tential applications include evaluation of simulators and displays.

The effectiveness of a simulator lies in the strength of the inferences that can be drawn about an operator's behavior in the real world. His decision-making in a simulator will differ from that in an actual situation if either 1) he perceives differently the causal relations among stimuli in the two environments, or 2) he has in the two cases entirely different utilities for real and simulated outcomes. Decision-making in the simulator for a given task is potentially the same as in the actual situation if the ROC for that task is the same in both environments. This, then, is a necessary though not sufficient condition for simulator validity. If it holds, useful results may be obtained even when it is not possible to induce subjects to act as if the utilities for outcomes were the same as in the real world since the range of operating points can be determined.

Displays may be effective in man-machine systems, either by helping the operator execute his control tasks more skillfully or by giving him information that helps him decide whether to try a task. An example of the latter is a display of the time remaining in the amber phase of a traffic light cycle. ROC analysis is appropriate for the experimental evaluation of such displays. If the display's function is mixed, it may enable the separation of the display's enhancement of the operator's control skill from the effect on his predictive ability.

References

[1] D. M. Green, "Psychoacoustics and detection theory," *J. Acoust. Soc. Am.*, vol. 32, pp. 1189–1203, 1960.
[2] D. M. Green and J. A. Swets, *Signal Detection Theory and Psychophysics*. New York: Wiley, 1966.
[3] J. A. Swets, W. P. Tanner, Jr., and T. G. Birdsall, "The evidence for a decision making theory of visual detection," Electronic Defense Group, University of Michigan, Ann Arbor, Tech. Rept. 40, 1955.
[4] J. P. Egan, A. I. Schulman, and G. Z. Greenberg, "Operating characteristics determined by binary decisions and ratings," *J. Acoust. Soc. Am.*, vol. 31, pp. 768–773, 1959.
[5] M. Toda, "Measurement of subjective probability distributions," Div. Math. Psychol., Inst. for Research, Pennsylvania State University, University Park, Rept. 3, 1963.

Resolved Motion Rate Control of Manipulators and Human Prostheses

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***Abstract*—The kinematics of remote manipulators and human prostheses is analyzed for the purpose of deriving resolved motion rate control. That is, the operator is enabled to call for the desired hand motion directly along axes relevant to the task environment. The approach suggests solutions to problems of coordination, motion under task constraints, and appreciation of forces encountered by the controlled hand.**

Introduction

RATE control is currently one of the two most common ways of controlling a remote manipulator. The operator seeks to specify the direction and speed with which the manipulator is to move using a joystick or a set of switches. It is used in most industrial applications where large force amplification is needed and where environmental constraints or distance between controller and manipulator dictate a nonmechanical control link. The other common control method is master-slave control in which the operator guides a (usually full-scale) model of the manipulator so that the remote slave will follow a specified path and come to rest at a specified point. It is usually used with purely mechanical linkages to operate "hot lab" manipulators for precise tasks and has the favorable attributes of spatial correspondence and force feedback [1].

Rate control seems the predominant mode in most prototype artificial arms to date. An exception is the "Boston arm."[1] This is a powered elbow prosthesis, which amplifies EMG signals to generate force output (with force and velocity feedback) in one degree of freedom.

When the manipulator is powered by electric motors, hydraulic actuators, or the like, rather than by the operator's own muscles, the problem of coordinating the actuators arises. The problem is solved automatically in master-slave control if master and slave are geometrically similar, since coordinated motor drive signals may

Manuscript received September 6, 1968; revised March 3, 1969. This work was supported in part by the U. S. Social and Rehabilitation Service of the Department of Health, Education and Welfare.
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[1] Joint project of Massachusetts Institute of Technology, Harvard Medical School, Massachusetts General Hospital, and Liberty Mutual Insurance Company.

be picked up directly from transducers on the master. With rate control, however, the problem is more formidable. The operator has an array of switches connected one-to-one to the motors. He is thus controlling in what might be termed "arm coordinates." More relevant to the task would be "world coordinates," such as horizontal, vertical, reach along the hand direction, and so on. Hand motions along all such directions are easily perceived visually and are concentrated at the hand, whereas the arm coordinates are associated with joints distributed along the arm, difficult to monitor, and singly irrelevant to the desired motion.

