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Design of fractional order predictive functional control for fractional industrial processes

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ABSTRACT

Many phenomena in practical processes cannot be accurately described by conventional differential equations, while fractional order differential equations can describe the characteristics of such processes more accurately. In this paper, the fractional order predictive functional control (FPFC) method is designed for a class of single-input single-output (SISO) fractional order linear systems. The Oustaloup approximation is employed to derive the approximate model of fractional order system. Meanwhile, the Grünwald–Letnikov (GL) definition and the fractional calculus operator are used in its cost function, which further extend the applications of fractional order calculus to the predictive functional control algorithm. And then the optimal control is obtained. Compared with traditional predictive functional control based on integer reduced order model, simulation results reveal that the fractional order controller yields improved control performance.

1. Introduction

The theory of fractional order calculus can be traced back to 300 years ago, but its application in the field of control engineering is nearly 20 years [1]. This study extends the use of calculus to not only integer order but also fractional order systems. One of the reasons maybe that a lot of phenomena cannot be accurately described by the integer order differential equations in realistic physical systems and the fractional order differential equation models can be more accurate than traditional integer order models to express the characteristics of the actual systems [2]. For this purpose, fractional order calculus has been used to model the complex industrial processes by several researchers [3–5]. Meanwhile, in [6], the recursive least-square method and recursive instrumental variable algorithm were used to estimate the parameters of fractional order models with generalized ARX structure. Madakyaru et al. proposed an approach to reconstruct the ARX models using the fractional order differential operators and orthonormal basis filters. In their paper, the models were identified from input–output perturbation data using a two-step nested optimization scheme and the experimental studies on practical benchmark heater-mixer setup shows the feasibility of this method [7].

In the field of control systems, the PD^{μ}controller [8], CRONE controller [9] and $PI^{\lambda}D^{\mu}$ controller [10] were proposed sequentially in the 20th century. With the wide use of PID controllers in industries, more and more studies involving fractional order calculus have been done to explore better design methods for fractional order control systems in recent years. For example, Yeroglu and Tan [11] presented the design techniques of fractional order PID controllers, and the Ziegler–Nichols method and Åström–Hägglund method were applied for the tuning of their controller parameters. In [12], a genetic algorithm was introduced to improve the accuracy of the designed fractional order $PI^{\lambda}D^{\mu}$ controller. A tuning graphical method of fractional order PID controller was studied in [13] and a tuning graphical method for fractional order $PI^{\lambda}D^{\mu}$ controllers was proposed on the basis of the sensitivity function constraint of the closed-loop transfer function in [14]. Jin et al. proposed a model reduction method and an explicit PID tuning rule for the PID auto-tuning based on fractional order system in [15]. In [16], the authors clearly came to the conclusion that a fractional order controller can obtain better performance than integer order controller for fractional order systems. As a consequence, a study of the fractional order control theory and its application in the field of practical process control is of great significance.

On the other hand, the motivation of the research on fractional order calculus is that fractional order models can fit the actual data more precisely and flexibly than integer order models and the outstanding merit of the fractional order model has laid a good foundation for model predictive control (MPC) based on process models. Predictive functional control (PFC) is one type of model predictive control technologies and it has been proposed by Richalet to control the dynamic system [17]. In this strategy, the control input is derived by solving the difference between the future predicted output and the desired trajectory by minimizing the cost function. PFC is the most popular one which has been widely used in research and practical control engineering that also

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reveal good control effect [18,19]. In [20], the online recursive least-squares identification and the self-adaptive predictive functional control algorithm were applied to the temperature control system. For complex nonlinear systems, the PFC methods based on fuzzy models were presented by lots of scholars [21–23]. Also, a predictive functional controller based on extended state space model was developed and it shows good control performance when it was compared with traditional state space PFC in [24,25]. The extended state space process predictive functional control algorithm based on genetic algorithm optimization is designed for batch processes under actuator faults in [26]. Based on the extended state space PFC method, Tao et al. designed a controller with the linear quadratic form to deal with the process under partial actuator failures in [27]. The extended state space PFC algorithms based on decoupling strategy that uses the adjugate matrix of the process [28] or partially decoupled scheme [29] have also been used for multivariable processes. In addition, based on the optimization idea, PFC was used to optimize the PI or PID controllers to obtain the advantages of both PFC and PID [30–32]. Though successful in process control, most PFC strategies are for integer order systems.

