Number System

General Representation

- A number is represented as

$$a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}a_{-4}$$

  - . - radix point
  - a_j – coefficients(symbols used in a number representation)
  - j – place value
- Example:
  - Decimal number: 567.28

Radix or Base

- In a positional numeral system(value of each symbol depends on the position), the **radix** or **base** is the number of unique digits, including the digit zero, used to represent numbers.
- Eg: Decimal number system:
  - Uses 10 symbols (0,1,2,3,4,5,6,7,8,9)
  - Hence radix or base = 10

Positional Number System

- Another representation of a number:

  $a_n r^n + a_{n-1} r^{n-1} + \ldots + a_1 r^1 + a_0 r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \ldots + a_{-m} r^{-m}$
- a_j – coefficients
- j – place value/ positional value
- r – radix or base
- Eg: 567.28(decimal number)
  - $567.28 = 5x10^2 + 6x10^1 + 7x10^0 + 2x10^{-1} + 8x10^{-2}$

Radix - 2 (Binary Numbers)

- Symbols Used: 0,1
- Base: 2

Radix - 8 (Octal Numbers)

- Symbols Used: 0,1,2,3,4,5,6,7
- Base: 8

Radix – 10 (Decimal Numbers)

- Symbols Used: 0,1,2,3,4,5,6,7,8,9
- Base: 10

Radix – 16 (Hexadecimal Numbers)

- Symbols Used: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Base: 16

Example

- The number “twenty-seven” can be represented in different ways :
  - IIIII IIIII IIIII IIIII IIIII II (sticks or unary code)
  - 27 (radix-10 or decimal code)
  - 11011 (radix-2 or binary code)
  - XXVII (roman numerals)

- The use of radix-2 (*binary) numbers became popular with* the onset of electronic computers,
  - binary digits or *bits, having only* two possible values 0 and 1, is compatible with electronic signals
- Radix-8 (*octal) and* radix-16 (*hexadecimal) numbers have been used as shorthand notation for binary* numbers.

General representation

- Usually first 10 symbols in a number system of radix r is represented by the symbols of decimal number system and for the 11^{th} symbol it starts with the symbols of English alphabets.
  - Eg:
    - Radix-6 , Symbols: 0,1,2,3,4,5
    - Radix-19, Symbols:0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F,G,H,I.
- To distinguish a number represented in a particular number system, the number is usually written by enclosing it in a parenthesis with a subscript of r.
  - Eg:
    - $(56)_8$ – octal
    - $(56)_{10}$ – decimal
    - $(1011)_2$ - binary

Conversion between Number Systems (Conversion from base-r to decimal)

- Conversion of any base-r number to decimal:
  - Multiply each digit with its weight(radix raised to its positional value) to get the resultant value of each symbol.
  - Add all the resultant symbol values.

Conversion between Number Systems (Binary to decimal number)

- 100.111

$$(100.111)_2 = 1x2^2+0x2^1+0x2^0+1x2^{-1}+1x2^{-2}+1x2^{-3}$$

$$= 4+0+0+0.5+0.25+0.125$$

$$= (4.875)_{10}$$

Conversion between Number Systems (Octal to decimal number)

- 517.35

$$(517.35)_8 = 5x8^2+1x8^1+7x8^0+3x8^{-1}+5x8^{-2}$$

$$= 320+8+7+0.375+0.078125$$

$$= (335.453125)_{10}$$

Conversion between Number Systems (Hexadecimal to decimal number)

- 786.BC

$$(786.BC)_{16} = 7x16^2 + 8x16^1 + 6x16^0 + Bx16^{-1} + Cx16^{-2}$$

$$= 1792 + 128 + 6 + 0.6875 + 0.046875$$

$$= (1926.734375)_{10}$$

Conversion between Number Systems (Conversion from decimal to base-r)

- Conversion of a decimal integer to a number in base r:
  - Divide the integer and all its successive quotients by r
  - accumulate the remainder in reverse order
  - Range of remainder: 0 to r-1
- Conversion of a decimal fraction to a number in base r:
  - Multiply the fraction by r and its successive remainder by r
  - accumulate the quotients in the same order
  - Range of coefficients: 0 to r-1

