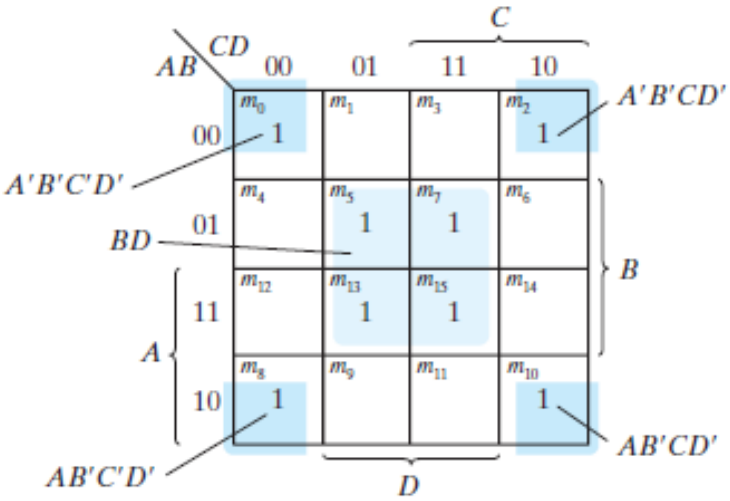
Prime Implicant & Essential Prime Implicant

- A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in the map.
  - This means that a single 1 on a map represents a prime implicant if it is not adjacent to any other 1's.
  - Two adjacent 1's form a prime implicant, provided that they are not within a group of four adjacent squares.
  - Four adjacent 1's form a prime implicant if they are not within a group of eight adjacent squares, and so on.
- If a minterm in a square is covered by only one prime implicant, that prime implicant is said to be essential.
  - The essential prime implicants are found by looking at each square marked with a 1 and checking the number of prime implicants that cover it.
  - The prime implicant is essential if it is the only prime implicant that covers the minterm.

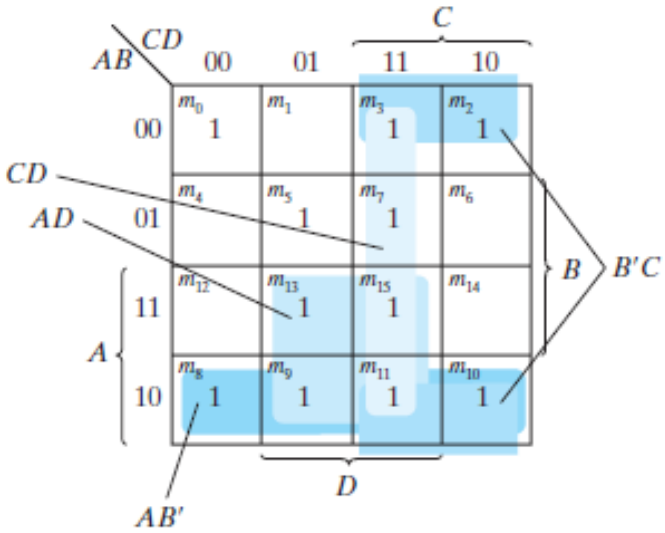
Prime Implicant & Essential Prime Implicant

- Consider the following four-variable Boolean function:
  - *F(A, B, C, D) = ∑(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)*