At present, several studies on the combination of fractional order calculus and MPC algorithm have been applied prosperously. The predictive functional controller based on state space model has been presented for fractional order systems and the two basic functions were considered in [33]. In [34], the GL definition was used to discretize the fractional order system and the fractional integral was considered in the cost function. The excellent performance of the designed non-minimal state space fractional order predictive functional controller for fractional order systems was displayed on the thermal fractional system. In [35], the adaptive generalized predictive control (GPC) algorithm was designed for fractional order dynamic model of solid oxide fuel cells. In [36–37], Romero et al. proposed a new GPC algorithm for fractional order systems and the fractional order operators were used in its cost function. The numerical approximation model and Oustaloup approximation model were used to predict the future dynamic output of the system, and the proposed MPC for fractional order control system achieved satisfactory performance by Rhouma et al. in [38–39]. Based on time domain, Guo et al. proposed some new control methods which combine the virtues of the fractional order PID algorithm and predictive control algorithm [40–42]. Moreover, Joshi et al. proposed an MPC method that can track the reference signals with limited uncertainties for fractional order systems. In particular, the Laplace transform of Caputo fractional order calculus and Mittag–Leffler function (MLF) were used to evaluate the process output. However, MPC was employed in a fractional order system with the fractional order α with $0<\alpha<1$ [43]. As shown above, the research on PFC for fractional order systems is limited in quantity and there are still requirements for new methods to achieve better performance of fractional order predictive controllers.

The main aim of our study is to develop a new approach to control the fractional order system and further improve the performance of the control system when compared with the integer order PFC since the industrial processes are fractional order systems in essence. In this paper, the proposed controller has been designed for the system described by SISO linear fractional differential equations. In addition, the fractional order derivative in the cost function of fractional PFC (FPFC) algorithm is expected to enhance the performance because of more tuning parameters. The proposed FPFC algorithm design is as follows. First, the input–output process model has been derived from the Oustaloup approximation of fractional order transfer function. Second, the predicted output is transformed into the matrix-form prediction. Then, the GL definition will be utilized to discrete the fractional order cost function. This method has such features as simple calculation, strong robustness, and strong anti-interference ability. Finally, FPFC shows good performance when it is successfully employed to practical heating furnace process.

The paper is organized as follows. In Section 2, the basic knowledge of fractional order calculus is described. In Section 3, the design of FPFC for fractional order systems is presented. Firstly, we use Oustaloup approximation method to approximate fractional order operator s^{α}, and the model of the fractional controlled process is obtained. Then, based on the obtained model, the integer order PFC is extended to the non-integer order predictive functional control to get the optimal control law. In Section 4, some simulation results are done to verify the performance of the FPFC controller on a heating furnace. Conclusion of the proposed method is drawn in Section 5.

2. Fractional order calculus

Fractional order calculus is expanded from traditional calculus, which allows the differential and integral equations to be of fractional orders instead of integer orders. With the rapid development of the theory of fractional calculus, several definitions available for fractional calculus are defined consecutively. There is no single definitions of fractional calculus so far. The three commonly used definitions are Grünwald–Letnikov (GL), Riemann–Liouville (RL), and Caputo definitions [44].

The GL definition of fractional order calculus is described as follows:

$$ {}_aD_t^{\beta} f(t) = \lim_{h\to 0} \frac{1}{h^{\beta}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\beta}{j} f(t-jh) \tag{1}$$

where, a is the initial time, h is the selected appropriate calculation step, $[(t-a)/h]$ is the integer part of the number $(t-a)/h, \omega_j^{(\beta)} = (-1)^j \binom{\beta}{j}$ is the polynomial coefficients and can be solved as

$$\omega_0^{(\beta)} = 1\ ,\ \ \omega_j^{(\beta)} = \left(1-\frac{\beta+1}{j}\right)\omega_{j-1}^{(\beta)},\ \ j=1,2,\cdots. \tag{2}$$

Considering the practical process and the short-term memory characteristics of fractional order calculus operator, the sample time T_s is substituted for calculating step h. For simplicity, we denote $D^{\beta} \equiv {}_aD_t^{\beta}$ with zero initial conditions, Eq. (1) can be converted into the following form:

$$D^{\beta} f(t) = \frac{1}{T_s^{\beta}} \sum_{j=0}^{n} (-1)^j \binom{\beta}{j} f((n-j)T_s) \tag{3}$$

where, $n = [(t-1)/T_s]$.

The RL definition of fractional order calculus can be defined as

$$D^{\beta} f(t) = \frac{1}{\Gamma(m-\beta)} \frac{\mathrm{d}^m}{\mathrm{d}t^m} \int_a^t \frac{f(\tau)}{(t-\tau)^{\beta-1}} \mathrm{d}\tau \tag{4}$$

where, $m-1<\beta<m,\ m\in \mathrm{N}$, and $\Gamma(\cdot)$ is the Euler's gamma function:

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt \tag{5}$$

The Caputo definition of fractional order calculus is described as

$$D^{\beta} f(t) = \frac{1}{\Gamma(m-\beta)} \int_a^t \frac{f^m(\tau)}{(t-\tau)^{1+\beta-m}} \mathrm{d}\tau \tag{6}$$

The Laplace transform of RL fractional calculus is:

$$\mathrm{L}\left\{D^{\beta} f(t)\right\} = s^{\beta} f(s) - \sum_{k=0}^{m-1} s^k D^{\beta-k-1} f(t)|_{t=0^+} \tag{7}$$

