Number System

Complement

- There are two types of complements for each base-r number system:
  - Radix complement (or) r's complement
  - Diminished radix complement (or) (r-1)'s complement
- Complements are used in digital computers for simplifying the subtraction operation.

r-1's Complement/ Diminished Radix Complement

- The (r-1)'s complement of N:

$$(r^n-1)-N$$

  - N – Number
  - r – radix /base
  - n – number of digits in N

- (r^n-1) is the largest number with n digits in base r
  - Hence, subtraction is between the number N from the largest number with n digits.

r-1's Complement of Binary Number/ 1's Complement

- **N = 1011000 ; r=2**

  n = 7 ; r – 1 = 1 (1's complement)

  1's complement $= (2^7\text{-}1)_{10}\text{-}(1011000)_2$

  $= (128 - 1)_{10} - (1011000)_2$

  $= (127)_{10} - (1011000)_2$

  $= (1111111)_2 - (1011000)_2$

  $= 0100111$

  Largest 7 digit binary number

- **N = 0101101 ; r=2**

  n =7

  1's complement = 1111111 – 0101101

  = 1010010

1’s complement – Another Method

- 1’s complement of a binary number is formed by
  - changing 1’s to 0’s and 0’s to 1’s
- Eg: N = 1011000
  - 1’s complement is 0100111

r-1's Complement of Octal Number/ 7's Complement

- N = 563 ; r = 8

  n= 3; r-1 = 7 (7's complement)

  7's complement $= (8^3\text{-}1)_{10}\text{-}(563)_8$

  $= (512\text{-}1)_{10}\text{-}(563)_8$

  $= (511)_{10}\text{-}(563)_8$

  $= (777)_8\text{-}(563)_8$

  $= 214$

Largest 3 digit octal number

r-1's Complement of Decimal Number/ 9's Complement

- N = 546700 ; r = 10

  n= 6; r-1 = 9 (9's complement)

  9's complement $= (10^6\text{-}1)_{10} - (546700)_{10}$

  $= (1000000\text{-}1)_{10} - (546700)_{10}$

  $= (999999)_{10} - (546700)_{10}$

  $= 453299$

Largest 6 digit decimal number

r-1's Complement of Hexdecimal Number/ 15's Complement

- N = C3DF ; r= 16

  n= 4; r-1 = 15 (15's complement)

  15's complement = $(16^4\text{-}1)_{10} - (C3DF)_{16}$

  = $(65536\text{-}1)_{10} - (C3DF)_{16}$

  = $(65535)_{10} - (C3DF)_{16}$

  = $(FFFF)_{16} - (C3DF)_{16}$

  = 3C20

Largest 4 digit hexadecimal number

r's Complement/Radix Complement

- The r's complement of N:

$$r^n - N \text{ for } N \neq 0$$

$$0 \quad \text{for } N = 0$$

  - N – Number
  - r – radix /base
  - n – number of digits in N
- r's complement is obtained by adding 1 to (r-1)'s complement:

$$r^n - N = [(r^n - 1) - N] + 1$$

- *Note: It is better to do (r-1)'s complement first for r's complement since it is easy to do the subtraction in (r-1)'s complement(no borrow problem!).*

r's Complement of Binary Number/ 2's Complement

- **N = 1011000 ; r=2**

n = 7 ; r = 2 (2's complement)

1's complement $= (1111111)_2 - (1011000)_2$

$= 0100111$

2's complement = 1's complement + 1

$= 0100111 + 1$

$= 0101000$

2's complement – Another Method

- 2's complement of a binary number is formed by
  - Scan the numbers from right to left
  - Till the first '1' is found write the digits as such.
  - After the first '1', invert all the digits.
- Eg: N = 1011000
  - Writing the digits till 1st '1' from right to left :

