Erasmus School of Economics

Seminar Financial Case Studies

Ortec Finance Group D

Author:	*Author:*	*Author:*	*Author:*
Tommaso Garutti	Ioannis Papazoglou	Themis Rallis	Sonia Singh
Student Number:	*Student Number:*	*Student Number:*	*Student Number:*
424192	477054	477783	479729

Supervisor:
Dr. Annika Schnücker

Abstract

This paper investigates the inclusion of macroeconomic variables in the context of both dynamic (statistical) factor models and state space models in order to accurately forecast the short rate, inflation, the output gap, and unemployment. We find that dynamic factor models are hard to beat, however for specific macroeconomic variables these are beaten by a simple state space benchmark. Over the coming weeks we wish to augment the state space specification to include the principal components, as well as unemployment.

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1 Introduction

Forecasting macroeconomic variables is of paramount importance to the policymakers of any economy. These are used to predict the future of the economy which guides the monetary and fiscal policies adopted by the central bank and central government respectively. Forecasts help policymakers determine the consequence of policy changes on the economy as well as other macroeconomic variables. Additionally, they provide vital information to retail and corporate investors who can align their future investment decisions with the interest rate changes. These changes can affect household expenditures, government borrowings, expansion of businesses and ultimately the overall economic state.[1] Macroeconomic relations study the aggregate behaviour of the economy; how different macro elements of the economy such as inflation, Gross Domestic Product (GDP), unemployment, interest rate, etc relate with and impact one another. In the United States, the Federal Reserve lays its focus on two macroeconomic goals - maximum sustainable employment and stable inflation. It achieves these two goals through the monetary policy instrument of the Federal funds rate. Adjustment of this rate (at which banks trade with each other on an overnight basis) creates a ripple effect in the economy.[2] It impacts the short rate at which commercial banks lend money to the economy, besides the long-term interest rates which are reflected as a change in the value of different asset classes. These changes have an overall impact on both the demand as well as the supply side of the economy. The lowering of short- and long-term interest rates increases the consumption power of economic agents, which is in turn countered by firms expanding their business by hiring more workforce ultimately stimulating production. This has a feedback effect on household income, further increasing their spending power. Consequently, increased demand leads to an increase in the price of goods and labour faced by firms, which drives inflation. Our contribution is to integrate unemployment in the context of a time series model to gain deeper insight into interest rate forecasts.

We intend to capture the following three macroeconomic relations to forecast the short-term interest rate: the Taylor Rule, Phillip's Curve and Okun's Law, which are briefly explained in Section 2. While the Taylor Rule establishes a relationship between the short rate and inflation and output gap, the Phillips curve models a negative relationship between inflation and unemployment, and Okun's Law links a negative relationship between output and unemployment. Usually, models tend to include accurate representations of the relations between the studied macroeconomic variables, but this comes at a severe cost of forecasting power. Rarely do these models outperform

[1] https://www.frbsf.org/economic-research/publications/economic-letter/2017/july/bridging-gap-forecasting-interest-rates-with-macroeconomic-trends/
[2] https://fred.stlouisfed.org/series/FEDFUNDS

even simple univariate benchmarks, such as autoregressive models of order one (AR(1)) or random walk models (Stock and Watson (2009)). While these simple benchmarks have powerful forecasts, none of them offer any deeper insight to the interconnectedness of the macroeconomy. Finding the right balance between forecasting power and a parsimoneous macrofinance model that includes sufficient macroeconomic information, particularly those of inflation, output and unemployment, is the main focus of this study. Specifically, we address the following research question: *Does the inclusion of unemployment in the macroeconomic relations of a dynamic factor/state space model for the short-term interest rate increase the predictive ability of the model?*

To answer the research question we use US macroeconomic variables data as well as the principal components provided to us by Ortec Finance ranging from 1985Q1 to 2018Q3. The choice of macroeconomic variables and sample period is explained in the Data section of the paper. For the purpose of estimation and forecasting we employ the following two model formulations: Dynamic Factor Model (DFM) and State Space Model (SSM). DFMs capture the linear relation between macroeconomic variables and the underlying latent factors driving them. These unobserved factors are time-varying and follow a Vector Auto-Regressive (VAR) process (Stock and Watson (2011)). On the other hand, SSMs describe the evolution of underlying states based on observations assumed to be noisy estimates of the states themselves. We use the Kalman filter to estimate the underlying state of the economy. Both the models as well as the filter are further described in Section 4.

The proposed method of estimation is in the spirit of Rudebusch and Wu (2008) (henceforth RW); where we deviate from this study by using a SSM. Additionally, we extend the originally developed model by including unemployment as an additional exogenous variable. We do this in an attempt to capture the macroeconomic relation that it has with output gap, inflation, and thus the short rate. We also incorporate the 10 principal components provided to us by Ortec Finance, the dynamics of which follow a VAR(1) process. The one difficulty regarding the macrofinance relation described by RW is that they contain expectational one-step-shead quantities, which do not align with the state space theory. We side-step this issue by rewriting the relations using the Sims (2002) algorithm and recognizing that these expectational quantities can be expressed as linear combinations of the observed variables, making their ommission from the SSM permissible. More information on this is given in Section 4.2.1.

1.1 Macrofinance Term Structure Models

Our work fits in the academic context of term structure models for the yield curve and the inclusion of macroeconomic variables to explain these. The earliest works regarding dynamic models of the term structure date back to the seminal works of Vasicek (1977), Nelson and Siegel (1987) and Cox et al. (1985), who condense the vast majority of macroeconomic data into a statistical factor model consisting of a few principal components. However, these models are limited by their ability to link the dynamic factors to the underlying driving forces of the macroeconomy. In their paper, Duffie and Kan (1996) create a generalised model of the yield curve under which the models created by Vasicek (1977), Nelson and Siegel (1987) and Cox et al. (1985) are special classes. These are called Gaussian Affine Term Structure Models (GATSMs) and allow for the inclusion of macroeconomic relations. Within this class of models, one can specify the macroeconomic variable of interest and relate it to the observed and latent macroeconomic variables used for forecasting. This can be done using either a DFM, which treat macroeconomic principal components as the underlying latent factors as in Stock and Watson (2002), Ang and Piazzesi (2003), Rudebusch and Wu (2008), Christensen et al. (2009), and Joslin et al. (2013), or a SSM, which specify the dynamics of the latent factors directly using macroeconomic relations as in Diebold et al. (2006) and Doshi et al. (2018).

DFMs have always been an attractive choice due to the computational ease they offer in terms of modelling time series data where the number of series N is of the order of the number of observations T. The central idea of a DFM is that a large part of the variability of the macrovariables can be attributed to a handful of unobserved dynamic factors while eliminating the average effect of the error associated with each particular time series, e.g. see Stock and Watson (2011). In their article, Stock and Watson (2002) study the forecasting of a macroeconomic time series variable with the help of a small number of factors (Diffussion Indexes (DIs)), which are estimated using an approximate dynamic factor model. Forecasts constructed by this model are compared to univariate and multivariate benchmark models at a forecasting horizon of 6-, 12- and 24-months. The DI forecasts were generally found to perform well in comparison to the benchmark models with a more pronounced outperformance at higher forecast horizons. The authors find that macroeconomic variability can be explained by a small number of significant sources which are sufficient for the purpose of forecasting as well. We build on this notion by dynamically incorporating the extra variable of unemployment which could be a significant source of information in explaining and forecasting the term structure of the interest rates.

One of the main challenges of this paper is the trade off between forecasting power and inclusion of macroeconomic relations. As pointed out by Stock and Watson (2009), even well-established macrorelations such as the Phillips curve may fail to hold in certain time periods. In particular, the Phillips Curve forecasts are found to outperform univariate forecasts during the late 1990's as opposed to the mid-1990's. Atkeson and Ohanian (2001) conclude that their four-quarter random walk benchmark model consistently outperforms a number of inflation forecasts over the sample period of 1985-1999. Future studies (e.g. Angeloni et al. (2006), Brissimis and Magginas (2008)) confirm their main findings while iterating that the performance of Phillips curve forecasts is sensitive to the sample period, even when a modernised New Keyensian model (Roberts (1995)) is used. More specifically, Stock and Watson (1999) study and compare forecasts of US inflation. Initially, the stability of US Phillips curve is characterized as statistically significantly unstable. As the Phillips curve is usually specified in terms of unemployment, it is observed that other variables do not significantly improve the forecasts of inflation, but forecasts based on other measures of aggregate activity can have same or better performance. They conclude that the most precise and trustworthy short-run forecasts of US inflation is produced by the Phillips curve. In our paper, we incorporate the information embedded within the Phillips curve model by linking inflation with unemployment in a similar fashion to the New Keyensian model; the interaction between inflation and unemployment has not been thoroughly explored in the academic literature in the context of (statistical) factor models.

