Functions in two Variables :

$\forall \varepsilon > 0, \exists \delta > 0 \ni |x - x_0| < \delta$, whenever

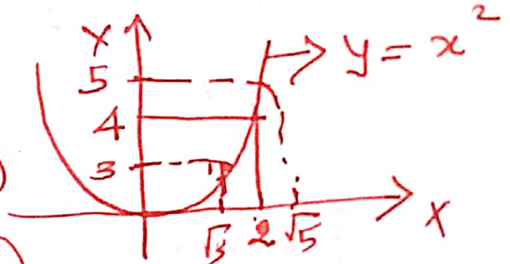
L

$|f(x) - f(x_0)| < \varepsilon$.

$x \rightarrow$ input, $f(x_0) \rightarrow$ output.

eg: $f(x) = x^2$; $x_0 = 2$

$L = 4$; $\varepsilon = 1$



$\varepsilon = 1$ (deviation in y)

$\delta = ?$ (deviation in x).

$|x^2 - 4| < 1$

$-1 < x^2 - 4 < 1$

$3 < x^2 < 5$ $\qquad \delta = \min(0.3, 0.5)$

$\sqrt{3} < x < \sqrt{5}$.

Now, $|x - 2| < \delta$ ...

$\sqrt{3} - 2 < x - 2 < \sqrt{5} - 2$

$-0.3 < x - 2 < 0.5$.

Domain, Range, level Curve, Open set, closed set.

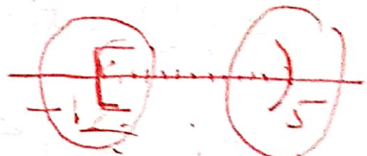
Domain:

To which the Collection of Points the function is defined is Called domain.

$$f : D \longrightarrow W$$

Domain — range

Interior Point:



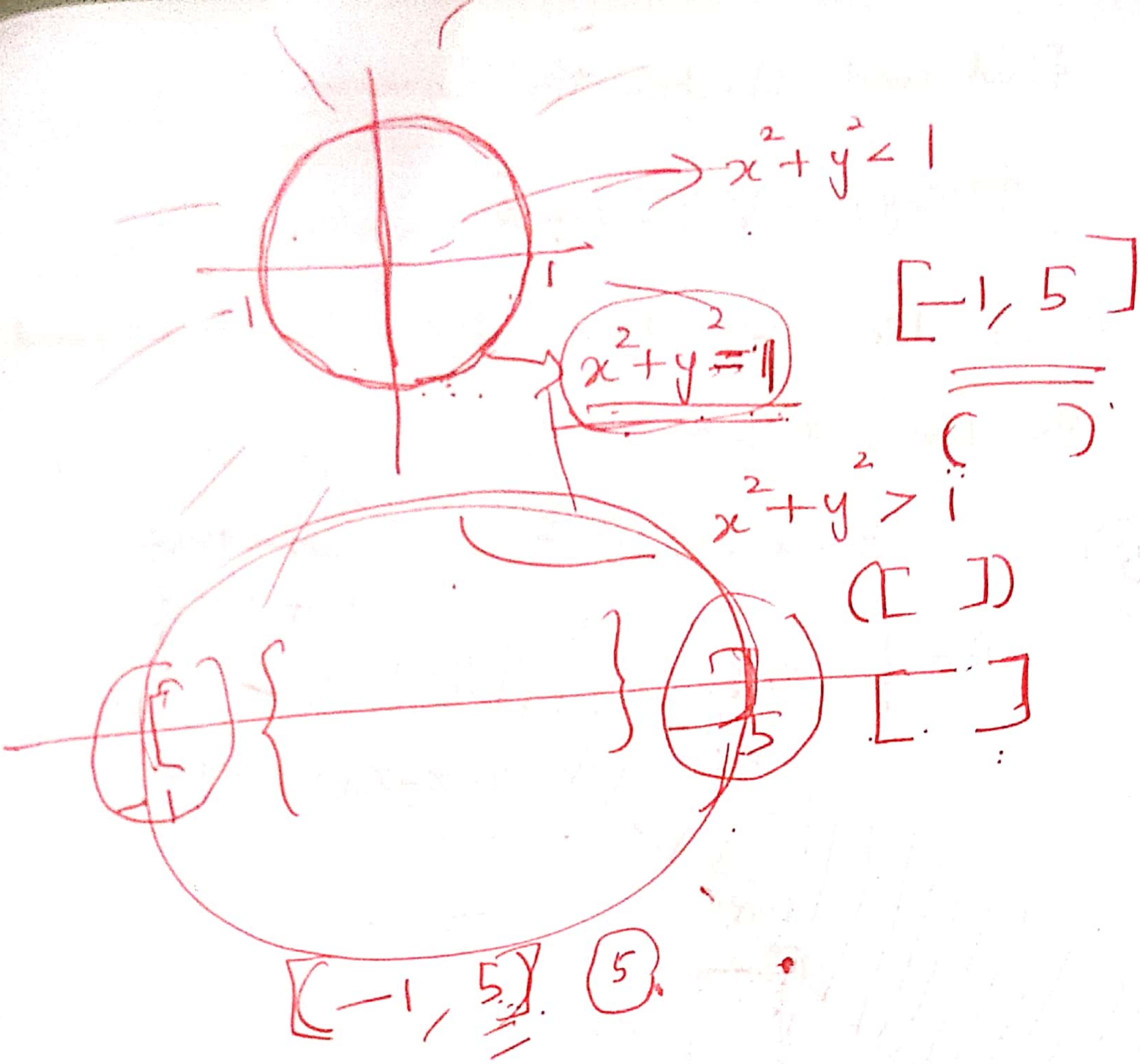
$$E = \{ [-1, 5] \cup \{7\} \cup [10, 15) \}$$

