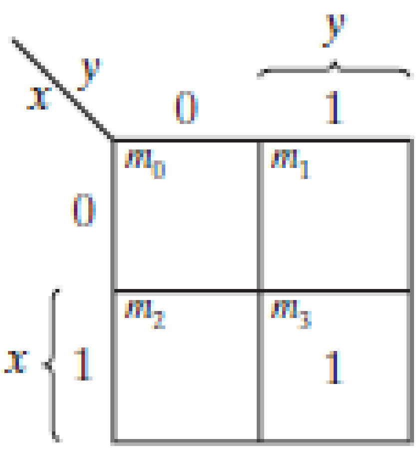
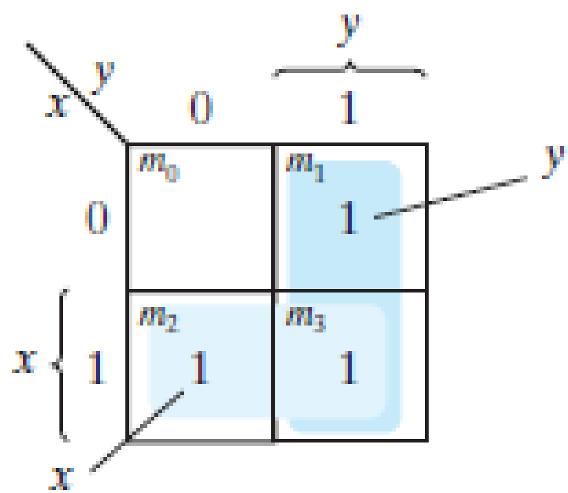
K-Map Simplification

- F1=xy
- F2=x+y = x(y+y’)+y(x+x’)=xy+xy’+xy+x’y=xy+xy’+x’y



xy

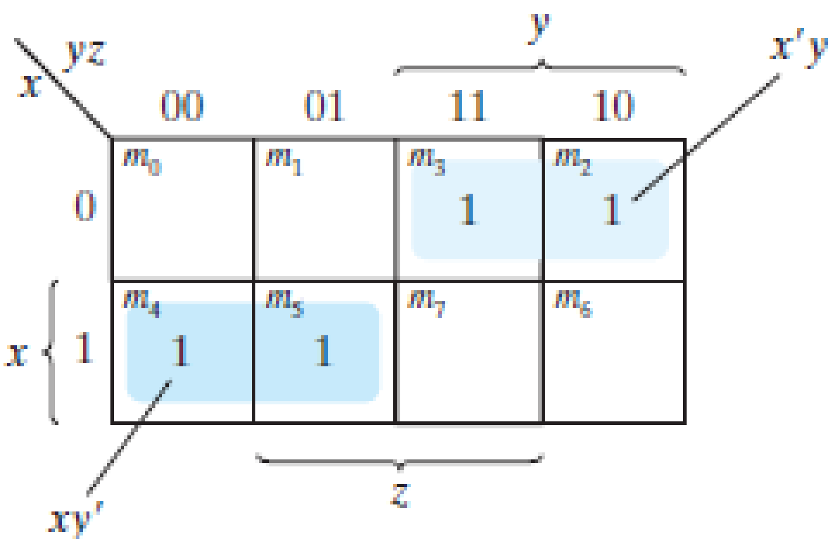


x + y

K-Map Simplification

- Simplify the Boolean function:

