Number System

General Representation

A number is represented as

- . radix point
- a_i coefficients(symbols used in a number representation)
- j place value

Example:

Decimal number: 567.28

Radix or Base

- In a positional numeral system(value of each symbol depends on the position), the radix or base is the number of unique digits, including the digit zero, used to represent numbers.
- Eg: Decimal number system:
 - Uses 10 symbols (0,1,2,3,4,5,6,7,8,9)
 - Hence radix or base = 10

Positional Number System

Another representation of a number:

$$a_n r^n + a_{n-1} r^{n-1} + ... + a_1 r^1 + a_0 r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} + ... + a_{-m} r^{-m}$$

- a_j coefficients
- j place value/ positional value
- r radix or base
- Eg: 567.28(decimal number)
 - $567.28 = 5x10^{2} + 6x10^{1} + 7x10^{0} + 2x10^{-1} + 8x10^{-2}$

Radix - 2 (Binary Numbers)

Symbols Used: 0,1

Radix - 8 (Octal Numbers)

• Symbols Used: 0,1,2,3,4,5,6,7

Radix – 10 (Decimal Numbers)

• Symbols Used: 0,1,2,3,4,5,6,7,8,9

Radix – 16 (Hexadecimal Numbers)

Symbols Used: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Example

• The number "twenty-seven" can be represented in different ways :

- The use of radix-2 (binary) numbers became popular with the onset of electronic computers,
 - binary digits or bits, having only two possible values 0 and 1, is compatible with electronic signals
- Radix-8 (octal) and radix-16 (hexadecimal) numbers have been used as shorthand notation for binary numbers.

General representation

 Usually first 10 symbols in a number system of radix r is represented by the symbols of decimal number system and for the 11th symbol it starts with the symbols of English alphabets.

— Eg:

- Radix-6, Symbols: 0,1,2,3,4,5
- Radix-19, Symbols:0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F,G,H,I.
- To distinguish a number represented in a particular number system, the number is usually written by enclosing it in a parenthesis with a subscript of r.

- Eg:

- $(56)_8$ octal
- $(56)_{10}$ decimal
- $(1011)_2$ binary

Conversion between Number Systems (Conversion from base-r to decimal)

- Conversion of any base-r number to decimal:
 - Multiply each digit with its weight(radix raised to its positional value) to get the resultant value of each symbol.
 - Add all the resultant symbol values.

Conversion between Number Systems (Binary to decimal number)

100.111

$$(100.111)_2 = 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$
$$= 4 + 0 + 0 + 0.5 + 0.25 + 0.125$$
$$= (4.875)_{10}$$

Conversion between Number Systems (Octal to decimal number)

• 517.35

```
(517.35)_8 = 5x8^2 + 1x8^1 + 7x8^0 + 3x8^{-1} + 5x8^{-2}
= 320+8+7+0.375+0.078125
= (335.453125)_{10}
```

Conversion between Number Systems (Hexadecimal to decimal number)

• 786.BC

```
(786.BC)_{16} = 7x16^{2} + 8x16^{1} + 6x16^{0} + Bx16^{-1} + Cx16^{-2}
= 1792 + 128 + 6 + 0.6875 + 0.046875
= (1926.734375)_{10}
```

Conversion between Number Systems (Conversion from decimal to base-r)

- Conversion of a decimal integer to a number in base r:
 - Divide the integer and all its successive quotients by r
 - accumulate the remainder in reverse order
 - Range of remainder: 0 to r-1
- Conversion of a decimal fraction to a number in base r:
 - Multiply the fraction by r and its successive remainder by r
 - accumulate the quotients in the same order
 - Range of coefficients: 0 to r-1

Conversion between Number Systems (decimal to binary number)