As Joslin et al. (2013) outline in their paper, a key problem faced by such Macroeconomic Term Structure Models (MTSMs) is the identification of the coefficients linking the short rate to inflation and the output gap, vis a vis the (modified) Taylor rule. What is typically hoped for is that these coefficients capture the reaction of central banks to changes in macrovariables. One may simply apply principal components to a data set and include any selection of these as latent variables that model the evolution of the short rate. However, this gives an unidentified system, one of a number of theoretically equivalent models that are indistinguishable from each other. Joslin et al. (2013) nonetheless study, what they call, canonical versions of the short rate equation where they "rotate" between risk factor portfolios. They find that the risk factor portfolio consisting of just the first principal component reveals factor loadings most consistent with the famous Taylor rules, but they emphasise that this is coincidental, as any type of rotation is observationally equivalent to any of the other uncountably infinite models. They point out that the main weakness of MTSMs is the underlying assumption that the studied macroeconomic variables are entirely spanned by the yield curve, which is not true considering a lot of information outside the yield curve also affect monetary policies (political shocks, environmental shocks etc.). Hence, they include unspanned macro risks in their MTSM family, which they find largely distorts the response of the spanned macro risks, highlighting the importance of inducing a more solid economic structure to the family of MTSMs. Stock and Watson (2002) sidestep this identification problem by "focusing on the forecasts implied by the factors rather than on the factors themselves". While we recognise this issue as a potential shortcoming, we try to underpin the correct coefficients by adding the information on unemployment and its relations to inflation and output, which should enforce a natural restriction on the possible coefficients.

The state-space formulation of GATSMs is popular in the literature for its flexibility in specifying the underlying state dynamics. Diebold et al. (2006) study a state-space model which includes the yield curve using latent factors (level, slope and curvature, Diebold and Li (2006)) and observable macroeconomic variables. They fit the yield curve at each point in time and estimate the underlying dynamics of the factors. Subsequently, a yields-macro model focused on the relation between latent factors of the yield curve and macroeconomy emerges. They conclude that the macroeconomy has more explanatory power in shaping the future yield curve rather than the reverse. More recently, Doshi et al. (2018) apply the state-space formulation to inflation and the output gap where the underlying states are the long-term inflation and output gap, which are unobserved. The most relevant study for this paper is the work of Rudebusch and Wu (2008), who develop a macro-finance model that captures the standard macroeconomic relationship between output and inflation in combination with a canonical no-arbitrage affine term structure model, both of which are estimated from the data using Maximum Likelihood (ML) with an intermediate step involving the Sims (2002) algorithm. The premise of their study is to provide a macroeconomic-based interpretation of the underlying factors affecting the short rate which leads to their final result: the level factor is interpreted as the perceived (medium-term) target inflation rate, whereas the slope factor as a cyclical monetary policy response to the economy. We make use of the original model as described in their 2003 working paper.[3] Our main contribution to the literature is to extend their model by including unemployment as one of the observabed variables, embedding the Phillips curve and Okun's law within our own state-space specification in the hopes of improving the forecasting power of the model.

The remainder of the paper is organised as follows: Section 2 describes the models and methodology, Section 3 describes the data, Section 4 describes the estimation techniques, Section 5 describes the forecasting method, and Section 6 concludes the paper with the results obtained.

[3] http://web.stanford.edu/~chadj/econ237/wu.pdf

2 Models

Below we outline the macroeconomic relations we intend to embed in our DFMs and SSMs. The econometric model we focus and elaborate upon is the macro-finance specification introduced by Rudebusch and Wu (2008). In our final model we intend to incorporate three macroeconomic relations: the Taylor rule, the Phillips curve and Okun's law. The Taylor rule is already taken in consideration by RW in their macro-finance specification, however we are interested in the effect of the additional inclusion of the Phillips curve and Okun's law relations in a similarly specified time series model.

2.1 Macroeconomic Relations

2.1.1 The Taylor Rule

In his seminal paper, Taylor (1993) proposes a simple rule that can model the central bank's decision making based on inflation and output gap. Bernanke (2015)[4] examines whether the Taylor rule can still be used as a benchmark for monetary policy. Initially, he relates the target for the federal funds rate to the current state of the economy, and then suggests a modified Taylor rule concluding that monetary policy should not be automatic, but systematic. Bernanke's modified version reads

$$i_t = r + \pi_t + 1.0y_t + 0.5(\pi_t - \pi_t^*), \tag{1}$$

where i_t is the federal fund rate, r is the short-term real interest rate, π_t is the inflation rate, π_t^* is the central bank's target inflation rate (= 2%), and y_t is the gap between actual and potential output. Bernanke comments on Taylor's critique of Fed policy and agrees that the rule is just a general guideline and does not include all the information related with a complex and dynamic economy for decision-making policies. For this reason it has become largely attractive to model monetary policy using this modified Taylor rule as a starting point, and then to add dynamics to the inflation and output gap.

2.1.2 The Phillips Curve

The second relation we are interested in is the Phillips (1958) curve. In his paper, Phillips investigates the relation between the unemployment rate or the rate of change of unemployment and the rate of change of money wage rates. The results identify an inverse relation between the

[4] https://www.brookings.edu/blog/ben-bernanke/2015/04/28/the-taylor-rule-a-benchmark-for-monetary-policy/

unemployment rate and the rate of change of wages. Since a change in wages implies a change in prices, the Phillips curve implies a negative relation between the unemployment rate and the inflation rate. Stock and Watson (2009) consider two formulations of the Phillips curve, one being an autoregressive distributed lag (ADL) model,

$$\pi_t - \pi_{t-1} = \mu + a(L)\Delta\pi_{t-1} + b(L)\nu_{t-1} + u_t,$$

where π_t is the inflation rate at time t, ν_t is the unemployment rate at time t, and $a(L)$ and $b(L)$ are lagged polynomials, for which the lag lengths are determined by the Akaike Information Criterion (AIC). For lag lengths p and q respectively the Phillips curve reads,

$$\pi_t - \pi_{t-1} = \mu + \Delta\pi_{t-1}\left(1 - \sum_{i=1}^{p} a_i L^i\right) + \nu_{t-1}\left(1 - \sum_{j=1}^{q} b_j L^j\right) + u_t.$$

In our paper we will consider the "triangle model" given by,

$$\pi_t = \mu + a(L)\pi_{t-1} + b(L)\nu_t + v_t, \tag{2}$$

where π_t is the inflation rate at time t, ν_t is the unemployment rate at time t, and $a(L)$ and $b(L)$ are lagged polynomials, for which the lag lengths are determined by the Akaike Information Criterion (AIC). This specification of the Phillips curve clearly shows a relation between the unemployment rate ν_t and the inflation rate π_t.

2.1.3 Okun's Law

The final macroeconomic relation we are interested in modelling is the Okun's law. Okun (1962) establishes a negative relation between unemployment and output. More specifically, Okun's law is a negative relation between the deviation of output from its potential, or the output gap, and the deviation of unemployment from the natural rate of unemployment. Throughout this paper we will use a simplification of the empirical model of Okun's Law developed by Guisinger et al. (2018):

$$y_t = c - d(\nu_t - \nu_t^*) + z_t, \tag{3}$$

where y_t is the output gap, ν_t is the unemployment rate and ν_t^* is the natural rate of unemployment at time t.

2.2 Macro-Finance Model

RW develop a macro-finance model for the short-term interest rate that considers both statistical properties and macroeconomic relations. This model combines a no-arbitrage yield-only model, which considers the statistical properties of the short-term interest rate, with a macroeconomic model of the short rate that interprets the short rate as a reaction function of the monetary policy.

The yield-only approach models the short rate in terms of two normalized latent factors $f_t = (L_t, S_t)'$,

$$i_t = \delta_0 + \delta_1' f_t = \delta_0 + L_t + S_t. \tag{4}$$

The unobserved latent factors are further endowed with autoregressive dynamics without taking into consideration any macroeconomic relations. On the other hand, the macroeconomic approach models the short rate in terms of a reaction function of the monetary policy. RW describe the short rate by means of the Taylor (1993) rule, which relates the short rate i_t with current economic growth and inflation,

$$i_t = G(X_t) + u_t = r + \pi_t + g_\pi(\pi_t - \pi_t^*) + g_y y_t + u_t, \tag{5}$$

where r is the short-term real interest rate, π_t is the cumulative inflation over the past year, π_t^* is the target inflation, and y_t is the output gap.

