

Number System

Signed Binary Number Representations

- Both signed and unsigned binary numbers consists of a string of bits when represented in computer.
- User determines whether a number is signed or not.
- Representation Types:
 - Signed-magnitude representation
 - Signed- Complement representation
 - 1's Complement
 - 2's Complement

Sign-Magnitude Representation

- The number consists of two parts:
 - Sign bit (leftmost bit)
 - Magnitude bits (other than leftmost bit)
- If the leftmost bit is
 - 0 – positive number
 - 1 – negative number
- The negative number has the same magnitude bits as the corresponding positive number but the sign bit is 1 rather than 0.
- Eg: 8-bit representation of 'fifteen'
 - +15 – 0 0001111
 - -15 – 1 0001111
- It is used in ordinary arithmetic but usually not in computer arithmetic, since sign and magnitude bits must be handled separately.

1's Complement Representation

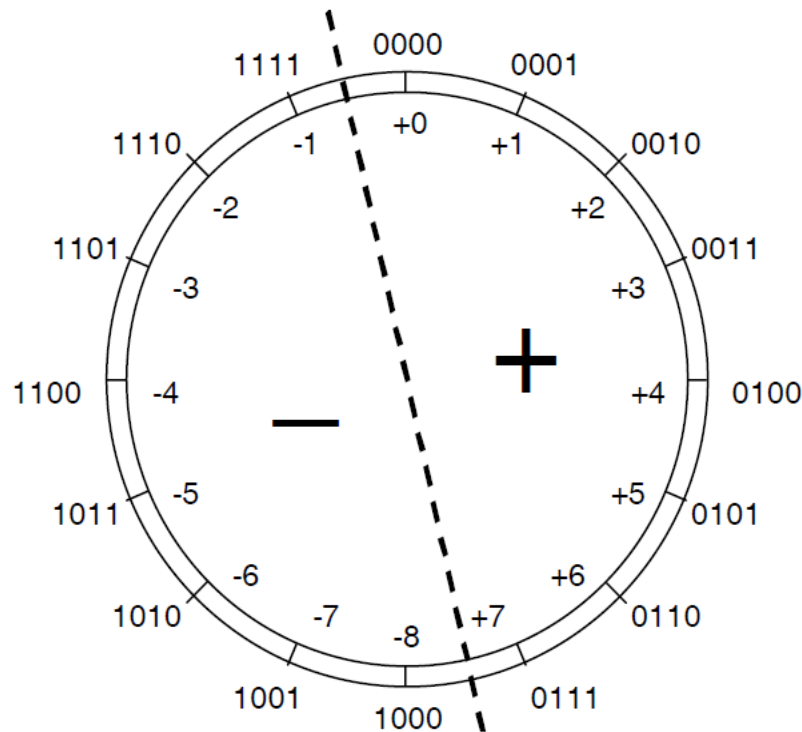
- The negative number is the 1's complement of the corresponding positive number.
- Has some difficulties while used for arithmetic operations.
- It is used in logical operations.
- There are two different representations for zero.(i.e) 0000 and 1111 (4 bit +0 and -0).
- Eg: 8-bit representation of 'fifteen'
 - +15 – 00001111
 - -15 – 11110000

2's Complement Representation

- The negative number is the 2's complement of the corresponding positive number.
- This is the most common representation used in computer arithmetic
- Eg: 8-bit representation of 'fifteen'
 - +15 – 0 0001111
 - -15 – 11110001

2's Complement Representation

Note: leftmost bit of the representation acts as the sign bit (0 for positive values, 1 for negative ones)



Schematic representation of 4-bit 2's-complement code for integers in $[-8, +7]$.

Conversion of decimal numbers to signed binary numbers

- Express decimal number -39 as 8-bit number in (a) sign-magnitude (b) 1's complement and (c) 2's complement representations.

- 8-bit representation for +39

00100111

First represent the corresponding positive number in the given number of bits. Else the minimum number of bits required to represent that particular number should be taken.

- (a) 8-bit Sign magnitude representation for -39:

10100111

- (b) 8-bit 1's complement representation for -39:

11011000

- (c) 8-bit 2's complement representation for -39:

11011001

Then use that number represented in the required number of bits to find the negative representation

Conversion of a signed binary number to decimal number

Determine the decimal value of signed binary number expressed in sign-magnitude representation.

- 10010101
 - Computing the weights of rightmost 7 bits:
 $0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 16 + 4 + 1 = 21$
 - Sign bit(leftmost bit) is 1.Hence it's a negative number
 - Therefore, the decimal number is -21

Conversion of a signed binary number to decimal number

Determine the decimal value of signed binary number expressed in 1's complement representation.

- 00010111
 - Computing the weights of the bits with the weight of the leftmost bit as negative:
$$-0 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 16 + 4 + 2 + 1 = +23$$
 - Therefore, the decimal number is +23
- 11101000 (*complement of the previous question*)
 - Computing the weights of the bits with the weight of the leftmost bit as negative:
$$-1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = -128 + 64 + 32 + 8 = -24$$
 - Adding 1 to the result (i.e) $-24 + 1 = -23$
 - Therefore, the decimal number is -23

Negative numbers alone add 1 if 1's complement representation

Conversion of a signed binary number to decimal number

Determine the decimal value of signed binary number expressed in 2's complement representation.

- 01010110
 - Computing the weights of the bits with the weight of the leftmost bit as negative:
$$-0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 64 + 16 + 4 + 2 = +86$$
 - Therefore, the decimal number is +86
- 10101010 (*complement of the previous question*)
 - Computing the weights of the bits with the weight of the leftmost bit as negative:
$$-1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = -128 + 32 + 8 + 2 = -86$$
 - Therefore, the decimal number is -86

Need not add 1 like 1's complement representation

3-bit representation of signed numbers

No	Possible 3-bit representations	If only positive numbers represented	If negative numbers also should be represented (sign-magnitude)	If negative numbers also should be represented (1's complement)	If negative numbers also should be represented (2's complement)
1	000	0	+0	+0	0
2	001	1	+1	+1	+1
3	010	2	+2	+2	+2
4	011	3	+3	+3	+3
5	100	4	-0	-3	-4
6	101	5	-1	-2	-3
7	110	6	-2	-1	-2
8	111	7	-3	-0	-1

With the available combinations of binary numbers for a given number of bits, positive and negative numbers must be represented(FOR SIGNED NUMBERS)!