DYNAMICS OF INTERNAL MOTION OF REDUNDANT MANIPULATORS

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ABSTRACT: In the paper the dynamic properties of the internal motion of redundant manipulators are considered. The motion is decoupled into the end-effector motion and the internal motion. Next, the dynamic model of a redundant manipulator is derived. A special attention is given to the inertial properties of the system in the space where internal motion is taking place; we define a *null space effective inertia* and its inverse. Finally, a control method is proposed which completely decouples the motion of the manipulator into the task space motion and the internal motion, and enables the selection of dynamic characteristics in both subspaces separately.

KEYWORDS: Redundant manipulators, kinematics, dynamics, internal motion, control

1. INTRODUCTION

One of the important issues of the new generation of robotic manipulators is the redundancy. The redundancy represents an enormous source of flexibility for robotic manipulators. A redundant manipulator has the ability to move the end-effector along the same task state using different configurations of the mechanical structure. The result is a significant increase in the dexterity of the system, which is essential to accomplish complex tasks. On the other hand, the redundancy has also an important influence on the dynamic behavior of the robotic system. An appropriate control of dynamic properties is essential for higher performance in robotic manipulation. Most of the research in the field of the dynamics of robotic manipulators has been devoted to the dynamics in the joint space. To control the dynamic properties of the system in the joint space different control methodologies have been proposed based on joint space dynamic models [8, 1]. As the next step, methods have been proposed where the control takes place in the task space [9]. These methods include the transformations between joint space trajectories and task space trajectories. However, in case of redundant manipulators these transformations are not unique. Different methodologies have been proposed to resolve the redundancy like optimization of a given performance criteria while satisfying primary task [11].

To overcome the limitations of control methods based on the joint space dynamics methods Khatib [5] proposed a new method for dealing dynamics and control in the task space. This method enables the description, analysis and control of the robot behavior in the task space, and can be used also for redundant manipulators when the dynamic behaviour of the end-effector is of interest. However, for the redundant manipulators the end-effector dynamics is only one part of the dynamics of the whole manipulator. The "rest" dynamics represents the dynamics of the internal motion of the manipulator. Recently, Park [12] proposed a decomposition of dynamics of kinematically redundant manipulators into the task space dynamics and null space dynamics based on minimally reparametrized homogenous velocity.

This paper presents a study of the dynamic properties of the internal motion of redundant manipulators without introducing new coordinates to describe the internal motion. We analyze what are the causes for the internal motion and how to use the internal motion to improve the performances of the overall system. Next, the influence of the selection of pseudo or generalized inverse on the internal motion is discussed. The paper reviews some basic methods to derive the model describing the dynamics of the manipulator in the task space. Next, methods to derive the model describing the dynamics of the internal motion are presented. A special attention is given to the inertial properties of the system in the space where internal motion is taking place; we define a *null space effective inertia* and its inverse. Finally, some control methods are presented which completely decouple the motion of the manipulator into the task space motion and the internal motion, and enable the selection of dynamic characteristics in both subspaces separately.

2. KINEMATICS

The robotic systems under study are n degrees of freedom (DOF) serial manipulators. We consider only the redundant systems which have more DOF than needed to accomplish the task, i.e. the dimension of the joint space n exceeds the dimension of the task space m. Let the configuration of the manipulator be represented by the vector $\boldsymbol{q}$ of n joint positions, and the end-effector position (and orientation) by m-dimensional vector $\boldsymbol{x}$ of task positions (and orientations). The joint and task positions are related by the following expression

$$\boldsymbol{x} = \boldsymbol{p}(\boldsymbol{q}) \tag{1}$$

where $\boldsymbol{p}$ is m-dimensional vector function representing the manipulator forward kinematics. Differentiating Eq. (1) we obtain the relation between joint and task space velocities

$$\dot{\boldsymbol{x}} = \mathbf{J}\dot{\boldsymbol{q}} \tag{2}$$

where $\mathbf{J} = \frac{\partial \mathbf{p}}{\partial \boldsymbol{q}}$ is the $m \times n$ manipulator Jacobian matrix. In the case of redundant manipulators there can exist also an internal motion which doesn't contribute to the motion of the end-effector. Hence, the general solution of Eq. (2) can be given as follows

