1.
$$\lim_{x \to c} f(x) = L$$

$$\lim_{x \to c} g(x) = M$$

① Sum rule :

$$\lim_{x \to c} \left[f(x) + g(x) \right] = L + M$$

② Difference rule

$$\lim_{x \to c} \left[f(x) - g(x) \right] = L - M$$

③ Product rule :

$$\lim_{x \to c} \left[f(x) \cdot g(x) \right] = L \cdot M$$

④ Constant Multiple rule

$$\lim_{x \to c} \left[c [f(x)] \right] = C \cdot M$$

⑤ Quotient rule :

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M} \quad , \quad g(x) \neq$$

⑥ Power rule :

$$\lim_{x \to c} (f(x))^{n/m} \quad \text{n and n are integers}$$

$$= (L)^{n/m}$$

$$\underset{x \to 2}{lt} \quad \frac{x^2 - 7x + 10}{x - 2}$$

$$= \underset{x \to 2}{lt} \quad \frac{(x-2)(x-5)}{(x-2)} = \frac{-3}{2}$$

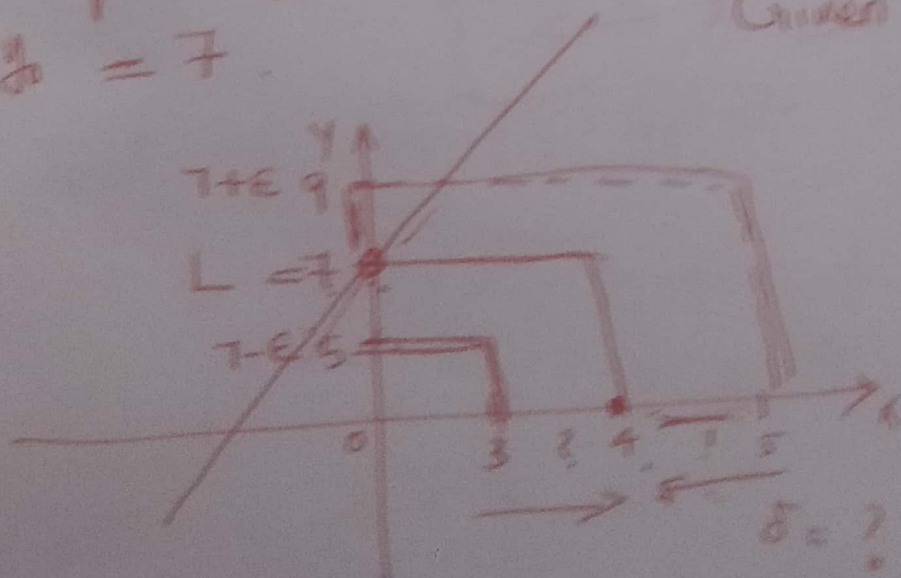## formal definition of limit.

$$\forall \varepsilon > 0 \quad \exists \delta > 0 \ni$$

$$0 < |x - x_0| < \delta \Rightarrow$$

$$|f(x) - L| \leq \varepsilon.$$

How close to $x_0 = 4$ must be hold the input x to be sure that the output $y = 2x - 1$ lies within 2 units of $y_0 = 7$.

Given $y = 2x - 1$

$L = 7$

$x_0 = 4$

$\varepsilon = 2$

$\delta = ?$

$y = 2(4) - 1 = 8 - 1 = 7$

$$\left| f(x) - L \right| < \varepsilon$$

$$\left| y - 7 \right| < 2$$

$$-2 < \left[\boxed{y} - 7 \right] < 2$$

$$-2 < 2x - 1 - 7 < 2$$

$$-1 < \boxed{x} - 4 < 1$$

$$\left| x - 4 \right| < 1$$
$$\left| x - x_0 \right| < \boxed{\delta}$$

$$\boxed{3 < x < 5}$$

find $\delta_2 > 0$ for the limit

$$\lim_{x \to 5} \sqrt{x-1} = 2 \text{ within}$$

$$\mathcal{E} = 1$$

$$f(x) = \sqrt{x-1} ; \quad L = 2$$

$$\mathcal{E} = 1$$

$$x_0 =$$

$$\forall \, \mathcal{E} > 0 \quad \exists \, \delta > 0 \ni$$

$$|x - x_0| < \delta \implies$$

$$|f(x) - L| < \mathcal{E}.$$

$$|\sqrt{x-1} - 2| < 1$$

$$-1 \leq \sqrt{x-1} \, \boxed{-2} < 1$$

$$1 < \sqrt{x-1} < 3$$

$$1 < x - 1 < 9 \qquad \delta = \min(3, 5)$$

$$2 < x < 10 \qquad \qquad \delta = 3$$

$$2 - 5 < x - 5 < 10 - 5$$

$$\tfrac{?}{?} < x - 5 < 5$$

$$0 < |x - 5| < 3$$

$$\forall \varepsilon = 1 > 0 \rightarrow \delta = 3 > 0$$

$$\ni \quad 0 < |x - 5| < 3 \implies$$

$$\left| \sqrt{x - 1} - 2 \right| < 1$$

Continuity

$$\underset{x \to a}{lt} \ f(x) = L$$

ie:-

$$\underset{x \to a}{lt} \ f(x) = f(a)$$

i) $f(a)$ exists ✓

ii) $\underset{x \to a}{lt} \ f(x)$ exists ✓

iii) $\underset{x \to a}{lt} \ f(x) = f(a)$