Resolved motion thus means that the motions of the various motors are combined and resolved into separately controllable hand motions along world coordinates. The implication is that *several* motors (perhaps all six in a six-degree-of-freedom device) must run simultaneously at different and time-varying rates in order to achieve steady hand motion along *one* world coordinate. It seems unlikely that a manipulator operator or an amputee will be able to specify speeds and directions for more than two or three motors at a time (one or two if he wants some attention free for other thoughts). If he were controlling in arm coordinates, the result would be inefficient or uncosmetic zig-zags. It makes more sense to let control be exerted directly in world coordinates, especially if it must be done along a few at a time.

The amputee's problems of coordinating the motion of several motors and of appreciating forces that oppose motion are similar to those encountered in remote manipulation. Often the perception of these forces, the discovery of directions along which the environment is stiff or along which it yields, is central to the accomplishment of tasks. The discussion that follows is relevant to manipulators and prostheses, and to generation of either motion or force along world coordinates.

Other Approaches

A previous approach to this problem involves mounting the individual motor switches on a joystick whose degrees of freedom correspond to those of the manipulator [3]. This has proven of value even when this correspondence is weak [4]. Drawbacks are the following:

1) true resolved motion does not result;
2) it is difficult to apply to redundant manipulators or to those that differ markedly from a human arm;
3) it is inapplicable to control of artificial limbs.

Of course, master–slave control solves the resolved motion problem automatically, but this method is not applicable in all cases. For example:

1) confined operator work space or great size disparity between master and slave;
2) human prostheses;
3) when the operator must perform other motor tasks simultaneously with manipulation;
4) if the manipulator is redundant, has offset joints, or has a continuous flex or snake structure.

Supervisory control of the manipulator [5] removes from the operator all need to specify motor speeds, but puts this requirement on a computer. Thus the problem remains.

Capabilities of Resolved Motion Rate Control

The following are some of the capabilities one could provide if resolved motion rate control were available.

1) Motion along an arbitrarily oriented straight line in space, such as parallel to a table top or blackboard, or in and out of a confined space. This should require no more than three controls, regardless of how many motors there are.
2) Motion of the hand while keeping its angular orientation in space fixed. Again three gets you six. This is useful in the archetypical spoon-to-mouth problem faced by arm amputees.
3) Angular reorientation of the hand and arm while keeping the hand's spatial location fixed. This is useful for twisting operations, prying, and scooping.
4) Motion along hand-oriented axes, the most useful of which would be a "reach" direction.
5) Motion along a mixture of hand- and wrist-oriented axes, such as reach (hand-oriented) plus lift and sweep (vertical and horizontal), the latter two wrist-oriented.

While any of these features appear useful, they represent an attempt on the whole to provide the user with natural control as far as possible. It is likely that operators will not adapt easily to the notions of axes, angles, and resolution. It is more natural to be able to select motions along directly relevant directions [especially those in 5) above]. One finds a target, points the hand, and then says "go," in some sense. Using visual feedback, he makes corrections on the fly. The manipulator or prosthesis becomes part of its own control mechanism. It might be that it would be easier to learn to control such a device. Capabilities such as 3), 4), and 5) allow one to parse tasks *and* control switches into *corresponding* subsets and deal with them separately and sequentially.

This is also relevant to the generation of control signals by an amputee. The Boston arm is blessed with the assumed availability of bicep and tricep muscles, from which EMG signals are taken. Since these are the muscles that operated the lost elbow, the correspondence is immediate and the operator learns quickly. A forequarter amputee (no arm stump at all) is not so fortunate. Muscles (or nerve fibres) once irrelevant to arm motion must now be used. When six or more muscle or nerve pairs are needed, it is worth speculating that the amputee will learn faster if the motors are lumped into task- or world-related groups, controllable by much less than six pairs at a time. (This in no way is meant to ignore the fact that, as children, we all learn to coordinate our arm muscles using arm coordinates. But this takes two to three years of very hard work.)