$$\dot{\boldsymbol{q}} = \mathbf{J}^{\#}\dot{\boldsymbol{x}} + \mathbf{N}\dot{\boldsymbol{q}} \tag{3}$$

where $\mathbf{J}^{\#}$ is the generalized inverse of $\mathbf{J}$ and $\mathbf{N}$ is $n \times n$ matrix representing the projection of $\dot{\boldsymbol{q}}$ into the null space of $\mathbf{J}$, $\mathbf{N} = (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})$. The first term on the right side of Eq. (3) represents the part of the joint space velocity necessary to perform the task and is denoted as $\dot{\boldsymbol{q}}_x$, $\dot{\boldsymbol{q}}_x = \mathbf{J}^{\#}\dot{\boldsymbol{x}}$. The second term denoted as $\dot{\boldsymbol{q}}_n$, $\dot{\boldsymbol{q}}_n = \mathbf{N}\dot{\boldsymbol{q}}$, represents the joint space velocity due to the internal motion. Actually, $\dot{\boldsymbol{q}}$ in the term $\dot{\boldsymbol{q}}_n$ can be an arbitrary velocity vector and is usually used to perform an additional subtask like optimization of different cost functions, obstacle avoidance, etc.

Differentiating Eq. (2), we obtain the relation between joint space accelerations and task space accelerations

$$\ddot{\boldsymbol{x}} = \mathbf{J}\ddot{\boldsymbol{q}} + \dot{\mathbf{J}}\dot{\boldsymbol{q}} \tag{4}$$

Considering also the accelerations in the null space of $\mathbf{J}$ the general solution of Eq. (4) is typically given in the form

$$\ddot{\boldsymbol{q}} = \mathbf{J}^{\#}(\ddot{\boldsymbol{x}} - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) + \mathbf{N}\ddot{\boldsymbol{q}} \tag{5}$$

To be able to decompose the joint accelerations $\ddot{\boldsymbol{q}}$ into accelerations subjected to the the task space motion and to the internal motion Eq. (5) has to be rewritten into the form

$$\ddot{\boldsymbol{q}} = \mathbf{J}^{\#}\ddot{\boldsymbol{x}} + \dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}} + \mathbf{N}\ddot{\boldsymbol{q}} + \dot{\mathbf{N}}\dot{\boldsymbol{q}} \tag{6}$$

The above equation can be obtained also by differentiating (3). The first two terms on the right side of Eq. (6) represent the joint space acceleration due to the task space motion, and last two terms represent the joint space acceleration due to the internal motion. The terms $\dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}}$ and $\dot{\mathbf{N}}\dot{\boldsymbol{q}}$ need a special attention. They describe the accelerations due to the change in the configuration of the manipulator and are required to maintain the task space and null space velocity, respectively [10].

3. STATICS

For the redundant manipulators the static relationship between the m-dimensional generalized force in task space $\boldsymbol{F} = \left[\boldsymbol{f}^T \vdots \boldsymbol{N}^T \right]^T$, where $\boldsymbol{f}$ represents the linear forces and $\boldsymbol{N}$ the moments, and the corresponding n-dimensional generalized joint space force $\boldsymbol{\tau}$ is

$$\boldsymbol{\tau} = \mathbf{J}^T\boldsymbol{F} + \mathbf{N}^T\boldsymbol{\tau}_n \tag{7}$$

where $\mathbf{N}^T$ is $n \times n$ matrix representing the projection into the null space of $\mathbf{J}^T$ and $\boldsymbol{\tau}_n$ is an arbitrary n-dimensional vector of joint torques.

