustrated fractional order PID controllers, the performance has been ameliorated due to its extra tuning parameters. Considering the improvement of fractional order PID and the better performance of MPC [28], the idea of designing MPC strategies for processes described as fractional order models has been proposed by several researchers [29–31]. Since traditional MPC strategies using integer order models cannot be employed directly for fractional order processes, some significant efforts have been made to fulfill this demand [32–34]. For example, Rhouma et al. provided an MPC method for fractional systems using numerical approximation to predict the future outputs [35]. Romero et al. designed new fractional-order generalized predictive controllers (GPC) that contained fractional order integral operators in the cost functions to enhance the performance [36,37]. In [38], non-minimal state space predictive functional control (PFC) was successfully used for fractional order two rods thermal bench and revealed the effectiveness of the fractional order predictive control. However, although fractional order models can be more accurate to predict the future model output than integer order models, the fact is that industrial control will have to cope with the common model/plant mismatches, which poses a challenge for fractional order MPC.

In this paper, extended non-minimal state space fractional order model predictive control is presented and tested on an industrial heating furnace model. First, the fractional order SISO system is discretized via GL definition. Next, the extended non-minimal state space fractional order model that contains the state variable and the fractional order output tracking error is formulated by choosing appropriate state variables. Then the fractional order integral is introduced into the cost function and the GL definition is used to obtain the discrete form of the continuous cost function. Finally, by minimizing the fractional order cost function, the control law is derived. The contributions of this paper are that the designed controller can achieve favorable dynamic set-point tracking as well as disturbance rejection even under model/plant mismatched cases, and simulation results show that the proposed algorithm is feasible for fractional order systems and provide theoretical basis for practical industrial applications.

The main aim of our study is to develop an approach to control the fractional order system and further improve the performance of the control system compared with the integer order MPC. Several scholars indicate that the fractional order models can fit the actual data more precisely and are flexible than integer order models, and the outstanding merit of the fractional order model has laid a good foundation for MPC. On the other hand, the ENMSS model has provided with some virtues, i.e., using the actual input-output process variables, the additional freedom is adopted to adjust the process response to obtain effective performance. Combined with these methods, the fractional order control system can benefit from the virtues of ENMSSMPC algorithm to yield improved performance.

This paper is organized as follows. In Section 2, the preliminary fractional order calculus is formulated and the problem is discussed. In Section 3, the extended non-minimal state space fractional order model is established and the predictive controller based on the cost function with non-integer order integral is presented. In Section 4, the simulation effectiveness of the proposed method is demonstrated on a fractional order temperature process. In Section 5, conclusion of this method is drawn.

2. Preliminary mathematical formulations

The theory of the fractional order calculus is the arbitrary order calculus theory, where the order can be integer or fractional, or even complex. In this section, Grünwald-Letnikov(GL) definition and Riemann-Liouville(RL) definition of fractional order calculus are formulated, and the process model of the main controlled object will be shown below.

2.1. Fractional order calculus

A lot of definitions of fractional order calculus have been derived by several scholars, and two of the commonly used fractional order definitions, i.e., Grünwald-Letnikov definition and Riemann-Liouville definition that coincide with each other under certain conditions [39], are employed in this paper.

For continuous function $f(t)$ in the integral interval $[t_0, t]$, the GL definition is defined as:

$$Dt\beta f(t) = \lim_{h\to 0}\frac{1}{h^\beta}\sum_{j=0}^{[(t-t_0)/h]}(-1)^j\binom{\beta}{j}f(t-jh) \tag{1}$$

where, t_0 is the lower limit of integral, h is the calculation step, $[\cdot]$ rounds down to the nearest integer, β is the order of fractional derivative, $\omega_j^{(\beta)} = (-1)^j\binom{\beta}{j}$ is the binomial coefficients and the numerical values can be recursively approximated as follows.

$$\omega_0^{(\beta)} = 1, \omega_j^{(\beta)} = (1 - \frac{\beta+1}{j})\omega_{j-1}^{(\beta)}, j = 1, 2, \cdots. \tag{2}$$

For simplicity, we denote $D^\beta \equiv Dt\beta$ at the initial time $t_0 = 0$ in this paper. The RL definition for β order derivative of function $f(t)$ is displayed as:

$$D_\beta f(t) = \frac{1}{\Gamma(n-\beta)}\frac{d^n}{dt^n}\int_0^t\frac{f(\xi)}{(t-\xi)^{\beta-1}}d\xi \tag{3}$$

where, $n-1<\beta<n$, $n \in N$, and the Euler-gamma function $\Gamma(x)$ is:

$$\Gamma(x) = \int_0^\infty e^{-\xi}t^{x-1}d\xi \tag{4}$$

By using the Laplace transform of RL definition under the zero initial conditions, the Laplace transform of the fractional order derivative is:

$$L[D^\beta f(t)] = s^\beta L[f(t)] = s^\beta F(s) \tag{5}$$

Due to the long memory characteristic of fractional order operators, the coefficients $\omega_j^{(\beta)}$ can slowly converge. But for the short-memory principle [36], only the recent past values of $f(t)$ is considered. Here h is replaced by the sampling time T_S and the initial time $t_0 = 0$ is considered, then Eq. (1) can be approximated as follows:

$$D_\beta f(t) \approx \frac{1}{{T_S}^\beta}\sum_{j=0}^{L_S}(-1)^j\binom{\beta}{j}f(t-jT_S) \tag{6}$$

where, L_S is the approximated memory length. The accurate evaluation will be obtained by increasing L_S, but the approximation will be computationally heavier.

2.2. Fractional order model

As we know, fractional order models can fit the actual data more precisely and flexibly than integer order models and the outstanding merit of the fractional order model has laid a good foundation for MPC based on these kinds of process models. For simplicity, the single-input single output linear fractional order system with dead time is considered in the following fractional order differential equation model:

$$TD^{\alpha}y(t)+y(t)=Ku(t-\tau) \tag{7}$$

where, α is the non-integer order of the fractional model, τ is the time delay and T, K are the time constant and proportionality coefficient of the process model respectively, and y, u are the output and input of the fractional order system respectively.

From Eq. (5), the Laplace transform of Eq. (7) is obtained as follows:

$$G(s)=\frac{Ke^{-\tau s}}{Ts^{\alpha}+1} \tag{8}$$

As traditional MPC methods are based on integer order systems, they cannot be directly employed for the aforementioned process described by fractional order models, which shows that a new method for fractional order system is needed.