Mathematics of Manipulators and the Proposed Rate Control Scheme

Six degrees of freedom, three positions and three rotations, are required to specify completely the location and orientation of an object in space. A manipulator therefore usually has six degrees of freedom; however, this may not be enough. Not every six-degree-of-freedom manipulator design allows the hand to be placed at *all* locations within the sphere of reach [6]. Also, in some applications, additional degrees of freedom may be needed due to environmental constraints. Such manipulators are called redundant. (The natural human arm is highly redundant.) They can reach more locations and reach them with smaller joint angle excursions than can nonredundant manipulators. This makes control easier since it avoids running into the stops and provides many alternate arm configurations that achieve the desired endpoint position, one of which may not be too different from the current arm configuration.

Let us group the external variables of interest, such as the six coordinates of the hand, plus "elbow" locations if desired, into a vector $\boldsymbol{x}$. Then, for any manipulator, $\boldsymbol{x}$ is related to the motor positions, grouped into the vector $\boldsymbol{\theta}$, by some usually complex vector relation

$$\boldsymbol{x} = f(\boldsymbol{\theta}). \tag{1}$$

The motor positions can be related to the joint angles in many ways. The relations are usually linear, involving gears and chains typically, and are assumed to be absorbed into (1). When $\boldsymbol{x}$ and $\boldsymbol{\theta}$ are of the same dimension, the manipulator is nonredundant. In a redundant manipulator, $\boldsymbol{\theta}$ is of higher dimension than $\boldsymbol{x}$. Since *we* choose $\boldsymbol{x}$, perhaps choosing different $\boldsymbol{x}$'s at different times during a manipulation task, we can make a nonredundant manipulator redundant solely by choosing not to control some portions of $\boldsymbol{x}$. Conversely, by arbitrarily freezing some motor positions or by adding more hand or "elbow" coordinates to $\boldsymbol{x}$, we may make a redundant manipulator nonredundant. In any case, (1) applies in some form.

If we differentiate (1) with respect to time, we obtain

$$\frac{d\boldsymbol{x}}{dt} = \dot{\boldsymbol{x}} = J(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{2}$$

where $J(\boldsymbol{\theta})$ is the Jacobian of f with respect to $\boldsymbol{\theta}$, a matrix with

$$[J]_{ij} = \frac{\partial f_i}{\partial \theta_j} \qquad \begin{matrix} 1 \le i \le n \\ 1 \le j \le m \end{matrix} \tag{3}$$

where n is the dimension of $\boldsymbol{x}$ and m that of $\boldsymbol{\theta}$. Thus if we are content to work with rates rather than positions, the relationship is linear.

If one were presented with a rate control switch box, one's first act in learning to operate the manipulator would probably be to activate each switch individually and observe the effect. If one does this in such a way that the affected motor runs at unit speed, then one has generated, in effect, the analog of one column of matrix J. This really will not help much, since resolved motion requires that several motors run simultaneously at different, nonunity, nonconstant speeds, by analogy with groups of muscles in one's arm. If $n = m$, we may synthesize these speeds by inverting $J(\boldsymbol{\theta})$, wherever the inverse exists, to obtain

$$\dot{\boldsymbol{\theta}} = J^{-1}(\boldsymbol{\theta})\dot{\boldsymbol{x}}. \tag{4}$$

The control strategy indicated by (4) is the following. Associate with each switch a component of $\dot{\boldsymbol{x}}$, feed the switch outputs through $J^{-1}(\boldsymbol{\theta})$, and feed the result to the motors as $\dot{\boldsymbol{\theta}}$, the motor speed, commands. Then, for example, if $\boldsymbol{x}$ contains both position and orientation coordinates of the hand, we may change orientation without changing position merely by specifying zero velocity for the position coordinates in $\dot{\boldsymbol{x}}$, and whatever we want for the orientation coordinates. Equation (4) does the rest. (It is interesting that (4) is analogous to the Newton–Raphson method for finding a $\boldsymbol{\theta}$ to correspond to a given $\boldsymbol{x}$. This is relevant to computer position control of manipulators.)

