Unraveling as Endogenous Market Thinness

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Outline

Law Clerkships

- The very best law school graduates want high-prestige clerkships
- The very best judges would like to hire the very best graduates as clerks
- All of these positions are filled after summer of 1st year. (Law school is 3 years)

Medical Residencies

- The very best medical school graduates want high-prestige residencies
- The very best residencies would like to hire the very best graduates as residents
- This market clears smoothly with an orderly centralized deferred acceptance process

Past Research

- Past modeling of unraveling uses:
  - Risk aversion
  - Cost of using match process (ie, frictions)
- Past empirical work appeals to stability of matching
  - It is a usually true various medical matching markets that if the process gives a stable allocation, it works
  - That said, some markets with stable allocations have still unraveled (eg, GI fellowships)

Past Research

- Fails to capture “endogenous market thinness”
- Do not explain why some stable clearinghouses fail

Model Goals

- Friction-less access to stable matching
- Stability at unraveled state
- Stable matching pareto preferred to unraveled state

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  - Buyers (eg. employers)
  - Sellers (eg. employees)

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- Second period DA with fixed contracts

Agents

- Sellers mass 1, indexed $i \in (0, 1]$
- Buyers mass 1, ex ante homogeneous, indexed $j \in (0, 1]$

Valuations

- When buyer and seller contract, a surplus of $i + \alpha\epsilon_{ij}$ is generated
- For each Buyer/Seller pair ϵ_{ij} is randomly drawn from Uniform $[0, 1]$

Mechanics

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- In period 1, buyers and sellers are matched randomly. Buyer has opportunity to make TIOLI offer. If they do not contract, they both move on to the next period.
- In period 2, remaining buyers and sellers engage in a deferred acceptance market. In this DA market, buyer receives a fixed share η of the surplus from trade

First Understand Period 2

Lemma

For any buyer and seller $\{s_i, b_j\}$ matched in period 2:

$$\epsilon_{ij} = 1$$

Definition

$\omega(i)$ is the probability that seller s_i has chosen to proceed to period 2

Period 2 Outcomes

Lemma

Each seller s_i who proceeds to period 2 expects a payoff of

$$\eta(i+\alpha) \tag{1}$$

Lemma

Each buyer expects a payoff of

$$\eta\left[\frac{\int_0^1 \hat{i}\omega(\hat{i})d\hat{i}}{\int_0^1 \omega(\hat{i})d\hat{i}}+\alpha\right] \tag{2}$$

Period 1 Outcomes

Lemma

Agents i and j matched in period 1 will contract early if:

$$i + \alpha\epsilon_{ij} > (1-\eta)(i+\alpha) + \eta\left[\frac{\int_0^1 \hat{i}\omega(\hat{i})di}{\int_0^1 \omega(\hat{i})d\hat{i}} + \alpha\right] \quad (3)$$

Guess and Verify

- First see that for each value of i, there is some value $\epsilon^*(i)$ such that inequality 3 is satisfied iff $\epsilon_{ij} > \epsilon^*(i)$

$$\epsilon^*(i) = (1-\eta) + \frac{\eta\left[\frac{\int_0^1 \hat{i}\omega(\hat{i})di}{\int_0^1 \omega(\hat{i})d\hat{i}} + \alpha - i\right]}{\alpha} \tag{4}$$

- Guess:

$$\epsilon^*(i) = a - \frac{\eta}{\alpha}i \tag{5}$$

for $a \in [0, 1 + \frac{\eta}{\alpha}]$

Elaborate on $\omega(\cdot)$

As we have specified that ϵ_{ij} $U[0, 1]$, we also can note that

$$\omega(i) = \begin{cases} 0|\epsilon^*(i) < 0 \\ \epsilon^*(i)|\epsilon^*(i) \in [0, 1] \\ 1|\epsilon^*(i) > 1 \end{cases} \tag{6}$$

The Part Where I Skip The Algebra

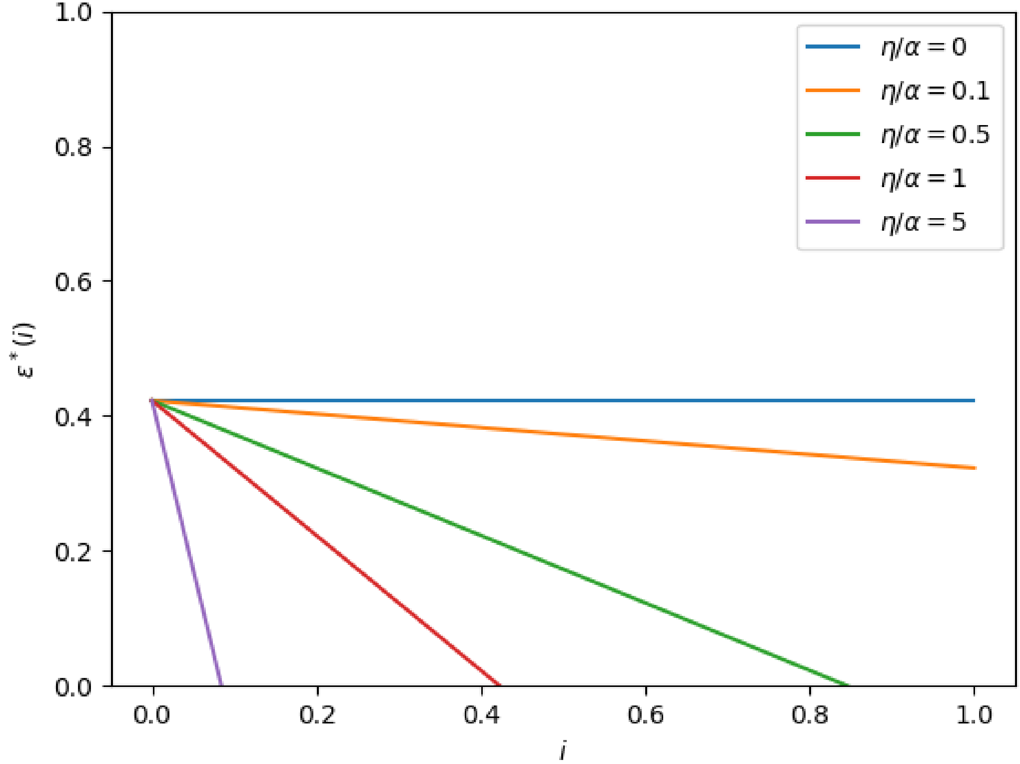
Combining equations 4, 5 and 6, I find:

$$\epsilon^*(i) = 1 - \frac{\sqrt{3}}{3} - \frac{\eta}{\alpha}i \tag{7}$$

(* This is almost but not quite true)

Outline

$\epsilon^*(i)$ is a sufficient statistic



Welfare

- TIOLI structure means sellers are unaffected by unraveling
- Buyers are worse off because of unraveling
  - Haven't quantified this yet, but since Sellers see no welfare change and the realized values of ϵ_{ij} are lower, this is clear

Endogenous Component

The next step here is to identify how much early contracting is caused by the endogenous of:

$$\eta \left[\frac{\int_0^1 \hat{i}\omega(\hat{i})d\hat{i}}{\int_0^1 \omega(\hat{i})d\hat{i}} + \alpha \right]$$

Defining Stability

A stable matching is a matching between the two sides of the market that is

1. feasible
2. individually rational
   - no agent would prefer to be unmatched
3. free of blocking pairs
   - no two agents would prefer to be matched to each other instead of their current assignments

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