Check in the nghbhood of the pt. If both pts exists in the set, then it is interior points.

Collection of interior pts is an 'open set'

Boundary pts:

Collection of exterior pts is an 'Boundary pts'

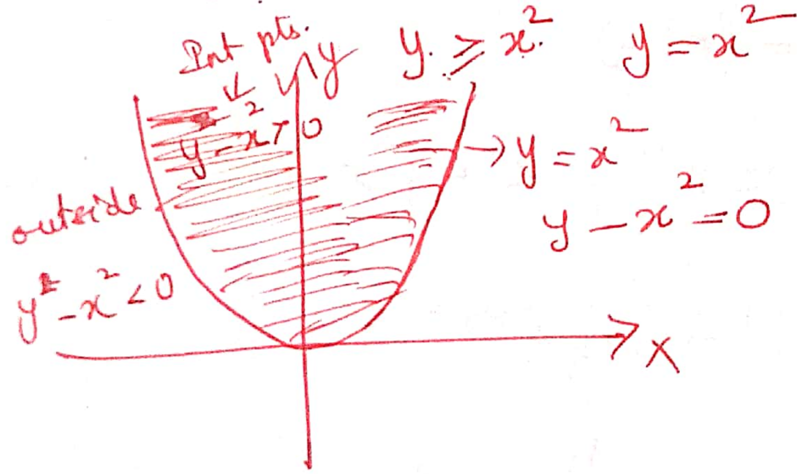


Describe the domain of the function

$f(x, y) = \sqrt{y - x^2}$ $\quad f: D \to \mathbb{R}$

$D: \{ (x, y) \in \mathbb{R}^2 ; \; y - x^2 \geq 0$

$y \geq x^2 \quad y = x^2$



Find and sketch the domains

$f(x,y) = \sqrt{y-x-2}$.

Since the function $g(x) = \sqrt{x}$ is defined only for $x \geq 0$.