If $m > n$, J^{-1} is not defined. If we do not wish to freeze arbitrary coordinates in $\boldsymbol{\theta}$ or add some "elbow" coordinates to $\boldsymbol{x}$, we may get around the difficulty by defining an optimality criterion, which the manipulator must satisfy while undergoing its motions. For example, minimize

$$G = \tfrac{1}{2} \int \dot{\boldsymbol{\theta}}^T A \dot{\boldsymbol{\theta}}\, dt \tag{5}$$

during the motion. Here, superscript T denotes vector transpose and A is a positive definite weighting matrix. Solution of such problems usually involves a great deal of computation. A simpler criterion is

$$G = \tfrac{1}{2}\dot{\boldsymbol{\theta}}^T A \dot{\boldsymbol{\theta}}. \tag{6}$$

That is, the assumed "cost" of motion is approximately the instantaneous weighted system kinetic energy. (See [7], where a different criterion was used.) Adjoining (2) to (6) with Lagrange multipliers and assuming, as before, that the desired $\dot{\boldsymbol{x}}$ is known, we obtain for the optimal $\dot{\boldsymbol{\theta}}$

$$\dot{\boldsymbol{\theta}}^T = \dot{\boldsymbol{x}}^T[J(\boldsymbol{\theta})A^{-1}J(\boldsymbol{\theta})^T]^{-1}J(\boldsymbol{\theta})A^{-1}, \tag{7}$$

which is directly analogous to (4). This method is equivalent to solving (4) via a pseudo-inverse in such a way that $\dot{\boldsymbol{\theta}}$ minimizes $[\dot{\boldsymbol{x}} - J\dot{\boldsymbol{\theta}}]^T A^{-1} [\dot{\boldsymbol{x}} - J\dot{\boldsymbol{\theta}}]$ [8]. The above derivation, however, makes plain the influence of A.

We may choose A so as to emphasize the role of some components of $\boldsymbol{x}$ and deemphasize others, for example, by heavily penalizing motions of the latter relative to the former. This is another way of obtaining some of the motion features listed above, which also protects the operator somewhat against errors of emphasis. To synthesize the required A for this purpose, we begin with the cost criterion

$$G' = \tfrac{1}{2}\dot{x}^T B \dot{x}. \tag{8}$$

B can usually be chosen by inspection to be positive definite and provide the desired relative emphases. For example, change hand orientation while keeping its location relatively fixed. Substituting (2) into (8) gives

$$G' = \tfrac{1}{2}\dot{\boldsymbol{\theta}}^T J^T(\boldsymbol{\theta}) B J(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}. \tag{9}$$

Comparing (9) with (6), we may identify

$$A = J^T(\boldsymbol{\theta}) B J(\boldsymbol{\theta}), \tag{10}$$

which synthesizes A. Note that this A is not necessarily positive definite. What is important, however, is that B be positive definite, and that the resulting A be nonsingular.

To obtain motion along hand-oriented axes, we need only note that, as far as rates are concerned, the hand-oriented axes are merely rotated from the fixed coordinate axes from which wrist position is measured. Then rates $\dot{\boldsymbol{h}}$ along the hand-oriented axes are related to the original $\dot{\boldsymbol{x}}$ by

$$\dot{\boldsymbol{x}} = R\dot{\boldsymbol{h}}. \tag{11}$$

Here, R is a partitioned matrix containing the usual rotation terms, the latter being trigonometric functions of hand orientation, and thus functions of $\boldsymbol{\theta}$. Substituting (11) into (4) gives

$$\dot{\boldsymbol{\theta}} = J^{-1}(\boldsymbol{\theta})R(\boldsymbol{\theta})\dot{\boldsymbol{h}}. \tag{12}$$

(Two remarks: 1) even though $\boldsymbol{h}$ is probably of lower dimension than $\boldsymbol{x}$, all $\boldsymbol{\theta}$'s will in general be changing when (12) is in force, since hand position is separately controllable; 2) equation (11) may equally well be substituted into equation (7) with similar results.)

