

# IFRS9 Business Case

---

FORECASTING DELINQUENCY RATES FOR A COVID-19 INSPIRED  
ECONOMIC SCENARIO

Tommaso Garutti  
tgarutti@gmail.com  
22 November 2022

**Contents**

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>Data</b>	<b>3</b>
<b>3</b>	<b>Methodology</b>	<b>3</b>
<b>4</b>	<b>Results</b>	<b>3</b>
4.1	Limitations . . . . .	4
<b>A</b>	<b>Appendix</b>	<b>5</b>
A.1	Data and Statistical Tests . . . . .	5
A.2	Estimation of the Markov-Switching Model . . . . .	6
A.2.1	Forecasting Performance Analysis . . . . .	6
A.2.2	Benchmark Models . . . . .	7

# 1 Introduction

The aim of this memo is to investigate the relationship between delinquency rates of different loan categories and macroeconomic variables. A Markov-Switching model is implemented to model delinquency rates based on historical macroeconomic data. The performance of the model is compared to several benchmark models which are well-known in the literature. The model is then applied generate forecasted delinquency rates for an economic scenario based on the current Covid-19 crisis. Finally, the limitations and further development opportunities of the approach are discussed.

# 2 Data

The dataset considered for the fitting of the model is comprised of the delinquency rates of different loan categories and the historical macroeconomic variables. The non-seasonally adjusted (NSA) delinquency rates for commercial banks from 1987 Q1 to 2020 Q1 are given for five different loan categories: real estate loans, consumer loans, leases, C&I loans and agricultural loans. The historical macroeconomic variables considered are the quarterly 10-year swap rate, 3-month interbank rate, and unemployment rate as well as the quarter-on-quarter (QoQ) annual change in the GPD and house price index. Due to a lack of historical house price index data, the QoQ annual change can only be retrieved from 1992 Q1. Hence, the dataset considered ultimately consists of 28 years of quarterly data from 1992 Q1 until 2020 Q1.

The NSA delinquency rates for the various loan categories are plotted in Figure 2. Several of these time series seem to exhibit pronounced seasonal patterns and moderate downward trends, which indicates that the series may be non-stationary. To test for non-stationarity, the augmented Dickey-Fuller (ADF) test is performed. Table 2 shows the ADF test results for the quarterly delinquency rates (Panel A) and QoQ rate changes (Panel B). The test confirms that the quarterly delinquency rates exhibit non-stationarity for all loan categories, while the QoQ rates are stationary. The QoQ rates are plotted in Figure 3 evidencing that the trend and seasonality patterns are not as pronounced anymore.

# 3 Methodology

The QoQ delinquency rates consistently rise during recession periods, hinting towards the use of a non-linear model to capture the different behaviour of the economic and financial factors during growth and recession periods. The Markov-Switching (MS) model of Hamilton (1989) is a widespread non-linear time series models that allows for the characterization of the time series behaviours in different regimes or states. By permitting the switching between these specifications, this model is able to capture more complex and dynamic patterns.

To model the QoQ delinquency rates, two regimes are assumed, namely economic expansions and recessions. The resulting univariate MS model (ARMSX(1)) can be defined as

$$y_t = \mu_{s_t} + \phi y_{t-1} + X_t \beta_{s_t} + \varepsilon_{s_t} \quad (1)$$

where  $y_t$  denotes the QoQ delinquency rate change at time  $t$ ,  $s_t$  is the state, and  $\mu_{s_t}$  is the state-dependent intercept,  $y_{t-1}$  is the first lag of the endogenous variable with a state-invariant coefficient  $\phi$ ,  $X_t$  is a matrix of exogenous variables with state-dependent coefficients  $\beta_{s_t}$ , and  $\varepsilon_{s_t} \sim N(0, \sigma_{s_t}^2)$  is an i.i.d. normal error term with a state-dependent variance. As exogenous variables, the quarterly 10-month swap rate, 3-month interbank rate and unemployment rate as well as quarter-on-quarter change in GDP and house price index are considered.