3. Extended non-minimal state space fractional order model predictive control

3.1. Extended non-minimal state space fractional order model

Since the control methods for fractional order models are much more difficult than those for integer order models, the numerical solution via approximation methods has been used to evaluate the fractional order models. In this paper, the GL definition is used to establish the discrete model of Eq. (8). With the sampling time T_S and the memory length L_S, the SISO fractional order model in Eq. (8) can be represented as:

$$y(k)+\mu\sum_{l=1}^{L_S}\omega_l^{(\alpha)}y(k-l)=Hu(k-d-1) \tag{9}$$

where,

$$\mu=\frac{T}{T_S^{\alpha}}\left(1+\frac{T}{T_S^{\alpha}}\right)^{-1},H=K\left(1+\frac{T}{T_S^{\alpha}}\right)^{-1}$$

$$\omega_0^{(\alpha)}=1,\ \omega_l^{(\alpha)}=\left(1-\frac{\alpha+1}{l}\right)\omega_{l-1}^{(\alpha)},\ l=1,2,\cdots L_S$$

The difference operator $\Delta=1-z^{-1}$ is added to Eq. (9), and then we have

$$\Delta y(k)=-F_1\Delta y(k-1)-F_2\Delta y(k-2)-\cdots-F_{L_S}\Delta y(k-L_S)+H\Delta u(k-1-d) \tag{10}$$

where,

$$F_l=\mu\omega_l^{(\alpha)}$$

Here $F_l(l=1,2,\cdots,L_S)$, H are the coefficients of the approximated model, the time delay $d=\tau/T_S$, $y(k)$ is the output of the fractional order discrete model at time instant k and $u(k-d-1)$ is the control input variable at time instant $k-d-1$.

The state space variable is denoted as

$$\Delta x_m(k)=[\Delta y(k),\Delta y(k-1),\cdots,\Delta y(k-L_S+1),\Delta u(k-1),\cdots,\Delta u(k-d)]^{\mathrm{T}} \tag{11}$$

Combining Eq. (10) with Eq. (11), the state space model is obtained as:

$$\begin{aligned}\Delta x_m(k+1)&=A_m\Delta x_m(k)+B_m\Delta u(k)\\ \Delta y(k+1)&=C_m\Delta x_m(k+1)\end{aligned} \tag{12}$$

with

$$A_m=\begin{bmatrix}-F_1 & -F_2 & \cdots & -F_{L_S-1} & -F_{L_S} & 0 & \cdots & 0 & H\\ 1 & 0 & \cdots & 0 & \cdots & 0 & \cdots & & 0\\ 0 & 1 & \ddots & & \ddots & & \ddots & & \\ \vdots & \ddots & \ddots & \ddots & & & & \ddots & \vdots\\ 0 & \cdots & 0 & 1 & 0 & & \ddots & & \\ 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & & 0\\ \vdots & \ddots & \ddots & & & 1 & \ddots & & \vdots\\ & & & & \ddots & \vdots & \ddots & & \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & \cdots & 1 & 0\end{bmatrix}$$

$$B_m=\begin{bmatrix}0 & \cdots & 0 & 1 & 0 & \cdots & 0\end{bmatrix}^T$$

$$C_m=\begin{bmatrix}1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\end{bmatrix}$$

where, the dimension of $\Delta x_m(k)$ is $(L_S+d)\times 1$.

Then the extended non-minimal state space fractional order model that contains the state variable and the fractional order output tracking error is formulated as:

$$z(k+1)=Az(k)+B\Delta u(k)+C\Delta r(k+1) \tag{13}$$

where,

$$z(k+1)=\begin{bmatrix}e(k+1)\\ \Delta x_m(k+1)\end{bmatrix},\ z(k)=\begin{bmatrix}e(k)\\ \Delta x_m(k)\end{bmatrix}$$

$$e(k)=y(k)-r(k)$$

$$e(k+1)=e(k)+C_mA_m\Delta x_m(k)+C_mB_m\Delta u(k)-\Delta r(k+1)$$

$$A=\begin{bmatrix}1 & C_mA_m\\ \mathbf{0} & A_m\end{bmatrix},\ B=\begin{bmatrix}C_mB_m\\ B_m\end{bmatrix},\ C=\begin{bmatrix}-1\\ \mathbf{0}\end{bmatrix}$$

and $r(k)$ is the tracking set-point trajectory at time instant k, $e(k)$ is the fractional order output tracking error, $\mathbf{0}$ is zero matrix of dimension $(L_S+d)\times 1$, and the dimensions of the matrices A, B, C are $(L_S+d+1)\times(L_S+d+1)$, $(L_S+d+1)\times 1$ and $(L_S+d+1)\times 1$ respectively.

3.2. Fractional order model predictive control based on extended non-sminimal state space model

Based on Eq. (13), the matrix form of the predicted output at time instant $k+i$ is derived as:

$$Z=Gz(k)+S\Delta U+\Psi\Delta R \tag{14}$$

where,

$$Z=\begin{bmatrix}z(k+1)\\ z(k+2)\\ \vdots\\ z(k+P)\end{bmatrix},\ S=\begin{bmatrix}B & & & O\\ AB & B & & \\ \vdots & \vdots & \ddots & \\ A^{P-1}B & A^{P-2}B & \cdots & A^{P-M}B\end{bmatrix},$$

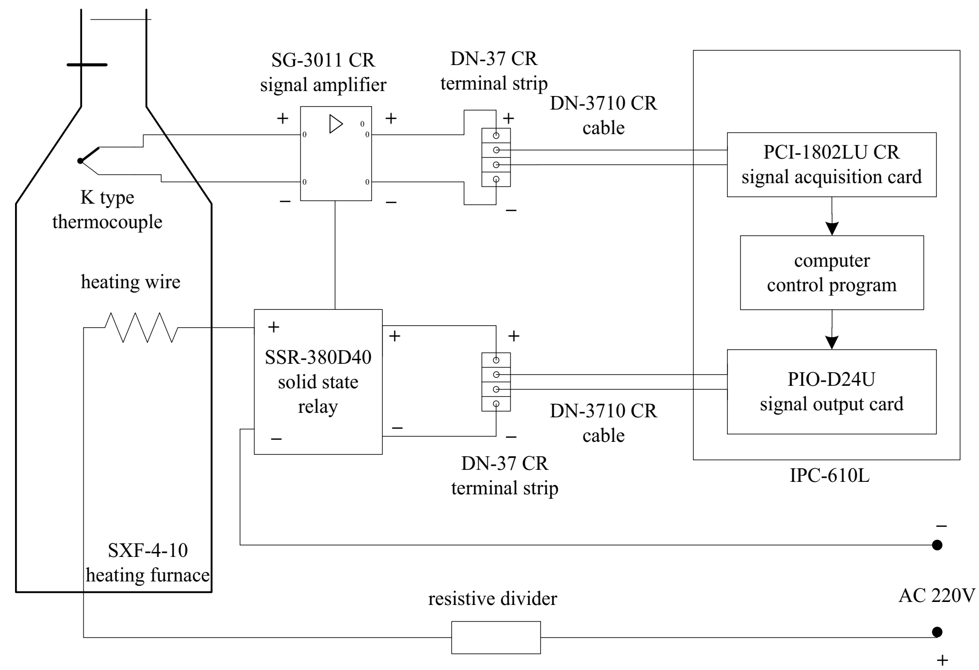

Fig. 1. The flow chart of SXF-4-10 heating furnace.