Examples

Fig. 1 shows a sketch of a typical industrial manipulator, together with a convenient coordinate system. This is actually a four-degree-of-freedom manipulator. Its links all lie in one plane, whose orientation is measured through angle d. The four degrees of freedom are the x, y, z coordinates of the hand plus angle c, the hand's orientation. These four quantities comprise $\boldsymbol{x}$. The four drive motor positions are denoted by θ_a, θ_b, θ_c and θ_d, and comprise $\boldsymbol{\theta}$. While θ_d drives angle d directly, the other θ's may be related to angles a, b, and c in many ways, summarized by

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = M \begin{bmatrix} \theta_a \\ \theta_b \\ \theta_c \end{bmatrix} + \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix} \tag{13}$$

where M is a 3×3 matrix, and a_0, b_0, and c_0 are the values of a, b, and c when θ_a, θ_b, and θ_c take on their (arbitrary) zero positions. A useful matrix M is

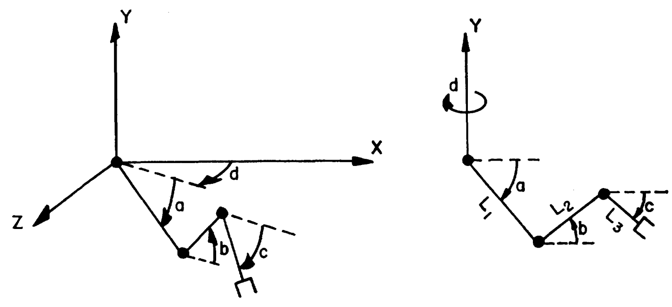

Fig. 1. (a) Sketch of a manipulator and its three-dimensional reference frame. (b) View of the manipulator in the plane of the linkage. Angles a, b, and c are measured from references in the X–Z plane.

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{14}$$

which makes each external angle independently controllable. This relation is mechanized in the manipulator attached to the oceanographic research vessel ALVIN [9]. Using $a_0 = b_0 = c_0 = 45°$, we get for $\boldsymbol{f}(\boldsymbol{\theta})$:

$$\begin{aligned} x &= \left[L_1 \cos\left(\theta_a + \frac{\pi}{4}\right) + L_2 \cos\left(\theta_b + \frac{\pi}{4}\right) + L_3 \cos\left(\theta_c + \frac{\pi}{4}\right)\right] \cos\theta_d \\ y &= -L_1 \sin\left(\theta_a + \frac{\pi}{4}\right) + L_2 \sin\left(\theta_b + \frac{\pi}{4}\right) - L_3 \sin\left(\theta_c + \frac{\pi}{4}\right) \\ z &= x \tan\theta_d \\ c &= \theta_c + \frac{\pi}{4}. \end{aligned} \tag{15}$$

When $\boldsymbol{\theta} = \boldsymbol{0}$, $L_1 = L_2 = 26$, $L_3 = 11.5$, we get for $J(\boldsymbol{\theta})$:

$$J(\boldsymbol{0}) = \begin{bmatrix} -18.51 & -18.4 & -8.14 & 0 \\ -18.51 & 18.4 & -8.14 & 0 \\ 0 & 0 & 0 & 45.1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{16}$$

Then

$$J^{-1}(\boldsymbol{0}) \cong \begin{bmatrix} -0.027 & -0.027 & 0 & -0.44 \\ -0.027 & 0.027 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.0222 & 0 \end{bmatrix}. \tag{17}$$

So, for example, if near $\boldsymbol{\theta} = \boldsymbol{0}$ we wished to tilt the hand without changing its location, we would call for

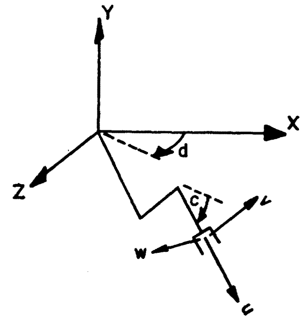

Fig. 2. The manipulator of Fig. 1 with a hand-oriented coordinate system *uvw*.

