SIMPLIFIED ROBOT ARM DYNAMICS FOR CONTROL

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Abstract

A brief summary and evaluation is presented on the use of symbolic state equation techniques in order to represent robot arm dynamics with sufficient accuracy for controlling arm motion. The use of homogeneous transformations and the Lagrangian formulation of mechanics offers a convenient frame for the derivation, analysis and simplification of complex robot dynamics equations. It is pointed out that simplified state equations can represent robot arm dynamics with good accuracy.

1. Introduction

The dynamic equations of a robot arm express the relationship between the motion of the arm and the forces or torques acting on it. More specifically, the dynamic equations relate the kinematic variables of the arm joints (position, velocity and acceleration) to the forces or torques at the joints which are required to produce and control a desired arm motion.

The dynamics of robot arms is complicated due to the complex dynamic interaction between the arm joints. Even for a rigid arm, the equations contain many terms which are related to the effective inertia, coupling inertia, centripetal and Coriolis forces, and gravity loading (see [1-2]). The main problem is the identification of those terms which are significant in controlling a robot arm.

There are two main approaches to obtaining solutions to the complex problem of robot arm dynamics: numerical and symbolic techniques. In the numerical solution techniques, the algorithms are treated as number generators. A recent survey of numerical solution techniques can be found in [3], and examples are described in [4]. In the symbolic techniques, the algorithms are employed to generate state equations. The state equations provide a higher-level insight into the control problem, are suited to perform control system analysis and synthesis, and offer a quantitative basis for simplifying the description of arm dynamics. In this paper we are mainly interested in symbolic solutions and state equations.

2. The Lagrangian Algorithm

The application of the Lagrangian formulation of mechanics together with the use of homogeneous transformations allows a relatively simple and compact symbolic description of the dynamic equations of robot arms. The algorithm is expressed by matrix operations which facilitates analysis. The derivation of the algorithm together with a detailed definition of the symbols can be found in [1,2,5,6]. The algorithm for an "n" degree-of-freedom arm is:

$$F_i = \sum_{j=1}^{n} D_{ij}\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q}_j\dot{q}_k + D_i \qquad (1)$$

where

$$D_{ij} = \sum_{p=\max i,j}^{n} \text{Trace}\left(U_{pj} J_p U_{pi}^T\right) \qquad (2)$$

$$D_{ijk} = \sum_{p=\max i,j,k}^{n} \text{Trace}\left(U_{pjk} J_p U_{pi}^T\right) \qquad (3)$$

$$D_i = \sum_{p=i}^{n} -m_p \bar{g}\, U_{pi} \bar{r}_p \qquad (4)$$

F_i	:	force or torque acting at joint "i"
$q_j, \dot{q}_j, \ddot{q}_j$	:	position, velocity and acceleration of joint "j"
m_p	:	mass of body (link) "p"
$\bar{r}_p$	:	mass center vector of body (link) "p" in the coordinate frame fixed in the same body, given as a 4 x 1 vector
$\bar{g}$	:	acceleration of gravity, given as a 1 x 4 vector
J_p	:	4 x 4 pseudo-inertia matrix for body (link) "p"

$$U_{pj} = \frac{\partial T_0^p}{\partial q_j} : \quad 4 \times 4 \text{ matrix}$$

$$U_{pjk} = \frac{\partial^2 T_0^p}{\partial q_j \partial q_k} : \quad 4 \times 4 \text{ matrix}$$

with $T_0^p = T_0^1 T_1^2 \ldots T_{p-1}^p$, $p \leq n$, concatenation of 4 x 4 joint coordinate transformation matrices.

The quantities defined by Eqs. (2-4) can be called "dynamic projection functions"; they are functions of the joint variables q_j. In particular, D_{ii} represents the effective inertia at joint "i"; D_{ij} represents the coupling inertia between joints "i" and "j"; D_{ijj} represents the centripetal force at joint "i" due to velocity at joint "j"; D_{ijk} represents the Coriolis force at joint

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"i" due to velocities at joints "j" and "k"; D_i represents the gravity loading at joint "i".

