TD Voltage Stability

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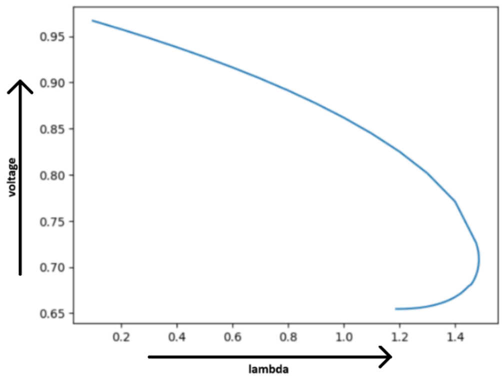
Contents

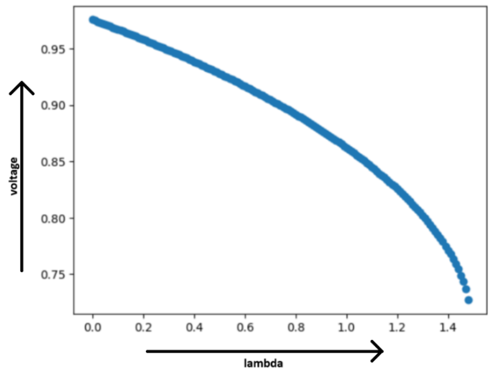
1 Simulation Code

1.1 Continuation Power Flow

To accurately determine the maximum value of load increment multiplier λ, the code for continuation power flow was implemented. Another way of determining the loadability limit, was to repeatedly run the Newton-Raphson power flow with increasing value of λ. The results of both these methods for IEEE 9-bus system are shown below:



(a) CPF plot: lamda(max)=1.485



(b) Alternate Method: lamda(max)=1.49

The code does not handle all issues related to matrix being singular, convergence and divergence of values, so for some systems, the CPF is not able to converge properly, or the plot is not smooth. Sometimes this can be improved by changing the step size of CPF, passed as an optional parameter to CPF function.

It needs to be mentioned that for each value of lamda, the P and Q values at each bus are $P_{orig}(1 + \lambda)$ and $Q_{orig}(1 + \lambda)$.

1.2 H-surface

As present in the paper [1], to compare whether the transmission system or the distribution system is limiting, the H-surface is plotted presenting the variation of loadability limit with sub-station voltage. This plot is generated by repeatedly repeating CPF for different substation voltages.

1.3 CPF with Capacitive Bank

If a capacitive bank is added at a bus, the plot of voltage with lambda max, needs to be generated such that lambda value is not multiplied with the power consumed by capacitor. Making change in CPF, such that it does not apply lambda to some bus, was a little difficult, and the results of CPF were coming similar to the method with repeated Newton-Raphson solutions. Thus for this part, the approach of repeated NR iterations was used, with lambda not being applied to bus on which capacitive bank was to be placed.

1.4 L-index

L-index is a voltage stability indicator. It is used to find the weak buses in the system.

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix}$$

where V_G and V_L are the complex bus voltages at generator and load buses, determined by the power flow solution.

Using this partitioned Y bus matrix, a parameter F_{LG} was defined.

$$F_{LG} = -[Y_{LL}]^{-1}[Y_{LG}] \tag{1}$$

The L-index is computed using F_{LG}.

$$L_j = \left| 1 - \sum_{i=1}^{g} F_{ji} \frac{V_i}{V_j} \right| \tag{2}$$

L-index can take values between 0 and 1. Values near 0 show that system is voltage stable while values near 1 show that voltage is near collapse. This is usually applied to only PQ buses only.

2 Systems Created

The IEEE 9 bus system was used for the transmission system, and the distribution system used had a radial topology. For most of the analysis, the distribution system was attached at bus 5 of transmission system. In IEEE 9 bus system, bus 5 consumes real power near to 90 MW. In initial distribution system, each consumed 9MW of power, so total of 10 distribution systems were attached in parallel. The transmission system has a base voltage of 345kV, and distribution system of 7.2kV. So a transformer is connected between them. Below we describe the topologies of different distribution systems tested.

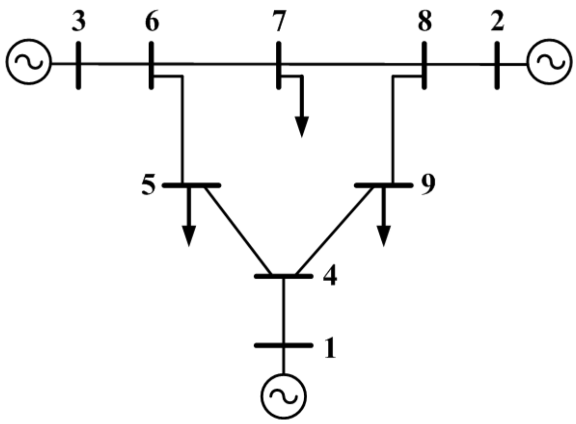


Figure 2: IEEE 9 Bus System, figure taken from [2]

Since multiple distribution systems are identical in parallel, they are combined into one system, with impedance divided and P,Q values multiplied by number of buses. This was done after comparing results of this system with original system, and observing same results.

2.1 Connection Topology 1

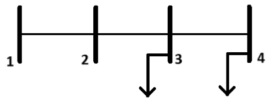
In this topology, the bus 1 of distribution system is connected to bus 5 of transmission, through an extra branch representing the transformer. The problem with this system is that when the combined system is compared with individual system, the extra branch added was not present in either of the two systems originally, so the comparison may not yield relevant results. But these systems are mentioned here due to some pattern observed in it. The following were tested in this:

1. 4 Bus
2. 6 Bus
3. 4 Bus with branching

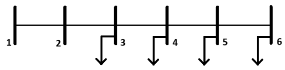
2.2 Connection Topology 2

In this topology, bus 5 of transmission system, and bus 1 of distribution system coincide. So bus 2 of distribution system is attached directly to bus 5 of transmission system. Thus no extra branch is needed here. The transformer is present between bus 5 (transmission system) and bus 2 (distribution system), and its per unit impedance is set to the per unit impedance of distribution line between bus 1 and 2.

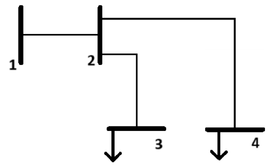
1. 4 Bus
2. 6 Bus



(a) 4 Bus



(b) 6 Bus



(c) 4 Bus with branching

Figure 3: Different Distribution Systems

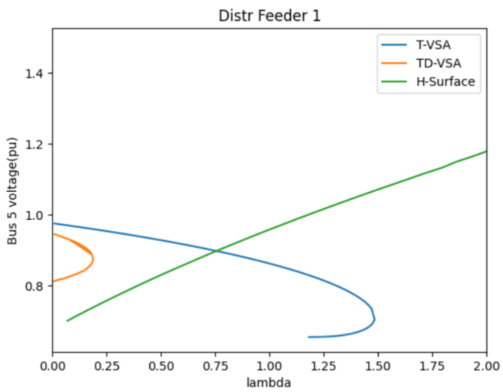
3 Replicating Paper Results

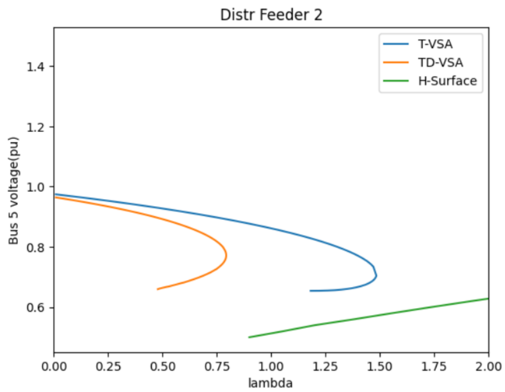
Initially, we tried to do analysis similar to that done in paper[1]. This was done on 4 bus system with connection topology 1. The distribution line impedance(pu) are the following:

	Feeder 1		Feeder 2	
Branch	R	X	R	X
1-2	0.136	0.324	0.054	0.08
2-3	0.102	0.243	0.04	0.036
3-4	0.17	0.405	0.068	0.12

Table 1: Distribution Line Data

The following plots were obtained for this:





According to H-surface analysis[1], in case of feeder 1, combined system is distribution limited, and with feeder 2, combined system is transmission limited.