In MS models, in addition to estimating the means, variances and other model parameters of each regime, the probability of regime change is estimated as well. Each period  $t$ , the state changes according to the following matrix of transition probabilities:

$$P(S_t = s_t | S_{t-1} = s_{t-1}) = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \quad (2)$$

where  $p_{ij}$  is the probability of transitioning from regime  $i$  to regime  $j$ . A more detailed explanation of the estimation procedure of the Markov-Switching model can be found in Appendix A.2.

# 4 Results

An out-of-sample forecast performance analysis is carried out in Appendix A.2.1, to compare the forecasting performance of the MS model against two well-known benchmarks. The MS model outperforms the two benchmarks in predicting the delinquency rates of four out of five of the considered loan categories.

Table 1 displays the forecasted delinquency rates using the MS model from 2020 Q2 until 2021 Q4. For each loan category, the forecast error is calculated as the mean of the absolute residuals of each fitted model.

This gives an indication of the potential estimation uncertainty of the MS model. On average, the forecasted delinquency rates deviated  $\pm 0.12\%$  from the actual values.

	Real Estate	Consumer	Leases	C&I	Agricultural
2020 Q2	1.48%	2.29%	0.77%	2.39%	1.58%
2020 Q3	0.94%	1.91%	1.01%	1.08%	1.70%
2020 Q4	1.54%	2.66%	1.20%	2.22%	1.87%
2021 Q1	1.40%	2.38%	1.27%	1.56%	2.49%
2021 Q2	0.87%	1.88%	0.65%	2.06%	1.65%
2021 Q3	0.25%	1.45%	0.87%	0.54%	1.77%
2021 Q4	0.93%	2.27%	1.07%	1.78%	1.95%
MAD	$\pm 0.12\%$	$\pm 0.08\%$	$\pm 0.10\%$	$\pm 0.14\%$	$\pm 0.14\%$

Table 1: The forecasted delinquency rates for all loan classes based on the economic scenario. The mean absolute deviation (MAD) is defined as the mean of the absolute residuals of each fitted MS model.

The forecasted delinquency rates based on Covid-19 inspired economic scenarios are displayed in Figure 1. The forecasted delinquencies of the agricultural and the C&I loans seem to exhibit a slight upward trend in the period of the economic scenario, as well as an increase in volatility. The forecasted delinquencies of the leases and the customer loans seem to display little change during the period of the economic scenario. Contrarily, the real estate loan defaults decreased over the period of the economic scenario. This downward trend in the real estate loan delinquency rates is especially unexpected, as the GDP in the economic scenario sharply decreases.

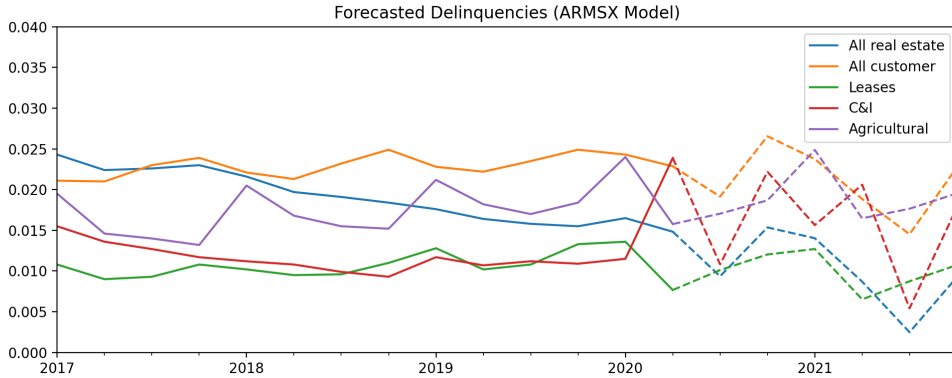


Figure 1: Delinquency rates of all loan categories. The actual delinquency rates are illustrated by the full line, while the forecasted delinquency rates by the dashed line.

#### 4.1 Limitations

The fact that there is little increase in the delinquency rates during the period of the Covid-19 inspired economic scenario, and each delinquency rate seems to follow a similar trend as in the previous years, suggests the autoregressive component of the MS model could be the main driver of the forecasts. A dominant autoregressive component would imply that the endogenous variable is less sensitive to changes in the exogenous economic variables, which would be significant especially in the state transition periods, when the economic variables exhibit significant changes.