Note that $D_{ij} = D_{ji}$ and $D_{ijk} = D_{ikj}$. Note also that the summation formalism of Eq. (3) defines functions which do not exist physically for any arm:

$$D_{ijk} \equiv 0 \text{ for } i = j \geq k \tag{5}$$

3. Simplifications

The simplification of robot arm dynamics is based on the evaluation of the dynamic functions defined by Eqs. (2-4). Two types of evaluations are possible: geometric and numeric.

The geometric evaluation of Eqs. (2-4) is quite general. In this evaluation one considers only the basic geometry of a given arm; that is, the nature of joints (linear or rotary joints) in the chain of joints. This can be accomplished by expanding Eqs. (2-4) into sub-matrices using the general rules of matrix algebra. For more details see [7,8]. It can be shown, e.g., that for an arm which has a linear joint (say, the j-th joint in the chain of "n" joints) with all other joints being rotary joints, the following dynamic projection functions do not exist:

$$D_{ijk} \equiv 0 \text{ for } j \leq k \tag{6}$$

Dependent upon the arm's basic geometry, other dynamic functions can be identically zero [7,8].

The numeric evaluation of the D_i, D_{ij} and D_{ijk} functions which are not zero for geometric reasons can be carried out for a given arm in two major steps. First, one derives the D_i, D_{ij} and D_{ijk} functions using only the zero/non-zero parametric symbols for the constant geometric and inertial properties of the given arm. This can be accomplished for all three types of functions (D_i, D_{ij} and D_{ijk}) by carrying out the matrix multiplications as shown in [7,8]. An alternative technique which works at least for the D_i and D_{ij} functions is the application of differential transformations as described in [6]. In the second step, one substitutes the numerical values of the geometric and inertial parameters into the state equations for D_i, D_{ij} and D_{ijk}. Some terms in the equations or even some of the equations can become zero for symmetry reasons. Then, the remaining terms can be evaluated according to their numerical significance in the force-torque equations.

The outcome of the geometric and numeric evaluations is a greatly simplified set of equations for D_i, D_{ij} and D_{ijk}. It is important to note that the reliability of numeric simplifications also depends on the accuracy in the values of the geometric and inertial constants of a given arm.

4. Examples

A good example is the dynamics of the Stanford-Scheinman arm. The exact dynamic equations for the first three joints of this arm — assuming only that the wrist joints are fixed — can be reduced to 48 multiplications, 26 additions and 2 trignometric functions as shown in [8]. These operations include all inertia, centripetal, Coriolis and gravity terms without numeric simplifications.

It is also easy to prove that, for the last four joints of the Stanford-Scheinman arm the following functions are identically zero: D_{333}, D_{344}, D_{366}, D_{444}, D_{455}, D_{466}, D_{555}, D_{566}, D_{644}, D_{666}, D_{446}, D_{556}, D_{334}, D_{335}, D_{336}, D_{345}, D_{346}, D_{356}, D_{434}, D_{435}, D_{436}, D_{534}, D_{535}, D_{536}, D_{634}, D_{635}, D_{636}, D_{433}, D_{533}, D_{633}, D_{45}, D_{56}, D_{34}, D_{36}, D_6 as shown in [8].

It is also shown in [7,6] that, e.g., the effective inertia function for the first joint of the Stanford-Scheinman arm, D_{11}, can be reduced from about 75 additions, 100 multiplications and 10 trigonometric functions to 4 additions, 6 multiplications and 2 trigonometric functions, without making any assumption on the motion of the full arm, and still retaining about 95 percent accuracy in the value of D_{11}.

5. Conclusion

In general, the significance of the various terms in the arm dynamic equations depends on the speed of motion. For slow motion, the centripetal and Coriolis forces are insignificant, and the significant terms for the effective and coupling inertia and gravity loading can be represented by simplified state equations with good accuracy. Even for fast motion of a six degree-of-freedom arm, the representation of all existing and significant terms in the dynamic equations can be reduced to simplified state equations with good accuracy.

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