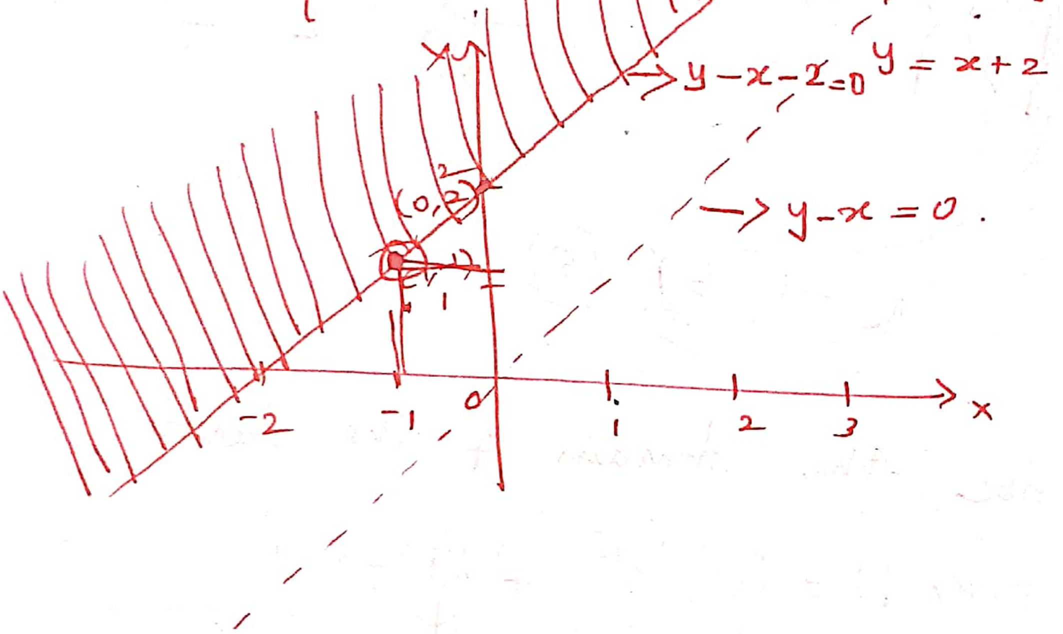
$x = -1$; $y = 1$

5) $y - x - 2 \geq 0$.

~~[illegible]~~

$1 + 1 - 2$

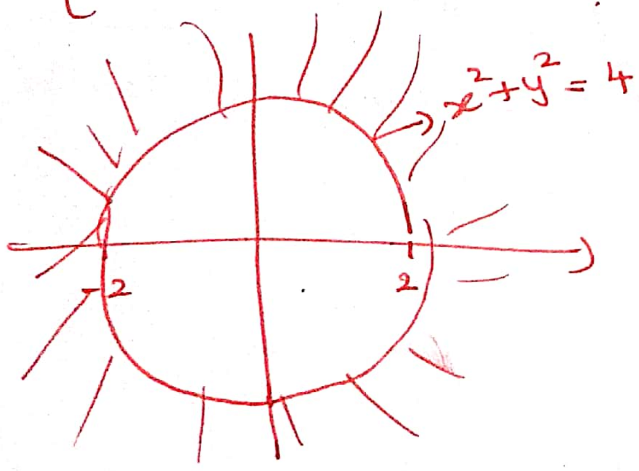
$D = \{ (x,y) \in \mathbb{R}^2 : y - x - 2 \geq 0 \}$



6) $f(x,y) = \ln(x^2 + y^2 - 4)$

$g(x) = \ln(x)$

$D = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 > 4 \}$



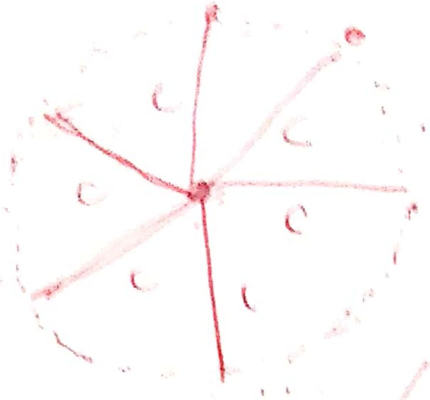
Scanned with CamScanner

Level Curve — 2D.

Level surface — 3D

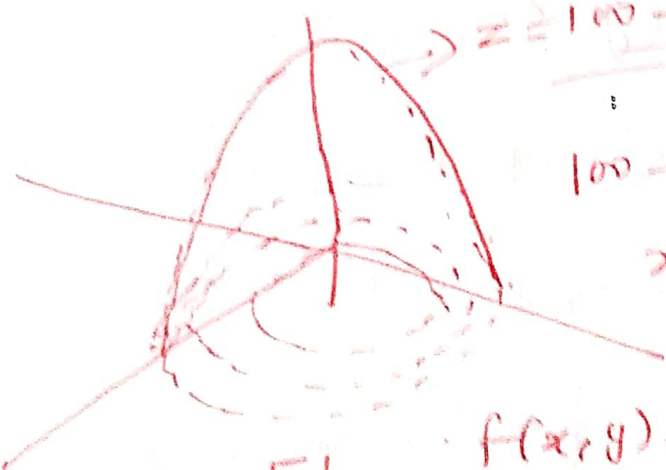
$y=1$

$f(x,y) = c$



$y=1$

$$f(x,y) = z = 100 - x^2 - y^2$$



$z = 100 - x^2 - y^2$

$100 - x^2 - y^2 = 0$

$x^2 + y^2 = 10^2.$

$f(x,y) = 75$

$f(x,y) = 51$

$100 - x^2 - y^2 = 51.$

$100 - 51 = x^2 + y^2 = C$

$x^2 + y^2 = 49.$

$100 - x^2 - y^2 = 75.$

$100 - 75 = x^2 + y^2$

$25 = x^2 + y^2$