Note: $A'B'C'D' + A'B'CD' = A'B'D'$
$AB'C'D' + AB'CD' = AB'D'$
$A'B'D' + AB'D' = B'D'$

(a) Essential prime implicants BD and $B'D'$

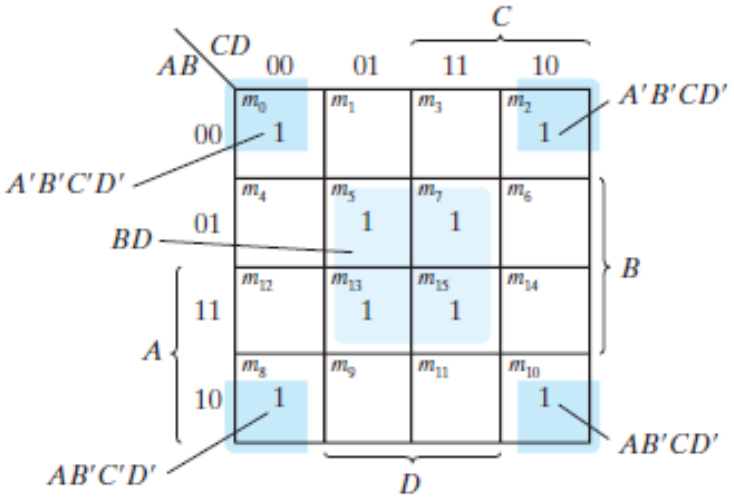


(b) Prime implicants CD, $B'C$, AD, and AB'

Prime Implicant & Essential Prime Implicant

Map1:

- Two essential prime implicants, each formed by collapsing four cells into a term having only two literals.
- One term is essential because there is only one way to include minterm *m0 within four adjacent squares.*
  - *These four squares define the* term *BD.*
- *Similarly, there is only one way that minterm m5 can be combined with four* adjacent squares,
  - This gives the second term *BD* .
- *The two essential prime implicants* cover eight minterms.
- The three minterms that were omitted from the partial map (*m3, m9, and m11) must be considered next.*

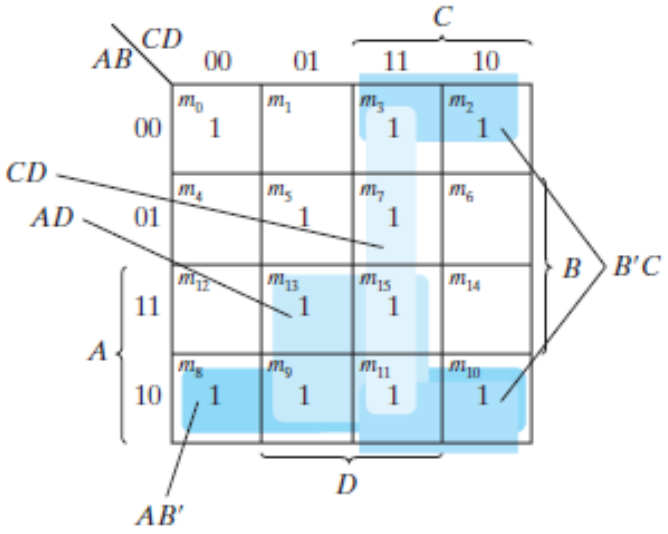


Note: $A'B'C'D' + A'B'CD' = A'B'D'$
$AB'C'D' + AB'CD' = AB'D'$
$A'B'D' + AB'D' = B'D'$

(a) Essential prime implicants BD and $B'D'$

Map 2:

- Shows all possible ways that the three minterms can be covered with prime implicants.
- Minterm *m3 can be covered with either prime implicant CD or prime* implicant *B'C.*
- *Minterm m9 can be covered with either AD or AB'.*
- *Minterm m11 is* covered with any one of the four prime implicants.
- The simplified expression is obtained from the logical sum of the two essential prime implicants and any two prime implicants that cover minterms *m3, m9, and m11.*



(b) Prime implicants CD, $B'C$, AD, and AB'

$$
\begin{aligned}
F &= BD + B'D' + CD + AD \\
&= BD + B'D' + CD + AB' \\
&= BD + B'D' + B'C + AD \\
&= BD + B'D' + B'C + AB'
\end{aligned}
$$

Disadvantages of map method

- Convenient when the number of variables does not exceed five or six.
- It is essentially trial and error procedure and relies on the ability of human user to recognize certain patterns.

Tabulation Method

- It is a specific step-by-step procedure that is guaranteed to produce a simplified standard-form expression for a function.
- Can be applied to problems with many variables.
- Suitable for machine computation.
- Formulated by Quine and later improved by McCluskey.
- It is also known as Quine-McCluskey method.
- Disadvantage:
  - Quite tedious for human use and is prone to mistakes due to its routine monotonous process.
- This method consists of two parts:
  - Find by an exhaustive search all the terms that are candidates for inclusion in the simplified function.(ie. Finding prime implicants)
  - The second operation is to choose among the prime implicants those that give an expression with the least number of literals.