4. SELECTION OF THE GENERALIZED INVERSE

Although there exist many generalized inverses $\mathbf{J}^{\#}$, for robotics systems $\mathbf{J}^{\#}$ can not be selected without imposing some restrictions on it. Most authors use the Moore-Penrose pseudoinverse [7] which is defined for $n > m$ as

$$\mathbf{J}^{+} = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}$$

or its "weighted" counterpart [3] defined as

$$\mathbf{J}^{+}_{w} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1}$$

where $\mathbf{W}$ is $n \times n$ weighting matrix. A special form of $\mathbf{J}^{+}_{w}$ is when $\mathbf{W} = \mathbf{H}$. Khatib [6] has proven that

$$\bar{\mathbf{J}} = \mathbf{H}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)^{-1} \tag{8}$$

is the only pseudoinverse which is dynamically consistent, i.e. the task space acceleration $\ddot{\boldsymbol{x}}$ is not affected by any arbitrary torques $\boldsymbol{\tau}_n$ applied through the associated null space, $\bar{\mathbf{N}}^T\boldsymbol{\tau}_n$, $\bar{\mathbf{N}}^T = (\mathbf{I} - \mathbf{J}^T\bar{\mathbf{J}}^T)$. Additionally, the dynamically consistent generalized inverse $\bar{\mathbf{J}}$ is the only generalized inverse which assures that an external force does not produce a null space acceleration [2].

5. MANIPULATOR DYNAMICS

Assuming the manipulator consists of rigid bodies the joint space equations of motion can be written in a form

$$\boldsymbol{\tau} = \mathbf{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \mathbf{g}(\boldsymbol{q}) - \boldsymbol{\tau}_E \tag{9}$$

where $\boldsymbol{\tau}$ is n-dimensional vector of control torques, $\mathbf{H}$ is $n \times n$ symmetric positive-definite inertia matrix, $\mathbf{h}$ is n-dimensional vector of Coriolis and centrifugal forces, and $\mathbf{g}$ is n-dimensional vector of gravity forces. Vector $\boldsymbol{\tau}_E$ summarizes effects of all external forces acting on the manipulator

Using the relation $\mathbf{I} = \mathbf{J}^T\mathbf{J}^{\#T} + \mathbf{N}^T$ and substituting Eqs. (5) and (7) into Eq. (9), the model (9) can be rewritten into the form

$$\begin{aligned}\mathbf{J}^T\boldsymbol{F} + \mathbf{N}^T\boldsymbol{\tau} = & \mathbf{J}^T\mathbf{J}^{\#T}\left(\mathbf{H}\mathbf{J}^{\#}(\ddot{\boldsymbol{x}} - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) + \boldsymbol{h} + \boldsymbol{g} - \boldsymbol{\tau}_E\right) + \\ & \mathbf{N}^T\left(\mathbf{H}\mathbf{N}\ddot{\boldsymbol{q}} + (\boldsymbol{h} + \boldsymbol{g}) - \boldsymbol{\tau}_E\right) + \\ & \left(\mathbf{J}^T\mathbf{J}^{\#T}\mathbf{H}\mathbf{N}\ddot{\boldsymbol{q}} + \mathbf{N}^T\mathbf{H}\mathbf{J}^{\#}(\ddot{\boldsymbol{x}} - \dot{\mathbf{J}}\dot{\boldsymbol{q}})\right)\end{aligned}$$

Note that the terms on the right side of the above equation are arranged into three groups. The first group includes the forces acting in the task space. The second group includes torques acting in the null space of $\mathbf{J}^T$. The third group represents the coupling forces and torques. To make the motion of the end-effector and the internal motion independent, it is necessary that the terms in the third group are always equal to zero

$$\mathbf{J}^{\#T}\mathbf{H}\mathbf{N} = \mathbf{N}^T\mathbf{H}\mathbf{J}^{\#} = \mathbf{0} \tag{10}$$

The only value of $\mathbf{J}^{\#}$ which satisfies the condition (10) is $\bar{\mathbf{J}}$ as defined in Eq. (8).