Fig. 2. The practical temperature data and open-loop step responses of models.

$$G = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^P \end{bmatrix}, \quad \Psi = \begin{bmatrix} C & & & O \\ AC & C & & \\ \vdots & \vdots & \ddots & \\ A^{P-1}C & A^{P-2}C & \cdots & C \end{bmatrix}$$

$$\Delta U = \begin{bmatrix} \Delta u(k) & \Delta u(k+1) & \cdots & \Delta u(k+M-1) \end{bmatrix}^T$$

$$\Delta R = \begin{bmatrix} \Delta r(k+1) & \Delta r(k+2) & \cdots & \Delta r(k+P) \end{bmatrix}^T$$

$$r(k+i) = \lambda^i y(k) + (1-\lambda^i)c(k)$$

and $c(k)$ is the set-point at the current time k, λ is the smoothing factor of the reference trajectory, P, M are the prediction horizon and control horizon respectively, and O is a zero diagonal matrix.

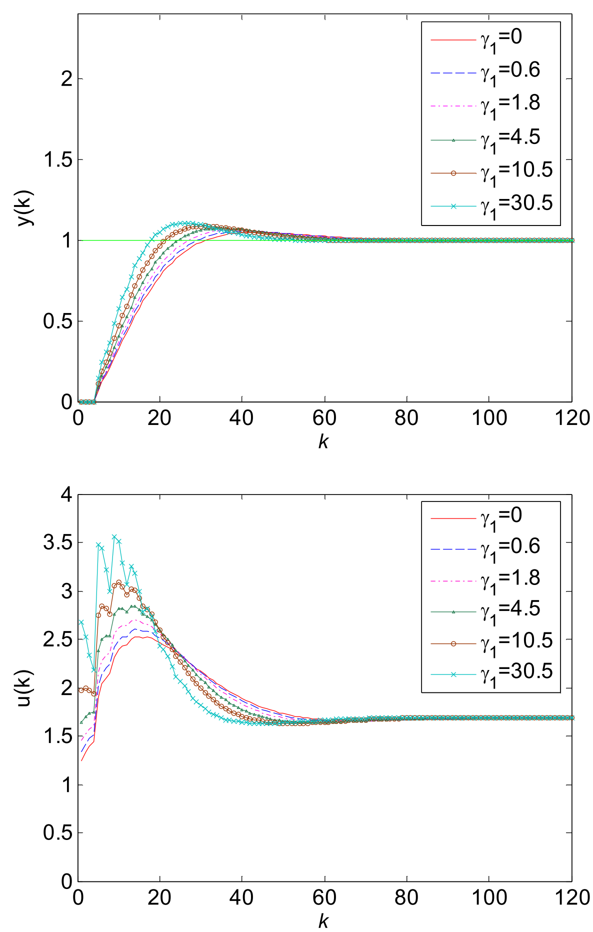

Fig. 3. Closed-loop responses of the proposed FMPC for variation in γ_1 ($\gamma_2 = 0$).

In general, the cost function is described by the single integral form of the quadratic function of the errors. In this paper, the integral will no longer be limited to the single integral, and the

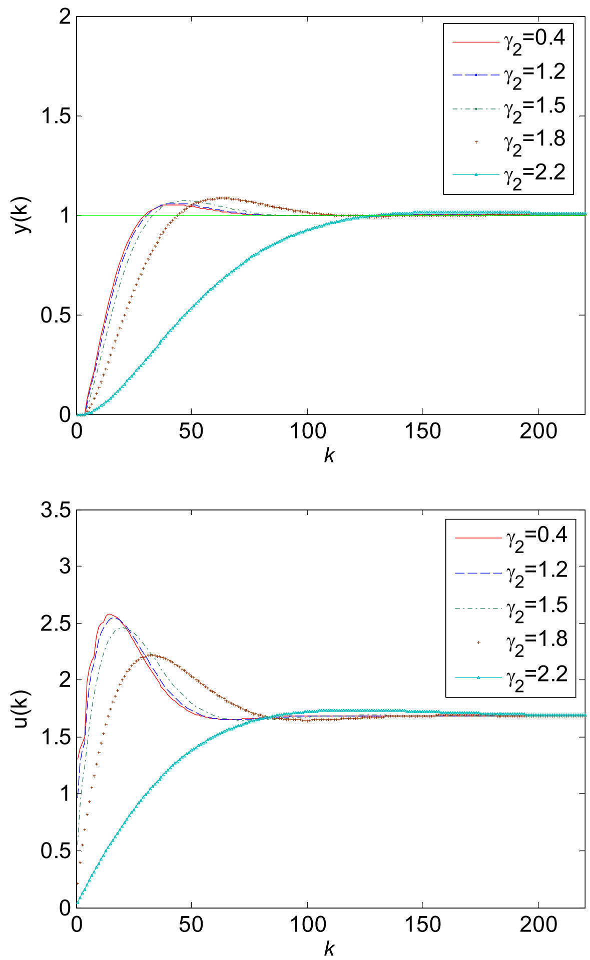

Fig. 4. Closed-loop responses of the proposed FMPC for variation in γ_2 ($\gamma_1 = 0$).