$$\dot{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{c} \end{bmatrix} \tag{18}$$

perhaps by pushing a switch labeled "hand orientation." Substituting (18) and (17) into (4), we find that the resulting motor speeds would be

$$\begin{aligned} \dot{\theta}_a &= -0.44\,\dot{c} \\ \dot{\theta}_b &= 0 \\ \dot{\theta}_c &= \dot{c} \\ \dot{\theta}_d &= 0 \end{aligned} \tag{19}$$

in the vicinity of $\boldsymbol{\theta} = \mathbf{0}$, varying suitably as $\boldsymbol{\theta}$ changed.

Fig. 2 shows a hand-oriented coordinate system attached to the manipulator of Fig. 1. The u direction is the reach direction, while v and w are mutually perpendicular and perpendicular to u. Then we have

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{c} \end{bmatrix} = \left[\begin{array}{c|c} R_{3\times 3} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{c} \end{bmatrix} \tag{20}$$

where

$$R = \begin{bmatrix} \cos d & 0 & -\sin d \\ 0 & 1 & 0 \\ \sin d & 0 & \cos d \end{bmatrix} \begin{bmatrix} \cos c & \sin c & 0 \\ -\sin c & \cos c & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{21}$$

Appreciation of Forces in Manipulation

Anyone who has operated a remote manipulator knows that there is more to manipulation than repositioning the manipulator's jaws. While some manipulation tasks are largely *rearrangement tasks,* others are largely *contact tasks* in which accomplishment of the task depends heavily on the operator's appreciation of the contact forces or impedances encountered by the manipulator's jaws as they strike or move in conjunction with portions of the task environment. The same applies to artificial arms.

Naturally, few real tasks are exclusively rearrangement or contact tasks. Yet while both features are involved, tasks can often be segmented into rearrangement subtasks separated by contact subtasks. The previous sections of this paper have been concerned with easing the accomplishment of the rearrangement subtasks. In this section, speculations are made on the applicability of this work to the accomplishment of contact tasks, especially in the area of powered prostheses.

Most prototype powered artificial arms are rate controlled. Except for what can be felt by the stump, there is no force feedback. This plus visual feedback may be sufficient for an elbow prosthesis, since only one degree of freedom is involved. The Boston arm is force controlled and has a local force feedback loop, which will cause the arm to drop if the load is increased and the amputee does not increase the EMG signal level. This loop has been shown effective in increasing the amputee's awareness of applied loads [10].

Contact tasks include at least two broad classes, passive tasks and active ones. A well-known example of the former is that of opening a door or turning a crank. People (and operators of force feedback manipulators) are greatly aided in such tasks by appreciating the increase in impedance to motion that accompanies a drift of the hand or jaw from the preferred circular trajectory. An example of the latter is cutting meat. This requires one to apply forces in a relatively freely chosen direction, to modify force and direction based on varying impedance, and to detect through varying impedance that the task has been completed.

Let us consider accomplishing such tasks with a six-degree-of-freedom powered arm having local force feedback loops at each joint similar to that in the Boston arm. Passive contact tasks might be possible, since the arm would certainly accommodate itself to the desired trajectory, provided that the amputee knew which EMG signals to strengthen or weaken (to keep the arm and load moving) and which to leave alone (to provide the accommodation). The first set corresponds to those motors whose motions can be resolved into hand motion *along* the desired trajectory, while the second set corresponds to those whose motions can be resolved into hand motion *normal* to that trajectory.[2] As the arm moves, the composition of these sets changes. One may speculate that visual feedback will be of little avail in determining which motors or muscle pairs are in which set. Rather, the operator should have resolved feedback: some combination of the strain gauge readings from the local feedback loops should be displayed to him so that he may determine which motors meet low impedance

[2] This is not meant to imply that the sets are mutually exclusive.

(the first set) and which meet high impedance (the second set). Active contact tasks may not be possible at all without such feedback.