Load Margin(MW)	Feeder 1	Feeder 2
T-VSA	133.538	133.538
TD-VSA	16.84	71.4

Table 2: Load Margin Estimation for Complete System

Load Margin(MW)	Feeder 1	Feeder 2
D-VSA	10.636	59.38
TD-VSA	1.684	7.14

Table 3: Load Margin Estimation for Distribution System

4 Jacobian Analysis

For 4 bus distribution system in connection topology 1, the system has following buses.

- Bus 2,3 are PV bus, and bus 4-9 are PQ bus, of transmission system.
- Thus for bus 2-9, P value is specified, and for bus 4-9, Q value is specified.
- Bus 10-13 are PQ bus of distribution system.
- The distribution system and transmission system are connected only by bus 5-bus 10 branch.

According to this, Jacobian has following general form

	P			Q		
Bus	2 … 9	10	11 … 13	4 … 9	10	11 … 13
2 ⋮ 9	T		0	T		0
10						
11 ⋮ 13	0		D	0		D
4 ⋮ 9	T		0	T		0
10						
11 ⋮ 13	0		D	0		D

Table 4: General Form of Jacobian

So many entries are 0 because, there is no direct connection between those buses.

As interchanging rows and columns does not change the magnitude of determinant of jacobian(which is considered a measure of closeness to collapse point), so rearranging jacobian by interchanging rows and columns gives this form:

Bus	2 … 9 4 … 9	10 10	11 … 13 11 … 13
2 ⋮ 9 4 ⋮ 9	T		0
10 10			
11 ⋮ 13 11 ⋮ 13	0		D

Table 5: Rearranged Form of Jacobian

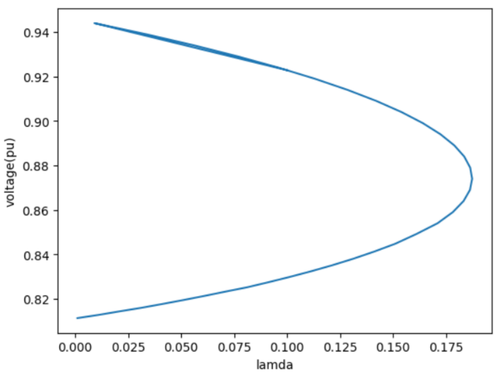
In above Jacobian, the T and D sub matrices can be considered to represent Transmission only and distribution only system. In further analysis, the Jacobian is divided into T and D parts based on this criteria. Note that bus 10 is not considered in any of the two parts, and is taken as a combining extra bus.

If connection topology 2 is used for a system, similar structure is obtained, with the difference that the sub-matrices marked 0, do not have all entries as 0. Also, the extra joining bus 10 is not there. Each bus is part of some sub-matrix.

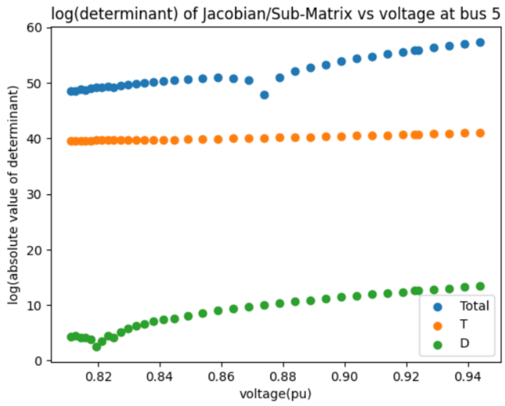
5 Determinant Of Jacobian

5.1 Using CPF

The system mentioned in section 3 feeder 1 was analyzed. The code for calculating CPF was executed, and at end of each iteration, the converged Jacobian was stored. This generated a list of converged Jacobian matrices. Each matrix in this list, was split into transmission and distribution sub-matrices and plotted determinant of each sub-matrix. This generated the below plot:



(a) CPF plot: V at max loadability=0.874



(b) Determinant of different matrices

- In CPF, the voltage at bus 5 corresponding to maximum loadability is 0.874. The dip in determinant of total jacobian matrix is also visible at voltage or 0.87. This verifies the fact that, near loadability limit, the jacobian determinant tends towards 0.
- The trend that determinant of D sub matrix tends to 0 at some other voltage was not investigated further.
- The determinant of total matrix was not becoming very small, as the step size was relatively large to observe that pattern. It was tried to get this by iterative analysis, present in next sub-section.

5.2 Iterative Newton-Raphson

In this method, the newton-raphson power flow was iteratively executed with increasing lambda values. When the loadability limit was crossed, the step size was decreased by factor of 10, and repeated. This was expected to give a finer visualization of determinant values near loadability limit.

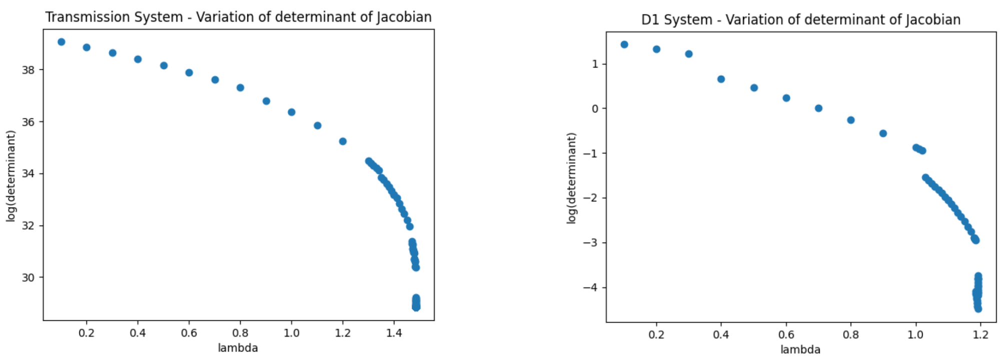


Figure 6: Feeder 1

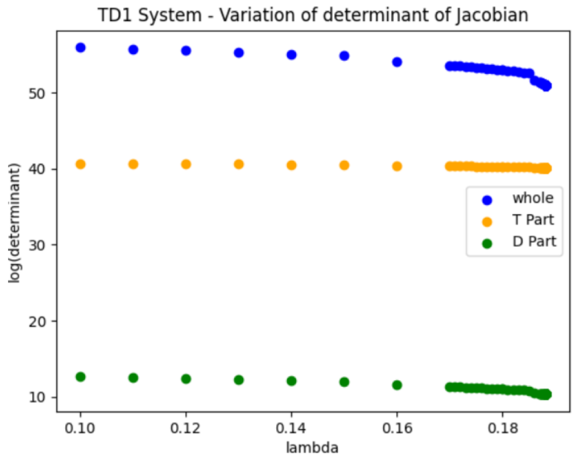
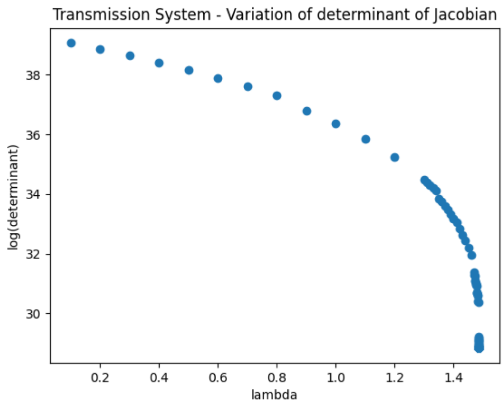


Figure 7: Feeder 1



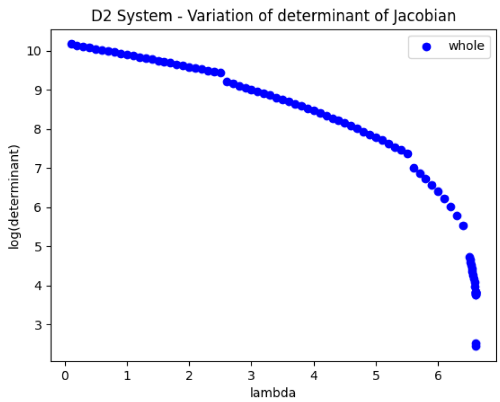


Figure 8: Feeder 2

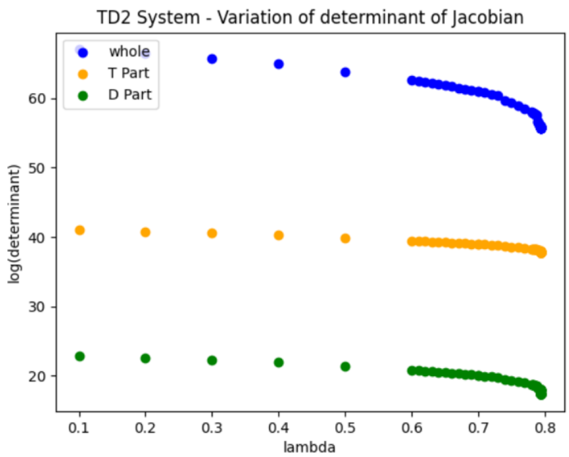


Figure 9: Feeder 2

6 Eigen Value & Condition Number Analysis

6.1 Introduction

Like the determinant analysis, we plotted variation of smallest eigen value of jacobian in CPF. The observed patterns presented below suggest that at the collapse point the smallest eigen value becomes zero. More specifically, at collapse point, the smallest eigen value crosses from right half to left half of complex plane. For the condition number, the numpy function np.linalg.cond(Matrix) was used.