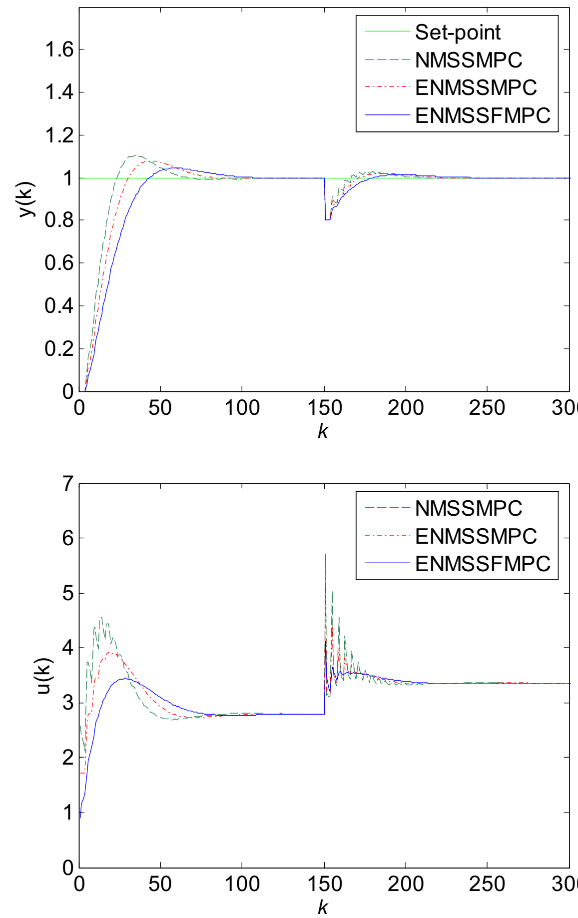

Fig. 5. Closed-loop responses under case 1.

order of the integral can be chosen as non-integer. For simplicity, we denote $I \equiv D^{-\gamma}$, where I is the fractional order integral notation and $D^{-\gamma}$ is the reverse derivative notation. Since the formulation $D^{-1}f(t) = \int_a^b f(t)dt$ holds, where $[a, b]$ is the continuous integral interval of $f(t)$, the cost function can be extended to fractional order as follows:

$$J = {}^{\gamma_1}I_{T_S}^{PT_S} z(t)^T z(t) + {}^{\gamma_2}I_{T_S}^{MT_S} \Delta u(t-1)^2 = \int_{T_S}^{PT_S} D^{1-\gamma_1} z(t)^T z(t)dt + \int_{T_S}^{MT_S} D^{1-\gamma_2} \Delta u(t-1)^2 dt \tag{15}$$

According to [40], the continuous cost function can be discretized by using GL definition with sample time T_S.

$$\begin{aligned} J \approx\ & T_S^{\gamma_1}[\omega_0^{(-\gamma_1)} z(k+P) + \omega_1^{(-\gamma_1)} z(k+P-1) + \cdots + (\omega_{P-1}^{(-\gamma_1)} - \omega_0^{(-\gamma_1)}) z(k+1) + (\omega_P^{(-\gamma_1)} - \omega_1^{(-\gamma_1)}) z(k) \\ & + (\omega_{P+1}^{(-\gamma_1)} - \omega_2^{(-\gamma_1)}) z(k-1) + \cdots] + T_S^{\gamma_2}[\omega_0^{(-\gamma_2)} \Delta u(k+M-1) + \omega_1^{(-\gamma_2)} \Delta u(k+M-2) + \cdots \\ & + (\omega_{M-1}^{(-\gamma_2)} - \omega_0^{(-\gamma_2)}) \Delta u(k) + (\omega_M^{(-\gamma_2)} - \omega_1^{(-\gamma_2)}) \Delta u(k-1) + \cdots] \\ =\ & (T_S^{\gamma_1} Z^{\mathrm{T}} \bar{\Lambda}_1 Z + T_S^{\gamma_2} \Delta U^{\mathrm{T}} \bar{\Lambda}_2 \Delta U) + T_S^{\gamma_1}[(\omega_P^{(-\gamma_1)} - \omega_1^{(-\gamma_1)}) z(k) + (\omega_{P+1}^{(-\gamma_1)} - \omega_2^{(-\gamma_1)}) z(k-1) + \cdots] \\ & + T_S^{\gamma_2}[(\omega_M^{(-\gamma_2)} - \omega_1^{(-\gamma_2)}) \Delta u(k-1) + (\omega_{M+1}^{(-\gamma_2)} - \omega_2^{(-\gamma_2)}) \Delta u(k-1) + \cdots] \end{aligned} \tag{16}$$

where,

$$\Lambda_1 = block\,diag\,[T_S{}^{\gamma_1}(w_{P-1}Q_1, w_{P-2}Q_2, \cdots, w_1 Q_{P-1}, w_0 Q_P)]$$

$$\Lambda_2 = T_S{}^{\gamma_2} diag(r_1 w_{M-1}, r_2 w_{M-2}, \cdots, r_{M-1} w_1, r_M w_0)$$

$$w_q = \omega_q^{(-\gamma_\varepsilon)} - \omega_{q-(X-1)}^{(-\gamma_\varepsilon)}$$

$\omega_0^{(-\gamma_\varepsilon)} = 1$, $\omega_q^{(-\gamma_\varepsilon)} = (1 - \frac{1-\gamma_\varepsilon}{q})\omega_{q-1}^{(-\gamma_\varepsilon)}$ for $\forall q > 0$ and $\omega_q^{(-\gamma_\varepsilon)} = 0$ for $q < 0$, $X = P$ for $\varepsilon = 1$ and $X = M$ for $\varepsilon = 2$, $Q_j = diag(q_e, q_{y1}, q_{y2}, \cdots, q_{yL_S}, q_{u1}, q_{u2}, \cdots, q_{ud})$, $R = \mathrm{diag}(r_1, r_2, \cdots, r_M)$.

As the minimization will never depends on the state variables and manipulated variables at the past time, they can be omitted and the cost function can be simplified further. In addition, the weighting matrix Q_j and R are introduced into the cost function, then the approximate discrete form is:

$$J' = Z^{\mathrm{T}} \Lambda_1 Z + \Delta U^{\mathrm{T}} \Lambda_2 \Delta U \tag{17}$$

For symmetrical weighted matrix $Q_j = diag(q_e, q_{y1}, q_{y2}, \cdots, q_{yL_S}, q_{u1}, q_{u2}, \cdots, q_{ud})$, the output error q_e must be considered and is not equal to 0. When considering $Q_j = diag(q_e, 0, 0, \cdots, 0, 0, 0, \cdots, 0)$, only the output error in the control system is taken into consideration, and the MPC based on ENMSS model reduces to the NMSS MPC. If $q_{y1}, q_{y2}, \cdots, q_{yL_S}$ and $q_{u1}, q_{u2}, \cdots, q_{ud}$ are not set as 0, the input increments and output