While the two specifications of the short rate are clearly different, RW and Ang and Piazzesi (2003) both conclude that there are in fact similarities between (4) and (5) that can be used to model the short rate. RW examine similarities between the latent factors L_t and S_t and the dependent variables of the Taylor Rule inspired short rate specification (5). Specifically, they find that the latent factor L_t of the yield-only specification (4) seems to follow the dynamics of nominal inflation over time, and can therefore be considered an approximation of nominal inflation $r + \pi_t$. Consequently, S_t is taken as an approximation of the remaining response $g_\pi(\pi_t - \pi_t^*) + g_y y_t$. RW's macro-finance specification defines the short rate as a sum of two latent factors L_t and S_t in a similar fashion to the yield-only model (4). However, the dynamics of the latent factors are developed as a weighted average of the lagged factor and its respective approximate dynamic,

$$
\begin{aligned}
L_t &= \rho_L L_{t-1} + (1-\rho_L)\pi_t + \varepsilon_{L,t}, \\
S_t &= \rho_S S_{t-1} + (1-\rho_S)(g_y y_t + g_\pi(\pi_t - L_t)) + u_{S,t}, \\
u_{S,t} &= \rho_u u_{S,t-1} + \varepsilon_{S,t},
\end{aligned} \tag{6}
$$

for the serially correlated shock to S_t, $u_{S,t}$. Under this specification, L_t thus approximates the target inflation rate and S_t captures the policy used to achieve the targeted inflation. RW complete the model by describing equations for the inflation rate π_t and the output gap y_t:

$$
\begin{aligned}
\pi_t &= \mu_\pi \mathbb{E}_t[\pi_{t+1}] + (1-\mu_\pi)\pi_{t-1} + \alpha_y y_t + \varepsilon_{\pi,t}, \\
y_t &= \mu_y \mathbb{E}_t[y_{t+1}] + (1-\mu_y)y_{t-1} - \beta_r(i_t - \mathbb{E}_t[\pi_{t+1}]) + \varepsilon_{y,t},
\end{aligned}
$$

with both π_t and y_t being influenced by a weighted average of their forward and backward looking behavior. For estimation purposes, additional lags are incorporated in the two specifications,

$$
\begin{aligned}
\pi_t &= \mu_\pi L_t + (1-\mu_\pi)[\alpha_1\pi_{t-1} + \alpha_2\pi_{t-2}] + \alpha_y y_{t-1} + \varepsilon_{\pi,t}, \\
y_t &= \mu_y \mathbb{E}_t[y_{t+1}] + (1-\mu_y)[\beta_1 y_{t-1} + \beta_2 y_{t-2}] - \beta_r(i_{t-1} - L_{t-1}) + \varepsilon_{y,t}.
\end{aligned} \tag{7}
$$

2.2.1 Adding Factor Dynamics to the Macro-Finance Model

It is simplistic to think that the dynamics of the macroeconomic variables inflation and output gap can be fully explained by several macroeconomic relations. Works by Sargent and Sims (1977) and Giannone et al. (2005) show that macroeconomic variables including inflation, output gap, unemployment and prices can be explained by just several dynamic factors. In order to more accurately explain these macroeconomic variables, we extend their dynamics (7) and include dynamic factors:

$$
\begin{aligned}
\pi_t &= \mu_\pi L_t + (1-\mu_\pi)[\alpha_1\pi_{t-1} + \alpha_2\pi_{t-2}] + \alpha_y y_{t-1} + \alpha_F' F_t + \varepsilon_{\pi,t}, \\
y_t &= \mu_y \mathbb{E}_t[y_{t+1}] + (1-\mu_y)[\beta_1 y_{t-1} + \beta_2 y_{t-2}] - \beta_r(i_{t-1} - L_{t-1}) + \beta_F' F_t + \varepsilon_{y,t},
\end{aligned} \tag{8}
$$

where F_t is a vector of latent factors. These factors are obtained from principal component analysis on a large amount of macroeconomic information. They are further endowed with autoregressive dynamics:

$$
F_t = \mathbf{\Phi} F_{t-1} + \varepsilon_{F,t}. \tag{9}
$$

2.2.2 Incorporating Unemployment

Unemployment is one of the key indicators of the overall health of an economy and an important feature of economic recession. A slump in economic growth in terms of production and output decreases the aggregate spending in the economy due to a decrease in aggregate demand. This gap between aggregate demand and supply causes firms to slash production and lay-off employees which further increases the rate of unemployment. This vicious cycle has important policy implications in terms of both monetary and fiscal policy. In order to boost the economy the Federal Reserve resorts to an expansionary monetary policy by reducing the Fed funds rate. The goal is to reduce the cost of borrowing for businesses and induce them to invest the borrowed funds to increase production. This increased production is supported by hiring more labour force. As a result the unemployment rate begins to rise as economic conditions are restored. Another measure taken by the government is an expansionary fiscal policy which entails increasing government spending on public projects thereby creating more jobs. These two measures play an important role in stabilising the economy due to the macro-relationships that exist between unemployment, inflation and output. Finally, we extend RW's macro-finance specification in order to include two additional macroeconomic relations: the Phillips curve and Okun's law. The Phillips curve is an inverse relation between the unemployment rate ν_t and the inflation rate π_t. Equation (2) provides a clear formulation that relates inflation to the unemployment ν_t at time t and further lags of unemployment. Okun's law specifies a positive relation between the output gap y_t and the deviation of unemployment from the natural rate of unemployment $(\nu_t - \nu_t^*)$. This relation is described in Equation (3).

We incorporate these two macroeconomic relations into the dynamics of the inflation rate π_t and the output gap y_t (8). For simplicity, we include only one lag of unemployment thus redefining the dynamics as follows,

$$\begin{aligned}
\pi_t &= \mu_\pi L_t + (1-\mu_\pi)[\alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2}] + \alpha_y y_{t-1} - \alpha_\nu \nu_t + \alpha_F' F_t + \varepsilon_{\pi,t}, \\
y_t &= \mu_y \mathbb{E}_t[y_{t+1}] + (1-\mu_y)[\beta_1 y_{t-1} + \beta_2 y_{t-2}] - \beta_r(i_{t-1} - L_{t-1}) - \beta_\nu(\nu_t - \nu_t^*) + \beta_F' F_t + \varepsilon_{y,t},
\end{aligned} \tag{10}$$

where ν_t is the unemployment rate and ν_t^* is the natural rate of unemployment at time t. Lastly, we define the dynamics of unemployment utilizing the same forward and backward looking dynamic as that used for inflation and output gap by RW:

$$\nu_t = \mu_\nu \mathbb{E}_t[\nu_{t+1}] + (1-\mu_\nu)[\psi_1 \nu_{t-1} + \psi_2 \nu_{t-2}] + \varepsilon_{\nu,t}. \tag{11}$$

A similar forward and backward looking dynamic is also used to model the natural rate of unemployment ν_t^*.

In addition to extending the RW macro-finance model, we may investigate the existence of an unemployment latent factor. RW assume that the short-term interest rate can be modeled as the sum of two latent factors L_t and S_t, which approximately correspond to the first two principal components of the yield curve data. We are interested in looking into the dynamics of the rest of the principal components and specifically if any are related to unemployment.

2.3 Dynamic Factor Model Benchmark

It has become popular to consider factor-augmented vector autoregressive (FAVAR) models to explain and identify monetary policy, see e.g. Bernanke et al. (2005) and Stefanovic (2015). FAVAR models have the ability to include vast amounts of information condensed into several statistical factors, known as principal components. These factors can be treated as "latent" driving forces that directly influence the observable variables. For our purposes we use the 10 principal components provided to us by Ortec Finance, whose dynamics we specify as a VAR(1) model:

$$F_t = \underset{[10\times10]}{\mathbf{\Phi}} F_{t-1} + \varepsilon_{F,t}, \tag{12}$$

where $\mathbf{\Phi}$ is the matrix of autoregressive coefficients for these factors F_t. The estimated autoregressive coefficients for such a VAR(1) model indicate the persistence between the factors and their lags as well as the relationship between factors and lags of the other factors. By collecting the observables within a 3×1 vector $X_t = (\pi_t, y_t, \nu_t)'$, the FAVAR model of order 1, FAVAR(1), is thus expressed as:

$$X_t = C + \underset{[3\times3]}{\mathbf{A}(L)} X_{t-1} + \underset{[3\times10]}{\mathbf{B}} F_t + u_t, \tag{13}$$

where C and u_t collect the constants and errors respectively into 3×1 vectors, $A(L)$ is a (collection) of matrix(es) containing autoregressive coefficients at the lag specified by L, and B is a matrix of coefficients that captures the effects of the 10 factors on the observed variables.

2.4 State Space Model Benchmark

As has been outlined earlier, SSMs are flexible in their specifications as they allow the underlying state dynamics to be modelled freely. These models are estimated using ML via the use of the Kalman filter, an iterative updating process based on the assumption that the underlying and observed states are jointly Gaussian. The general specification of such a model states that the unobserved variables ξ_t have autoregressive dynamics

$$\xi_{t+1} = \mathbf{\Pi}\xi_t + \varepsilon_{\xi,t}$$

where $\varepsilon_{\xi,t} \sim N(0, \mathbf{R})$. The observed variables, x_t, which are directly driven by the unobserved states have general dynamics

$$x_t = \mathbf{Q}\xi_t + \varepsilon_{x,t}$$

where $\varepsilon_{x,t} \sim N(0, \mathbf{S})$. Our beliefs of the underlying state are updating based on the actual observed variables, up until ML estimation reaches optimality for the coefficients in $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{S}$ and $\mathbf{\Pi}$. For our benchmark, we wish to incorporate the macroeconomic variables of inflation and output gap within the observation equation, further including lags of these. For the underlying states, we specify them to be the unobserved inflation L_t and underlying state of the business cycle S_t, while including lags of the exogenous observed variables in the state transition equation as well. More information of the estimation process and full form of the SSM can be found in Section 4.

2.5 Model Summary

For completeness, the table below summarises the set of models that will be considered in this research. Table 1 outlines these. So far, the only two models we have not estimated yet are SSM 1 and SSM 2. The specifications described in Section 4 will be eventually augmented to include the additional exogenous variables. We specifically address the difference between DFM 1 and DFM 2. For DFM 1, we allow all AR parameters of the observed variables to be estimated, whereas in DFM 2, we specifically restrict these parameters to exactly reflect the macrofinance relations given in Section 2.2:

$$\begin{bmatrix} \pi_t \\ y_t \\ \nu_t \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & 0 \\ \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ \nu_{t-1} \end{bmatrix}.$$

Table 1: Summary of all models.