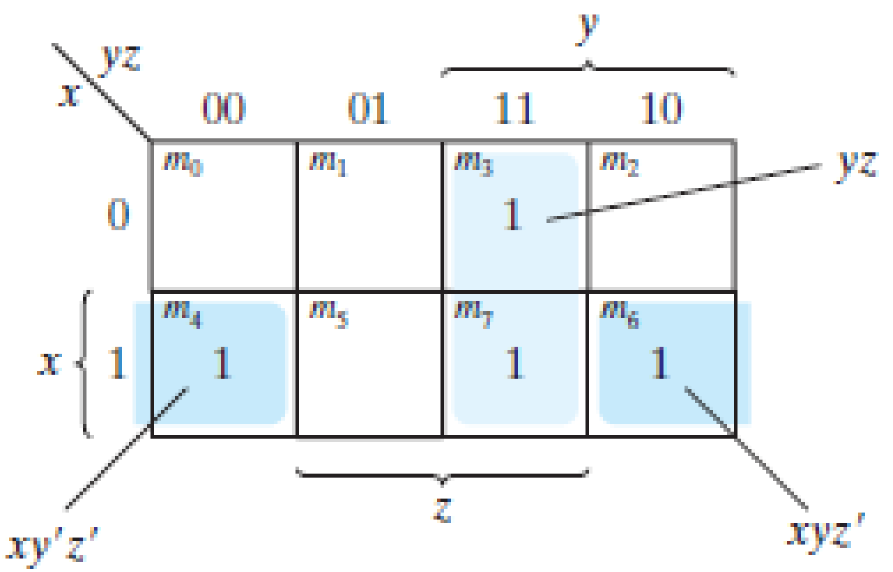
$$F\ (x, y, z) = \sum(2, 3, 4, 5)$$



- F = x’y + xy’

K-Map Simplification

- Simplify the Boolean function
  - $F(x, y, z) = \sum(3, 4, 6, 7)$



- $F = yz + xz'$

K-Map Simplification

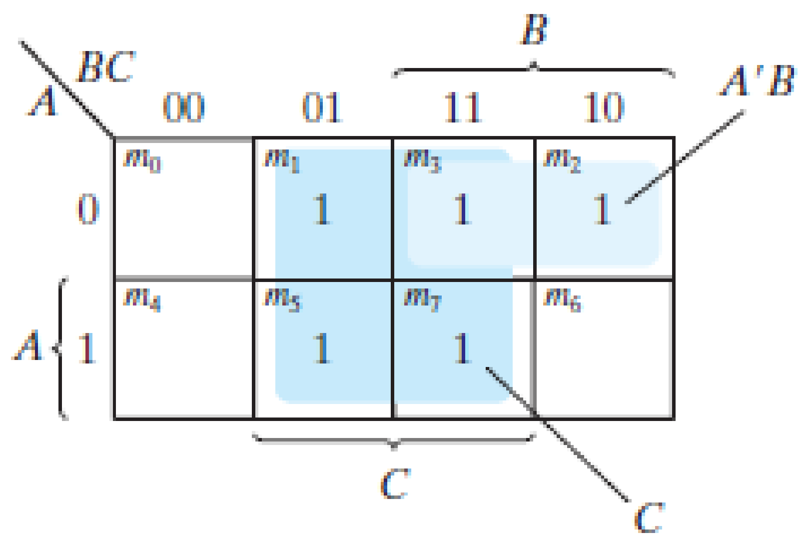
- For the Boolean function

$$F = A'C + A'B + AB'C + BC$$

(a) Express this function as a sum of minterms.

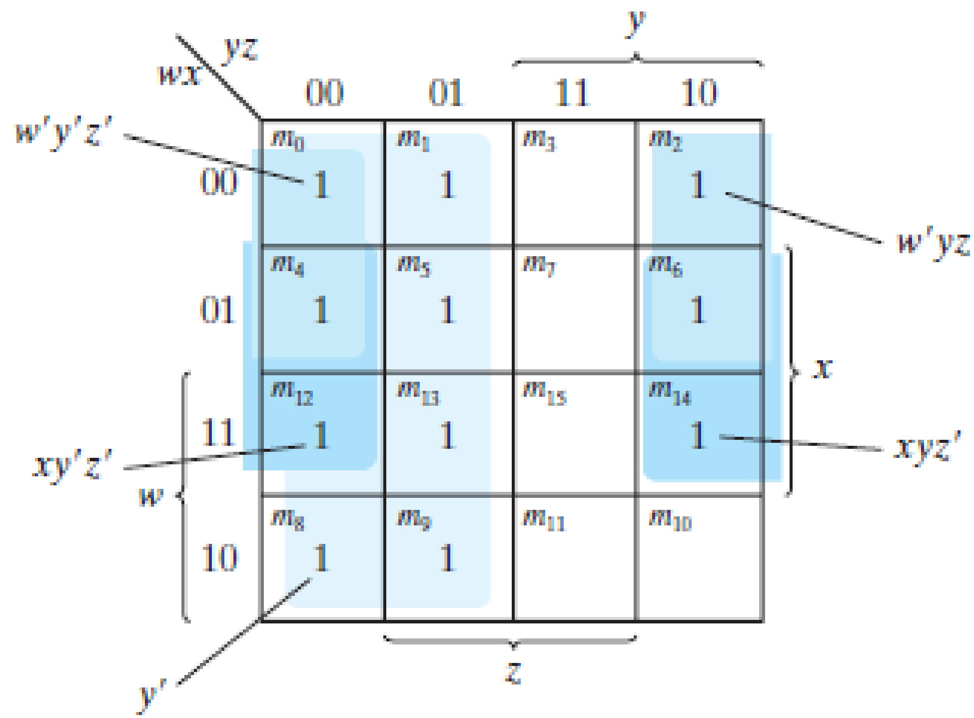
(b) Find the minimal sum-of-products expression.