The Laplace transform of Caputo definition is given by

$$\mathrm{L}\left\{D^{\beta}f(t)\right\}=s^{\beta}f(s)-\sum_{k=0}^{m-1}s^{\beta-k-1}\left[\frac{\mathrm{d}^k f(t)}{\mathrm{d}t^k}\right]_{t=0^+} \tag{8}$$

For convenience, the Laplace transform of the RL fractional order derivative under zero initial condition is given by

$$\mathrm{L}\left\{D^{\beta}f(t)\right\}=s^{\beta}\mathrm{L}\{f(t)\} \tag{9}$$

A fractional order SISO linear system can be characterized by fractional differential equation (FDE) as follows:

$$\begin{aligned}&y(t)+a_1D^{\alpha_1}y(t)+a_2D^{\alpha_2}y(t)+\cdots+a_{n1}D^{\alpha_{n1}}y(t)\\&=b_0D^{\beta_0}u(t)+b_1D^{\beta_1}u(t)+\cdots+b_{n2}D^{\beta_{n2}}u(t)\end{aligned} \tag{10}$$

where, $y(t)$,$u(t)$ are the output and input of the controlled system, $a_i(i=1,2,\cdots,n_1)$,$b_j(j=0,1,\cdots,n_2)$ are the coefficients, $\alpha_1<\alpha_2<\cdots<\alpha_{n1}$, $\beta_0<\beta_1<\cdots<\beta_{n_2}$.

Combining the RL definition and its Laplace transform in (9), the fractional order system under zero initial condition can be written as the following transform function model:

$$G(s)=\frac{b_0s^{\beta_0}+b_1s^{\beta_1}+\cdots+b_0s^{\beta_{n_2}}}{1+a_1s^{\alpha_1}+a_2s^{\alpha_2}+\cdots+a_{n_1}s^{\alpha_{n_1}}} \tag{11}$$

In this paper, the Laplace transform of the RL definition is used to describe the fractional order system, while GL definition is applied to derive the discrete form of the system.

3. Design of fractional order predictive functional controller

In this section, predictive functional control (PFC) algorithm is introduced to control the fractional order system, where the obtained controller has the same basic characteristics as traditional PFC algorithm used for integer order system.

Since most of the designed controller algorithms are used for the integer order system, it is very difficult to derive algorithm directly for fractional order systems. In this paper, the Oustaloup approximation [45]will be adopted for numerical solution of fractional order differential equation.

The Oustaloup approximation method is shown as follows:

$$s^{\alpha}\approx K\prod_{k=1}^{N}\frac{s+w_k'}{s+w_k} \tag{12}$$

where $K=w_h^{\alpha}$, $w_k^{'}=w_bw_u^{(2k-1-\alpha)/N}$, $w_k=w_bw_u^{(2k-1+\alpha)/N}$, $w_u=\sqrt{w_h}$ $/^h w_b$ and $0<\alpha<1$, the approximation limits N is chosen beforehand, w_b,w_h are the lower and higher frequency approximation interval. In the above approximation, the value of N depends on the order of the integer order transfer function which is approximated by the fractional order derivative operator $s^{\alpha}(0<\alpha<1)$. The ripple caused by Oustaloup approximation may be practically eliminated by increasing N, but the approximation will be computationally heavier. The determination of the N value always depends on the computations and the practical requirements.

Combining with Eqs. (11) and (12), the fractional order system can be approximated to high order integer system. The process model can be transformed into a corresponding difference equation model by adding a zero-order holder at the sample time T_S.

$$\begin{aligned}&A(z^{-1})y(k)=B(z^{-1})u(k)\\&A(z^{-1})=1+F_1z^{-1}+\cdots+F_{n_1}z^{-n_1}\\&B(z^{-1})=H_1z^{-1}+\cdots+H_{n_2}z^{-n_2}\end{aligned} \tag{13}$$

where, z^{-1} is the backward shift operator, n_1,n_2 are the orders of y and u, $F_1,F_2,\cdots,F_{n_1},H_1,H_2,\cdots,H_{n_2}$ are the coefficients of the derived discrete process model.

In the PFC algorithm, the control law is represented in the form of a linear combination of a set of basic functions. Based on the characteristics of step function and structure of the control variable, we select a basic function for the step function in the following form:

$$u(k+i)=u(k)\ ,\ \ i=1,\ 2,\cdots \tag{14}$$

Let P denote the prediction horizon, the predicted output of the model can be calculated recursively by assuming a constant manipulated signal $u(k)$:

$$\begin{aligned}y(k+1)&=-F_1y(k)-F_2y(k-1)-\cdots-F_{n_1}y(k-n_1+1)\\&\quad+H_1u(k)+H_2u(k-1)+\cdots+H_{n_2}u(k-n_2+1)\\y(k+2)&=-F_1y(k+1)-F_2y(k)-\cdots-F_{n_1}y(k-n_1+2)\\&\quad+H_1u(k+1)+H_2u(k)+\cdots+H_{n_2}u(k-n_2+2)\\&\ \vdots\\y(k+P)&=-F_1y(k+P-1)-F_2y(k+P-2)-\cdots-F_{n_1}y(k+P-n_1)\\&\quad+H_1u(k+P-1)+H_2u(k+P-2)+\cdots+H_{n_2}u(k+P-n_2)\end{aligned} \tag{15}$$

where, $y(k+i)$is the output prediction of the process model for time instant $k+i$made at time instant k, $i=1,2,\cdots,P$.