Model Type	Forecasted Variables	Exogenous Variables	Additional Info
DFM Benchmark	$[i_t, \pi_t, y_t, F_t]$	$[F_t]$	Purely statistical
DFM 1	$[i_t, \pi_t, y_t, F_t]$	$[\pi_t, y_t, F_t]$	No parameter restrictions
DFM 2	$[i_t, \pi_t, y_t, \nu_t, F_t]$	$[\pi_t, y_t, \nu_t, F_t]$	With macro restrictions
SSM Benchmark	$[i_t, \pi_t, y_t]$	$[\pi_t, y_t]$	No PCs or ν_t
SSM 1	$[i_t, \pi_t, y_t]$	$[\pi_t, y_t, F_t]$	Without ν_t
SSM 2	$[i_t, \pi_t, y_t, \nu_t]$	$[\pi_t, y_t, \nu_t, F_t]$	Full hybrid model with PCs

Note: SSM 1 and 2 are considered hybrid SSM/DFM models as they contain the PC factors as explanatory exogenous variables, but are not a (direct) part of the Kalman estimation and are described by their own VAR process.

Inflation is restricted to be explained by its own lag and the lagged value of output gap, output gap restricted to be explained by the lagged values of itself, inflation and unemployment and lastly, unemployment restricted to be explained by its own lag alone. As seen in the data section the (negative) correlation between output gap and unemployment is extremely strong which empirically supports Okun's Law. The weak correlation between inflation and unemployment motivates our choice of restricting the effect of the latter on output gap to be zero in the DFM with unemployment. This is in line with the literature which suggests that Phillip's curve is sensitive to the sample period.

3 Data

In the late 1970's the US economy entered a state of stagflation characterised by growing inflation and stagnant business activity resulting in a state of recession throughout 1982. Monetary tightening by the Federal Reserve Board in the beginning of 1979 was instrumental in easing the inflation rate and igniting a period of sustained growth all through the 1980's until the end of the century. Unemployment and inflation rates remained at reasonable levels which was reflected in the overall economic state. In this paper we intend to capture the previously mentioned macroeconomic relations: Taylor Rule, Phillip's Curve and Okun's Law. Therefore we make use of (quarterly) US macroeconomic variables such as Personal Consumption Expenditure (PCE) inflation, output gap, short rate, unemployment and NAIRU for the estimation and forecasting of the short rate ranging from 1985Q1 to 2018Q4. In preliminary data analysis we find that demeaning inflation data from 1974Q4 to 2018Q4 misrepresents the data post 1980. This motivates our choice of the sample period in order to obtain meaningful estimation results.

For estimating the latent factors we use US yield curve data for the same sample period.

We observe a number of stylised facts from the following representation of yield data in Table 8 mentioned in the Appendix. On average the yield curve increases with an increase in time to maturity. The yield dynamics show a high level of persistence for the first three lags across maturities of all lengths. The short end of the yield curve is more volatile than the long end.

Table 2: Descriptive statistics of macroeconomic data.

	Mean	Std. Dev.	Max.	Min.	$\hat{\rho}(1)$	$\hat{\rho}(2)$	$\hat{\rho}(3)$	$\hat{\rho}(10)$
Output gap	-0.016	0.018	0.022	-0.066	0.957	0.888	0.797	0.178
PCE core inflation	0.022	0.009	0.047	0.009	0.956	0.908	0.855	0.659
Modified Taylor Rule	0.028	0.024	0.079	-0.042	0.955	0.888	0.800	0.285
Short rate (3 months)	0.033	0.026	0.088	0.000	0.967	0.931	0.885	0.532
Unemployment	0.060	0.015	0.099	0.039	0.970	0.918	0.848	0.179
NAIRU	0.053	0.004	0.060	0.046	0.971	0.932	0.886	0.424

Note: Descriptive statistics of macroeconomic data from the United States during the sample period 1985Q1-2018Q4.

Table 2 presents the summary statistics for US macrovariables. Almost all the variables with the exception of equity return and GDP growth show strong persistence with their immediate lagged values, the highest being for Unemployment and Non-accelerating Inflation rate of Unemployment (NAIRU). This persistence weakens with an increase in the number of lags across all variables. Output gap is the difference between actual and potential output. A negative mean value indicates that on average the the economic output is below its potential value. The average rate of unemployment for the entire sample is 6% which is higher than the 'normal' unemployment rate between 4 and 5%. On average, both the CPI as well as the PCE inflation rates are around 2% which is in line the target inflation rate of the Fed.[5]

Table 3: Correlations of macroeconomic data.

	Output gap	PCE inflation	Taylor Rule	Short rate	Unemp-loyment	NAIRU
Output gap	1	0.079	0.810	0.542	-0.898	-0.257
PCE inflation	0.079	1	0.648	0.741	0.014	0.627
Taylor Rule	0.810	0.648	1	0.850	-0.678	0.172
Short rate	0.542	0.741	0.850	1	-0.372	0.521
Unemployment	-0.898	0.014	-0.678	-0.372	1	0.529
NAIRU	-0.257	0.627	0.172	0.521	0.529	1

Note: Correlations of macroeconomic data from the United States during the sample period 1985Q1-2018Q4.

Table 3 shows the correlation between the different macrovariables. Short rate shows strong

[5] https://www.federalreserve.gov/faqs/economy_14424.htm

positive correlations with both output gap and inflation. A strong negative correlation between unemployment and output gap motivates our choice of capturing Okun's Law in the proposed model. The absence of a negative correlation between unemployment and inflation empirically supports the stable macroeconomic environment in the US during the sample period.

3.1 Investigation of the Taylor Rule

Preliminary regressions using Equation (5) for the full sample period and two subsample periods (pre- and post-Taylor (1993)) are shown in Table 4 below. The coefficients given by Equation (5) were backed out using the regressed coefficients via:

$$i_t = a + b_1\pi_t + b_2 y_t + u_t$$

$$\implies g_\pi = b_1 - 1,\ g_y = b_2 \text{ and } r = a + g_\pi \pi_t^*,$$

where it is used that $\pi_t^* = 0.02$.

Table 4: Regression coefficients of short rate on inflation, output gap and a constant.

	Full-Sample (1985:$Q$1−2018:$Q$3)	Pre-Taylor (1985:$Q$1−1992:$Q$4)	Post-Taylor (1993:$Q$1−2018:$Q$3)
g_π	0.929	0.893	0.405
g_y	0.691	0.494	0.741
r	0.020	0.019	0.019

Note: Splitting the full sample into pre- and post-Taylor allows to us to see if the Taylor (1993) paper has had a significant effect on monetary policy, which looks to be the case.

It is clear from the coefficients in Table 4 that the parameters of the Taylor rule regarding largely vary before and after the proposed Taylor rule in 1993. The post-Taylor coefficient for output gap and inflation is comparable to the (Bernanke) modified Taylor rule, whereas over the full and pre-Taylor sample period the short rate does not seem to adhere closely to either the regular or the modified Taylor rule.

3.2 Principal Component Analysis of US Yield Curve Data

We perform principal component analysis on the US yield curve data to obtain the latent factors used by RW in their Macro-Finance specification. The first latent factor L_t corresponds to the first principal component, while S_t corresponds to the second.

Figure 1 displays the first principal component of the US yield curve L_t as well as the inflation rate. The similarity between the two series supports the statement of RW and Ang and Piazzesi (2003) that the dynamics of L_t are clearly driven by the dynamics of inflation. This is further supported by the correlation between the first latent factor L_t and inflation, which, for the sample period 1985Q1-2018Q4, is 0.77.

Figure 1: Plot of the first principal component L_t (green), against inflation (black) for sample period 1985Q1-2018Q4.

Figure 2 below displays the second latent factor S_t plotted against its fitted value $\hat{S}_t$. The dynamics of the two series are in line with the dynamics of S_t mentioned earlier in (6) where it is approximated by output gap and the deviation of inflation from its target level.

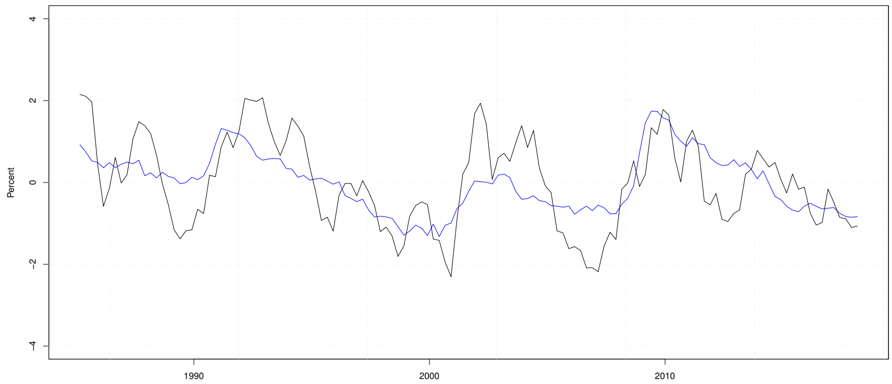

Figure 2: Plot of second principal component S_t (blue) against fitted S_t (black) for sample period 1985Q1-2018Q4.