The equation of the end-effector motion subjected to generalized task forces $\boldsymbol{F}$ is given in the form [5]

$$\boldsymbol{F} = \mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{x}} + \boldsymbol{\mu}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{\gamma}(\boldsymbol{q}) - \boldsymbol{F}_E$$

where $\mathbf{M}$, $\boldsymbol{\mu}$, $\boldsymbol{\gamma}$ and $\boldsymbol{F}_E$ are, respectively, $m \times m$ symmetric positive-definite matrix describing the inertial properties of the manipulator in the task space, m-dimensional vector of Coriolis and centrifugal forces, m-dimensional vector of gravity forces, and m-dimensional vector of external forces, all acting in the task space

$$\mathbf{M} = \bar{\mathbf{J}}^T \mathbf{H} \bar{\mathbf{J}} = (\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)^{-1}$$

$$\boldsymbol{\mu} = \bar{\mathbf{J}}^T \boldsymbol{h} - \mathbf{M}\dot{\mathbf{J}}\dot{\boldsymbol{q}}, \quad \boldsymbol{\gamma} = \bar{\mathbf{J}}^T \boldsymbol{g}, \quad \boldsymbol{F}_E = \mathbf{J}^{\#T}\boldsymbol{\tau}_E$$

The equation of the internal motion of the manipulator subjected to the torque applied through the null space of $\mathbf{J}^T$ is given in the form

$$\bar{\mathbf{N}}^T\boldsymbol{\tau} = \bar{\mathbf{N}}^T\mathbf{H}\bar{\mathbf{N}}\ddot{\boldsymbol{q}} + \bar{\mathbf{N}}^T(\mathbf{h}+\mathbf{g}) - \bar{\mathbf{N}}^T\boldsymbol{\tau}_E$$

The matrix, which premultiplies $\ddot{\boldsymbol{q}}$ and is defined as

$$\mathbf{H}_n = \bar{\mathbf{N}}^T\mathbf{H}\bar{\mathbf{N}} = \mathbf{H} - \mathbf{J}^T\mathbf{M}\mathbf{J}$$

will be denoted as the *null space effective inertia matrix*. The matrix $\mathbf{H}_n$ describes the inertial properties of the system in the null space. As $\mathbf{H}_n$ has not a full rank, $\mathrm{rank}(\mathbf{H}_n) < n$, we define the generalized inverse of the null space effective inertia matrix $\mathbf{H}_n$ as

$$\mathbf{H}_n^{\ddagger} = \bar{\mathbf{N}}\mathbf{H}^{-1}\bar{\mathbf{N}}^T$$

Note that $\mathbf{H}_n^{\ddagger}\mathbf{H}_n\mathbf{H}_n^{\ddagger} = \mathbf{H}_n^{\ddagger}$, $\mathbf{H}_n\mathbf{H}_n^{\ddagger}\mathbf{H}_n = \mathbf{H}_n$, and $\mathbf{H}_n^{\ddagger} = (\mathbf{H}_n^{\ddagger})^T$.

6. CONTROL ALGORITHMS

Most tasks performed by a redundant manipulator are broken down into several subtasks with different priorities. In the following it is assumed that the subtask with the highest priority, referred to as the main task, is associated with the positioning of the end-effector in the task space and the secondary task is to track a prescribed null space velocity.

Utilizing a formulation of the inverse kinematics at the acceleration level a general formulation of the control law is given in the form

$$\boldsymbol{\tau} = \mathbf{H}(\mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_c - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) + \mathbf{N}(\boldsymbol{\phi} + \dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}}) + \mathbf{h} + \mathbf{g} \tag{11}$$

where $\ddot{\boldsymbol{x}}_c$ and $\boldsymbol{\phi}$ represent the control law for the task motion and internal motion, respectively. Combining Eqs. (9) and (11) yields

$$\mathbf{H}\ddot{\boldsymbol{q}} - \boldsymbol{\tau}_E = \mathbf{H}(\mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_c - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) + \mathbf{N}(\boldsymbol{\phi} + \dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}}))$$

which simplifies to

$$\mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_c - \ddot{\boldsymbol{x}}) + \mathbf{N}(-\ddot{\boldsymbol{q}} + \boldsymbol{\phi} + \dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}}) = -\mathbf{H}^{-1}\boldsymbol{\tau}_E \tag{12}$$