When the predictive horizon $P>n_2$ is considered, the predicted output depends on the manipulated variable$u(k)$ from time instant $k+n_2$. Submitting Eq. (14) into Eq. (15), the formula can be represented in the following form:

$$\begin{aligned}&\begin{bmatrix}1&0&\cdots&&\cdots&0\\F_1&1&&&&\\\vdots&\ddots&\ddots&\ddots&&\\F_{n_1}&&\ddots&\ddots&&\vdots\\0&\ddots&&\ddots&\ddots&\ddots\\\vdots&&\ddots&&\ddots&\ddots\;0\\0&\cdots&0&F_{n_1}&\cdots\;F_1&1\end{bmatrix}\begin{bmatrix}y(k+1)\\y(k+2)\\\\\vdots\\\vdots\\\\y(k+P)\end{bmatrix}\\&=\begin{bmatrix}-F_1&-F_2&\cdots&-F_{n_1}\\-F_2&&\therefore&0\\\vdots&\therefore&\therefore&\\-F_{n_1}&0&\cdots&\vdots\\0&\cdots&\cdots&0\\\vdots&&&\vdots\\0&\cdots&\cdots&0\end{bmatrix}\begin{bmatrix}y(k)\\y(k-1)\\\\\vdots\\\vdots\\\\y(k-n_1+1)\end{bmatrix}+\begin{bmatrix}H_1\\H_1+H_2\\\vdots\\\sum_{i=1}^{n_2}H_i\\\vdots\\\sum_{i=1}^{n_2}H_i\end{bmatrix}u(k)\\&+\begin{bmatrix}H_2&H_3&\cdots&H_{n_2}\\H_3&&\therefore&0\\\vdots&\therefore&\therefore&\\H_{n_2}&0&\cdots&\vdots\\0&\cdots&\cdots&0\\\vdots&&&\vdots\\0&\cdots&\cdots&0\end{bmatrix}\begin{bmatrix}u(k-1)\\u(k-2)\\\\\vdots\\\vdots\\\\u(k-n_2+1)\end{bmatrix}\end{aligned} \tag{16}$$

The corresponding model is further written as follows:

$$AY=BY_{past}+Cu(k)+DU_{past} \tag{17}$$

with

$$\begin{aligned}Y&=[y(k+1),y(k+2),\cdots,\ y(k+P)]^{\mathrm{T}}Y_{past}\\&=[y(k),y(k-1),\cdots,\ y(k-n_1+1)]^{\mathrm{T}}U_{past}\\&=[u(k-1),u(k-2),\cdots,\ u(k-n_2+1)]^{\mathrm{T}}\end{aligned}$$

$$A=\begin{bmatrix} 1 & 0 & \cdots & & \cdots & 0 \\ F_1 & 1 & & & & \\ \vdots & \ddots & \ddots & \ddots & & \\ F_{n_1} & & \ddots & \ddots & & \vdots \\ 0 & \ddots & & \ddots & \ddots & \ddots \\ \vdots & & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & F_{n_1} & \cdots & F_1 \;\; 1 \end{bmatrix},\quad B=\begin{bmatrix} -F_1 & -F_2 & \cdots & -F_{n_1} \\ -F_2 & & \iddots & 0 \\ \vdots & \iddots & \iddots & \\ -F_{n_1} & 0 & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} H_1 \\ H_1+H_2 \\ \vdots \\ \sum_{i=1}^{n_2} H_i \\ \vdots \\ \sum_{i=1}^{n_2} H_i \end{bmatrix},\quad D=\begin{bmatrix} H_2 & H_3 & \cdots & H_{n_2} \\ H_3 & & \iddots & 0 \\ \vdots & \iddots & \iddots & \\ H_{n_2} & 0 & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}$$

where the dimensions of the matrices A,B,C,D are $P\times P$, $P\times n_1$, $P\times 1$ and $P\times(n_2-1)$.

Combining the above formulations in Eq. (17), the predicted output model is given as

$$Y=\overline{B}Y_{past}+\overline{C}u(k)+\overline{D}U_{past} \tag{18}$$

where $\overline{B}=A^{-1}B, \overline{C}=A^{-1}C, \overline{D}=A^{-1}D$.

In practical processes, the model/plant mismatch, noise, and other factors widely exist, so we need to compensate for these situations:

$$e(k+i)=y_p(k)-y(k), i=1,2,\cdots,P \tag{19}$$

where, $y_p(k)$,$y(k)$ are the measured current process output and predictive model output at time instant k.

