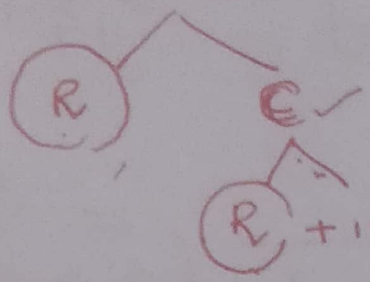
Number System

N — Natural, {1, ...}
W — whole, {0, ...}
Z — Integers, {-∞, ∞}
Q = P/q — Rational, Irrational
0.2323|..



Partial fractions

Nr / Dr . degree of Dr > degree of Nr

$$Dr \overline{)\, Nr}\ \ Q \qquad Nr = \frac{Q \times R}{D}$$

Type 1:

$$\frac{1}{(a_1x+b_1)(a_2x+b_2)(a_3x+b_3)\cdots a_1x+b_1} = \frac{A}{a_2x+b_2} + \frac{B}{a_3x+b_3} + \frac{C}{\ }$$

Type 2:

$$\frac{1}{(a_1x+b_1)^2(a_2x+b_2)} = \frac{A}{a_1x+b_1} + \frac{B}{(a_1x+b_1)^2} + \frac{C}{(a_2x+b_2)}$$

Type 4

$$\frac{1}{(a_1x^2+b_1x+c_1)} = \frac{Ax+B}{a_1x^2+b_1x+c_1}$$

$$\frac{1}{(a_1x^2+b_1x+c_1)(a_2x^2+b_2x+c_2)}$$

$$\frac{Ax+B}{a_1x^2+b_1x+c_1} + \frac{Cx+D}{a_2x^2+b_2x+c_2}$$

(1) Find the Partial fraction decomp

$$\frac{x-29}{(x-4)(x+1)}$$

$$\frac{x-29}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$$

$$= \frac{A(x+1)+B(x-4)}{(x-4)(x+1)}$$

$$x-29 = Ax+Bx+A-4B$$

$$= (A+B)x + A-4B$$

$A + B = 1$ ——— (1)

$A - 4B = -29$ ——— (2)

i.e., $-A + 4B = 29$.

$\boxed{5B = 30} \Rightarrow \boxed{B = 6}$

$A + 6 = 1$

$\Rightarrow \boxed{A = -5}$.

$$\frac{x - 29}{(x-4)(x+1)} = \frac{-5}{(x-4)} + \frac{6}{(x+1)}$$

(2) $$\frac{2x^2 + x}{(x-1)^2 (x+1)^2}$$

$$= \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} + \frac{D}{(x+1)^2}$$

$$= \frac{A(x-1)(x+1)^2 + B(x+1)^2 + C(x+1)(x-1)^2 + D(x-1)^2}{(x-1)^2 (x+1)^2}$$

$$= A(x^3 + 2x^2 + x - x^2 - 2x - 1) + B(x^2 + 2x + 1) + C(x^3 - 2x^2 + x + x - 2x + 1) + D(x^2 - 2x + 1)$$

$$A(x^3+x^2-x-1)+B(x^2+2x+1)$$
$$+C(x^3-x^2-x+1)$$
$$+D(x^2-2x+1)$$

Equating

x^3, $A+C=0$ — (1)

x^2, $A+B-C+D=2$ — (2)

x, $-A+2B-C-2D=1$ — (3)

Constant, $-A+B+C+D=0$ — (4)

$A=-C$ $\quad C=1/2$ $\quad B=3/4$

$D=1/4$

Other method:

$$2x^2 + x = A(x-1)(x+1)^2 + B(x+1)^2 + C(x+1)(x-1)^2 + D(x-1)^2.$$

Put $x = 1$.

$$2(1) + 1 = B(1+1)^2$$

$$3 = 4B \Rightarrow \boxed{B = 3/4}$$ ✓

Put $x = -1$

$$2(+) - 1 = D(-2)^2$$

$$2 - 1 = 4D$$

$$1 = 4D \Rightarrow \boxed{D = 1/4}$$ ✓

Put $x = 0$

$$A + C = 0.$$

$$-A + C = -1$$

$A = 1/2$

$C = -1/2$

$$0 = A(-1) + B(1) + C(1) + D(1)$$

$$-A + C = -1.$$

$$\frac{10-x}{x^2+10x+25} = \frac{10-x}{(x+5)^2}$$

$$= \frac{A}{x+5} + \frac{B}{(x+5)^2}$$

$$10-x = A(x+5) + B$$

$$10-x = Ax + 5A + B$$

Equating:

x ; $A = -1$.

$5A + B = 10 \Rightarrow \boxed{B = +15}$

$$\int u\,dv = uv - \int v\,du.$$

$$\int u\,dv = uv - u'v_1 + u''v_2 + \ldots\ldots$$

"I LATE"

$\sin(n)x$ $\cos(n)x$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$x^2 + 4$$
$$x^2 + 99$$