An additional limitation of the MS model is that the same functional form is used for each loan category, as well as for each state. Adjusting the hyperparameters such as the number of lags of the endogenous variable, or testing further which exogenous variables should be included with each loan category could yield better results. Furthermore, it is assumed that the business cycle consists of only two states; further research could be performed to extend the MS model for a higher number of states. Alternatively, since the VARX(1) model outperforms the ARX(1) model for almost all loan categories, as shown in Table 3, this suggests that extending the univariate MS model to a multivariate MS model could capture the correlation between delinquency rates of different loan categories. While there are numerous opportunities for further developing the model, the results of the out-of-sample forecast analysis show that the MS model still outperforms several widely recognized benchmarks in forecasting delinquency rates.

## A Appendix

### A.1 Data and Statistical Tests

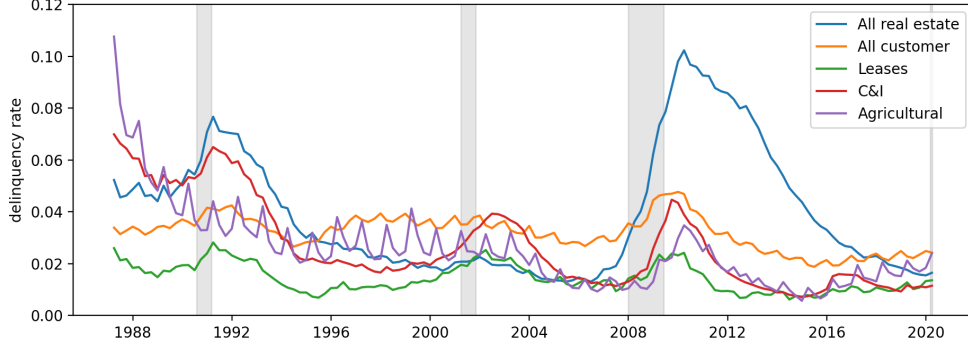


Figure 2: Delinquency Rates (NSA) for all U.S. commercial banks from 1987 to 2020. The shaded grey areas indicate NBER-based recession periods.

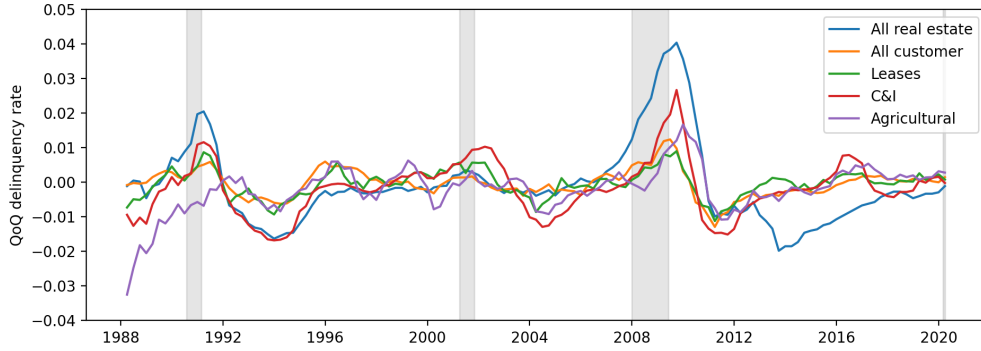


Figure 3: Quarter-on-quarter Delinquency Rates for all commercial banks from 1987 to 2020. The shaded grey areas indicate NBER-based recession periods.

Panel A: Quarterly rates					
	Real Estate	Consumer	Leases	C&I	Agricultural
ADF Statistics	-2.463	-2.092	-2.472	-1.934	-2.395
p-value	0.125	0.248	0.122	0.316	0.143
Panel B: Quarter-on-quarter change					
	Real Estate	Consumer	Leases	C&I	Agricultural
ADF Statistics	-2.226	-3.765	-3.498	-3.370	-3.521
p-value	0.197	0.003***	0.008**	0.012**	0.007***

Table 2: The ADF test is a unit root stationarity test without structural breaks. The results of the ADF test performed on the original quarterly delinquency rates data and the quarter-on-quarter annual change delinquency rates. All quarterly delinquency rates are found to be non-stationary, while all but one QoQ transformed loan category are stationary with at least a 5% (\*\*) confidence level. The final loan category (Real Estate QoQ) is found to be stationary, at a 5% confidence level, using the Zivot-Andrews (ZA) test, which, similarly to the ADF test, tests for a unit root, but includes a single structural break in the dataset.