Then, we need to calculate the corrected process output model as follows:

$$\begin{aligned}\tilde{Y}&=Y+E\\ &=\overline{B}Y_{past}+\overline{C}u(k)+\overline{D}U_{past}+E\\ E&=[e(k+1),e(k+2),\cdots,e(k+P)]^{\mathrm{T}}\end{aligned} \tag{20}$$

We choose the reference trajectory $y_r(k+i)$ as

$$y_r(k+i)=\lambda^i y_p(k)+\left(1-\lambda^i\right)c(k)\ \ ,\ \ i=0,1,\cdots P \tag{21}$$

where λ is the smoothing factor of the reference trajectory, $c(k)$ is the set-point at time instant k.

As for the integer order system, the cost function is selected when considering the receding horizon optimization problem:

$$J_{PFC}=\min\sum_{i=1}^{P}[y_r(k+i)-y(k+i)-e(k+i)]^2 \tag{22}$$

Eq. (22) is the general cost function of the predictive functional control and the continuous generalization is $J=\int_a^b[f(t)]^2dt$, where $[a,b]$ is the continuous integral interval. In general, the cost function is described by the single integral form of the quadratic function of the errors. In this paper, the integral will no longer be limited to the single integral, and the order of the integral can be chosen as non-integerγ. For simplicity, we denote ${}^{\gamma}I\equiv D^{-\gamma}$, where ${}^{\gamma}I$ is the fractional order integral notation and$D^{-\gamma}$ is the derivative notation of the negative. Since the formulation $D^{-1}[f(t)]^2=\int_a^b[f(t)]^2dt$ holds, the cost function can be extended to fractional order $J_F={}^{\gamma}I_a^b[f(t)]^2=\int_a^b D^{1-\gamma}[f(t)]^2dt$. Then, the continuous form of the cost function of fractional order PFC can be written as follows:

$$\begin{aligned}J_{FPFC}&={}^{\gamma}I_{T_S}^{PT_S}[y_r(t)-y(t)-e(t)]^2\\ &=\int_{T_S}^{PT_S}D^{1-\gamma}[y_r(t)-y(t)-e(t)]^2\mathrm{d}t\end{aligned} \tag{23}$$

As is seen in [46], the fractional order integral operator can be discretized by using the GL definition:

$$\begin{aligned}{}^{\gamma}I_a^b f(t)&=\int_a^b\left[D^{1-\gamma}f(t)\right]\\ &\cong T_S^{\gamma}\sum_{r=0}^{b/T_S}(-1)^{\gamma}\binom{-\gamma}{r}f(b-rT_S)-T_S^{\gamma}\sum_{r=0}^{a/T_S}(-1)^{\gamma}\binom{-\gamma}{r}f(a-rT_S)\\ &={T_S}^{\gamma}WF^{\mathrm{T}}\end{aligned} \tag{24}$$

where,

$$W=[\omega_b-\omega_a,\omega_{b-1}-\omega_{a-1},\cdots,\omega_{b-a}-\omega_0,\omega_{b-a-1},\cdots,\omega_1,\omega_0]$$
$$F=[f(0),f(T_S),\cdots,f(a-T_S),f(a),\cdots,f(b-T_S),f(b)]$$

Thus, according to Eqs. (23) and (24), the discrete form of the cost function can be derived as

$$J_{FPFC}\cong\left(Y_r-\tilde{Y}\right)^T\Lambda(T_S,\gamma)\left(Y_r-\tilde{Y}\right) \tag{25}$$

where,

$$Y_r=[y_r(k+1),y_r(k+2),\cdots,y_r(k+P)]^T$$

$$\Lambda(T_S,\gamma)=T_S diag(m_{P-1},m_{P-2},\cdots,m_1,m_0)$$

with $m_j=\omega_j^{(-\gamma)}-\omega_{j-(P-1)}^{(-\gamma)}$,${\omega_0}^{(-\gamma)}=1$,$\omega_j^{(-\gamma)}=(1-\frac{1-\gamma}{j})\omega_{j-1}^{(-\gamma)}$ for $\forall j>0$, and $\omega_j^{(-\gamma)}=0$ for $j<0$.

In order to find the minimum value of Eq. (25), $\frac{\partial J_{FPFC}}{\partial u(k)}=0$ is considered. And we obtain the optimal control law:

$$u(k)=\left(\overline{C}^{\mathrm{T}}\Lambda(T_S,\gamma)\overline{C}\right)^{-1}\overline{C}^{\mathrm{T}}\Lambda(T_S,\gamma)(Y_r-\overline{B}Y_{past}-\overline{D}U_{past}-E) \tag{26}$$

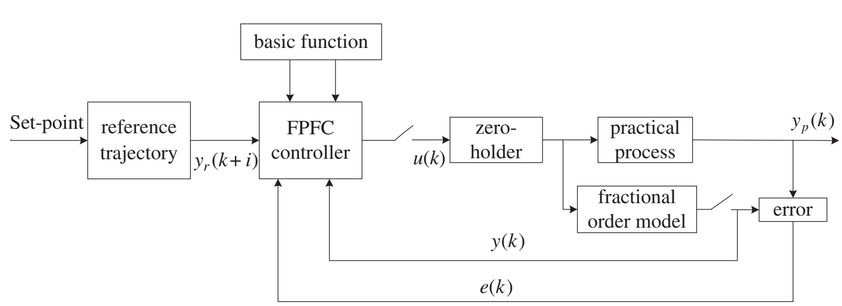

Fig. 1. The flow chat of the PFC based on fractional order model.

