

30/11/20.

Domain, open set, closed set, baby pts....

$$\forall \varepsilon > 0, \exists \delta > 0 \Rightarrow |x - x_0| < \delta \text{ whenever } |f(x) - f(x_0)| < \varepsilon$$

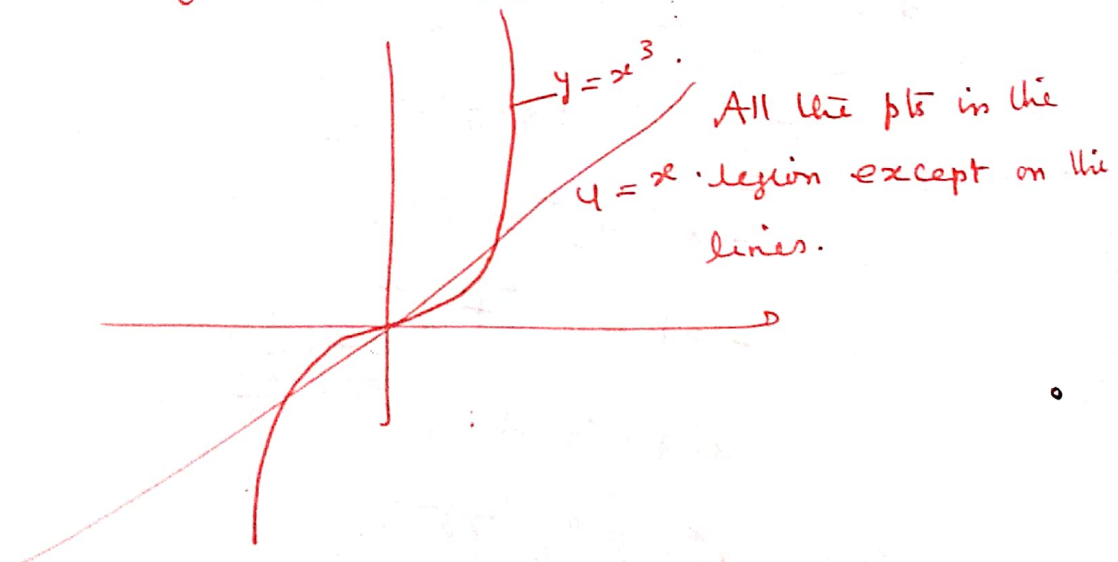
Find and sketch the domains for each function.

5) $f(x, y) = \sqrt{y - x - 2}$.

7) $f(x, y) = \frac{(x-1)(y+2)}{(y-x)(y-x^3)}$

$$D = \left\{ (x, y) \in \mathbb{R}^2 : (y-x) \neq 0 \cap (y-x^3) \neq 0 \right\}$$

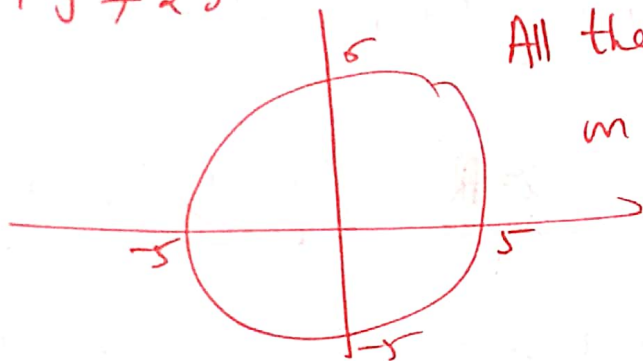
$y-x \neq 0 \Rightarrow y \neq x$ and $y \neq x^3$