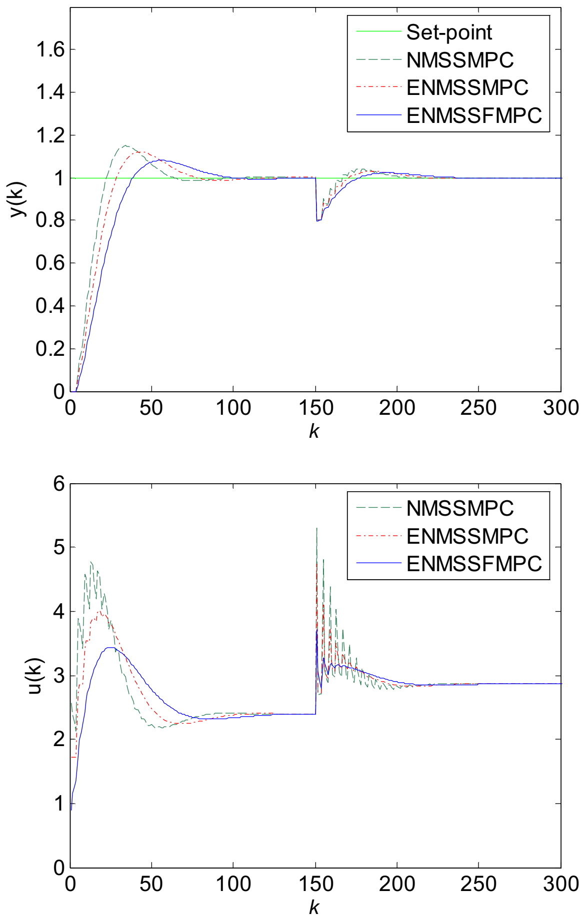

Fig. 6. Closed-loop responses under case 2.

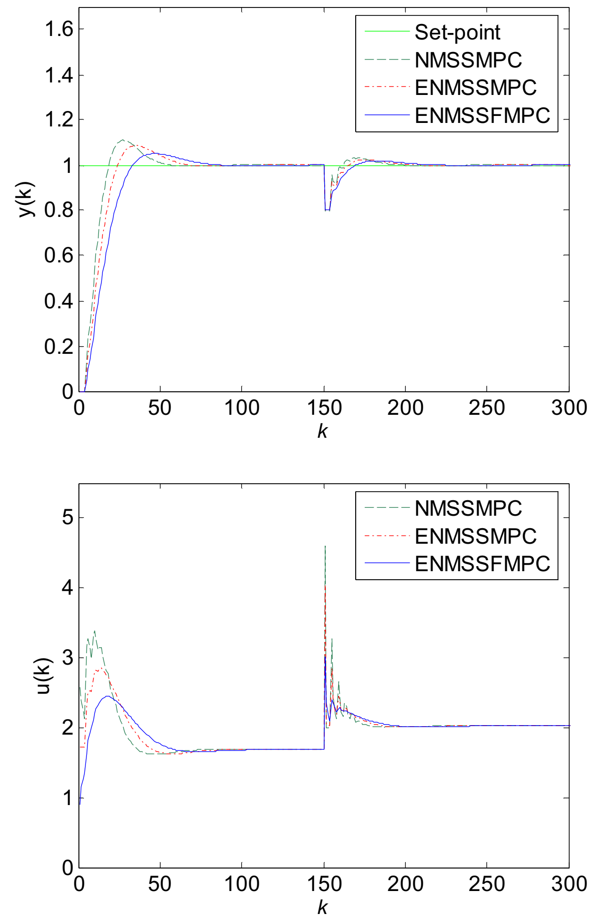

Fig. 7. Closed-loop responses under case 3.

increments as well as the output tracking error are considered in the control system.

The control law can be derived by minimizing Eq. (17) as follows,

$$\Delta U = -(S^T \Lambda_1 S + \Lambda_2)^{-1} S^T \Lambda_1 (Gz(k) + \Psi \Delta R) \tag{18}$$

$$u(k) = [\,1 \quad 0 \quad \cdots \quad 0\,] \Delta U + u(k-1) \tag{19}$$

4. Case study: heating furnace temperature control process

4.1. The process description of industrial heating furnace

The flow chart of SXF-4-10 heating furnace process is shown in Fig. 1. The temperature of the heating furnace is controlled by on-off control of the SSR-380D40 solid state relay.

The K type thermocouple is used to detect the practical furnace temperature and transfer the temperature signal to the corresponding voltage signal. Due to the fact that the voltage is too small to be recognized by the PCI-1802LU CR signal acquisition card, the SG-3011 CR signal amplifier is added to the control circuit. Then the voltage signal is sent to the electronic terminal strip DN-37CR. The PCI-1802LU CR signal acquisition card of the industrial personal computer (IPC-610L) is used to recognize the signal transferred from DN-3710 CR cable and send the signal to the computer. The voltage will be converted into the corresponding temperature by the computer programs and the output digital signal is transmitted to the PIO-D24U signal card. When the input digital signal is set to 0×01, the +5 V voltage will be obtained from the PIO-D24U signal card. The DN-3710 CR cable and the DN-37CR terminal strip is used to send the +5 V voltage to the SSR-380D40 solid state relay. Then the SSR-380D40 relay is on-state and the SFX 4-10 furnace is under the heating state. On the contrary, if the input digital signal is 0×00, the voltage of the DN-37CR terminal strip is 0 V and the SSR-380D40 is at off-state. Then the circuit of the heating furnace is cut off and the heat is dissipating.

For the control computer (IPC-610L), the error signal of the temperature is the input and the sampling time is set as $T_S = 10s$. Within the set nT_S heating time, the duty factor dr will be performed. In other words, SSR-380D40 is on-state if the input digital signal is 0×01 at the time $dr{\cdot}T_S$, while the relay is at off-state when the input signal is 0×00 during the remaining time. Then the computer will calculate the output of the duty factor by comparing the set-point with the practical measured temperature at the instant. The signal of the duty factor will be obtained by the solid state relay and the next loop will start again.

4.2. The process model

In this process, the temperature in the SXF-4-10 heating furnace is the variable to be controlled to track the set-point. The control variable is the duty factor of heating time. To simulate the control effect, the fractional order model and integer first order plus dead time model have been modeled via the measured temperature of the heating furnace.