4 Estimation

In this section we outline the estimation methods that were used for the purposes of this report. We begin with the DFM, followed by the SSM.

4.1 Dynamic Factor Model

Six different FAVAR models and their information criteria are presented in Table 5, with the model with the lowest value of the information criteria being selected. It is evident that the FAVAR(2) model yields the lowest AIC, but is only slightly beaten by the FAVAR(1) model in terms of BIC. We note that the inclusion of both unemployment and the principal components adds valuable information into the model. In order to preserve parsimony of the model, we will henceforth be using the FAVAR(1) specification for modelling and forecasting.

Table 5: Information criteria for various models.

Model	AIC	BIC
VAR(1) (without PCs)	−4448.2	−4381.9
FAVAR(1) (without ν_t)	−2880.8	−2798.6
FAVAR(1) (including ν_t)	−4530.2	−4397.3
FAVAR(2) (including ν_t)	−4557.1	−4396.0
FAVAR(3) (including ν_t)	−4525.0	−4335.8
FAVAR(4) (including ν_t)	−4519.1	−4302.0

Note: Six factor models have been estimated by regressing the vector of variables with and without factors on its own lag(s). The sample period is from 1985:Q1-2018:Q4. The FAVAR(2) model with the variables including the factors and unemployment yields the lowest value of the information criteria.

The strongest persistence exists in the first order lags of each of the three observed variables, which is not surprising. Forecasts of the short rate i_t using both Equations (1) and (5) will be given using the FAVAR(1). On the one hand, we directly use the formulation of the modified Taylor rule as per (1), and on the other, we perform a rolling window regression updating the coefficients of g_π and g_y for the resultant short rate forecasts as per (5).

4.2 State Space Model

We now provide a different estimation approach to forecasting macroeconomic relations. By considering underlying (unobserved) latent factors that are postulated to drive these relations, a state space formulation of the problem can be created, with the underlying states being estimated via the Kalman filter. Let the unobserved states be collected into a vector $\xi_t = (L_t, S_t)'$, where L_t represents the actual underlying inflation, and S_t represents the (unobserved) business cycle (i.e, the fluctuations in an economy's output). The dynamics of these unobserved factors are specified by the macro-relations given by RW, as seen in Equations (6) and (7). For estimation purposes here, we note two deviations from the RW macrofinance model:

- the serially correlated error in S_t is not considered, which we thus replace with the error $\varepsilon_{S,t}$;
- we remove any expected future variables, that is, the quantities $\mathbb{E}_t[\nu_{t+1}]$ and $\mathbb{E}_t[y_{t+1}]$ are postulated to have no effect on the underlying state ξ_t.

Let $z_t = (\pi_t, y_t)'$ represent the exogenous variables that the state ξ_t is dependent on. (In the coming weeks, we aim to augment this specification to include unemployment dynamics, as well as principal components within the observation equation to get a full, factor-augmented state space formulation.) We may retain all other relations regarding inflation, output gap and unemployment by thus defining the following state transition equation:

$$\xi_{t+1} = \underset{[2\times 2]}{\mathbf{\Pi}}\, \xi_t + \underset{[2\times 2]}{\mathbf{\Theta_1}}\, z_t + \underset{[2\times 2]}{\mathbf{\Theta_2}}\, z_{t-1} + \underset{[2\times 2]}{\mathbf{\Sigma}}\, v_{t+1}, \tag{14}$$

where $\mathbf{\Pi}$, $\mathbf{\Theta_1}$ and $\mathbf{\Theta_2}$ are matrices containing the information from the coefficients in Equation (6). We assume that $\varepsilon_{L,t}, \varepsilon_{S,t} \sim N(0,1)$. This means that the state error vector $v_t = (\varepsilon_{L,t}, \varepsilon_{S,t})' \sim N(0, \mathbf{R})$ where $\mathbf{R} = \mathbf{\Sigma\Sigma}'$. Now, we define the observed variables that depend on the unobserved state as $x_t = (i_t, \pi_t, y_t)'$. The observation equation is thus given as:

$$x_t = c + \underset{[3\times 2]}{\mathbf{Q}}\, \xi_t + \underset{[3\times 2]}{\mathbf{H_1}}\, z_{t-1} + \underset{[3\times 2]}{\mathbf{H_2}}\, z_{t-2} + \underset{[3\times 3]}{\mathbf{J}}\, w_t \tag{15}$$

where, as in the state equation, the $[3\times 1]$ vector c and matrices $\mathbf{Q}$, $\mathbf{H_1}$ and $\mathbf{H_2}$ contain information pertaining to the coefficients in Equation (7). In this formulation, we specify that the short rate is given with measurement error $\varepsilon_{i,t} \sim N(0,1)$, and is solely dependent on the unobserved factors,

$$i_t = \delta_0 + \delta_L L_t + \delta_S S_t + \varepsilon_{i,t},$$

which implies that the first row of $\mathbf{H_1}$ and $\mathbf{H_2}$ contains zeros. The error vector for all observations is constructed as $w_t = (\varepsilon_{i,t}, \varepsilon_{\pi,t}, \varepsilon_{y,t})' \sim N(0, \mathbf{S})$ where $\mathbf{S} = \mathbf{JJ}'$, noting that $\varepsilon_{\pi,t}, \varepsilon_{y,t} \sim N(0,1)$.

4.2.1 Initialisation

The Kalman filter requires adequate initial estimates. The best way to ensure that no forward-looking bias is contained in these estimates, information relating only up to the (in-sample) information set $\mathcal{I}_T$ for $t = 1, \ldots, T$ is used. Preliminary regressions of Equations (6) and (7) for $t = 1, \ldots, T$ are calculated, where the first two principal components are used in place of L_t and S_t. In their formulation, RW define the expectational error in forecasting y_t as $\zeta_t = y_t - \mathbb{E}_{t-1}[y_t] \sim N(0, 1)$. For our regressions containing expectational quantities, we generate new time series for these quantities via $\mathbb{E}_t[y_{t+1}] = y_t + \zeta_{t+1}$, where ζ_{t+1} is a random standard normal variable drawn at every time $t = 1, \ldots, T$. In order to use all of these regressed estimates, we must first write the problem in linear rational expectations (LRE) form, that is, in the form:

$$\mathbf{\Gamma_0} Y_t = \mathbf{\Gamma_1} Y_{t-1} + \mathbf{\Psi} \epsilon_t + \mathbf{\Lambda} \eta_t. \tag{16}$$

Here, Y_t is a vector of observables (containing the expectational quantities), ϵ_t is a vector of (possibly serially correlated) exogenous random disturbances, and η_t is a vector that collects all the expectational errors satisfying $\mathbb{E}_t[\eta_{t+1}] = 0$. For the purposes of this analysis, $Y_t = [\pi \; \pi_{t-1} \; y_t \; y_{t-1} \; L_t \; S_t \; \mathbb{E}_t[y_{t+1}]]'$ and $\epsilon = [\varepsilon_{\pi,t}, \varepsilon_{y,t}, \varepsilon_{L,t}, \varepsilon_{S,t}]'$. See the appendix for details of the full form of the remaining matrices. The system described by Equation (16) is solved using the Sims (2002) algorithm to give

$$Y_t = \underset{[7\times 7]}{\mathbf{\Gamma}} Y_{t-1} + \underset{[7\times 4]}{\mathbf{\Omega}} \epsilon_t. \tag{17}$$

We may now use the coefficients from the matrices $\mathbf{\Gamma}$ and $\mathbf{\Omega}$ to initialise the Kalman filter. More specifically,

$$\mathbf{\Pi}^{(0)} = \begin{bmatrix} \mathbf{\Gamma}_{5,5} & \mathbf{\Gamma}_{5,6} \\ \mathbf{\Gamma}_{6,5} & \mathbf{\Gamma}_{6,6} \end{bmatrix}, \; \mathbf{\Theta_1}^{(0)} = \begin{bmatrix} \mathbf{\Gamma}_{5,1} & \mathbf{\Gamma}_{5,3} \\ \mathbf{\Gamma}_{6,1} & \mathbf{\Gamma}_{6,3} \end{bmatrix}, \; \mathbf{\Theta_2}^{(0)} = \begin{bmatrix} \mathbf{\Gamma}_{5,2} & \mathbf{\Gamma}_{5,4} \\ \mathbf{\Gamma}_{6,2} & \mathbf{\Gamma}_{6,4} \end{bmatrix}, \text{ and } \mathbf{\Sigma}^{(0)} = \begin{bmatrix} \mathbf{\Omega}_{5,3} & \mathbf{\Omega}_{5,4} \\ \mathbf{\Omega}_{6,3} & \mathbf{\Omega}_{6,4} \end{bmatrix}$$

for the state transition equation. We note that since the Sims output yields coefficients for lagged states, we use the regression estimates of the window to initialise $\mathbf{Q}$ matrix. Regarding the $\mathbf{J}$ matrix, we note that the Sims algorithm will give ones in certain places as it is rewriting the macroeconomic relations. Thus, the actual initialisation of the remaining coefficients are given as