$$8) \quad f(x, y) = \frac{\sin(xy)}{x^2 + y^2 - 25}$$

$$D = \left\{ (x, y) \in \mathbb{R}^2 / (x^2 + y^2 - 25) \neq 0 \right\}$$

$$x^2 + y^2 \neq 25$$



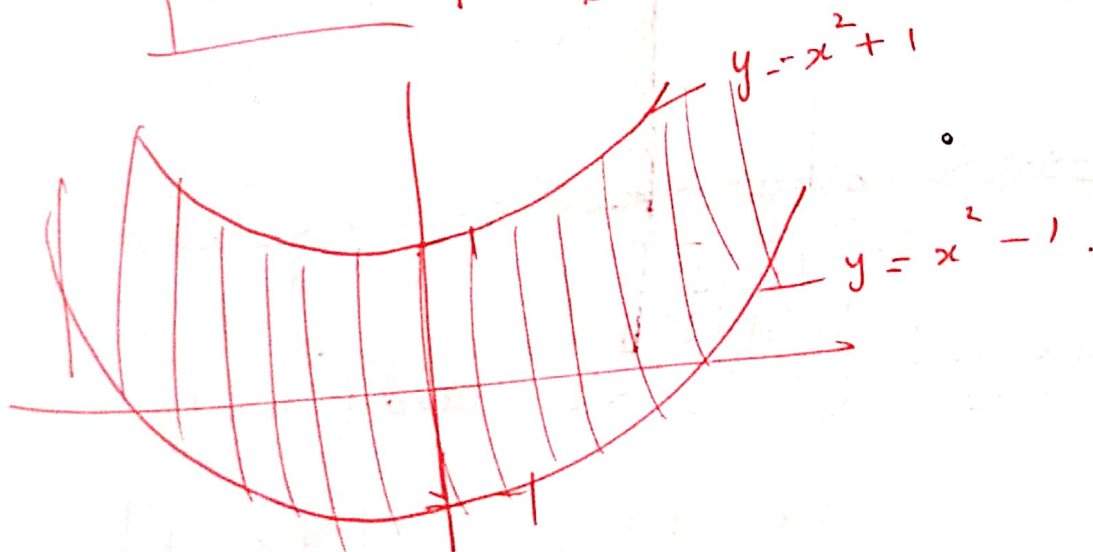
All the points except
on the bdry.

$$9) \quad f(x, y) = \cos^{-1}(y - x^2)$$

$$|y - x^2| \leq 1 \quad \rightarrow \quad -1 \leq y - x^2 \leq 1$$

$$y - x^2 = 1 \quad ; \quad y - x^2 = -1$$

$$\boxed{y = x^2 + 1} \quad ; \quad \boxed{y = x^2 - 1}$$



$$10) f(x, y) = \ln(\underline{xy + x - y - 1})$$

$$xy + x - y - 1 > 0$$

$$x(y+1) - 1(y+1) > 0$$

$$(x-1)(y+1) > 0$$

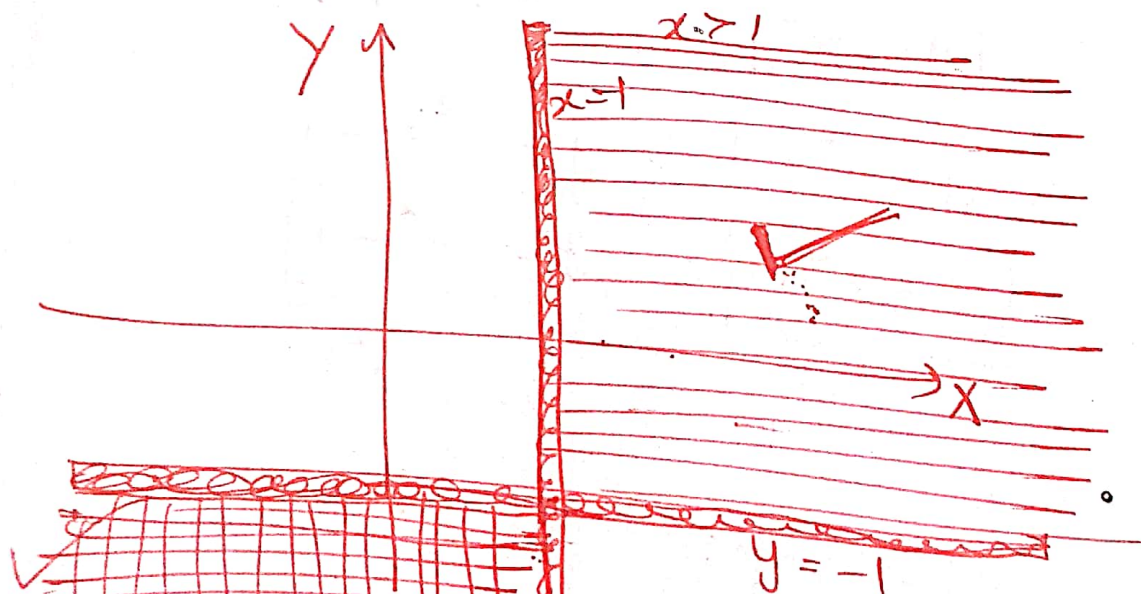
$$D: \left\{ \underline{(x, y) \in \mathbb{R}^2}, \underline{(x-1)(y+1) > 0} \right\}$$