4. Examples

In this section, we present the simulation results of FPFC on a practical process to validate the performance through MATLAB simulation. First, the flow chat of the PFC based on fractional order model is shown in Fig. 1. In the flow chart, the reference trajectory and the basic function are chosen beforehand. The error between the model output $y(k)$ and measured process output $y_p(k)$ is used to correct the predictive model output. The FPFC controller based on fractional order model is developed to control the mentioned process. As the fractional order model cannot be employed directly to traditional integer order PFC controller, the Oustaloup approximation has been used to describe the fractional order system.

In [47], the fractional order model of the heating furnace is developed by the measured current process output and the system can be described as follows:

$$c_2 y^{(\alpha_2)}(t) + c_1 y^{(\alpha_1)}(t) + c_0 y(t) = u(t) \tag{27}$$

Based on Eq. (27) and the calculated parameters, the fractional order model of the heating furnace is obtained as follows:

$$G(s) = \frac{1}{14994.3s^{1.31} + 6009.52s^{0.97} + 1.69} \tag{28}$$

As for the fractional order system described by the Oustaloup approximation system, the order of the derivative operator $s^{\alpha}(0<\alpha<1)$ is $N=4$, the lower and higher frequency approximation intervals are $w_b=10^{-6}$ and $w_h=10^{6}$, and the transfer function is obtained as follows:

$$G(s) = \frac{P1(s)}{N1(s)} \tag{29}$$

where,

$$\begin{aligned} P1(s) &= s^8 + 9.948\times10^5 s^7 + 8.417\times10^{10} s^6 + 8.283\times10^{13} s^5 + 7.001 \\ &\quad \times 10^{15} s^4 + 6.889\times10^{15} s^3 + 5.823\times10^{14} s^2 + 5.725\times10^{11} s \\ &\quad + 4.786\times10^7 \\ N1(s) &= 1.086\times10^6 s^9 + 9.961\times10^{11} s^8 + 1.189\times10^{16} s^7 + 1.015 \\ &\quad \times 10^{19} s^6 + 1.506\times10^{20} s^5 + 1.4\times10^{20} s^4 + 5.002\times10^{18} s^3 \\ &\quad + 6.527\times10^{15} s^2 + 1.375\times10^{12}\, s + 8.132\times10^7 \end{aligned}$$

In this paper, we choose the sample time $T_S=1$. Then the model with zero-holder is as follows:

$$G(z) = \frac{P2(z)}{N2(z)} \tag{30}$$

where,

$$\begin{aligned} P2(z) &= 4.656\times10^{-5} z^8 - 1.515\times10^{-4} z^7 + 1.767\times10^{-4} z^6 - 8.401 \\ &\quad \times 10^{-5} z^5 + 1.116\times10^{-5} z^4 + 1.097\times10^{-6} z^3 - 1.481 \\ &\quad \times 10^{-16} z^2 + 5.485\times10^{-33} z + 2.044\times10^{-50} \\ N2(z) &= z^9 - 4.348z^8 + 7.414z^7 - 6.154z^6 + 2.459z^5 - 0.3703z^4 \\ &\quad + 2.787\times10^{-7} z^3 - 1.989\times10^{-24} z^2 + 8.905\times10^{-41} z \\ &\quad + 2.398\times10^{-59} \end{aligned}$$

For the above high order system, we use the optimal reduced order algorithm to get its reduced model. Since the first order model can approximately approach to the original fractional order model well, it is employed for the traditional PFC controller. Hence, the fractional order model can be transformed into the integer reduced order model and the derived first order model is as follows:

$$G(s) = \frac{0.5889}{5138.7s + 1} \tag{31}$$

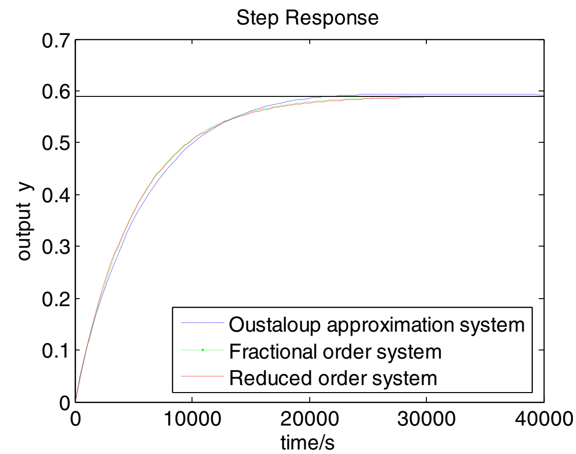

Fig. 2. The open-loop step responses of the heating furnace model.

The step responses of the original fractional order model, the Oustaloup approximation model and the reduced order model are shown in Fig. 2. The approximation results of the two models are satisfied, though the permissible errors till exist in the Oustaloup approximation model and first order model.