The integer order model is derived by using step-response modeling method [2] as:

$$G(s) = \frac{0.57}{500s + 1} e^{-240s} \tag{20}$$

The fractional order model is derived from the integer order step-response model. Based on the parameters of $T_0 = 500$, $\alpha_0 = 1$,

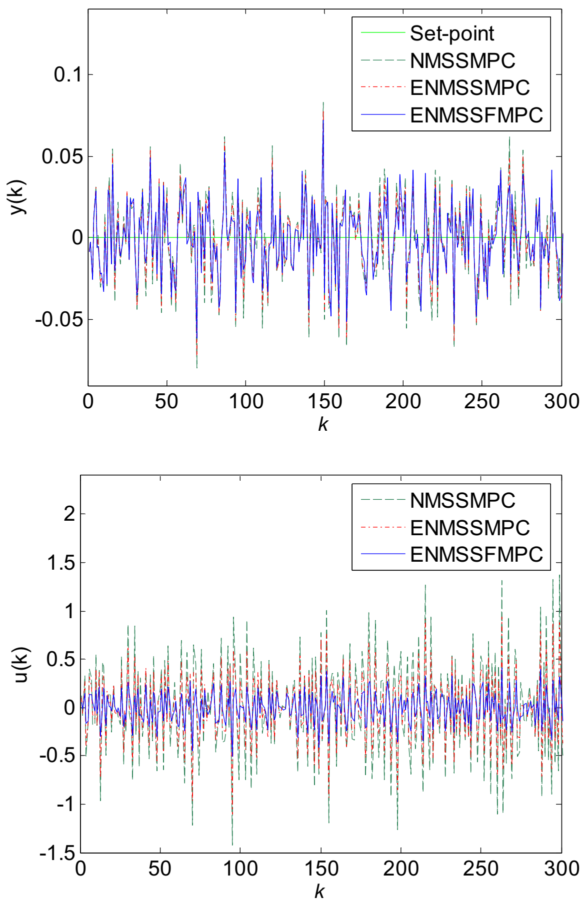

Fig. 8. Closed-loop responses with noise under case 1.

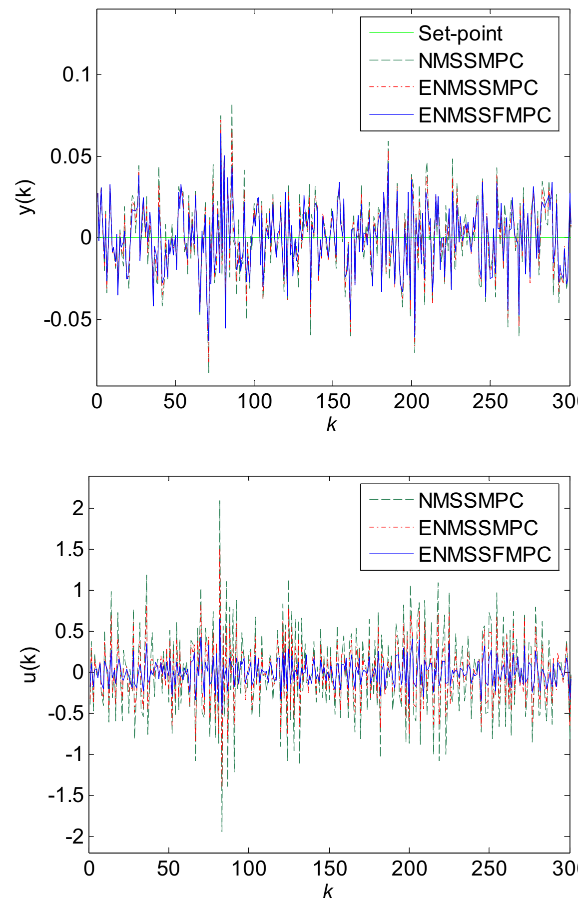

Fig. 9. Closed-loop responses with noise under case 2.

$K_0 = 0.57$ of the integer order model, the parameters of the fractional order model were adjusted along with the variances between the step-response output of the model and practical process output of the heating furnace. The least square method is adopted to derive a more accurate model than the integer order model, and this fractional order mode has been verified in several experiment of the heating furnace. The fractional order model of heating furnace process is:

$$G(s) = \frac{0.585}{300s^{0.92}+1}e^{-240s} \tag{21}$$

To evaluate the two models of the process, the variances of errors between the models and the sampled temperature are shown in Table 1. The open-loop step response of the two models and the practical temperature data are shown in Fig. 2. In the open-loop step response of the models, the output of the model will be closed to the steady state value at 3200s. Thus the sampling time is chosen as $T_S = 80$ and the approximate memory length is $L_S = 40$. Then the coefficients of the process model in Eq. (10) are obtained as:

Table 1
The variances of the output temperature.

Transfer function $G(s)$	variances
First order plus dead time model	1.7592
Fractional order model	1.4856

$$\begin{array}{llllll}
\{F_1, F_2, \cdots, F_{L_S}\} & = \{-955/1233 & -191/6165 & -47/4214 & -71/12242 & -107/29950 \\
-59/24286 & -69/39137 & -25/18658 & -52/49333 & -27/31702 & -47/66854 \\
-11/18627 & -5/9934 & -12/27631 & -40/105623 & -26/78017 & -10/33827 \\
-48/181757 & -19/80033 & -12/55915 & -19/97441 & -13/73045 & -8/49045 \\
-7/46646 & -16/115489 & -22/171459 & -13/109073 & -12/108095 & -12/115759 \\
-11/113368 & -14/153813 & -12/140255 & -21/260609 & -13/170985 & -11/153077 \\
-11/161701 & -3/46514 & -4/65319 & -10/171753 & -11/198454\} &
\end{array}$$

and $H = 90/973$.