Case 1: $x-1 > 0$

$$y+1 > 0$$

$$x > 1$$

$$y > -1$$



Case 2: $(x-1) < 0$ and $(y+1) < 0$

$$\boxed{x < 1}$$

$$\boxed{y < -1}$$

$$1) f(x, y) = \sqrt{(x^2 - 4)(y^2 - 9)}$$

$$\text{Def: } (x^2 - 4)(y^2 - 9) \geq 0.$$

$$D = \{(x, y) \in \mathbb{R}^2 : (x^2 - 4)(y^2 - 9) \geq 0\}.$$

$$\text{Case (i)} \quad x^2 - 4 \geq 0 \quad ; \quad y^2 - 9 \geq 0.$$

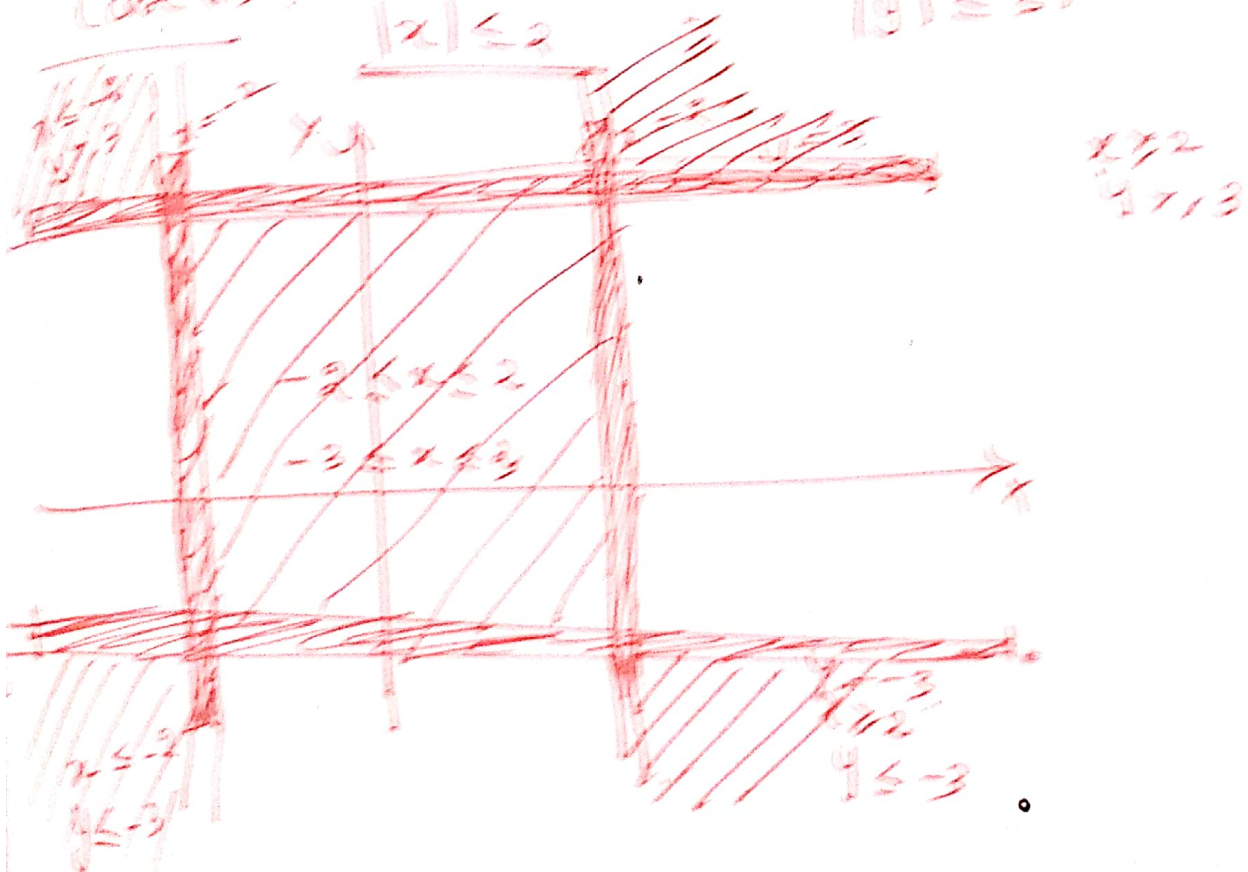
$$x^2 \geq 4 \quad ; \quad y^2 \geq 9.$$