In order to verify the control performance of the proposed method, a comparison with standard integer order PFC is made. Here, the FPFC controller is designed based on the Oustaloup approximation model, while the integer PFC controller is based on the first order model. Then, both controllers are applied to control the fractional order model described by the Oustaloup approximation system. The set-point is changed from 0 to 1 at time instant $k=0$, besides, output disturbance with amplitude of -0.1 is added to the process at time instant $k=300$.

Under the model/plant match case, the performance of the FPFC controlled system associated with the parameters change is shown in Figs. 3-4. And Fig. 5 shows the comparison with the standard PFC and the FPFC control system under the model/plant match case. In Fig. 3, the effect of the variable γ is examined by changing it from 0.2 to 5.5. It has been observed that by increasing γ, the settling time of the response is decreased and better performance is obtained. Furthermore, the response rate of the FPFC system cannot be faster after the parameter γ reaches a certain extent. In this figure, FPFC controller parameters are chosen as $\lambda=0.95$, $P=10$. In Fig. 4, the settling time of the output

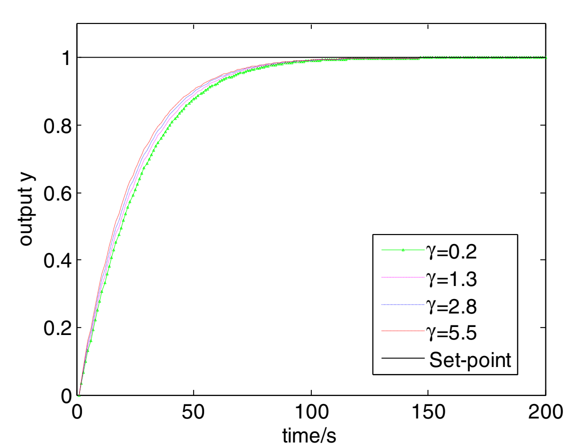

Fig. 3. The closed-loop response of the proposed FPFC for variation in γ.

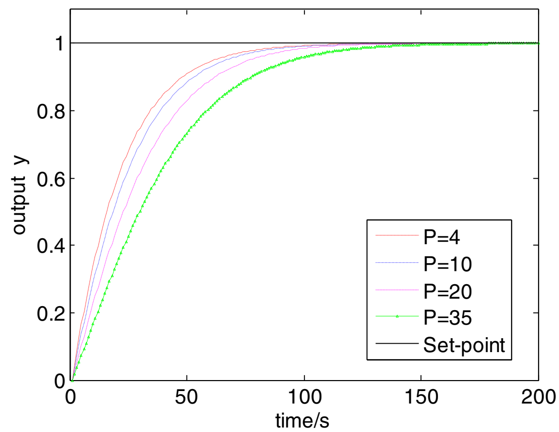

Fig. 4. The closed-loop response of the proposed FPFC for variation in *P*.

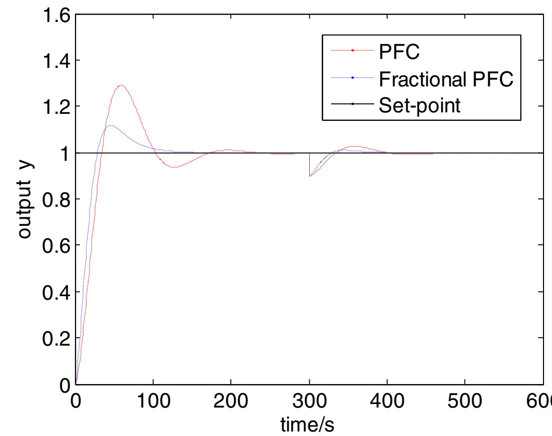

Fig. 6. The closed-loop response under Case 1.

response of the FPFC system becomes longer when P changes from 4 to 35 with $\lambda = 0.95, \gamma = 0.8$.

In Fig. 5, the response of the FPFC has been compared with response of the integer order PFC. The control parameters for FPFC controller have been chosen as $\lambda = 0.95, P = 10, \gamma = 0.8$. For integer order PFC, the common parameters are the same as the FPFC. It shows that the proposed FPFC method can obtain better performance than standard PFC. The response rate of the FPFC controller has been accelerated and the settling time decreases with no overshoots and is quite effective in suppressing disturbance. Meanwhile, the tracking performance, the rise time and the recovery to the set-point under disturbance of FPFC controller are smoother than those of integer PFC.