    - - - 1 0 0 0
  - Inverting the rest of the numbers

    0 1 0 1 0 0 0
  - Hence 2's complement is : 0101000
- Eg: N = 1011001
  - Writing the digits till 1st '1' from right to left :

    - - - - - - 1
  - Inverting the rest of the numbers

    0 1 0 0 1 1 1
  - Hence 2's complement is : 0100111

Easy method to directly write the 2's complement of a given number !

r's Complement of Octal Number/ 8's Complement

- N = 563 ; r = 8

  n= 3; r = 8 (8's complement)

  7's complement = $(777)_8$-$(563)_8$

  = 214

  8's complement = 7's complement + 1

  = 214 +1

  = 215

r's Complement of Decimal Number/ 10's Complement

- N = 546700 ; r = 10

  n= 6; r = 10 (10's complement)

  9's complement = $(999999)_{10} - (546700)_{10}$

  = 453299

  10's complement = 9's complement + 1

  = 453299 + 1

  = 453300

r's Complement of Hexadecimal Number/ 16's Complement

- N = C3DF ; r= 16

  n= 4; r = 16 (16's complement)

  15's complement = $(FFFF)_{16} - (C3DF)_{16}$

  = 3C20

  16's complement = 15's complement + 1

  = 3C20 + 1

  = 3C21

Complement of a number with radix point

- If the original number N contains a radix point,
  - Temporarily remove the point to perform complement.
  - The radix point is then restored to the complemented number in the same relative position.

Complement with radix point (Base- 2)

- N = 1101.011

  1101.011= 1101011×2^{-3}

  1's complement of 1101011=1111111-1101011

  = 0010100

  2's complement of 1101011 = 0010100 + 1

  = 0010101

  1's complement of 1101.011= 0010100×2^{-3}

  = 0010.100

  2's complement of 1101.011 = 0010101×2^{-3}

  = 0010.101

Complement with radix point (Base- 8)

- N = 323.64

  $323.64 = 32364 \times 8^{-2}$

  7's complement of 32364 $= 77777 - 32364$

  $= 45413$

  8's complement of 32364 $= 45413 + 1$

  $= 45414$

  7's complement of 323.64 $= 45413 \times 8^{-2}$

  $= 454.13$

  8's complement of 323.64 $= 45414 \times 8^{-2}$

  $= 454.14$

Complement with radix point (Base- 10)

- N = 325.93

  $325.93 = 32593 \times 10^{-2}$

  9's complement of 32593 $= 99999 - 32593$

  $= 67406$

  10's complement of 32593 $= 67406 + 1$

  $= 67407$

  9's complement of 325.93 $= 67406 \times 10^{-2}$

  $= 674.06$

  10's complement of 325.93 $= 67407 \times 10^{-2}$

  $= 674.07$

Complement with radix point (Base- 16)

- N = ABC.3E2

  ABC.3E2 = ABC3E2 x 16^{-3}

  15's complement of ABC3E2 =FFFFFF- ABC3E2

  = 543C1D

  16's complement of ABC3E2 = 543C1D + 1

  = 543C1E

  15's complement of ABC.3E2 = 543C1D x 16^{-3}

  = 543.C1D

  16's complement of ABC.3E2 = 543C1E x16^{-3}

  = 543.C1E

Subtraction with Complements
r's Complement Subtraction

- The subtraction of 2 n-digit unsigned numbers M-N in base r can be done as follows:
  - Add the minuend M to the r's complement of the subtrahend N. This performs: $M+(r^n-N)=M-N+r^n$
  - If $M \geq N$, the sum will produce an end carry,r^n, which can be discarded. Hence the result is M-N
  - If $M < N$, the sum does not produce an end carry and is equal to $r^n-(N-M)$, which is the r's complement of N-M

r's Complement Subtraction(Base-2)