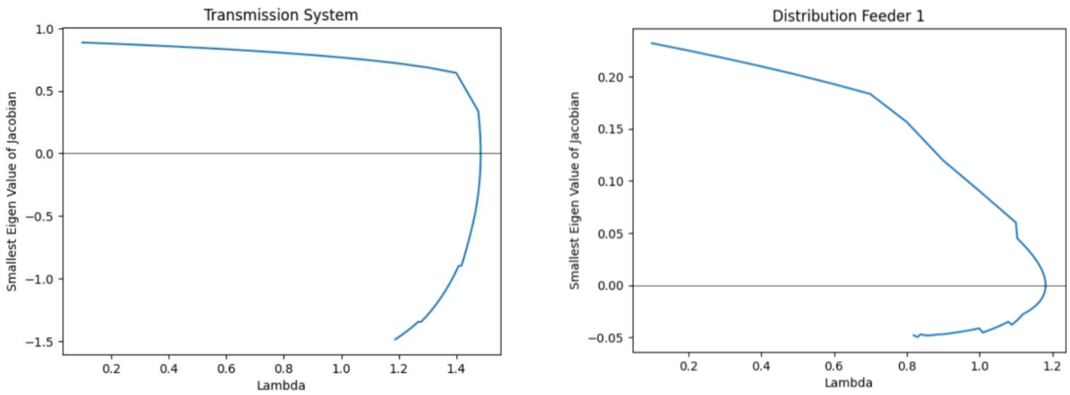
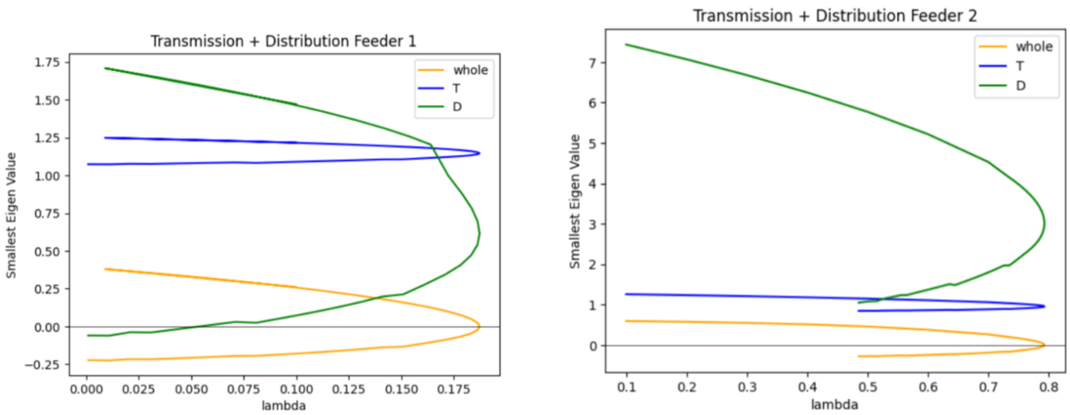


Figure 10: Eigen value analysis for IEEE 9 bus transmission system, and 4 bus distribution system

For more testing of this patter, the plots were generated for combined TD system, for both feeder 1 and feeder 2. Also, the smallest eigen value of T and D sub-matrices was also plotted.



(a) With Feeder 1

(b) With Feeder 2

Figure 11: Eigen value analysis for combined TD System

- It is observed that the plot for combined system crosses y=0 line at the collapse point, suggesting that the pattern of smallest eigen value is satisfied.
- In case of feeder 1, at collapse point, eigen value of D is smaller than eigen value of T. Also in this case combined system is distribution limited(as per H-surface).
- With feeder 2, at collapse point, eigen value of T is smaller than eigen value of D. In this case combined system is transmission limited(as per H-surface).
- This suggested a pattern, that the sub-matrix corresponding to the limiting system, may have smaller smallest eigen value at collapse point. This pattern was further analyzed.

6.2 Adding Impedance Division Factor

To create multiple combined TD Systems, such that they gradually move from distribution limiting to transmission limiting, the variable **impedance division factor** was introduced. For each distribution line:

$$\text{Distribution Line Impedance} = \frac{\text{Base Impedance}}{\text{Impedance Division factor}}$$

The following plot shows H-surface for various impedance division factor, for system in section 3.

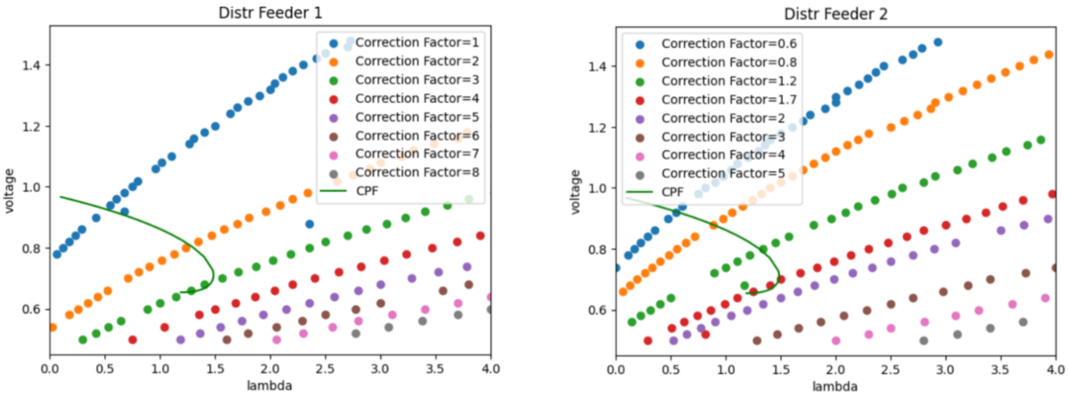


Figure 12: H-Surface and T-VSA for multiple distribution systems

So according to H-surface analysis,

	Critical Impedance Division Factor
Feeder 1	3
Feeder 2	1.7

6.3 Eigen Value & Condition Number variation with Impedance Division Factor

The Jacobian corresponding to lamda equal to loadability limit was found for distribution systems with varying impedance division factor. Each jacobian was divided into transmission and distribution sub-matrices as described above and smallest eigen value of each was plotted. The following plots were obtained:

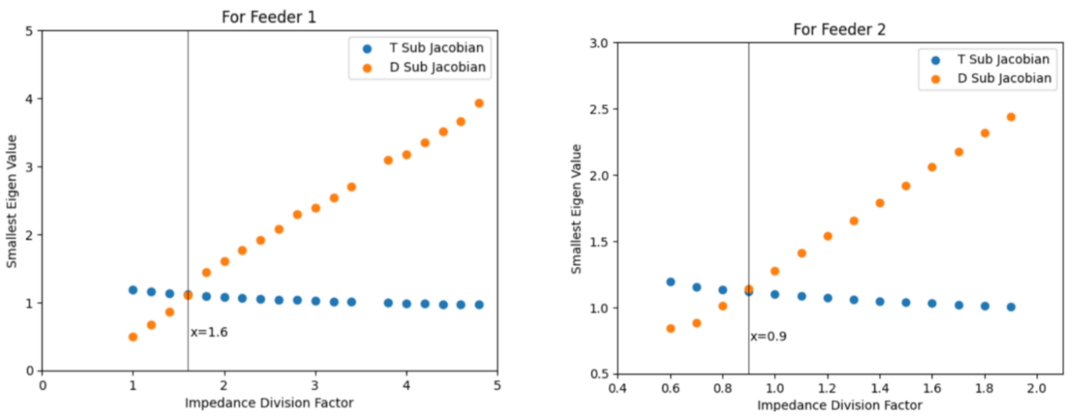


Figure 13: Variation of eigen value with Impedance Division Factor

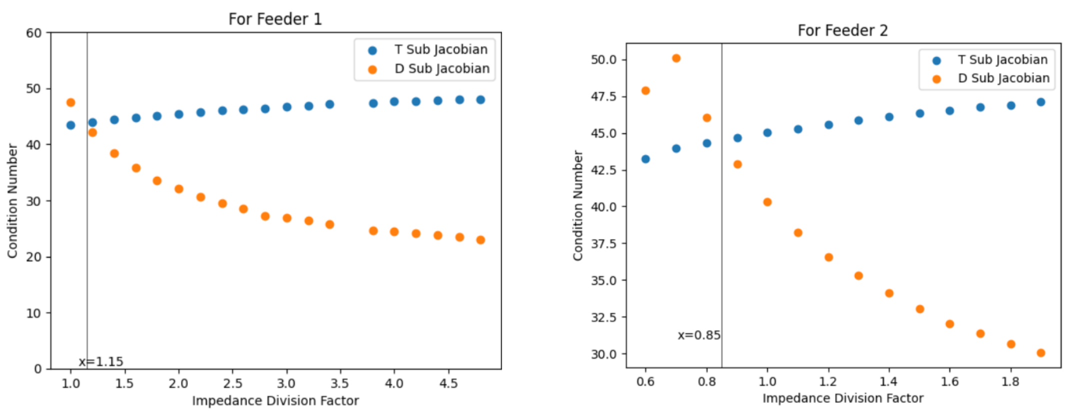


Figure 14: Variation of condition number with Impedance Division Factor

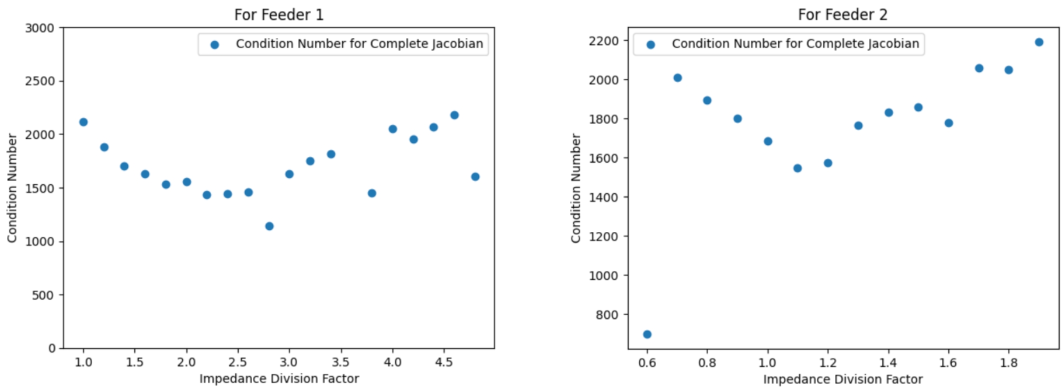


Figure 15: Variation of condition number of complete Jacobian with Impedance Division Factor

If we consider the point where D sub matrix's eigen value becomes greater than T sub matrix's, as the point where combined system changes from distribution limiting to transmission limiting, then we get the following:

	Critical Impedance Division Factor
Feeder 1	1.6
Feeder 2	0.9

6.4 Variation of Loadability Limit with Impedance division factor

We plotted the maximum value of lamda for different combined TD systems, by varying the impedance division factor.

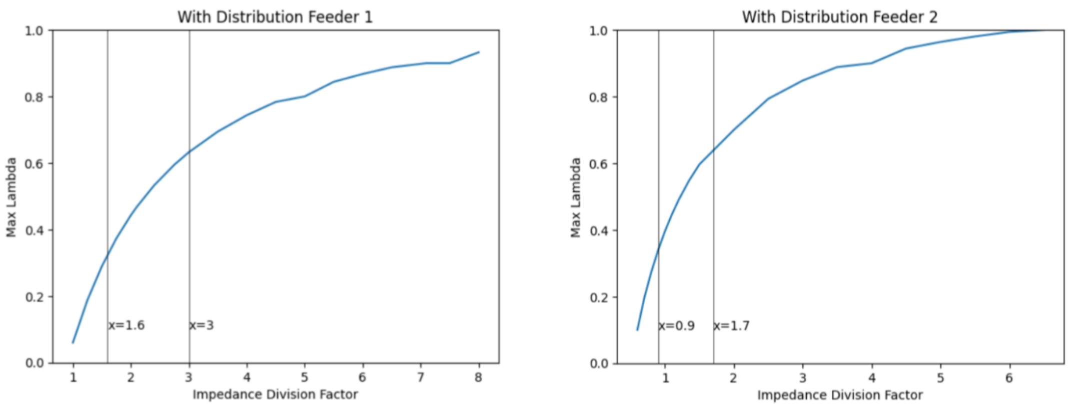


Figure 16: Variation of lamda max value with Impedance Division Factor

7 Relevance of Eigen Values of sub-matrices

As we observed in the above plots, the critical value from H-surface analysis and from eigen value analysis were not matching. The relevance of critical value from eigen value analysis was seen in a different manner.

The plots presented below are for the T-VSA, TD-VSA and H-surface, for different impedance division factor. For critical impedance division factor obtained from eigen value analysis, it was seen that the peek of the combined TD system CPF curve aligned in voltage with the point of intersection of H-surface and transmission only CPF curve.