$$c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{Q}^{(0)} = \begin{bmatrix} 0.1 & 0.1 \\ \hat{\mu}_\pi & \mathbf{\Gamma}_{1,6} \\ \mathbf{\Gamma}_{3,5} & -\hat{\beta}_r \end{bmatrix}, \ \mathbf{H_1}^{(0)} = \begin{bmatrix} 0 & 0 \\ \mathbf{\Gamma}_{1,1} & \mathbf{\Gamma}_{1,3} \\ \mathbf{\Gamma}_{3,1} & \mathbf{\Gamma}_{3,3} \end{bmatrix}, \ \mathbf{H_2}^{(0)} = \begin{bmatrix} 0 & 0 \\ \mathbf{\Gamma}_{1,2} & \mathbf{\Gamma}_{1,4} \\ \mathbf{\Gamma}_{3,2} & \mathbf{\Gamma}_{3,4} \end{bmatrix},$$

$$\text{and } \mathbf{J}^{(0)} = \begin{bmatrix} \hat{\sigma}_i & 0 & 0 \\ 0 & \mathbf{\Omega}_{1,1}\hat{\sigma}_\pi & \mathbf{\Omega}_{1,2}\sqrt{\hat{\sigma}_{\pi,y}} \\ 0 & \mathbf{\Omega}_{3,1}\sqrt{\hat{\sigma}_{\pi,y}} & \mathbf{\Omega}_{3,2}\hat{\sigma}_y \end{bmatrix}$$

for the observation equation. The subscripts $\{i, j\}$ refer to the matrix element of row i and column j. As we wish to preserve the structure of the macroeconomy in the observation equation for macroeconomic forecasting, we note that the above matrices represent the following:

$$c = \begin{bmatrix} \delta_0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{Q} = \begin{bmatrix} \delta_L & \delta_S \\ \mu_\pi & 0 \\ 0 & -\beta_r \end{bmatrix}, \mathbf{H}_1 = \begin{bmatrix} 0 & 0 \\ (1-\mu_\pi)\alpha_1 & \alpha_y \\ 0 & (1-\mu_y)\beta_1 \end{bmatrix}, \mathbf{H}_2 = \begin{bmatrix} 0 & 0 \\ (1-\mu_\pi)\alpha_2 & 0 \\ 0 & (1-\mu_y)\beta_2 \end{bmatrix}.$$

For this reason, within the Kalman filter, only the matrix elements that contain macroeconomic parameters are estimated, all others are restricted to 0. We initialise the measurement error of the short rate to simply be the standard deviation of the in-sample short rate data. The resulting initial state covariance matrix is thus $\mathbf{R}^{(0)} = \mathbf{\Sigma}^{(0)}\mathbf{\Sigma}^{(0)\prime}$ and initial observation covariance matrix is $\mathbf{S}^{(0)} = \mathbf{J}^{(0)}\mathbf{J}^{(0)\prime}$. For the state itself, we use a diffuse initialisation $\xi_t \sim (0, \infty)$. One final remark concerns the Sims output. As the linear solver may find a solution which is either not unique, or approximates a solution if it finds one does not exist, throughout the estimation windows certain iterations yield near-singular covariance matrices giving divergent loglikelihoods. To bypass this issue, before initialisation of the Kalman filter a check was done on each parameter. If Sims assigns an unwanted zero to a parameter (within a specified tolerance set to user preference), a uniform random number $u \sim U(0.3, 0.8)$ is assigned instead. For details on the structure of the filter itself, please find this in the Appendix.

5 Forecasting

This section explains the forecasting techniques used for both the DFM and SSM families of models. For the SSM formulation, forecasting many steps ahead is considerably more complicated due to the existence of lagged exogenous information, which may be unknown if the step is large enough. We evaluate the forecasts using two loss functions, the mean absolute error (MAE) and the root mean square error (RMSE). If the estimation window size is w and the step ahead forecast is s, then the out-of-sample forecasting window size is $T-w-s$. These loss functions are thus defined as:

$$\mathrm{MAE}_i = \frac{1}{T-w-s}\sum_{t=1}^{T-w-s} |\hat{x}_{i,t+s} - x_{i,t+s}|, \quad \mathrm{RMSE}_i = \sqrt{\frac{1}{T-w-s}\sum_{t=1}^{T-w-s} (\hat{x}_{i,t+s} - x_{i,t+s})^2},$$

where $i = 1, 2, 3$ relates to the observed macrovariables being forecasted.

5.1 FAVAR Forecasting

Forecasting a factor augmented vector autoregressive model involves two fundamental steps: first, forecasting the factors by means of their specified dynamics; second, forecasting the observable variables using the forecasted augmenting factors. Following the specifications of the dynamic factor model benchmark in Equations (12) and (13), the one-step ahead forecast equations are given by

$$\begin{aligned}\hat{F}_{T+1} &= \mathbf{\Phi} F_T,\\ \hat{X}_{T+1} &= \mathbf{A} X_T + \mathbf{B}\hat{F}_{T+1},\end{aligned}$$

where F_T are the augmenting factors at the estimation horizon $t = T$, and X_T are the observable variables. We note that since we are only including one lagged observation, we drop the lag operator L for the matrix $\mathbf{A}$. In a similar fashion, $s-$step ahead forecast equations are given by:

$$\begin{aligned}\hat{F}_{T+s} &= \mathbf{\Phi}\hat{F}_{T+s-1},\\ \hat{X}_{T+s} &= \mathbf{A}\hat{X}_{T+s-1} + \mathbf{B}\hat{F}_{T+s}\\ &= \mathbf{A}^s X_T + \mathbf{B}\left(\sum_{i=1}^{s} \mathbf{A}^{s-i}\mathbf{\Phi}^i\right) F_T.\end{aligned}$$

5.2 Kalman Forecasting

Using the Kalman filter for forecasting is generally straightforward, since the predicted states are forecasts themselves. However, since the specifications of the model given by Equations (14) and (15) have lagged observational values, forecasts far into the future become increasingly complicated. Firstly, we note that the exogenous vector is a subset of the observation vector, $z_t \in x_t$. This means that forecasted observation values can be iteratively used to forecast future unobserved states. We outline this process step-by-step below. The superscript (k) that is used in the Appendix regarding filtering is dropped on the estimated matrices for notational convenience, as it is implied here that these forecasts happen at the end of loglikelihood maximisation with forecasts only computed using the final ML parameters. Let s denote the $s-$step ahead forecast, and $t = T$ denote the final observation of the estimation window. Then the first $(s = 1)$ forecasted state is

$$\hat{\xi}_{T+1|T} = \mathbf{\Pi}\hat{\xi}_{T|T} + \mathbf{\Theta_1} z_T + \mathbf{\Theta_2} z_{T-1},$$

which provides us with the first observational forecast:

$$\hat{x}_{T+1|T} = \mathbf{Q}\hat{\xi}_{T+1|T} + \mathbf{H_1} z_T + \mathbf{H_2} z_{T-1}.$$

As was mentioned earlier, this observational forecast includes forecasts for z_t, that is,

$$\hat{z}_{T+1|T} = \begin{bmatrix} \hat{x}_{T+1|T,2} \\ \hat{x}_{T+1|T,3} \end{bmatrix}$$

where the indices 2 and 3 indicate the element of the vector x_t. We may use this information for the $s = 2$ state forecast,

$$\hat{\xi}_{T+2|T} = \mathbf{\Pi}^2\hat{\xi}_{T|T} + \mathbf{\Theta_1}\hat{z}_{T+1|T} + \left(\mathbf{\Pi\Theta_1} + \mathbf{\Theta_2}\right) z_T + \mathbf{\Pi\Theta_2} z_{T-1},$$

which in turn gives us the $s = 2$ observational forecast:

$$\hat{x}_{T+2|T} = \mathbf{Q}\hat{\xi}_{T+2|T} + \mathbf{H_1}\hat{z}_{T+1|T} + \mathbf{H_2} z_T.$$

For $s \geq 3$, it is possible to generalise the state forecasts:

$$\hat{\xi}_{T+s|T} = \begin{cases} \mathbf{\Pi}\hat{\xi}_{T|T} + \mathbf{\Theta_1} z_T + \mathbf{\Theta_2} z_{T-1}, & \text{for } s = 1; \\ \mathbf{\Pi}^2\hat{\xi}_{T|T} + \mathbf{\Theta_1}\hat{z}_{T+1|T} + \left(\mathbf{\Pi\Theta_1} + \mathbf{\Theta_2}\right) z_T + \mathbf{\Pi\Theta_2} z_{T-1}, & \text{for } s = 2; \\ \mathbf{\Pi}^s\hat{\xi}_{T|T} + \left(\mathbf{\Pi}^{s-1}\mathbf{\Theta_1} + \mathbf{\Pi}^{s-2}\mathbf{\Theta_2}\right) z_T + \sum_{i=2}^{s} \mathbf{\Pi}^{s-i}\mathbf{\Theta_1}\hat{z}_{T+i-1|T} & \\ + \sum_{i=3}^{s} \mathbf{\Pi}^{s-i}\mathbf{\Theta_2}\hat{z}_{T+i-2|T} + \mathbf{\Pi}^{s-1}\mathbf{\Theta_2} z_{T-1}, & \text{for } s \geq 3. \end{cases} \tag{18}$$

The forecasts for the macrovariables in the observation equation can be generalised to:

$$\hat{x}_{T+s|T} = \mathbf{Q}\hat{\xi}_{T+s|T} + \mathbf{H_1}\hat{z}_{T+s-1|T} + \mathbf{H_2}\hat{z}_{T+s-2|T}. \tag{19}$$

6 Results

In this section we discuss the forecasting power of the models described in the previous sections. We start with the benchmark DFM.