We can relate the mathematics of this type of feedback to the discussions in the previous sections as follows. If the muscle pairs (or nerves or whatever) are connected to the motors of a six-degree-of-freedom Boston arm[3] one-to-one, with no resolved motion, then a vector of EMG signals is related to the vector of motor speeds by

$$\mathbf{EMG} = \dot{\boldsymbol{\theta}} \tag{22}$$

if the arm is moving unopposed by the environment. If the EMG signals are strong enough that a six vector of external force $\boldsymbol{F}_e$ caused by contacting part of the environment does not activate any of the local feedback loops, then $\boldsymbol{F}_e$, $\boldsymbol{\Delta x}$, the arm displacement caused by $\boldsymbol{F}_e$, and K, the stiffness matrix of the arm, are related by

$$\boldsymbol{F}_e = K\boldsymbol{\Delta x}. \tag{23}$$

This $\boldsymbol{\Delta x}$, which may be quite small and thus difficult to utilize for visual feedback, will correspond to a $\boldsymbol{\Delta\theta}$ of

$$\boldsymbol{\Delta\theta} = J^{-1}\,\boldsymbol{\Delta x} \tag{24}$$

if we assume that the limbs are much stiffer than the joints. At each joint there is a strain gauge whose reading is related only to the corresponding $\boldsymbol{\Delta\theta}$ so that a vector of strain gauge readings $\boldsymbol{\varepsilon}$ is given by

$$\boldsymbol{\varepsilon} = D\,\boldsymbol{\Delta\theta} \tag{25}$$

where D is a diagonal matrix. Then

$$\boldsymbol{\varepsilon} = DJ^{-1}K^{-1}\boldsymbol{F}_e. \tag{26}$$

If a feedback signal s is sent to each muscle pair (applied by some kind of "tickler"), then it would be logical to set

$$\boldsymbol{s} = \boldsymbol{\varepsilon} \tag{27}$$

so that

$$\boldsymbol{s} = DJ^{-1}K^{-1}\boldsymbol{F}_e. \tag{28}$$

And if $D = dI$, where d is a scalar, then

$$\boldsymbol{s} = dJ^{-1}K^{-1}\boldsymbol{F}_e \tag{29}$$

$$= d\,\boldsymbol{\Delta\theta} \quad \text{(compare to (22))} \tag{30}$$

so that, as expected, the tickled muscles correspond to those whose activation causes the external force to be resisted.

If the muscles are connected to the motors so as to produce resolved motion, then

$$\mathbf{EMG} = J\dot{\boldsymbol{\theta}} = \dot{\boldsymbol{x}}. \tag{31}$$

Now, it makes sense to set

$$\boldsymbol{s} = J\boldsymbol{\varepsilon} \tag{32}$$

so that

$$\boldsymbol{s} = JDJ^{-1}K^{-1}\boldsymbol{F}_e \tag{33}$$

$$= dK^{-1}\boldsymbol{F}_e \tag{34}$$

$$= d\,\boldsymbol{\Delta x} \quad \text{(compare to (31))} \tag{35}$$

if $D = dI$. Thus again the tickled muscles correspond to those whose motion causes $\boldsymbol{F}_e$ to be counteracted. Note that we could set

$$\boldsymbol{s} = KJ\boldsymbol{\varepsilon}, \tag{36}$$

which leads to

$$\boldsymbol{s} = d\boldsymbol{F}_e. \tag{37}$$

Now the external force vector itself is being displayed to the muscles.

Equation (30) shows that unresolved muscle-to-motor and strain gauge-to-muscle hookups will enable the amputee to detect which components of $\boldsymbol{\theta}$ can be changed with little resistance from the environment and which will be resisted. Equation (35) shows that resolved hookups enable the amputee to detect which components of $\boldsymbol{x}$ can be changed with little environmental resistance and which meet larger resistance. Inasmuch as the force information in the latter hookup can be correlated directly with visual data concerning the environment and the hand (perhaps limited to motions along low-resistance directions), it may be that the latter hookup would be preferable.

Since no experiments have been performed, however, it is premature to say which of these (or other) schemes would be of the most assistance to an amputee. It is fair to say, though, that something of this sort will be better than nothing.