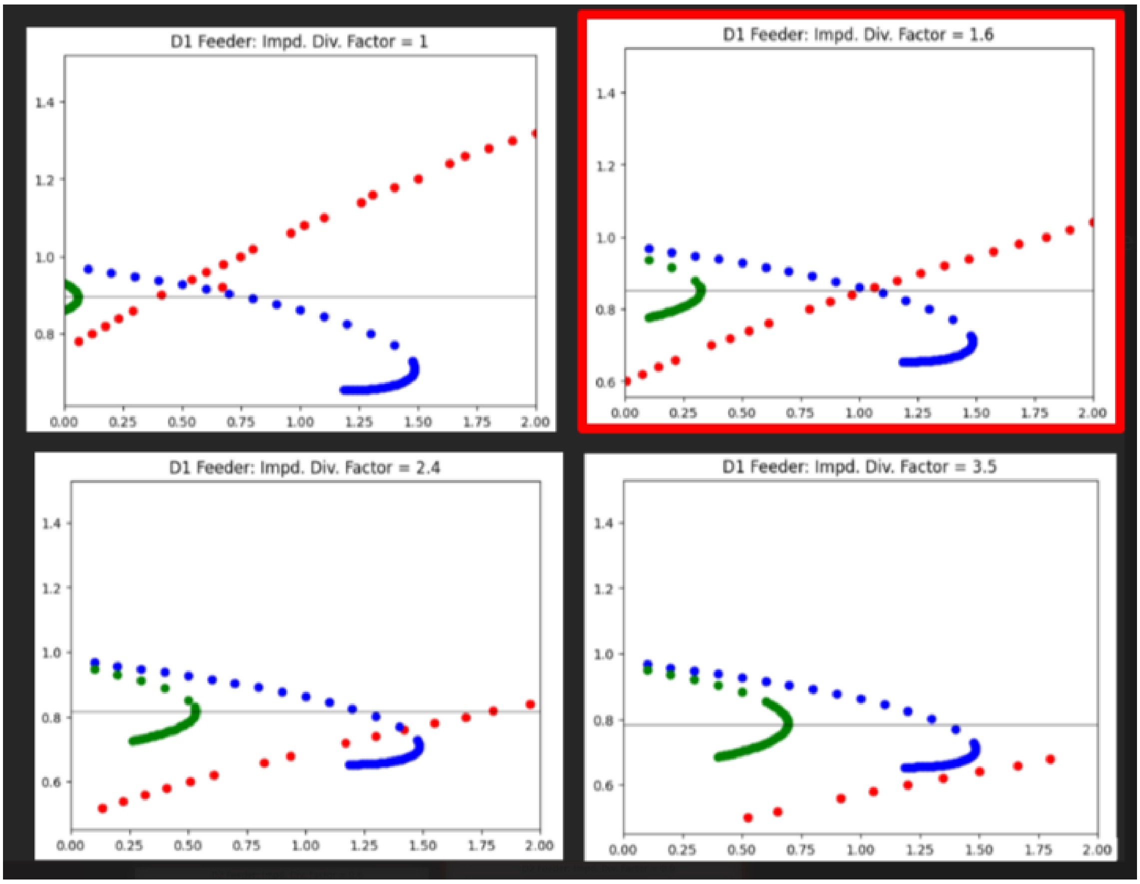


Figure 17: Feeder 1

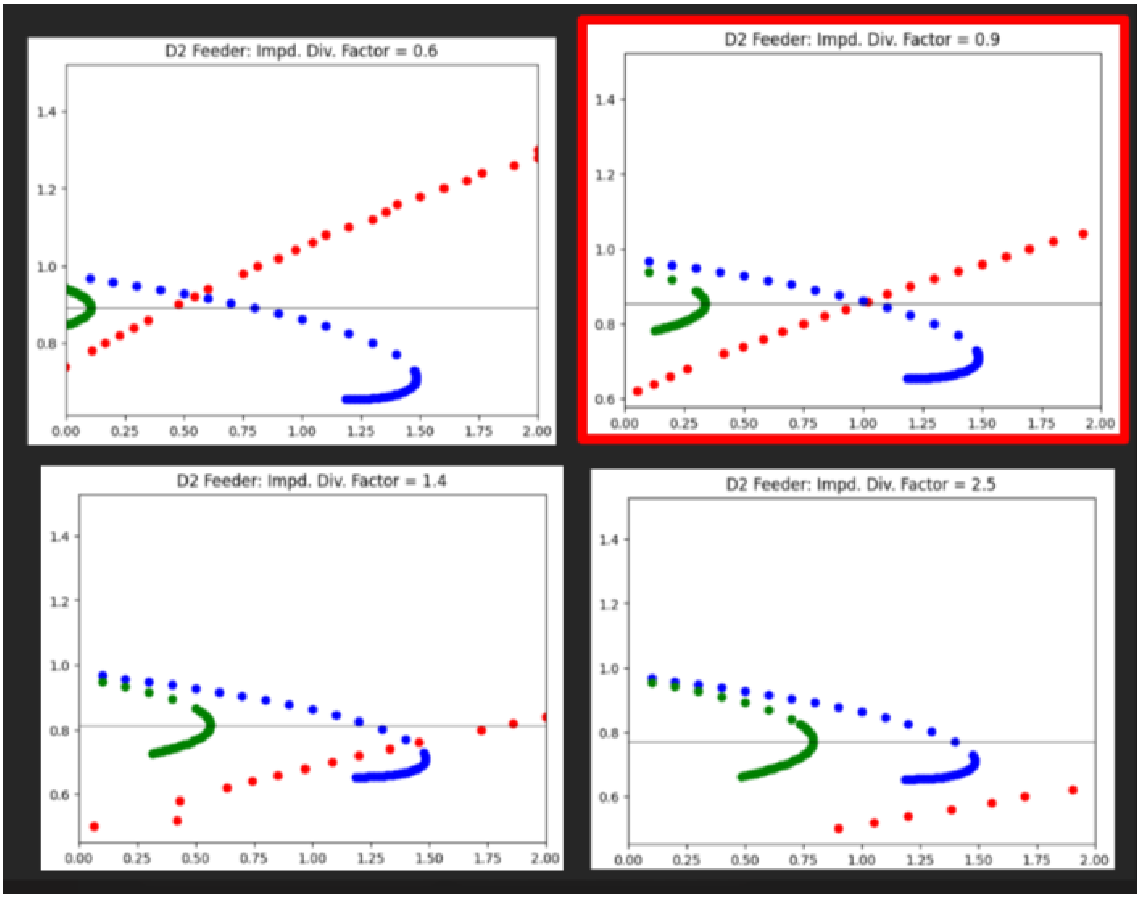


Figure 18: Feeder 2

This trend was visible for both distribution feeders, so it was thought that it may have some significance.

7.1 Testing this for different topology

To further test this idea, the same analysis was performed for 6 bus and 4 bus with branching distribution systems. But this pattern was not visible in them, so this idea was not investigated further.

8 Analyzing Connection Topology 2

In this connection topology, no extra branch was added to connect the transmission and distribution system, i.e., all branches were originally part of either transmission only or distribution only system.

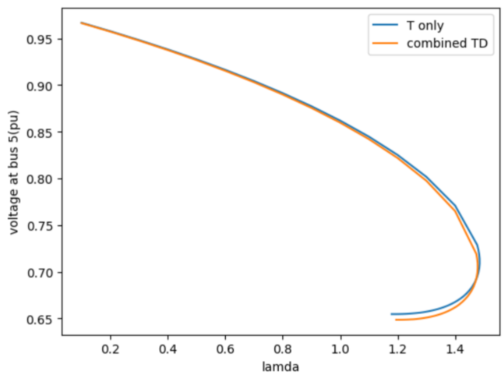
8.1 4 Bus Distribution System

With varying the impedance division factor, it was seen by H-surface analysis, that :

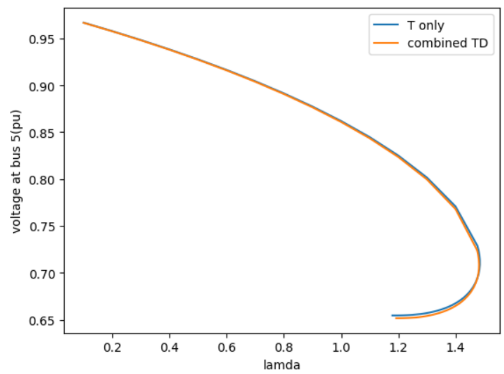
	Critical Impedance Division Factor
Feeder 1	3
Feeder 2	1.7

8.1.1 Comparing T and TD Systems

If the distribution line impedance is very small, then the distribution system should behave like a lumped system. So the CPF curve of combined TD system should align with that of transmission only system. This is also observed in the simulations as shown below:



(a) With Feeder 1



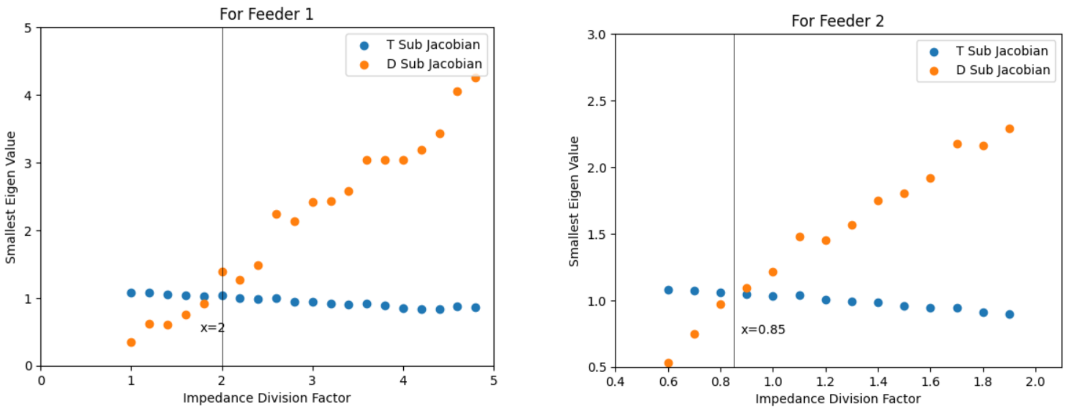
(b) With Feeder 2

Figure 19: CPF plots for T and TD systems, for Impedance Division Factor=50

8.1.2 Eigen Value & Condition Number Variation

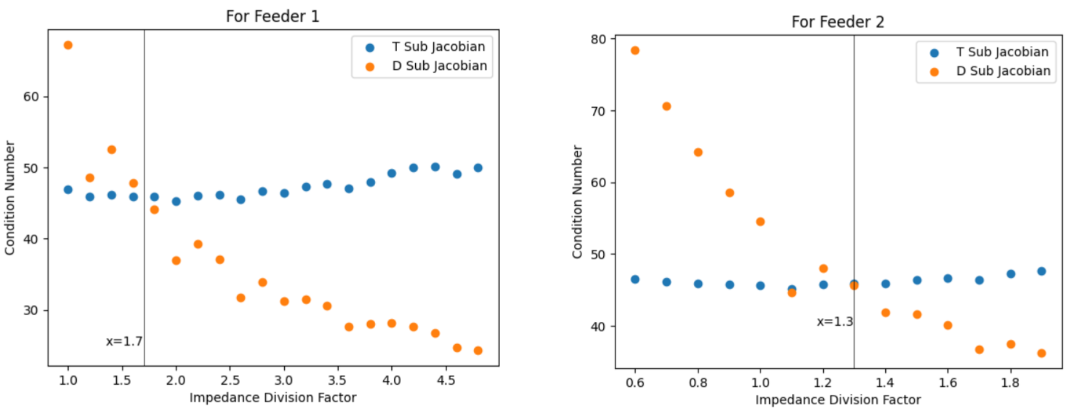
As there was not extra bus in this, the matrix was split into T and D sub-matrices in two ways, one where each bus was part of one sub-matrix, and one in which a junction bus was excluded from both sub-matrices.

1. **Matrix Split 1 :** In this each bus was part of one sub-matrix. The eigen value plots obtained were:



(a) With Feeder 1 (b) With Feeder 2

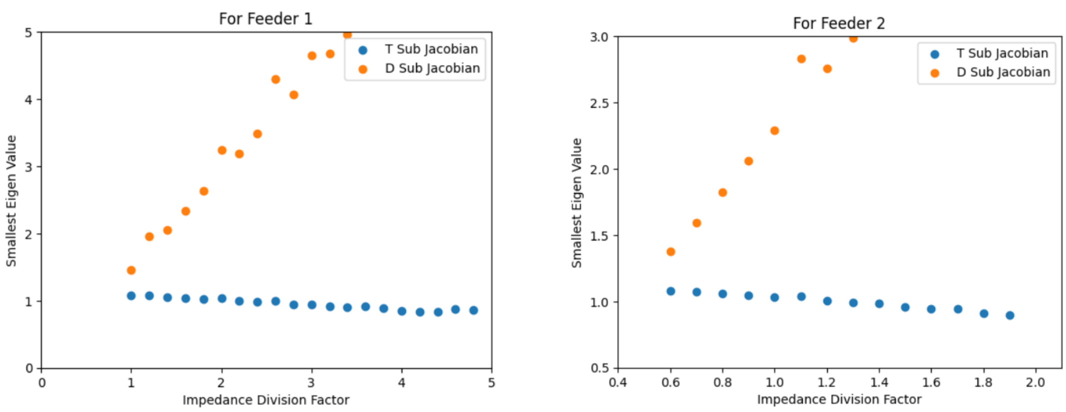
Figure 20: Variation Of eigen value with Impedance Division Factor



(a) With Feeder 1 (b) With Feeder 2

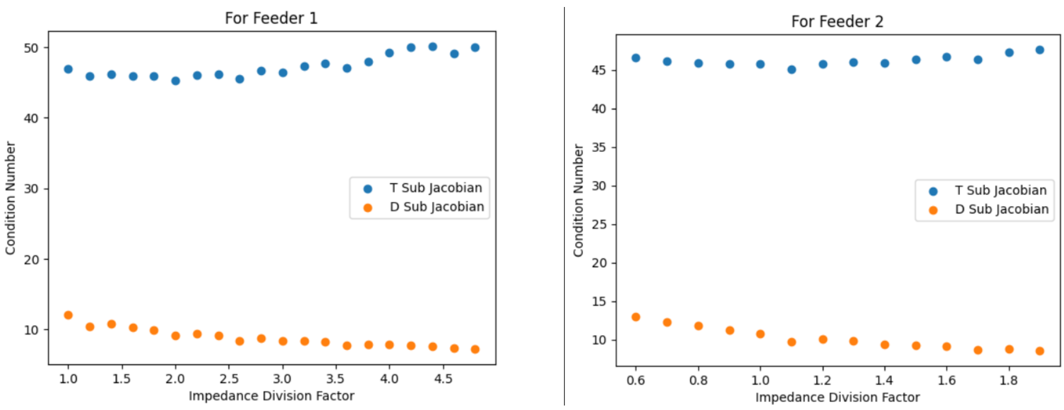
Figure 21: Variation Of Condition Number with Impedance Division Factor

2. **Matrix Split 2 :** In this bus 10 was taken as the junction bus and excluded from both sub-matrices.



(a) With Feeder 1 (b) With Feeder 2

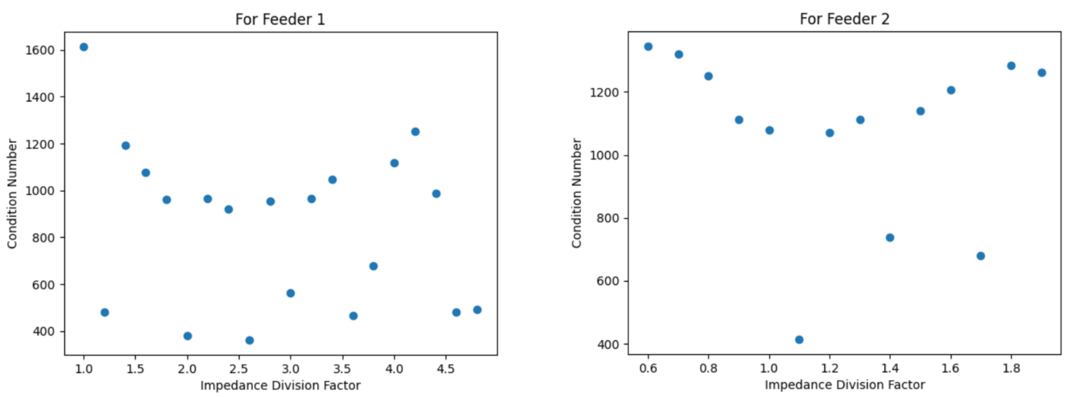
Figure 22: Variation Of eigen value with Impedance Division Factor



(a) With Feeder 1 (b) With Feeder 2

Figure 23: Variation Of Condition Number with Impedance Division Factor

3. **Condition Number of Complete Jacobian:**

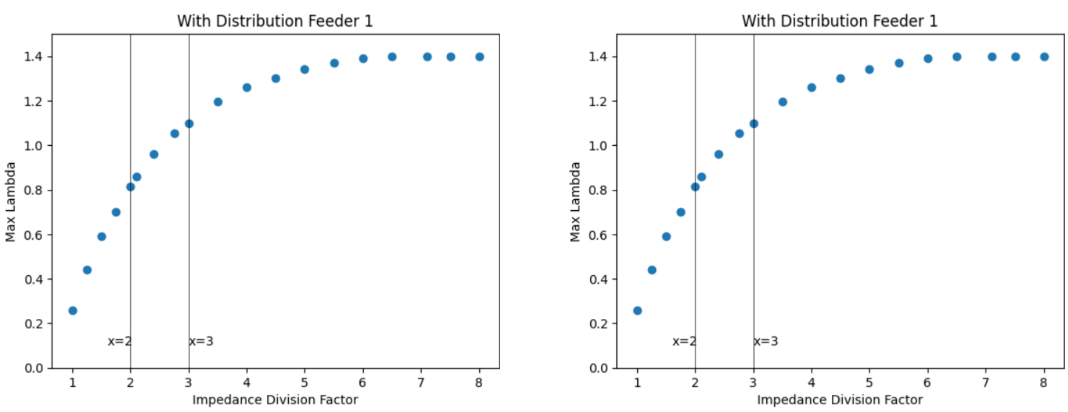


(a) With Feeder 1 (b) With Feeder 2

Figure 24: Variation Of Condition Number with Impedance Division Factor

8.1.3 Loadability Limit Variation

The variation of loadability limit of the combined system with impedance division factor of distribution system was seen. The loadability limit starts to saturate, suggesting that combined system moves from distribution limiting to transmission limiting.