- $F(A, B, C) = \sum(1, 2, 3, 5, 7)$
- $F = C + A'B$



K-Map Simplification

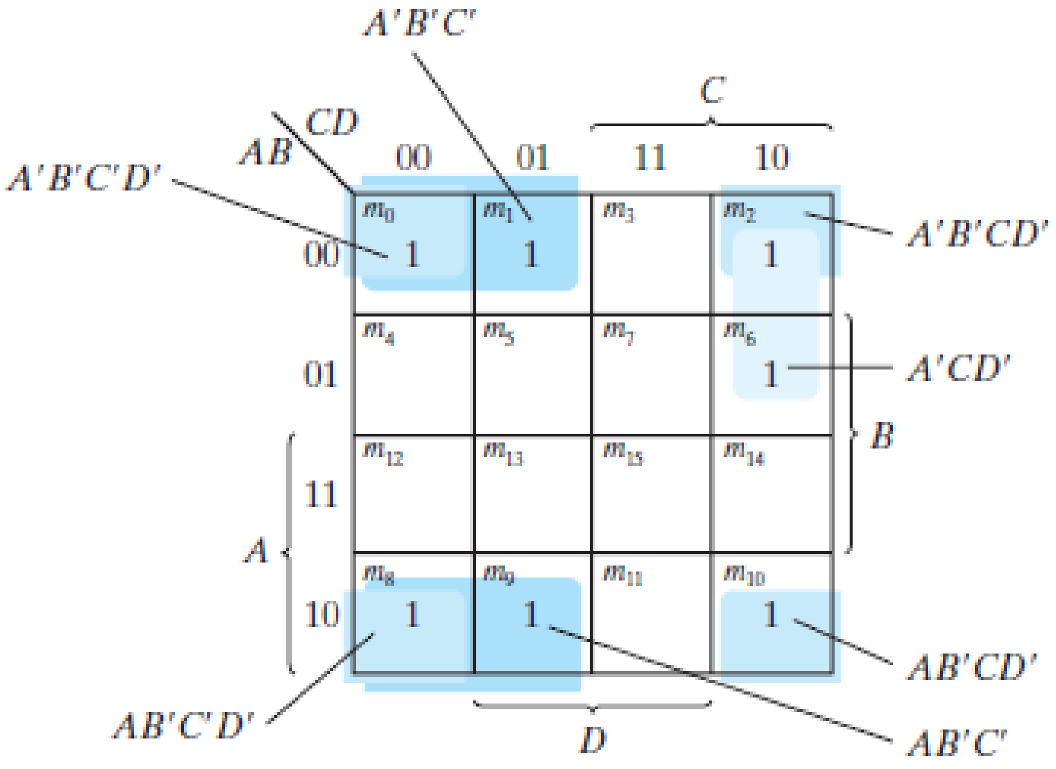
- Simplify the Boolean function
  - *F (w, x, y, z) =* $\sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$



- $F = y' + w'z' + xz'$

K-Map Simplification

- Simplify the Boolean function
  - $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



- $F = B'D' + B'C' + A'CD'$

K-map!

- In choosing adjacent squares in a map, we must ensure that
  - (1) all the minterms of the function are covered when we combine the squares,
  - (2) the number of terms in the expression is minimized
  - (3) there are no redundant terms (i.e., minterms already covered by other terms).

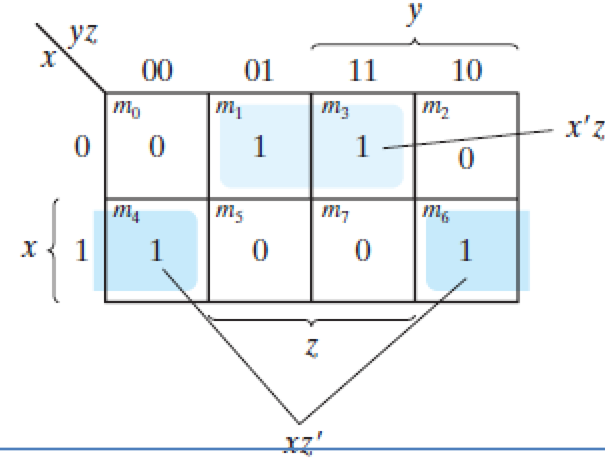
Complement using K-map

- Complement of a function is represented in the map by the squares not marked by 1's.
- Mark the empty squares by 0's and combine them into valid adjacent squares to obtain simplified sum-of-products expression of the complement of the function (i.e., of *F '*).