The integer order model in Eq. (20) can be discretized as:

$$G(z) = \frac{0.08428}{z^4 - 0.8521z^3} \tag{22}$$

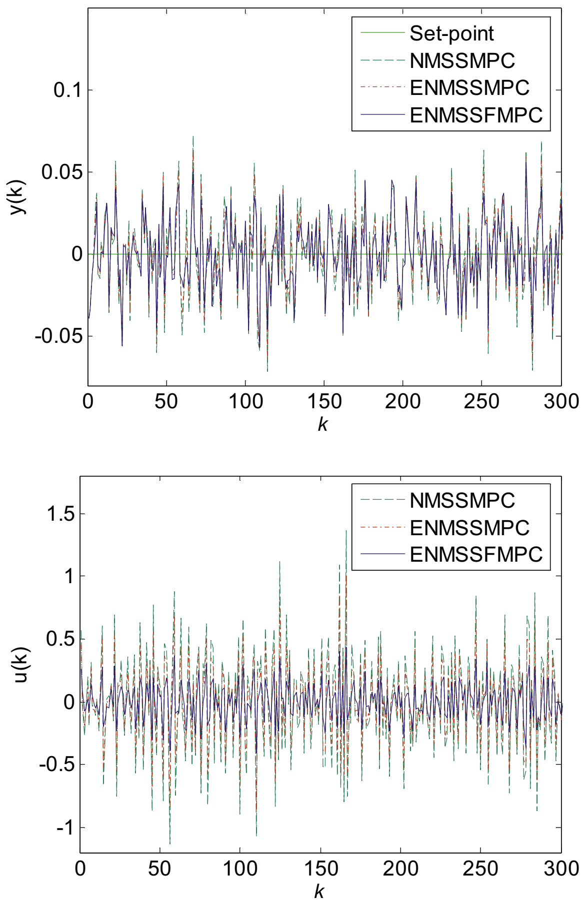

Fig. 10. Closed-loop responses with noise under case 3.

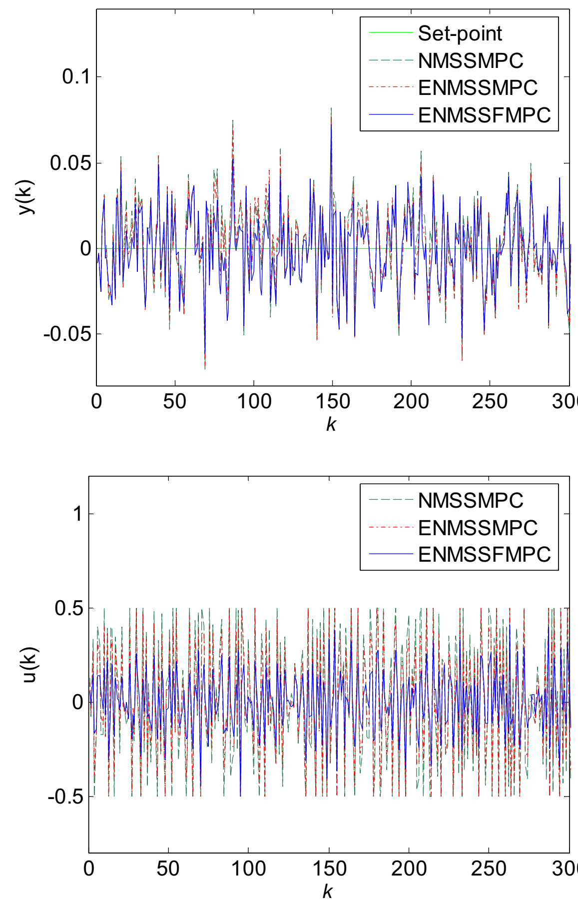

Fig. 11. Closed-loop responses with input hard constraints and measured noise under case 1.

Table 2
The parameters of controllers.

Parameters	ENMSS FMPC	ENMSS MPC	NMSS MPC
P	15	15	15
M	1	1	1
λ	0.6	0.6	0.6
Q_j	$diag(1, 30, 6\cdots, 6, 0, 0, 0)$	$diag(1, 30, 0, 0, 0)$	$diag(30, 0, 0, 0)$
r_1	0.01	0.01	0.01
γ_1, γ_2	0.4,1.2	\	\

Table 3
Standard deviations of output under measured noise.

Standard deviations			
Mismatched cases	ENMSS FMPC	ENMSS MPC	NMSS MPC
Case 1	0.021692	0.023859	0.025961
Case 2	0.020118	0.022002	0.024276
Case 3	0.020674	0.023617	0.026017

Table 4
Standard deviations of output under input hard constraints and measured noise.

Standard deviations			
Mismatched cases	ENMSS FMPC	ENMSS MPC	NMSS MPC
Case 1	0.021689	0.023656	0.024448
Case 2	0.020137	0.021323	0.020137
Case 3	0.020674	0.023641	0.024839

4.3. Performance of the proposed fractional order controller

The model in Eq. (22) is the nominal plant for integer order model predictive controller while the model in Eq. (10) with the coefficients will be the nominal plant for fractional order model predictive controller. Then the manipulated variables of controllers will be employed to control the fractional order model which is more accurate to predict the output of the process.

To verify the performance of the controller, the proposed extended non-minimal state space fractional order model predictive control (ENMSSFMPC) will be compared with integer order MPC methods based on ENMSS model as well as the NMSS model under the same conditions. For a fair comparison, the same parameters of the controllers have been considered in Table 2. In addition, the extra tuning parameters γ_1, γ_2 and weight coefficients in Q_j of the fractional order MPC will make it more flexible.

In practical industrial processes, the precise process model cannot be obtained at all, which shows that uncertainty in industrial processes may cause the model/plant mismatches and affect the stability and performance of controllers. It is very important to evaluate the performance of controllers under the model/plant mismatched case. We use the Monte Carlo simulation to get the parameters of the fractional order process in Eq. (21), where the nominal variables are displayed as $K = 0.585$, $T = 300$, $\alpha = 0.92$, $\tau = 240$. The three random model/plant mismatched cases with the maximum of 30% uncertainty from the original plant described by Eq. (21) are generated and the parameters are listed as follows.

$$\text{Case 1}: \quad K = 0.4149, T = 258.9938, \alpha = 0.7537, \tau = 275.5371$$

$$\text{Case 2}: \quad K = 0.51435, T = 370.9, \alpha = 0.75392, \tau = 263.25$$

$$\text{Case 3}: \quad K = 0.70407, T = 333, \alpha = 0.76133, \tau = 258.54$$

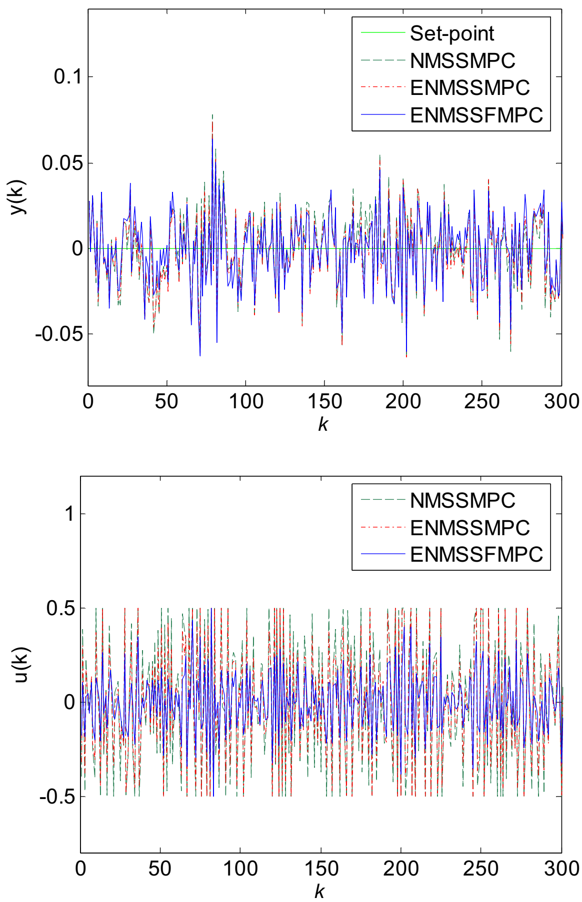

Fig. 12. Closed-loop responses with input hard constraints and measured noise under case 2.