(a) With Feeder 1 (b) With Feeder 2

Figure 25: Variation Of loadability limit with Impedance Division Factor

Note: The plots mentioned in section 7 were also generated for this system, and the pattern was not observed in them.

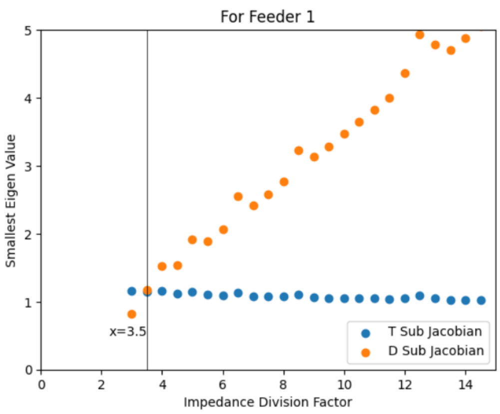
8.2 6 Bus Distribution System

With varying the impedance division factor, it was seen by H-surface analysis, that :

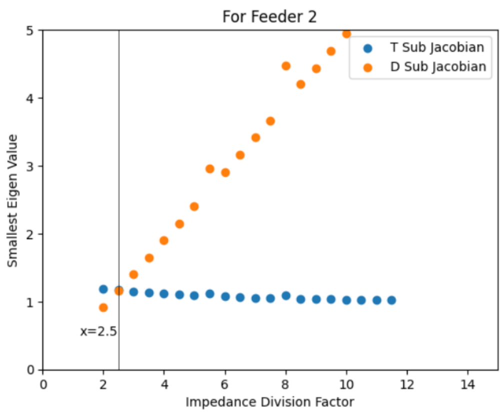
	Critical Impedance Division Factor
Feeder 1	7
Feeder 2	6

8.2.1 Eigen Value & Condition Number Variation

1. **Matrix Split 1 :** In this each bus was part of one sub-matrix. The eigen value plots obtained were:

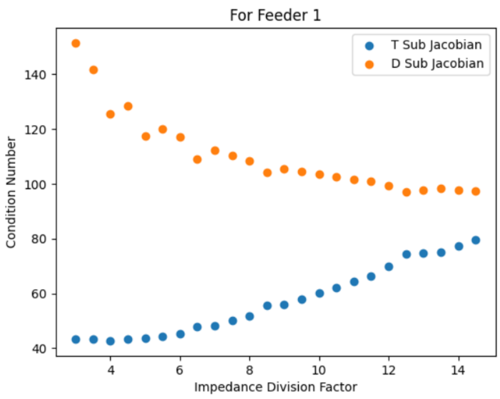


(a) With Feeder 1

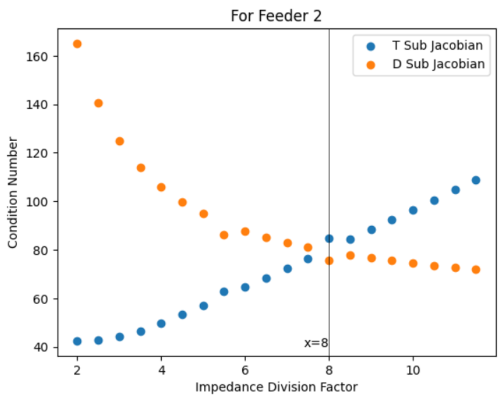


(b) With Feeder 2

Figure 26: Variation Of eigen value with Impedance Division Factor



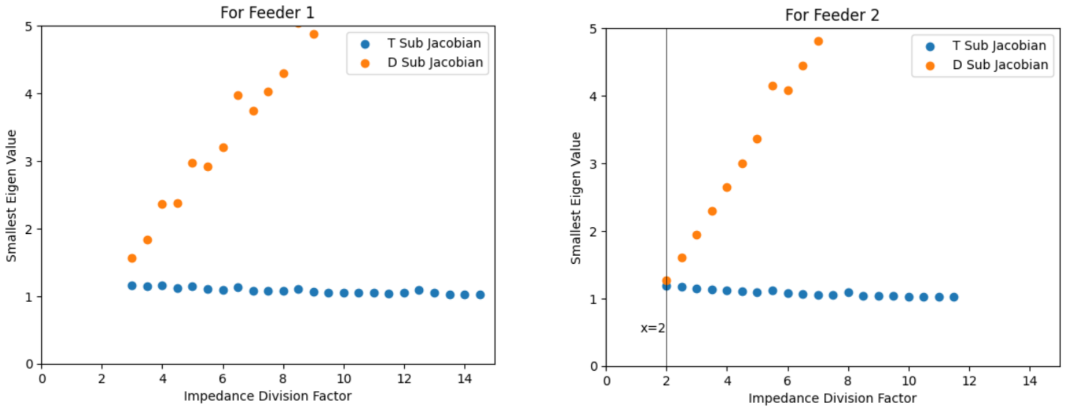
(a) With Feeder 1



(b) With Feeder 2

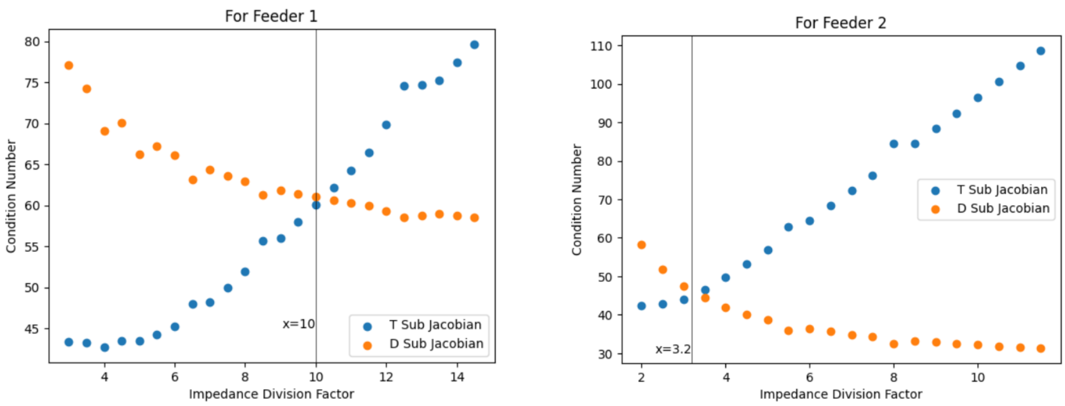
Figure 27: Variation Of Condition Number with Impedance Division Factor

2. **Matrix Split 2 :** In this bus 10 was taken as the junction bus and excluded from both sub-matrices.



(a) With Feeder 1 (b) With Feeder 2

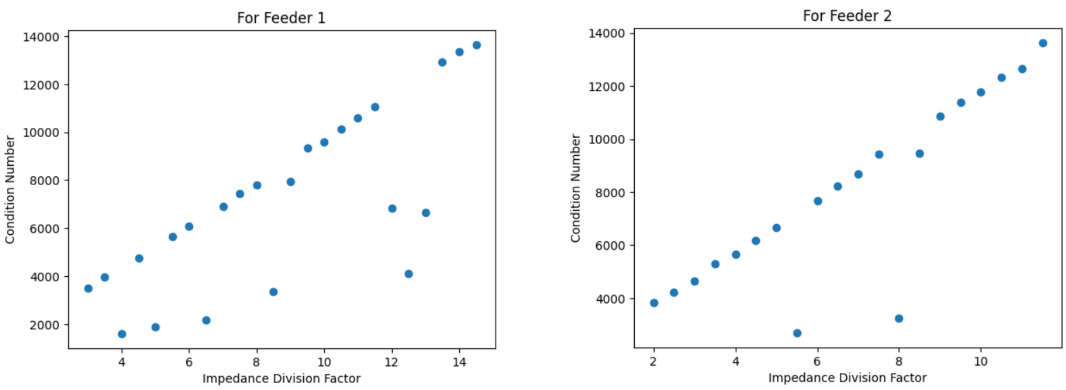
Figure 28: Variation Of eigen value with Impedance Division Factor