However, the completely precise model of the process cannot be obtained. The model/plant match is the case that we assumed it to be the ideal state and it never exists. Since uncertainty in practice may lead to model/plant mismatches and impair the performance of controllers, it is particularly important to study control performance under model/plant mismatches. It means the model in Eq. (28) may not precisely describe the output response of the heating furnace and the difference between the model and plant will never be eliminated. In order to simulate the practical process, the Monte Carlo simulation is used to generate parameters randomly at the same time, where the nominal variables are shown in Eq. (27) as $\alpha_1 = 0.97, \alpha_2 = 1.31, c_0 = 1.69, c_1 = 6009.52, c_2 = 14994.3$. The three random model/plant mismatch cases are chosen with a maximum of 30% uncertainty from the original plant in Eq. (28). and the three practical process transfer functions are obtained as

Case 1.

$$G(s) = \frac{1}{19310s^{1.6053} + 5185.4s^{1.1081} + 1.4389} \tag{32}$$

Case 2.

$$G(s) = \frac{1}{18036s^{1.2153} + 4277.5s^{1.1631} + 1.8738} \tag{33}$$

Case 3.

$$G(s) = \frac{1}{19044s^{1.2990} + 5040.1s^{1.1977} + 1.7983} \tag{34}$$

The simulation results are presented by designing the controller using the nominal process model Eq. (27) to control the three practical processes. In the three cases, the parameters for the PFCs are: $\lambda = 0.95, P = 10, \gamma = 0.8$. Figs. 6–8 show the responses of the three cases. From the results of the comparisons, it is easy to find that the performance of the proposed FPFC controller can still maintain satisfactory

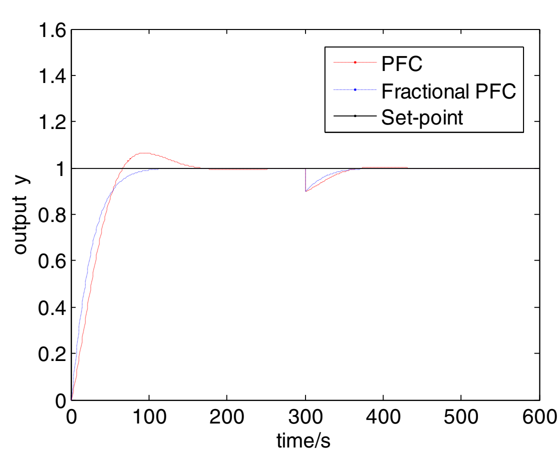

Fig. 5. The closed-loop response under model/plant match case.

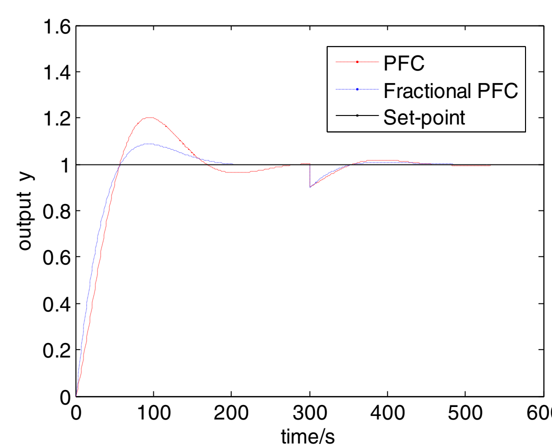

Fig. 7. The closed-loop response under Case 2.

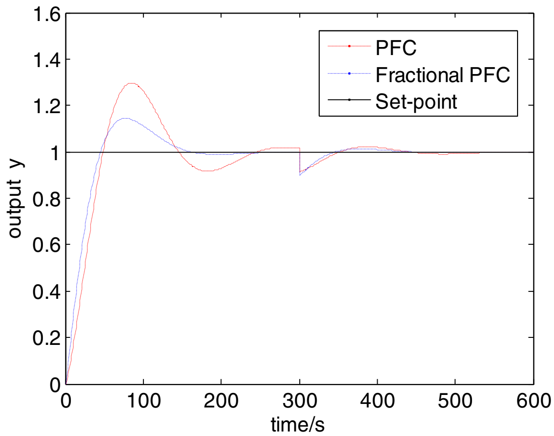

Fig. 8. The closed-loop response under Case 3.

results. In Fig. 6, the output responses of the proposed FPFC controller successfully track the set-point with smaller overshoot than those of integer PFC. On the other hand, there will be a little greater oscillations and longer settling time in the output response of integer PFC control system. In Figs. 7 and 8, the simulations results are the same as those in Fig. 6, the performance of the proposed FPFC controller is better and the overshoots and oscillations are smaller. In general, performance in terms of reference tracking and disturbance rejection is improved based on the proposed FPFC.

5. Conclusion

In this paper, a new approach to PFC using the fractional order system and further improve the performance of the control system is proposed. For this purpose, the proposed FPFC controller based on input–output model has been designed for the system described by SISO linear fractional differential equation. First, the input–output process model was derived from the Oustaloup approximation of fractional order transfer function. Second, the predicted output was transformed into the matrix-form prediction. Then, the GL definition was utilized to discrete the fractional order cost function. Finally, an example of heating furnace has been adopted to verify the performance of the proposed FPFC algorithm. By comparing the responses of FPFC with integer PFC, it is revealed that improved control performance can be achieved. Besides, the fractional order derivative in the cost function of FPFC is proved to enhance the performance because of the additional tuning parameters.

Conflict of interest

The authors declare that there are no conflict of interest.

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