$$|x| \geq 2 \quad ; \quad |y| \geq 3.$$

$$\text{Case (ii)}: -2 \leq x \leq 2 \quad ; \quad -3 \leq y \leq 3.$$

$$|x| \leq 2$$

$$|y| \leq 3.$$

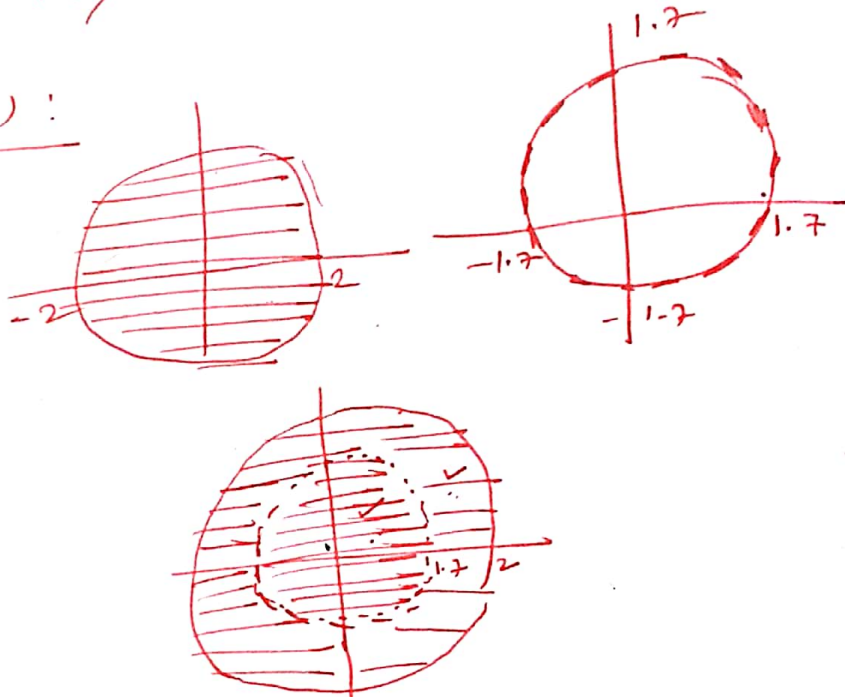


$$12) f(x, y) = \frac{1}{\ln(4 - x^2 - y^2)}$$

$$D: \left\{ x, y \in \mathbb{R}^2 : 4 - x^2 - y^2 > 0, \text{ and } 4 - x^2 - y^2 \neq 1 \right\}$$

$$\text{ie., } x^2 + y^2 < 4 \text{ and } x^2 + y^2 \neq 3.$$

Case (i) :



level Curve $f(x, y) = \underline{\underline{C}}$.

$$13) f(x, y) = x + y - 1. \quad C = -3, -2, -1, 0, 1, 2, 3.$$

$$x + y - 1 = -3, \quad x + y - 1 = -2, \quad x + y - 1 = -1, \quad x + y - 1 = 0, \quad x + y - 1 = 1, \quad x + y - 1 = 2, \quad x + y - 1 = 3.$$