## A.2 Estimation of the Markov-Switching Model

The univariate Markov-Switching model is described as a two-state process in Equation (1), where the sets of state-dependent parameters differ in correspondence of the regime of economic expansion and the regime of economic recession. The two states are not fixed with a predefined period for each state, but taken as a discrete unobserved random variable  $s_t$ , where the transition from state to state is driven by an ergodic Markov chain.

Therefore, for each period of the dataset, the state follows a Markov transition probability matrix, exhibiting the Markov process (or memoryless property). That is, the transition probability of the future state depends solely on the present state, not on the sequence of past observations. For a two-state MS model, the transition probabilities are defined as in Equation (2), where  $p_{ij} = P(S_t = j | S_{t-1} = i)$  is the probability of transitioning from regime  $i$  to regime  $j$ . Since the state of each observation is unknown, the Hamilton filter is applied in order to infer and assign a state to each period of the dataset. The Hamilton filter is a recursive procedure, that uses the transition probabilities and the information of the previous state to derive the log-likelihood function. Maximizing the log-likelihood results in the optimized state-parameters and transition probabilities, as well as the filtered state probabilities for each period  $t$ ,  $P(S_t = j | \mathcal{I}_{t-1})$  and the smoothed probabilities  $P(S_t = j | \mathcal{I}_T)$ , where  $\mathcal{I}_t$  is the information set at time  $t$ .

The estimated smoothed probabilities of the recession state are illustrated in Figure 4. It is clear from the figures that a state mimicking the recession is not captured in all cases; there are several cases where the probability of a recession is deemed to be high even when the NBER data does not indicate a recession period. However, the smoothed probabilities graphs of all loan categories indicate a recession period in 2008.