PRODUCT-OF-SUMS & SUM-OF-PRODUCTS SIMPLIFICATIONS

- Consider, the truth table that defines the function *F*
  - In sum-of-minterms form F is expressed as
    - *F (x, y, z) =∑ (1, 3, 4, 6)*
  - In product-of-maxterms form F is expressed as
    - *F (x, y, z) = ∏(0, 2, 5, 7)*
  - For the sum of products, we combine the 1's to obtain
    - *F = x'z + xz'*
  - For product of sums, we combine the 0's to obtain the simplified complemented function
    - *F' = xz + x'z'*
  - Taking the complement of *F', we obtain the simplified function in* product-of-sums form:
    - *F = (x' + z')(x + z)*

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



Instead of finding F' and complementing it, can directly group 0s in K-map and write it in simplified product-of-sums form directly

PRODUCT-OF-SUMS & SUM-OF-PRODUCTS SIMPLIFICATIONS

- Simplify the following Boolean function into

  (a) sum-of-products form and

  (b) product-of-sums form

  $$F(A, B, C, D) = \sum(0, 1, 2, 5, 8, 9, 10)$$

  - (a) sum-of-products form:

    $$F = B'D' + B'C' + A'C'D$$

  - (b) product-of-sums form:

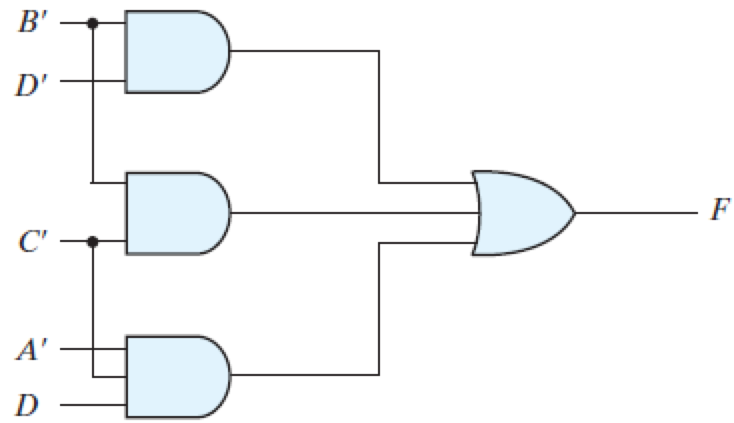
    $$F = (A' + B')(C' + D')(B' + D)$$

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

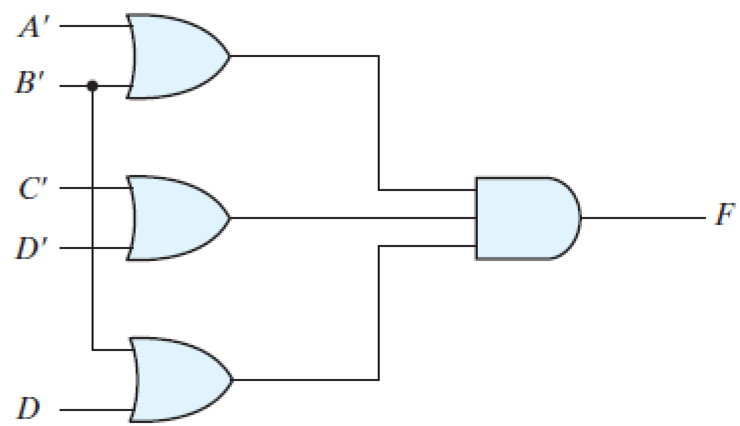
AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

PRODUCT-OF-SUMS & SUM-OF-PRODUCTS SIMPLIFICATIONS

A B C D	F
0 0 0 0	1
0 0 0 1	1
0 0 1 0	1
0 0 1 1	0
0 1 0 0	0
0 1 0 1	1
0 1 1 0	0
0 1 1 1	0
1 0 0 0	1
1 0 0 1	1
1 0 1 0	1
1 0 1 1	0
1 1 0 0	0
1 1 0 1	0
1 1 1 0	0
1 1 1 1	0



$F = B'D' + B'C' + A'C'D$



$F = (A' + B')(C' + D')(B' + D)$