Premultiplying Eq. (12) with $\mathbf{J}$ yields

$$\ddot{\boldsymbol{x}}_c - \ddot{\boldsymbol{x}} = -\mathbf{J}\mathbf{H}^{-1}\boldsymbol{\tau}_E \tag{13}$$

since $\mathbf{J}\mathbf{J}^{\#} = \mathbf{I}$. Similarly, premultiplying Eq. (12) with $\mathbf{N}$ and rewriting it yields

$$\mathbf{N}(-\ddot{\boldsymbol{q}} + \boldsymbol{\phi} + \dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}}) = -\mathbf{N}\mathbf{H}^{-1}\boldsymbol{\tau}_E \tag{14}$$

since $\mathbf{N}$ is idempotent.

If $\mathbf{J}^{\#} = \bar{\mathbf{J}}$ is used in Eq. (11), then Eq. (13) can be rewritten into the form

$$\mathbf{M}(\ddot{\boldsymbol{x}}_c - \ddot{\boldsymbol{x}}) = -\boldsymbol{F}_E \tag{15}$$

and Eq. (14) can be rewritten into the form

$$\bar{\mathbf{N}}(-\ddot{\boldsymbol{q}} + \boldsymbol{\phi} + \dot{\bar{\mathbf{J}}}\dot{\boldsymbol{x}}) = -\mathbf{H}_n^{\ddagger}\boldsymbol{\tau}_E \tag{16}$$

6.1. Task space controller

Let $\ddot{\boldsymbol{x}}_c$ be selected as

$$\ddot{\boldsymbol{x}}_c = \ddot{\boldsymbol{x}}_d + \mathbf{K}_v\dot{\boldsymbol{e}} + \mathbf{K}_p\boldsymbol{e} \tag{17}$$

where $\boldsymbol{e}$, $\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x}$, is the tracking error, $\ddot{\boldsymbol{x}}_d$ is the desired task space acceleration, and $\mathbf{K}_v$

and $\mathbf{K}_p$ are $n\times n$ constant gain matrices. The selection of $\mathbf{K}_v$ and $\mathbf{K}_p$ can be based on the desired task space impedance. Substituting Eq. (17) for $\ddot{\boldsymbol{x}}_c$ in Eq. (15), yields

$$\mathbf{M}\ddot{\boldsymbol{e}} + \mathbf{M}\mathbf{K}_v\dot{\boldsymbol{e}} + \mathbf{M}\mathbf{K}_p\boldsymbol{e} = -\boldsymbol{F}_E$$

As we can see the task space impedance can not be chosen freely. However, if the desired inertial properties are not so important and may be set to $\mathbf{M}$, then by selecting $\mathbf{K}_v = \mathbf{M}^{-1}\mathbf{D}_d$ and $\mathbf{K}_p = \mathbf{M}^{-1}\mathbf{K}_d$ the following task space impedance can be achieved

$$\mathbf{M}\ddot{\boldsymbol{e}} + \mathbf{D}_d\dot{\boldsymbol{e}} + \mathbf{K}_d\boldsymbol{e} = -\boldsymbol{F}_E$$

$\mathbf{D}_d$ and $\mathbf{K}_d$ are the desired task space damping and stiffness matrices, respectively.

6.2. Null space controller

Besides the main task, i.e. the tracking of a trajectory in the task space, a redundant system can perform an additional subtask by selecting an appropriate vector $\boldsymbol{\phi}$ in the control law (11) which moves the manipulator toward the desired configuration. Let $\dot{\boldsymbol{\varphi}}$ be the desired null space velocity. To obtain good tracking of $\dot{\boldsymbol{\varphi}}$ in the null space, the following $\boldsymbol{\phi}$ is proposed

$$\boldsymbol{\phi} = \ddot{\boldsymbol{\varphi}} + \mathbf{K}_n\dot{\boldsymbol{e}}_n \qquad (18)$$

where $\dot{\boldsymbol{e}}_n = \bar{\mathbf{N}}(\dot{\boldsymbol{\varphi}} - \dot{\boldsymbol{q}})$ and $\mathbf{K}_n$ is $n\times n$ diagonal gain matrix. The above controller is similar to the one given in [4] except that in our case the pseudoinverse of the Jacobian matrix can be a weighted pseudoinverse.