6.1 Forecast Evaluation

The two tables in this section present the relative performance of the proposed models (DFM 1, DFM 2 and SSM) in comparison to the DFM benchmark model in forecasting the short rate, inflation and output gap. We report the results as ratios of MAE and RMSE of the proposed models to those of the purely statistical benchmark DFM for forecast horizons of s =1-, 2-, 4-, 8-, 12- and 16-quarters.

Table 6: Forecast evaluation - Mean Absolute Error (MAE)

Variable	Model	Forecast Horizon					
		1	2	4	8	12	16
Short Rate	DFM Benchmark	1	1	1	1	1	1
	DFM 1	1.094	1.025	0.985	0.959	0.984	1.016
	DFM 2	1.406	1.222	1.059	0.879	0.843	0.827
	SSM Benchmark	0.638	0.698	0.767	0.521	0.381	0.339
		1	2	4	8	12	16
Inflation	DFM Benchmark	1	1	1	1	1	1
	DFM 1	0.667	0.931	1.114	0.896	0.700	0.449
	DFM 2	0.667	0.931	1.114	0.875	0.686	0.436
	SSM Benchmark	1.889	3.000	2.571	2.208	1.586	1.808
		1	2	4	8	12	16
Output Gap	DFM Benchmark	1	1	1	1	1	1
	DFM 1	0.274	0.388	0.630	0.912	1.094	1.489
	DFM 2	0.259	0.362	0.543	0.766	0.902	1.185
	SSM Benchmark	0.481	0.743	0.975	0.824	0.812	0.920

Note: Short rate, inflation and output gap forecasts are evaluated in terms of (relative) Mean Absolute Error. It is computed as a ratio of the MAE obtained from forecasts of the proposed models: DFM 1, DFM 2 and SSM to the MAE obtained from the DFM benchmark forecasts. A value above 1 indicates that the proposed model outperforms the benchmark whereas a value below 1 indicates the opposite.

In terms of both MAE and RMSE, the DFM's outperform the SSM in forcasting inflation and output gap. However, SSM marks a significant improvement as compared to the DFM in forecasting the short rate especially in the long-run. We observe that short rate is best forecast using an SSM across all forecast horizons as compared to a DFM. Within the class of DFM's,including unemployment in the model while restricting its impact on inflation and output gap slightly improves long-run forecasts. This result captures the Taylor rule which describes the macroeconmic relation between short rate and output gap and inflation. Additionally, it also supports the empirical failure of the Phillip's curve in the chosen sample period. The DFM's consistently outperform the SSM in forecasting inflation and perform exceedingly well in producing long-run forecasts. It is interesting to note that DFM's 1 and 2 perform quite similarly thereby nullifying the effect of unemployment in forecasting inflation. This provides further empirical evidence that Phillip's curve is indeed sensitive to the sample period chosen. SSM performs worse than the benchmark DFM in forecasting inflation. The three models perform well in forecasting output gap for horizons upto 8 quarters beyond which their performance deteriorates. We observe that including unemployment in DFM 2 better forecasts output gap as compared to DFM 1 across all forecast horizons. This empirical supports the Okun's Law that we intend to capture in this paper. SSM consistently beats the benchmark as opposed to the DFM's 1 and 2 which only outperform the benchmark at shorter horizons.

Table 7: Forecast evaluation - Root Mean Squared Error (RMSE)

Variable	Model	Forecast Horizon					
		1	2	4	8	12	16
Short Rate	DFM Benchmark	1	1	1	1	1	1
	DFM 1	1.367	1.161	0.965	0.832	0.799	0.772
	DFM 2	1.367	1.152	0.947	0.805	0.761	0.723
	SSM Benchmark	0.783	0.871	0.940	0.517	0.394	0.316
		1	2	4	8	12	16
Inflation	DFM Benchmark	1	1	1	1	1	1
	DFM 1	0.727	1.027	0.948	0.540	0.320	0.174
	DFM 2	0.697	0.973	0.862	0.520	0.331	0.174
	SSM Benchmark	4.576	6.622	3.069	1.680	0.904	0.798
		1	2	4	8	12	16
Output Gap	DFM Benchmark	1	1	1	1	1	1
	DFM 1	0.279	0.420	0.667	1.082	1.403	1.818
	DFM 2	0.267	0.394	0.601	0.970	1.243	1.562
	SSM Benchmark	0.733	1.399	1.376	0.925	0.944	0.975

Note: Short rate, inflation and output gap forecasts are evaluated in terms of (relative) Root Mean Squared Error. It is computed as a ratio of the MAE obtained from forecasts of the proposed models: DFM 1, DFM 2 and SSM to the MAE obtained from the DFM benchmark forecasts. A value above 1 indicates that the proposed model outperforms the benchmark whereas a value below 1 indicates the opposite.

7 Discussion

8 Conclusion and Future Outlook

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Appendix

A Linear Rational Expectations Form

Defining $Y_t = [\pi\ \pi_{t-1}\ y_t\ y_{t-1}\ L_t\ S_t\ u_{S,t}\ \mathbb{E}_t[y_{t+1}]]'$, the full matrices that are given to the Sims (2002) algorithm are given as:

$$\mathbf{\Gamma_0} = \begin{bmatrix} 1 & 0 & 0 & 0 & -\mu_\pi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\mu_y \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \rho_L - 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ (\rho_S - 1)g_\pi & 0 & (\rho_S - 1)g_y & 0 & (1-\rho_S)g_\pi & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{\Psi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{\Lambda} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{\Gamma_1} = \begin{bmatrix} (1-\mu_\pi)\alpha_1 & (1-\mu_\pi)\alpha_2 & \alpha_y & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\mu_y)\beta_1 & (1-\mu_y)\beta_2 & 0 & -\beta_r & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_S & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_u & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

We do not show the full form of the matrices that were used for SSM 1 and 2, as these increase the dimensions of the matrices to [24 × 24] and we believe the above specification gives enough insight into how they were constructed.

B The Kalman Filter

B.1 Prediction and Updating Steps

The prediction step of the Kalman filter is given by

$$\hat{\xi}_{t+1|t} = \mathbf{\Pi}^{(k)}\hat{\xi}_{t|t} + \mathbf{\Theta_1}^{(k)}z_t + \mathbf{\Theta_2}^{(k)}z_{t-1},$$

where the superscript (k) signifies the k^{th} ML iteration estimate of $\mathbf{\Pi}$, $\mathbf{\Theta_1}$ and $\mathbf{\Theta_2}$. The existence of exogenous (observed) variables in the state equation provides valuable information for the state itself. It is also convenient that the observed quantities appear at (lags of) the current time t, meaning it is assumed there is no uncertainty in these quantities, hence they contribute nothing to the variance of the state. Thus, the predicted covariance is simply

$$\mathbf{P}_{t+1|t} = \mathbf{\Pi}^{(k)}\mathbf{P}_{t|t}\mathbf{\Pi}^{(k)\prime} + \mathbf{R}^{(k)}.$$

We may now write the expected value and covariance of the observed state as well, which are given by

$$\begin{aligned}\mathbb{E}_t[x_{t+1}] &= \mathbf{Q}^{(k)}\hat{\xi}_{t+1|t} + \mathbf{H_1}^{(k)}z_t + \mathbf{H_2}^{(k)}z_{t-1}\\ \mathrm{Var}_t[x_{t+1}] &= \mathbf{Q}^{(k)}\mathbf{P}_{t+1|t}\mathbf{Q}^{(k)\prime} + \mathbf{S}^{(k)}.\end{aligned}$$

The covariance of the state with the observation is derived as

$$\begin{aligned}\mathrm{Cov}_t[\hat{\xi}_{t+1}, x_{t+1}] &= \mathrm{Cov}_t[\hat{\xi}_{t+1}, \mathbf{Q}^{(k)}\hat{\xi}_{t+1} + \mathbf{H_1}^{(k)}z_t + \mathbf{H_2}^{(k)}z_{t-1} + \mathbf{J}w_t]\\ &= \mathrm{Var}_t[\hat{\xi}_{t+1}]\mathbf{Q}^{(k)\prime}\\ &= \mathbf{P}_{t+1|t}\mathbf{Q}^{(k)\prime}.\end{aligned}$$