Tabulation Method

- **Step 1** − Arrange the given min terms in an **ascending order** and make the groups based on the number of ones present in their binary representations. So, there will be **at most 'n+1' groups** if there are 'n' Boolean variables in a Boolean function or 'n' bits in the binary equivalent of min terms.
- **Step 2** − Compare the min terms present in **successive groups**. If there is a change in only one-bit position, then take the pair of those two min terms. Place this symbol '_' in the differed bit position and keep the remaining bits as it is.
- **Step 3** − Repeat step2 with newly formed terms till we get all **prime implicants**.
- **Step 4** − Formulate the **prime implicant table**. It consists of set of rows and columns. Prime implicants can be placed in row wise and min terms can be placed in column wise. Place 'X' in the cells corresponding to the min terms that are covered in each prime implicant.
- **Step 5** − Find the essential prime implicants by observing each column. If the min term is covered only by one prime implicant, then it is **essential prime implicant**. Those essential prime implicants will be part of the simplified Boolean function.
- **Step 6** − Reduce the prime implicant table by removing the row of each essential prime implicant and the columns corresponding to the min terms that are covered in that essential prime implicant. Repeat step 5 for Reduced prime implicant table. Stop this process when all min terms of given Boolean function are over.

Tabulation Method

- Determine the prime implicants of the function
  - $F(w,x,y,z)=\sum(1,4,6,7,8,9,10,11,15)$

Grouping the minterms according to the number of 1s:

	w x y z			**w x y z**			**w x y z**
(1)	0 0 0 1	✓	(1,9)	- 0 0 1		(8,9,10,11)	1 0 - -
(4)	0 1 0 0	✓	(4,6)	0 1 - 0		(8,9,10,11)	1 0 - -
(8)	1 0 0 0	✓	(8,9)	1 0 0 -	✓		
			(8,10)	1 0 - 0	✓		
(6)	0 1 1 0	✓					
(9)	1 0 0 1	✓	(6,7)	0 1 1 -			
(10)	1 0 1 0	✓	(9,11)	1 0 - 1	✓		
			(10,11)	1 0 1 -	✓		
(7)	0 1 1 1	✓					
(11)	1 0 1 1	✓	(7,15)	- 1 1 1			
			(11,15)	1 – 1 1			
(15)	1 1 1 1	✓					

The prime implicants are:
x'y'z, w'xz', w'xy, xyz, wyz, wx'

The sum of prime implicants need not be the expression with minimum number of terms!!

Tabulation Method

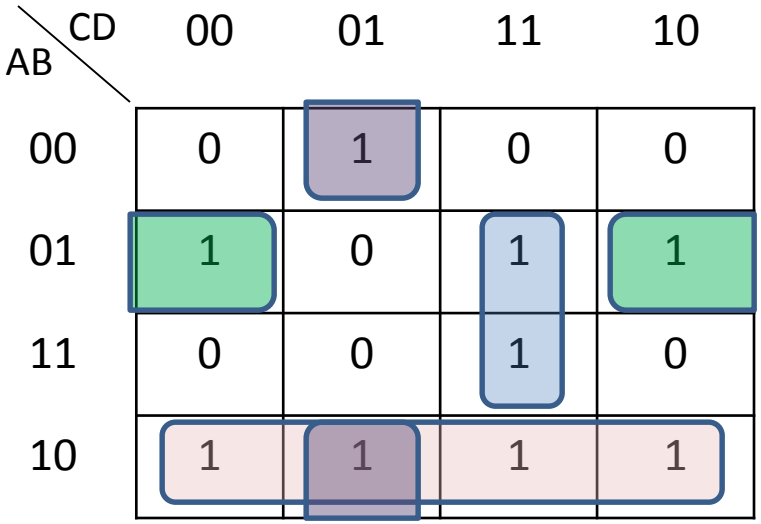
Selection of prime implicants to form the minimized function using prime implicant table:

			1	4	6	7	8	9	10	11	15
✓	x’ y’ z	(1,9)	X					X			
✓	w’ x z’	(4,6)		X	X						
	w’ x y	(6,7)			X	X					
	x y z	(7,15)				X					X
	w y z	(11,15)								X	X
✓	w x’	(8,9,10,11)					X	X	X	X	
			✓	✓	✓		✓	✓	✓	✓	

Minimized Expression:

F= x’y’z+w’xz’+wx’+xyz

Verification of Tabulation method using K-map!!