Substituting (18) into (16) yields

$$\bar{\mathbf{N}}(\ddot{\boldsymbol{\varphi}} - \ddot{\boldsymbol{q}}) = \bar{\mathbf{N}}(-\mathbf{K}_n\dot{\boldsymbol{e}}_n - \dot{\bar{\mathbf{J}}}\dot{\boldsymbol{x}}) - \mathbf{H}_n^{\ddagger}\boldsymbol{\tau}_E \qquad (19)$$

Differentiating $\dot{\boldsymbol{e}}_n$ results in

$$\ddot{\boldsymbol{e}}_n = \bar{\mathbf{N}}(\ddot{\boldsymbol{\varphi}} - \ddot{\boldsymbol{q}}) + \dot{\bar{\mathbf{N}}}(\dot{\boldsymbol{\varphi}} - \dot{\boldsymbol{q}}) \qquad (20)$$

Using Eq. (19) in the above equation yields

$$\ddot{\boldsymbol{e}}_n = -\bar{\mathbf{N}}\mathbf{K}_n\dot{\boldsymbol{e}}_n + \dot{\bar{\mathbf{N}}}\dot{\boldsymbol{\varphi}} - \dot{\bar{\mathbf{N}}}\dot{\boldsymbol{q}} + \bar{\mathbf{N}}\dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}} - \mathbf{H}_n^{\ddagger}\boldsymbol{\tau}_E \qquad (21)$$

Note that $\dot{\boldsymbol{\varphi}}$ belongs to the null space of $\mathbf{J}$ and $\bar{\mathbf{N}}\dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}} = \bar{\mathbf{N}}\dot{\bar{\mathbf{N}}}\dot{\boldsymbol{q}}$. Hence, Eq. (21) can be rewritten into the form

$$\ddot{\boldsymbol{e}}_n = -\bar{\mathbf{N}}\mathbf{K}_n\dot{\boldsymbol{e}}_n - (\mathbf{I} - \bar{\mathbf{N}})\dot{\bar{\mathbf{N}}}\dot{\boldsymbol{q}} - \mathbf{H}_n^{\ddagger}\boldsymbol{\tau}_E \qquad (22)$$

Next, we show that for $\boldsymbol{\tau}_E = 0$ the proposed control method (18) assures asymptotic stability of the system in the null space and that the $\dot{\boldsymbol{e}}$ converges to zero. Let the Lyapunov function be defined as $v = 1/2\,\dot{\boldsymbol{e}}_n^T\mathbf{H}\dot{\boldsymbol{e}}_n$. Differentiating v and substituting Eq. (22) for $\ddot{\boldsymbol{e}}$ yields

$$\begin{aligned}\dot{v} =& \dot{\boldsymbol{e}}_n^T\mathbf{H}\ddot{\boldsymbol{e}}_n + \tfrac{1}{2}\dot{\boldsymbol{e}}_n^T\dot{\mathbf{H}}\dot{\boldsymbol{e}}_n \\ =& -\dot{\boldsymbol{e}}_n^T\mathbf{H}\bar{\mathbf{N}}\mathbf{K}_n\dot{\boldsymbol{e}}_n + \dot{\boldsymbol{e}}_n^T\mathbf{H}(\mathbf{I} - \bar{\mathbf{N}})\dot{\bar{\mathbf{N}}}\dot{\boldsymbol{q}} + \tfrac{1}{2}\dot{\boldsymbol{e}}_n^T\dot{\mathbf{H}}\dot{\boldsymbol{e}}_n \\ =& -\dot{\boldsymbol{e}}_n^T(\mathbf{H}\mathbf{K}_n - \tfrac{1}{2}\dot{\mathbf{H}})\dot{\boldsymbol{e}}_n\end{aligned} \qquad (23)$$

since $\mathbf{H}\bar{\mathbf{N}} = \bar{\mathbf{N}}^T\mathbf{H}$, $\dot{\boldsymbol{e}}_n^T\bar{\mathbf{N}}^T = \dot{\boldsymbol{e}}_n^T$ and $\bar{\mathbf{N}}(\mathbf{I} - \bar{\mathbf{N}}) = 0$. From Eq. (23) it follows that the lower bound of $\mathbf{K}_n$ depends on the requirement that $(\mathbf{H}\mathbf{K}_n - \frac{1}{2}\dot{\mathbf{H}})$ is a positive definite matrix.