Therefore, we may write the full joint distribution of the state and observations as a bivariate normal with the following characteristics:

$$\begin{pmatrix} x_{t+1}\\ \hat{\xi}_{t+1}\end{pmatrix}\Bigg|\,\mathcal{I}_t \sim N\left(\begin{bmatrix}\mathbf{Q}^{(k)}\hat{\xi}_{t+1|t} + \mathbf{H_1}^{(k)}z_t + \mathbf{H_2}^{(k)}z_{t-1}\\ \mathbf{\Pi}^{(k)}\hat{\xi}_{t|t} + \mathbf{\Theta_1}^{(k)}z_t + \mathbf{\Theta_2}^{(k)}z_{t-1}\end{bmatrix}, \begin{bmatrix}\mathbf{Q}^{(k)}\mathbf{P}_{t+1|t}\mathbf{Q}^{(k)\prime} + \mathbf{S}^{(k)} & \mathbf{Q}^{(k)}\mathbf{P}_{t+1|t}\\ \mathbf{P}_{t+1|t}\mathbf{Q}^{(k)\prime} & \mathbf{\Pi}^{(k)}\mathbf{P}_{t|t}\mathbf{\Pi}^{(k)\prime} + \mathbf{R}^{(k)}\end{bmatrix}\right).$$

From this specification, we define the Kalman gain as

$$\underset{[2\times 3]}{\mathbf{G}^{(k)}} = \mathbf{P}_{t+1|t}\mathbf{Q}^{(k)\prime}\left(\mathbf{Q}^{(k)}\mathbf{P}_{t+1|t}\mathbf{Q}^{(k)\prime} + \mathbf{S}^{(k)}\right)^{-1},$$

from which the Kalman updating steps follow immediately:

$$\begin{aligned}
\hat{\xi}_{t+1|t+1} &= \hat{\xi}_{t+1|t} + \mathbf{G}^{(k)}\left(x_{t+1} - \mathbf{Q}^{(k)}\hat{\xi}_{t+1|t} - \mathbf{H_1}^{(k)}z_t - \mathbf{H_2}^{(k)}z_{t-1}\right),\\
\mathbf{P}_{t+1|t+1} &= \left(\mathbf{I} - \mathbf{G}^{(k)}\mathbf{Q}^{(k)}\right)\mathbf{P}_{t+1|t},
\end{aligned}$$

where $\mathbf{I}$ is the $[2\times 2]$ identity matrix. We note that the term in the parenthesis in the updating step of the state vector is the discrepancy between the observed values and the predicted observations, which is simply the forecasting error.

B.2 Loglikelihood

As is typical of Markovian processes, the loglikelihood of the this system can only be expressed in terms of prediction error decomposition. If we collect all parameters estimated in a vector θ, we can write the loglikelihood as

$$\begin{aligned}
\text{LogL} &= \sum_{t=3}^{T}\log f(x_t|\mathcal{I}_{t-1};\theta)\\
&= -\frac{3(T-2)}{2}\log(2\pi) - \frac{1}{2}\sum_{t=3}^{T}\log\left(\left|\mathbf{Q}^{(k)}\mathbf{P}_{t|t-1}\mathbf{Q}^{(k)\prime} + \mathbf{S}^{(k)}\right|\right)\\
&\quad - \frac{1}{2}\sum_{t=3}^{T}\left(x_t - \mathbf{Q}^{(k)}\hat{\xi}_{t|t-1} - \mathbf{H_1}^{(k)}z_{t-1} - \mathbf{H_2}^{(k)}z_{t-2}\right)'\left(\mathbf{Q}^{(k)}\mathbf{P}_{t|t-1}\mathbf{Q}^{(k)\prime} + \mathbf{S}^{(k)}\right)^{-1}\\
&\qquad\qquad \left(x_t - \mathbf{Q}^{(k)}\hat{\xi}_{t|t-1} - \mathbf{H_1}^{(k)}z_{t-1} - \mathbf{H_2}^{(k)}z_{t-2}\right).
\end{aligned}$$

We have the factor of 3 in the first constant due to the dimensionality of the observation equation, and we also remove 2 dofs due to the lagged values of the exogenous variables z_t.

C Summary Statistics

	Mean	Std. Dev.	Maximum	Minimum	$\hat{\rho}(1)$	$\hat{\rho}(2)$	$\hat{\rho}(3)$	$\hat{\rho}(10)$
3	0.034	0.027	0.092	0.000	0.967	0.932	0.884	0.525
6	0.036	0.027	0.095	0.000	0.964	0.928	0.883	0.534
12	0.037	0.027	0.096	0.001	0.961	0.926	0.882	0.569
24	0.041	0.028	0.104	0.002	0.956	0.919	0.877	0.631
36	0.043	0.027	0.108	0.003	0.951	0.912	0.872	0.664
60	0.047	0.026	0.113	0.006	0.945	0.901	0.861	0.705
84	0.050	0.025	0.116	0.010	0.942	0.894	0.851	0.719
120	0.052	0.023	0.117	0.015	0.939	0.888	0.842	0.725

Table 8: Descriptives US Yield Curve, maturities from 3 to 120 months.

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	Mean	Std. Dev.	Maximum	Minimum	$\hat{\rho}(1)$	$\hat{\rho}(2)$	$\hat{\rho}(3)$	$\hat{\rho}(10)$
PC1	0.423	0.673	1.309	-1.222	0.951	0.917	0.883	0.684
PC2	0.007	1.113	2.130	-4.716	0.244	0.120	0.110	0.061
PC3	-0.257	0.869	1.298	-2.113	0.873	0.847	0.842	0.682
PC4	-0.061	0.990	1.759	-3.889	0.515	0.330	0.181	-0.022
PC5	0.075	1.050	3.287	-2.372	0.796	0.751	0.659	0.246
PC6	-0.028	0.878	1.630	-1.852	0.891	0.837	0.811	0.495
PC7	0.021	0.866	2.375	-2.261	0.226	0.336	0.155	0.088
PC8	-0.052	0.994	2.868	-2.592	0.425	0.338	0.297	0.021
PC9	0.073	0.895	3.328	-2.048	0.294	0.483	0.190	0.046
PC10	-0.087	1.001	3.145	-2.423	0.139	0.382	0.051	0.163

Table 9: Descriptives of PC Ortec.

	Mean	Std. Dev.	Maximum	Minimum	$\hat{\rho}(1)$	$\hat{\rho}(2)$	$\hat{\rho}(3)$	$\hat{\rho}(10)$
PC1	0.000	0.073	0.105	-0.172	0.957	0.920	0.880	0.646
PC2	0.000	0.011	0.022	-0.023	0.858	0.672	0.495	-0.126
PC3	0.000	0.003	0.006	-0.009	0.686	0.436	0.293	0.035
PC4	0.000	0.001	0.004	-0.002	0.651	0.581	0.451	-0.070
PC5	0.000	0.001	0.002	-0.002	0.461	0.337	0.328	-0.086
PC6	0.000	0.000	0.002	-0.001	0.259	0.206	0.072	-0.148
PC7	0.000	0.000	0.002	-0.001	0.582	0.402	0.372	-0.020
PC8	0.000	0.000	0.001	-0.001	0.159	0.137	0.080	-0.116

Table 10: Descriptives of principal components from yield curve.

	Equity return	GDP growth	Output gap	CPI inflation	PCE inflation	Taylor Rule	Short rate	Long rate	Unemployment	NAIRU
PC1	-0.030	-0.055	-0.004	-0.619	-0.903	-0.534	-0.757	-0.875	-0.154	-0.791
PC2	0.826	0.487	0.170	0.009	0.107	0.193	0.227	0.221	-0.068	0.092
PC3	0.153	-0.209	-0.253	-0.428	-0.587	-0.539	-0.747	-0.824	0.106	-0.582
PC4	-0.265	0.374	0.516	0.351	0.086	0.445	0.272	0.151	-0.523	-0.216
PC5	0.034	0.041	0.202	0.155	0.230	0.289	0.292	0.261	-0.124	0.247
PC6	-0.021	0.002	-0.309	-0.197	0.017	-0.226	-0.315	-0.270	0.365	0.091
PC7	-0.091	-0.038	-0.077	0.087	-0.174	-0.161	-0.120	-0.099	0.014	-0.107
PC8	-0.025	0.051	0.405	0.024	-0.165	0.213	0.101	-0.030	-0.346	-0.150
PC9	0.070	-0.099	0.365	0.218	0.196	0.394	0.310	0.133	-0.337	-0.066
PC10	-0.027	0.224	0.132	-0.210	-0.267	-0.056	-0.111	-0.109	-0.178	-0.176

Table 11: Correlation between the principal components (Ortec) and the US data.

	Equity return	GDP growth	Output gap	CPI inflation	PCE inflation	Taylor Rule	Short rate	Long rate	Unemployment	NAIRU
PC1	-0.096	-0.286	-0.467	-0.625	-0.772	-0.811	-0.982	-0.968	0.292	-0.577
PC2	0.022	0.057	-0.574	-0.044	0.162	-0.343	-0.175	0.238	0.584	0.356
PC3	0.052	-0.197	-0.298	0.185	0.215	-0.102	0.063	0.060	0.348	0.334
PC4	-0.068	0.015	-0.123	0.239	0.142	-0.011	-0.024	0.015	0.231	0.257
PC5	0.044	0.104	-0.107	-0.111	0.048	-0.054	0.001	0.011	0.189	0.331
PC6	-0.011	-0.001	-0.026	0.064	0.034	0.000	0.000	0.006	0.133	0.147
PC7	-0.021	0.063	0.093	-0.161	-0.205	-0.049	-0.002	-0.009	-0.060	0.024
PC8	-0.055	0.014	-0.053	-0.082	0.019	-0.029	-0.003	-0.006	0.060	0.008

Table 12: Correlation between the principal components (data) and the US data.