F= x’y’z+w’xz’+wx’+xyz

Tabulation Method

- Determine the prime implicants of the function
  - $F(w,x,y,z)=\sum(0,3,5,6,7,10,12,13)$
  - $d(w,x,y,z)=\sum(2,9,15)$

Grouping the minterms according to the number of 1s:

	w x y z			**w x y z**			**w x y z**
(0)	0 0 0 0	✓	(0,2)	0 0 - 0		(2,3,6,7)	0 - 1 -
						(2,3,6,7)	0 - 1 -
(2)	0 0 1 0	✓	(2,3)	0 0 1 -	✓		
			(2,6)	0 - 1 0	✓	(5,7,13,15)	- 1 - 1
(3)	0 0 1 1	✓	(2,10)	- 0 1 0		(5,7,13,15)	- 1 - 1
(5)	0 1 0 1	✓					
(6)	0 1 1 0	✓	(3,7)	0 - 1 1	✓		
(9)	1 0 0 1		(5,7)	0 1 - 1	✓		
(10)	1 0 1 0	✓	(6,7)	0 1 1 -	✓		
(12)	1 1 0 0	✓	(5,13)	- 1 0 1	✓		
			(9,13)	1 - 0 1			
(7)	0 1 1 1	✓	(12,13)	1 1 0 -			
(13)	1 1 0 1	✓					
			(7,15)	- 1 1 1	✓		
(15)	1 1 1 1	✓	(13,15)	1 1 - 1	✓		

The prime implicants are:
w'x'z', x'yz', wy'z, wxy', w'y, xz

Tabulation Method

Selection of prime implicants to form the minimized function using prime implicant table:

			0	3	5	6	7	10	12	13
✓	w'x'z'	(0,2)	X							
✓	w' y z'	(2,10)						X		
	wy'z	(9,13)								X
✓	wxy'	(12,13)							X	X
✓	w'y	(2,3,6,7)		X		X	X			
✓	xz	(5,7,13,15)			X		X			
			✓	✓	✓	✓	✓	✓	✓	✓

Minimized Expression:

F= w'x'z' + x'yz' + wxy' +w'y + xz

Tabulation Method

- Simplify the following expression to product of sum using Tabulation method
  - $F(a,b,c,d)=\prod(1,3,5,7,13,15)$

Grouping the maxterms according to the number of 1s:

	a b c d			**a b c d**			**a b c d**
(1)	0 0 0 1	✓	(1,3)	0 0 - 1	✓	(1,3,5,7)	0 - - 1
			(1,5)	0 - 0 1	✓	(1,3,5,7)	0 - - 1
(3)	0 0 1 1	✓					
(5)	0 1 0 1	✓	(3,7)	0 - 1 1	✓	(5,7,13,15)	- 1 - 1
			(5,7)	0 1 - 1	✓	(5,7,13,15)	- 1 - 1
(7)	0 1 1 1	✓	(5,13)	- 1 0 1	✓		
(13)	1 1 0 1	✓					
			(7,15)	- 1 1 1	✓		
(15)	1 1 1 1	✓	(13,15)	1 1 - 1	✓		

The prime implicants are: (a+d’), (b’+d’)

Tabulation Method

Selection of prime implicants to form the minimized function using prime implicant table:

			1	3	5	7	13	15
✓	(a+d')	(1,3,5,7)	X	X	X	X		
✓	(b'+d')	(5,7,13,15)			X	X	X	X
			✓	✓			✓	✓

Minimized Expression:

F= (a+d')(b'+d')

- 1,3,4,5,10,12,13,15