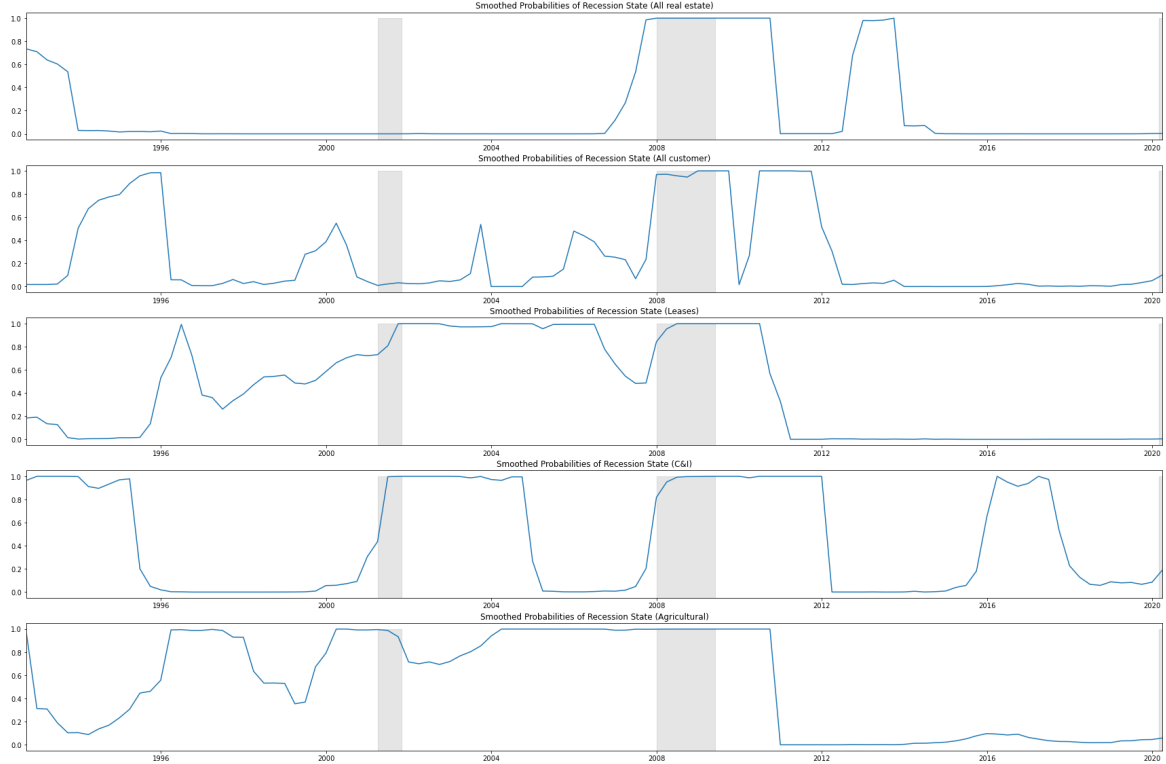


Figure 4: Smoothed probabilities of the recession state for all different loan categories. The shaded grey areas indicate NBER-based recession periods.

### A.2.1 Forecasting Performance Analysis

A forecasting performance analysis was performed to compare the forecasting performance of the Markov-Switching model (ARMSX) to several benchmarks: a univariate AR(1) model with exogenous variables - ARX(1); a multivariate VAR(1) model with exogenous variables - VARX(1). The benchmark models are further explained in Appendix A.2.2.

The same dataset, as explained in Section 2, was used to perform this analysis, however it was split into train and test datasets to perform out-of-sample forecasts. The train dataset is defined from 1992 Q1 to 2017 Q1, and the test dataset from 2017 Q2 to 2020 Q1. The top half of Table 3 below shows the mean squared forecast errors (MSFE) of the delinquency rates forecasts for the MS model (ARXMS(1)) and the benchmarks.

The second half of the table gives the MSFE of the models as a ratio of the MSFE of the ARX(1) benchmark, to illustrate the performance of the models as a comparison to the benchmark.

	Real Estate	Consumer	Leases	C&I	Agricultural
ARX(1)	3.07E-04	3.17E-05	4.77E-05	3.83E-04	2.00E-05
VARX(1)	3.66E-04	1.69E-05	2.07E-05	1.07E-04	9.57E-06
ARXMS(1)	9.56E-05	5.33E-06	8.21E-06	4.55E-05	5.66E-05
	Real Estate	Consumer	Leases	C&I	Agricultural
ARX(1)	1	1	1	1	1
VARX(1)	1.193	0.535	0.433	0.279	0.479
ARXMS(1)	0.312	0.168	0.172	0.119	2.835

Table 3: Panel A: the MSFE for all loan categories for the ARMSX(1) model and the benchmarks. Panel B: the MSFE for all loan categories as a ratio of each model and the ARX(1) benchmark.

It is evident from the results that the MS model outperforms both benchmarks in forecasting the delinquency rates for all loan categories apart from the Agricultural loans, where the VARX(1) model performs best.

### A.2.2 Benchmark Models

To evaluate the performance of the MS model, several benchmark models which are widely used in the current literature are considered. The first benchmark model is the univariate autoregression model with exogenous variables, ARX(1), which is specified as

$$y_t = \mu + \phi y_{t-1} + X_t \beta + \varepsilon_t \quad (3)$$

where the model parameters  $\mu$ ,  $\phi$  and  $\beta$  are invariant. The error terms are i.i.d and  $\varepsilon_t \sim N(0, \sigma^2)$

The second benchmark model is the VARX(1). This model is the multivariate AR(1) model. VAR (vector autoregression) is a generalization of AR (autoregressive model) for multiple time series, identifying the linear relationship between them.

$$Y_t = \mu + \phi Y_{t-1} + X_t \beta + \varepsilon_t \quad (4)$$

where the model parameters  $\mu$ ,  $\phi$  and  $\beta$  are invariant. The error terms are i.i.d and  $\varepsilon_t \sim N(0, \sigma^2)$

## References

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Journal of Econometric Society*, 57(2):357–384.