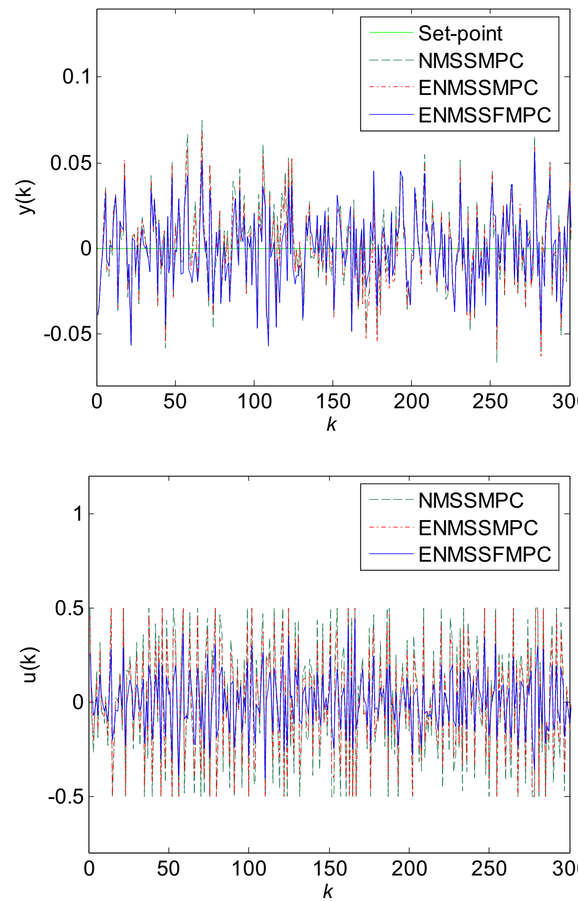

Fig. 13. Closed-loop responses with input hard constraints and measured noise under case 3.

4.3.1. Fractional orders

In this part, the comparative experiment results under different fractional orders are shown in Figs. 3 and 4. For the simple and effective consideration, one of the above three mismatched cases has been discussed and Case 3 is chosen.

In this paper, the parameters γ_1, γ_2 are arbitrary orders. For simplicity, the closed-loop responses of the proposed FMPC shown in Fig. 3 are simulated for variation in γ_1 under $\gamma_2 = 0$, and the simulations shown in Fig. 4 are for variation in γ_2 under $\gamma_1 = 0$. In Fig. 3, the effect of the parameter is checked by changing γ_1 from 0 to 30.5. It can be shown that the response time of the output can decrease and meanwhile the fluctuation of the input will be increased correspondingly. In Fig. 4, the relatively appropriate small value of γ_2 may be appreciated for the response rate and the incremental input. In view of the effect of the two parameters, $\gamma_1 = 0.4$ and $\gamma_2 = 1.2$ are selected to benefit for the expected control performance. The other parameters can be seen in Table 2.

4.3.2. Output disturbance case

The set-point is 1 from the time instant $k = 0$, and the output disturbance with amplitude of -0.2 is added to the process. Figs. 5–7 show the output responses and the manipulated input of the control systems under the three model/plant mismatches. As can be seen from these figures, the output responses of FMPC controllers can well track the set-point with smaller fluctuations, overshoots and oscillations than those of the other MPC controllers. Though the faster response rate of MPC control system is witnessed, tremendous oscillations and overshoots are obtained. In FMPC algorithm, the improved overall performance with smoother stable process output, input and better disturbance rejection is obtained at the expense of responsiveness.

4.3.3. Measurement noise case

In practice, the measurement noise is also one of the factors that should be considered to evaluate the performance of the proposed controller. The random white noise with the value of standard deviation of 0.02 is added to the actual process output of the three model/plant mismatched cases, while the set-point is set as $c(k) = 0$.

Figs. 8–10 show the output responses of the three model/plant mismatches under the measurement noise. From the overall comparison, the ensemble performance of the proposed FMPC controller is better than that of the other MPC controllers. Under the measurement noise, the standard deviation and oscillation of the proposed FMPC controller are smaller than those of the other MPC controllers. Also, the standard deviations in Table 3 indicated this aspect further.

4.3.4. Input hard constraints case plus measurement noise

Due to the upper and lower limits in industrial process actuator, the performance of controllers with the input hard constraints and measurement noise is studied in this section. The random white noise with the value of standard deviation of 0.02 is still added to the process output, and the set-point is set as $c(k) = 0$. The limited range of control input variables is $-0.5 < u(k) < 0.5$. The results of the comparison between the proposed fractional order MPC and the integer order MPC controllers with input hard constraints and measurement noise are shown in Figs. 11–13 . We can obviously realize that the control input of the integer order MPC controllers act drastically, while the proposed fractional order MPC can maintain the smooth control signals. The detail standard deviations are shown in Table 4, which also demonstrate clearly the better performance of the proposed fractional order controller.

5. Conclusion

In this paper, an approach of fractional order model predictive control is designed and improved performance in terms of both dynamic and steady process performance is obtained. Firstly, the fractional order SISO system is discretized by GL definition. Secondly, the extended non-minimal state space fractional order model that contains the state variable and the fractional order output tracking error is formulated by choosing appropriate state variables. Thirdly, the fractional order integral is introduced into the cost function and the GL definition is used to obtain the discrete form of the continuous cost function. Lastly, the simulation on the fractional order temperature control system is illustrated to reflect the performance of the proposed fractional order MPC method. The comparison between FMPC and MPC indicates the improvement of the control performance. Besides, the successful use of the proposed method also promotes the application of the fractional order calculus on MPC.

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