- 1010100-1000011

  2's complement of 1000011=1111111-1000011+1 = 0111101

	1 0 1 0 1 0 0
	0 1 1 1 1 0 1
1	0 0 1 0 0 0 1

Answer =0010001

- 1000011-1010100

  2's complement of 1010100=1111111-1010100+1 = 0101100

1 0 0 0 0 1 1
0 1 0 1 1 0 0
1 1 0 1 1 1 1

Answer =1101111 (or) - 0010001

r’s Complement Subtraction(Base-8)

- 342-614

  8’s complement of 614=777-614+1 = 164

	3 4 2	
	1 6 4	
	5 2 6	

Answer =526 (or) -252

- 614-342

  8’s complement of 342=777-342+1 = 436

	6 1 4	
	4 3 6	
1	2 5 2	

Answer =252

r’s Complement Subtraction(Base-10)

- 72532-3250

  10’s complement of 03250=99999-03250+1 = 96750

	7	2	5	3	2
	9	6	7	5	0
1	6	9	2	8	2

Answer =69282

- 3250-72532

  10’s complement of 72532=99999-72532+1 = 27468

0	3	2	5	0
2	7	4	6	8
3	0	7	1	8

Answer =30718 (or) -69282

r's Complement Subtraction(Base-16)

- CB2-672

  16's complement of 672=FFF-672+1 =98E

	C B 2
	9 8 E
1	6 4 0

Answer =640

- 672-CB2

  16's complement of CB2=FFF-CB2+1 = 34E

6 7 2
3 4 E
9 C 0

Answer =9C0 (or) - 640

Subtraction with Complements
r-1's Complement Subtraction

- The subtraction of 2 n-digit unsigned numbers M-N in base r can be done as follows:
  - Add the minuend M to the r's complement of the subtrahend N.
  - If M≥ N, the sum will produce an end carry, which is added to the result since it produces a sum that is 1 less than the correct difference(only if carry is generated).
    - Removing the end carry and adding 1 to the sum is referred to as an end-around carry.
  - If M< N, the sum does not produce an end carry,which is the r-1's complement of N-M

r-1’s Complement Subtraction(Base-2)

- 1010100-1000011

  1’s complement of 1000011=1111111-1000011 = 0111100

```
  1 0 1 0 1 0 0
  0 1 1 1 1 0 0
-------------------
1 | 0 0 1 0 0 0 0
              1
-------------------
  0 0 1 0 0 0 1
-------------------
```

Answer =0010001

- 1000011-1010100

  1’s complement of 1010100=1111111-1010100 = 0101011

```
  1 0 0 0 0 1 1
  0 1 0 1 0 1 1
-----------------
  1 1 0 1 1 1 0
-----------------
```

Answer =1101110 (or) -0010001

r-1’s Complement Subtraction(Base-8)

- 402-314

  7’s complement of 314=777-314 = 463

  4 0 2

  4 6 3

  1 | 0 6 5

  1

  0 6 6

  Answer =066

- 314-402

  7’s complement of 402=777-402 = 375

  3 1 4

  3 7 5

  7 1 1

  Answer =711 (or) - 066

r-1’s Complement Subtraction(Base-10)

- 4567-1234

  9’s complement of 1234=99999-1234= 8765

```
  4 5 6 7
  8 7 6 5
-------------
1 | 3 3 3 2
      1
-------------
  3 3 3 3
-------------
```

Answer =3333

- 1234-4567

  9’s complement of 4567=99999-4567= 5432

```
  1 2 3 4
  5 4 3 2
-----------
  6 6 6 6
-----------
```

Answer =6666 (or) -3333

r-1's Complement Subtraction(Base-16)

- B06-C7C

  15's complement of C7C=FFF-C7C = 383

```
  B 0 6
  3 8 3
-------
  E 8 9
-------
```

Answer =E89 (or) -176

- C7C-B06

  15's complement of B06=FFF-B06 = 4F9

```
    C 7 C
    4 F 9
-----------
1 | 1 7 5
      1
-------
  1 7 6
-------
```

Answer =176