$$x + y = -2$$

$$x + y = -1$$

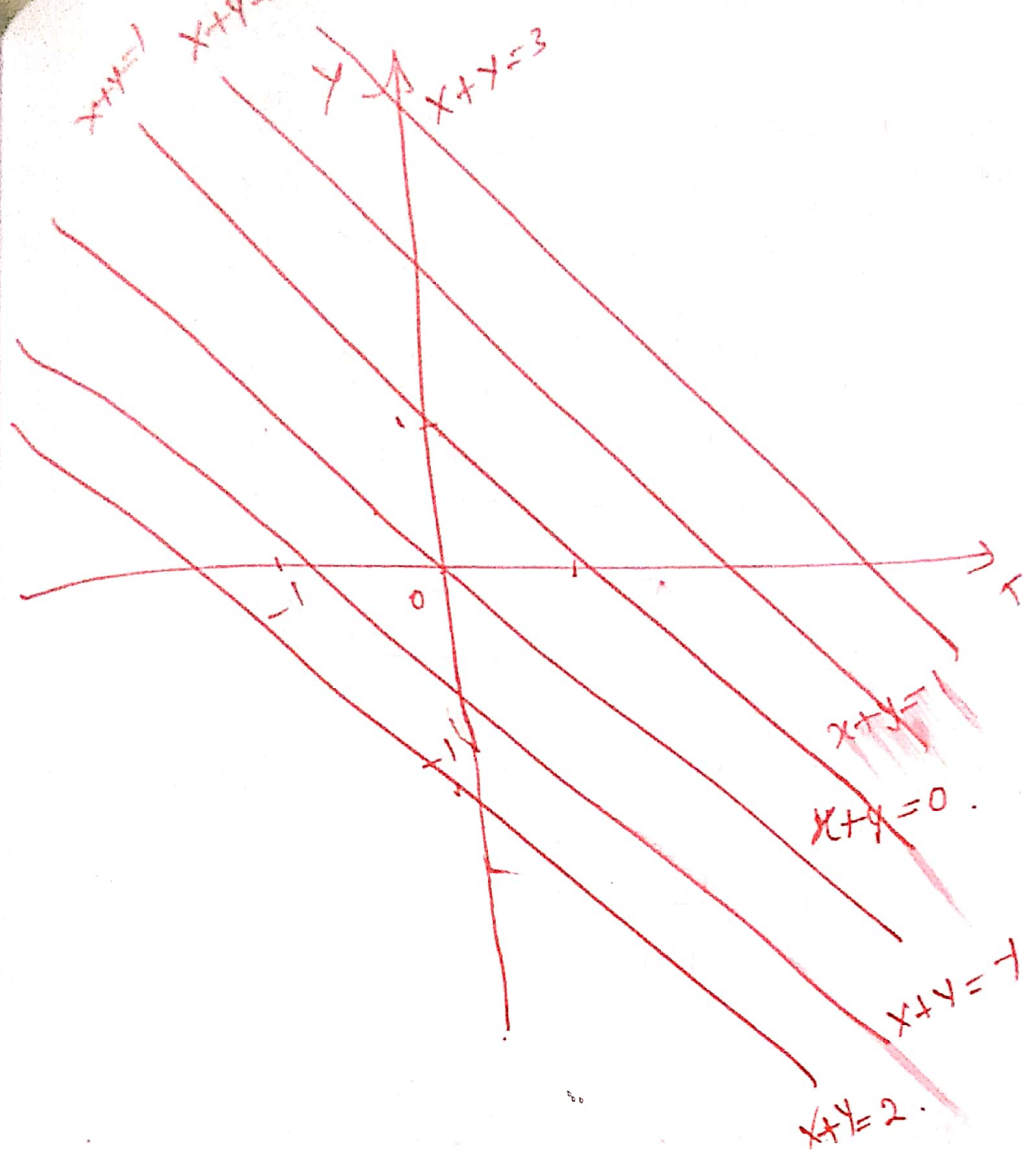
$$x + y = 0$$

$$x + y = 1$$

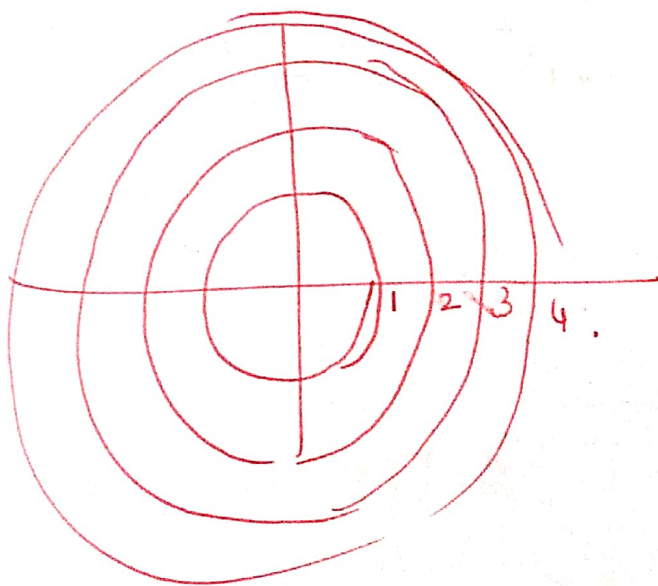
$$x + y = 2$$

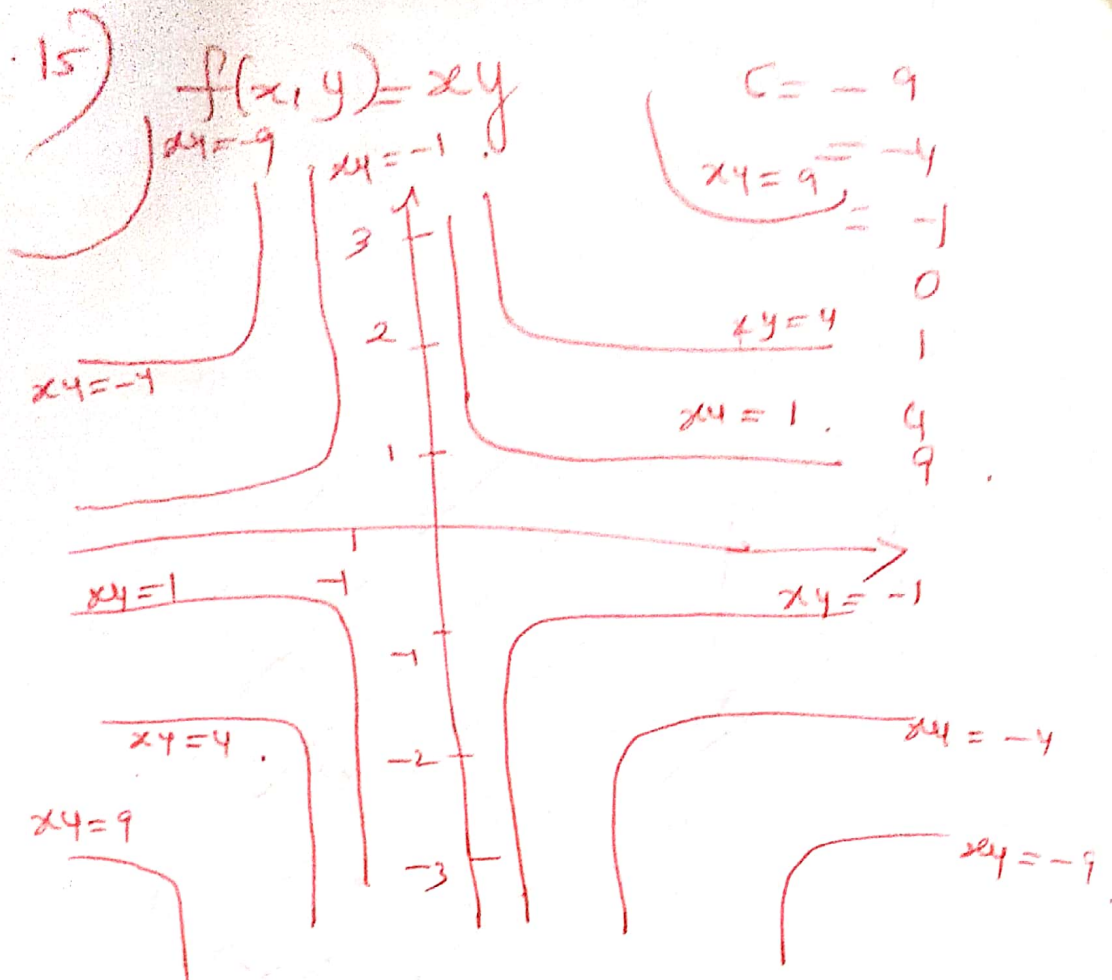
$$x + y = 3$$

$$x + y = 4$$



14) $f(x, y) = x^2 + y^2$ $C = 0, 1, 4, 9, 16, 25$





16) $f(x, y) = \sqrt{25 - x^2 - y^2}$ $C = 0, 1, 2, 3, 4$

$$x^2 + y^2 = 5^2$$

$$x^2 + y^2 = (\sqrt{24})^2 ; x^2 + y^2 = (\sqrt{21})^2$$