Finally, premultiplying it with $\mathbf{H}_n$ provides a description of the null space dynamics in the form

$$\mathbf{H}_n\ddot{\boldsymbol{e}}_n + \mathbf{H}_n\mathbf{K}_n\dot{\boldsymbol{e}}_n = -\bar{\mathbf{N}}^T\boldsymbol{\tau}_E \qquad (24)$$

since $\bar{\mathbf{N}}(\mathbf{I} - \bar{\mathbf{N}}) = 0$. Summarizing, the control method (18) enables the selection of the damping by selecting $\mathbf{K}_n$.

For illustration Fig. 1 shows time responses of a 4-DOF planar manipulator for the step change of the desired task space position ($\boldsymbol{x} : [-0.04, 0.48]^T \to [-0.54, 0.68]^T$) and for the step change in the desired null space velocity ($\dot{\boldsymbol{\varphi}} : [0,0,0,0]^T \to \bar{\mathbf{N}}[5,0,0,0]^T$). The controller parameters are: $\mathbf{K}_p = 1000\mathbf{I}\mathrm{s}^{-2}$, $\mathbf{K}_v = 80\mathbf{I}\mathrm{s}^{-1}$ and $\mathbf{K}_n = 20\mathbf{I}\mathrm{s}^{-1}$. The obtained responses have exactly the desired close loop dynamics, i.e. as a linear system

$$\frac{1000}{s^2 + 80s + 1000}$$

in the task space and as a linear system

$$\frac{20}{s + 20}$$

in the null space.

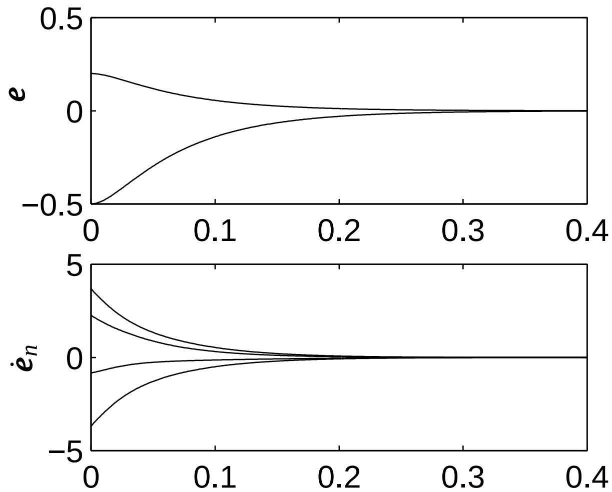

Figure 1. 4-DOF planar manipulator: Task position error $\boldsymbol{e}$ and null space velocity error $\dot{\boldsymbol{e}}_n$ versus time for step inputs

7. CONCLUSION

The paper considers the dynamic properties of a redundant manipulator. A special attention is given to the dynamic decoupling and the inertial properties of the system in the space where internal motion is taking place; we define a *null space effective inertia* and its inverse. In contrast to some other authors ([12]) in our case the internal motion is characterized only by the selection of the pseudoinverse. We do not introduce new coordinates to describe the internal motion (like a minimally reparametrized homogenous velocity). Hence, it is easier to generate the desired internal motion using different measures which are functions of $\boldsymbol{q}$.

Finally, we propose a control algorithm (11) which decouples the motion of the manipulator into the end-effector motion and the internal motion. The controller enables the selection of dynamic characteristics in both subspaces separately. We show also that for good tracking of the null space velocity ($\dot{\boldsymbol{e}} \rightarrow 0$) it is necessary to compensate the accelerations in the null space due to the task space motion (term $\dot{\mathbf{J}}^{\#}\dot{\boldsymbol{x}}$ in control law (11)).

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