Conversion between Number Systems (decimal to binary number)

- 343.392

2	343		
2	171	-	**1**
2	85	-	**1**
2	42	-	**1**
2	21	-	**0**
2	10	-	**1**
2	5	-	**0**
2	2	-	**1**
	1	-	**0**

$(343)_{10} = (101010111)_2$

0.392 x 2 = **0** .784
0.784 x 2 = **1** .568
0.568 x 2 = **1** .136
0.136 x 2 = **0** .272
0.272 x 2 = **0** .544
0.544 x 2 = **1** .088

$(0.392)_{10} = (0.011001\ldots)_2$

Ans: $(343.392)_{10} = (101010111.011001)_2$

Conversion between Number Systems (decimal to octal number)

- 153.513

8	153		
8	19	-	**1**
	2	-	**3**

$(153)_{10} = (231)_8$

0.513 x 8 = **4** .104
0.104 x 8 = **0** .832
0.832 x 8 = **6** .656
0.656 x 8 = **5** .248
0.248 x 8 = **1** .984
0.984 x 8 = **7** .872

$(0.513)_{10} = (0.406517\ldots)_8$

Ans: $(153.513)_{10} = (231.406517)_8$

Conversion between Number Systems (decimal to hexadecimal number)

- 487.365

16	487		
16	30	-	**7**
	1	-	**E**

$(487)_{10} = (1E7)_{16}$

0.365 x 16 = **5** .84
0.84 x 16 = **13** .44
0.44 x 16 = **7** .04
0.04 x 16 = **0** .64
0.64 x 16 = **10** .24
0.24 x 16 = **3** .84

$(0.365)_{10} = (0.5D70A3\ldots)_{16}$

Ans: $(487.365)_{10} = (1E7.5D70A3)_{16}$

Conversion from Binary to Octal and Octal to Binary

- Conversion of a binary number to a octal number:
  - Keep splitting the binary number into 3 bits from right to left before radix point and left to right after radix point.
  - If the leftmost part before radix point has lesser than 3 bits, add 0s to fill the places.
  - If the rightmost part after radix point has lesser than 3 bits, add 0s to fill the places.
  - Write the corresponding octal symbol for each 3 bits and accumulate them.
- Conversion of a octal number to a binary number:
  - Write the 3 bit binary equivalent of each octal number and accumulate them.

Conversion from Binary to Octal

- 10110001101011.111100000110
  - Splitting the numbers into 3 bits:

    010 110 001 101 011 . 111 100 000 110

    2 6 1 5 3 . 7 4 0 6
  - Answer:
    $(10110001101011.111100000110)_2=(26153.7406)_8$

Conversion from Octal to Binary

- 673.124
  - Writing 3-bit binary equivalent for each number.

6	7	3	.	1	2	4
110	111	011		001	010	100

  - Answer: $(673.124)_8=(110111011.001010100)_2$

Conversion from Binary to Hexadecimal and Hexadecimal to Binary

- Conversion of a binary number to a hexadecimal number:
  - Keep splitting the binary number into 4 bits from right to left before radix point and left to right after radix point.
  - If the leftmost part has lesser than 4 bits, add 0s to fill the places.
  - If the rightmost part after radix point has lesser than 4 bits, add 0s to fill the places.
  - Write the corresponding hexadecimal symbol for each 4 bits and accumulate them.
- Conversion of a hexadecimal number to a binary number:
  - Write the 4 bit binary equivalent of each hexadecimal number and accumulate them.

Conversion from Binary to Hexadecimal

- 101100011010 11.111100000110
  - Splitting the numbers into 4 bits:

    0010 1100 0110 1011 . 1111 0000 0110

    2 C 6 B . F 0 6
  - Answer:
    $(101100011010 11.111100000110)_2=(2C6B.F06)_{16}$

Conversion from Hexadecimal to Binary

- 306.D
  - Writing 4-bit binary equivalent for each number.

    3 0 6 . D

    0011 0000 0110 . 1101
  - Answer: $(306.D)_{16}=(001100000110.1101)_2$

Binary	Decimal	Hexadecimal
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F