• 343.392

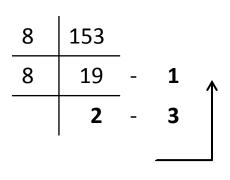
343									
171	-	1			0.392	X	2	=	0 .784
85	- -	1			0.784	X	2	=	1 .568
42	- -	1			0.568	X	2	=	1 .136
21	- -	0			0.136	X	2	=	0 .272
10	- -	1			0.272	X	2	=	0 .544
5	-	0			0.544	X	2	=	1 .088
2	-	1 ↑							
1	-	0		(0.39	$(2)_{10} = (0)_{10}$	0.02	110	01	.) ₂
	171 85 42 21 10 5	171 - 85 - 42 - 21 - 10 - 5 - 2 -	171 - 1 85 - 1 42 - 1 21 - 0 10 - 1 5 - 0 2 - 1	171 - 1 85 - 1 42 - 1 21 - 0 10 - 1 5 - 0 2 - 1 ↑	171 - 1 85 - 1 42 - 1 21 - 0 10 - 1 5 - 0 2 - 1 ↑	171 - 1 0.392 85 - 1 0.784 42 - 1 0.568 21 - 0 0.136 10 - 1 0.272 5 - 0 0.544 2 - 1 (0.392) = (0.392)	171 - 1 0.392 x 85 - 1 0.784 x 42 - 1 0.568 x 21 - 0 0.136 x 10 - 1 0.272 x 5 - 0 0.544 x 2 - 1 (0.392) = (0.07) $(0.392) = (0.07)$ $(0.392) = (0.07)$	171 - 1 0.392 x 2 85 - 1 0.784 x 2 42 - 1 0.568 x 2 21 - 0 0.136 x 2 10 - 1 0.272 x 2 5 - 0 0.544 x 2 2 - 1 (0.392) $_{10}$ = (0.0110)	171 - 1 0.392 x 2 = 85 - 1 0.784 x 2 = 42 - 1 0.568 x 2 = 21 - 0 0.136 x 2 = 10 - 1 0.272 x 2 = 5 - 0 0.544 x 2 = 2 - 1 (0.392) = (0.011001)

 $(343)_{10} = (101010111)_2$

Ans: $(343.392)_{10} = (101010111.011001)_2$

Conversion between Number Systems (decimal to octal number)

• 153.513



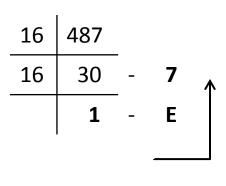
$$(153)_{10} = (231)_8$$

$$0.513 \times 8 = 4.104$$
 $0.104 \times 8 = 0.832$
 $0.832 \times 8 = 6.656$
 $0.656 \times 8 = 5.248$
 $0.248 \times 8 = 1.984$
 $0.984 \times 8 = 7.872$

Ans: $(153.513)_{10} = (231.406517)_8$

Conversion between Number Systems (decimal to hexadecimal number)

• 487.365



$$(487)_{10} = (1E7)_{16}$$

$$0.365$$
 x $16 = 5.84$
 0.84 x $16 = 13.44$
 0.44 x $16 = 7.04$
 0.04 x $16 = 0.64$
 0.64 x $16 = 10.24$
 0.24 x $16 = 3.84$
 $(0.365)_{10} = (0.5D70A3...)_{16}$

Ans: $(487.365)_{10} = (1E7.5D70A3)_{16}$

Conversion from Binary to Octal and Octal to Binary

- Conversion of a binary number to a octal number:
 - Keep splitting the binary number into 3 bits from right to left before radix point and left to right after radix point.
 - If the leftmost part before radix point has lesser than 3 bits, add 0s to fill the places.
 - If the rightmost part after radix point has lesser than 3 bits, add 0s to fill the places.
 - Write the corresponding octal symbol for each 3 bits and accumulate them.
- Conversion of a octal number to a binary number:
 - Write the 3 bit binary equivalent of each octal number and accumulate them.

Conversion from Binary to Octal

- 10110001101011.111100000110
 - Splitting the numbers into 3 bits:

```
010 110 001 101 011 . 111 100 000 110
```

2 6 1 5 3 . 7 4 0 6

- Answer:

 $(10110001101011.111100000110)_2 = (26153.7406)_8$

Conversion from Octal to Binary

- 673.124
 - Writing 3-bit binary equivalent for each number.

```
6 7 3.1 2 4
110 111 011 001 010 100
```

- Answer: $(673.124)_8 = (110111011.001010100)_2$

Conversion from Binary to Hexadecimal and Hexadecimal to Binary

- Conversion of a binary number to a hexadecimal number:
 - Keep splitting the binary number into 4 bits from right to left before radix point and left to right after radix point.
 - If the leftmost part has lesser than 4 bits, add 0s to fill the places.
 - If the rightmost part after radix point has lesser than 4 bits, add 0s to fill the places.
 - Write the corresponding hexadecimal symbol for each 4 bits and accumulate them.
- Conversion of a hexadecimal number to a binary number:
 - Write the 4 bit binary equivalent of each hexadecimal number and accumulate them.

Conversion from Binary to Hexadecimal

- 10110001101011.111100000110
 - Splitting the numbers into 4 bits:

```
0010 1100 0110 1011 . 1111 0000 0110
```

C 6 B . F 0

– Answer:

```
(10110001101011.1111100000110)_{2} = (2C6B.F06)_{16}
```

Conversion from Hexadecimal to Binary

- 306.D
 - Writing 4-bit binary equivalent for each number.

```
3 0 6 D 0011 0000 0110 1101
```

- Answer: $(306.D)_{16} = (001100000110.1101)_2$

Binary	Decimal	Hexadecimal
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	Α
1011	11	В
1100	12	C
1101	13	D
1110	14	E
1111	15	F