(a) With Feeder 1 (b) With Feeder 2

Figure 29: Variation Of Condition Number with Impedance Division Factor

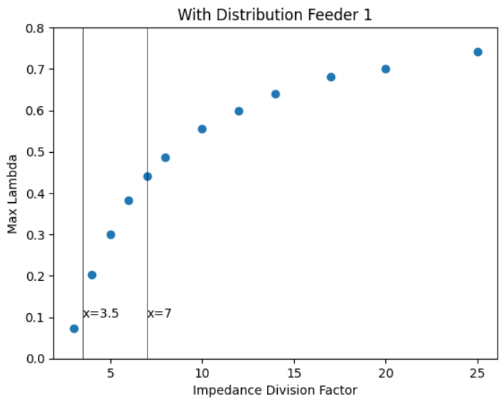
3. **Condition Number of Complete Jacobian :**



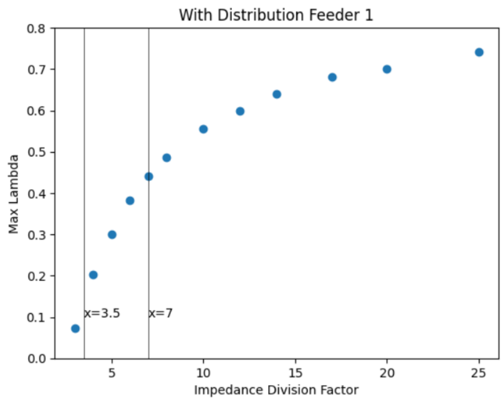
(a) With Feeder 1 (b) With Feeder 2

Figure 30: Variation Of Condition Number of Complete Jacobian with Impedance Division Factor

8.2.2 Loadability Limit Variation



(a) With Feeder 1



(b) With Feeder 2

Figure 31: Variation Of loadability limit with Impedance Division Factor

Note: The plots mentioned in section 7 were also generated for this system, and the pattern was not observed in them.

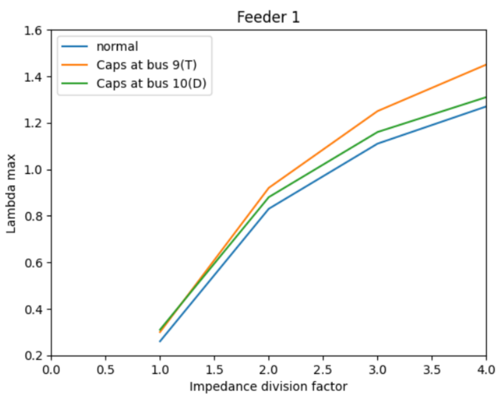
9 Adding Capacitive Bank

In this part, it was tried to observe adding capacitive bank to which bus would give maximum increase in loadability limit. To add the capacitive bank, a power consumption of P=0, Q=-10pu, was added at that bus.

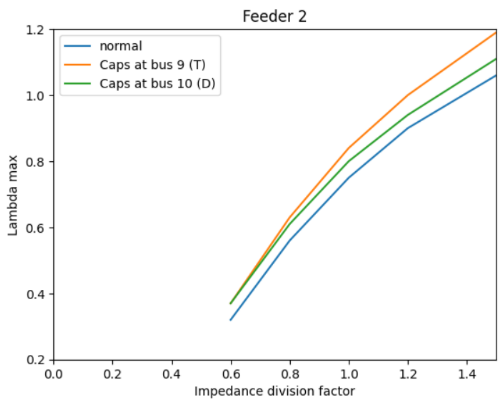
9.1 In Transmission and Distribution Buses

For the combined TD system, we added the capacitive bank at bus 9 (part of transmission system) and bus 10 (part of distribution system) separately, and the effect on loadability limit was varying the impedance division factor.

The system used was with connection topology 2 and 4 bus distribution system.



(a) With Feeder 1



(b) With Feeder 2

Figure 32: Variation Of loadability limit with Impedance Division Factor for different Capacitive Bank Addition

It was seen that as system moved from distribution limiting to transmission limiting, the increase in loadability limit by placing capacitor at transmission system is more than at distribution system.

It was expected that for low impedance division factors, adding at distribution system will give more benefit. But this was not seen, probably due to the fact that the capacitive bank was added at bus 10, which is very close to the transmission system (it is directly connected to bus 5).

9.2 In Limiting and Non-limiting Distribution Buses

- In this part, two distribution system were used. One with Impedance Division Factor as 1, and one with this as 7. This was done for feeder 1 only, and as seen in 25a for 1, combined system is distribution limited, and for 7 it is transmission limited.
- Then 3 type of combined systems were created each with 10 distribution lines attached.
  - System 1: All 10 distribution system had impedance division factor=1, so all were distribution limited.
  - System 2: All 10 distribution system had impedance division factor=6, so all were transmission limited.
  - System 3: 5 distribution limited and 5 transmission limited.
- CPF plots for each of the three system are:

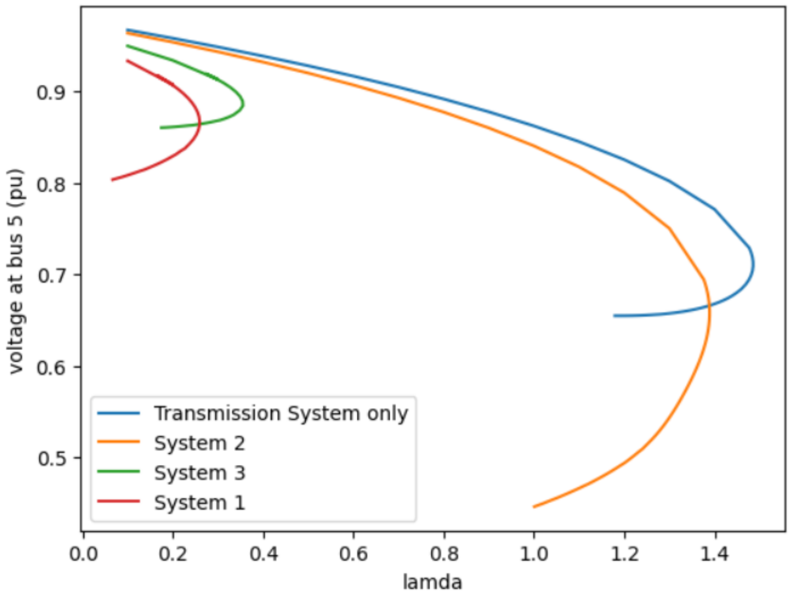


Figure 33: CPF plots for different systems

As expected, as we move from transmission limited to distribution limited, the loadability limit moves away from that of transmission only value.

- For System 3, once capacitor bank was added at each distribution system with factor=1, and once at each bus with factor=6. The loadability limit for them are:

Impedance Division Factor of System where Caps is placed	Maximum Lamda
No Caps	0.36
1	1.42
6	0.45

Table 6: Loadability limit by placing capacitor

9.3 In Transmission System

The method of finding change in loadability limit by adding capacitive, was also use on transmission system only, to determine which bus could be weak. The idea was, the bus that gives maximum increase in loadability limit by placing capacitive bank would be the weakest. This was tested for IEEE 9 bus system, with P+jQ=0-10j, being placed for capacitive bank. In this regard the following data was observed.

Condition	Loadability Limit
No Capacitive Effect	1.49
Caps at Bus 5	1.55
Caps at Bus 7	1.1
Caps at Bus 9	2.07

Table 7: Loadability Limit for different Capacitor Placement

So according to this bus 9 is the weakest bus. Another point observed in this is that adding capacitor at bus 7, leads to decrease in loadability limit, which was not expected.

10 L-index Analysis

We found the L-index values of the buses in IEEE-9 bus system.

Bus Number	L-index value
5	0.9345
7	0.9567
9	0.9135

From this, we got that bus 5 weaker than bus 9.

References

[1] A. Singhal and V. Ajjarapu, "Long-term voltage stability assessment of an integrated transmission distribution system," 2017 North American Power Symposium (NAPS), Morgantown, WV, USA, 2017, pp. 1-6, doi: 10.1109/NAPS.2017.8107402.

[2] Small-disturbance angle stability analysis of microgrids: A graph theory viewpoint - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Diagram-of-the-IEEE-9-bus-test-system_fig2_303381482