Conclusions

The mathematics of multi-degree-of-freedom manipulators and prostheses has been analyzed. If we are content to work with rates, then the problem is linear, regardless of the arm configuration, provided that no motor variables hit their stops. We show that the operator can obtain control of motion easily along "world coordinates" if the control actions are modified by the inverse of the arm's Jacobian matrix. This allows us to choose among several interesting coordinate systems in which to control. A redundant arm can be "programmed" to obey certain useful and relevant constraints during motion, constraints which would be difficult to obey with conventional one-to-one rate control. The above formulations are also relevant in providing force feedback to amputees with EMG controlled prostheses and we speculate that multi-degree-of-freedom prostheses would be easier to learn and more applicable to everyday contact tasks if so designed.

[3] This is hypothetical, of course, since no such arm exists at this time.

References

[1] E. Johnson and W. Corliss, "Teleoperators and human augmentation," NASA, SP-5047, p. 57.
[2] R. A. Rothchild and R. W. Mann, "An EMG controlled force sensing proportional rate elbow prosthesis," *Proc. 1966 Symp. on Biomedical Engrg.* (Milwaukee, Wisc.), vol. 1, 1966.
[3] E. Johnson and W. Corliss, *op. cit.*, p. 91.
[4] W. Verplank, "Symbolic and analogic command hardware for computer-aided manipulation," S.M. thesis, Dept. of Mech. Engrg., Massachusetts Institute of Technology, Cambridge, 1967.
[5] W. R. Ferrell and T. B. Sheridan, "Supervisory control of remote manipulation," *IEEE Spectrum*, vol. 4, pp. 81–88, October 1967.
[6] D. Pieper, "Kinematics of computer controlled manipulators." Ph.D. dissertation, Stanford University, Stanford, Calif., 1968.
[7] H. Mergler and P. Hammond, "A path optimization scheme for a numerically controlled remote manipulator," presented at the 1965 6th Annual Symp. of the IEEE Human Factors in Electronics Group, May 1965.
[8] R. Penrose, "The best approximation to the solution of matrix equations," *Proc. Cambridge Phil. Soc.*, vol. 52, pp. 17–19, 1956.
[9] Woods Hole Oceanographic Inst., Ref. 67–67.
[10] R. W. Mann, personal communication.

Operator-Centered Adaptive Compensation of Continuous Manual Control Systems

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***Abstract*—The subject of this paper is an adaptive compensator, which has the function of minimizing the compensation required of the human operator in a continuous single-dimensional manual control system. A random input and a compensatory display are assumed. Use of this compensator does not require detailed knowledge of system dynamics since its adaptive action is based solely upon the operator's control actions. A quasi-linear operator model and the parameter-tracking model method of measurement are used to provide the required on-line mathematical description of the operator's control actions.**

Design of the compensator is based upon past results, which show the human operator to be an adaptive controller who introduces compensation, via his control actions, similar to that which would be chosen by a servo-engineer in minimizing the square of the system error. The compensator's function results from the established fact that an operator can control a system most accurately when the least compensation is required of him.

Experimental results demonstrate that the compensator reduces system error while simultaneously simplifying the operator's task. They also establish some of the limitations of the compensation scheme as currently implemented.

Introduction

In recent years, an extensive amount of research has been devoted to modeling the human operator of a continuous manual control system. Representative of the results from this research are the widely used quasi-linear model [1]–[6], a nonlinear analog proposed by Diamontides [7], a sampled-data model formulated by Bekey [8] and an adaptive model studied by Fu and Gould [9]. These results and others are discussed and compared in several papers [10]–[14].

Operator models are instrumental in learning more about the manner in which an operator performs and the interactions that exist between man and the machine portion of a system. Furthermore, they are required when using conventional techniques to synthesize compensators for a given manual control system. In spite of their many uses, operator models have not been employed as a functional part of an adaptive system. The intent of this paper is to show one way in which they can be used to advantage in an adaptive compensation scheme.

The objective of compensation in manual control systems is to minimize or simplify the operator's task. Such simplification enables an operator to do a more accurate job of controlling the system of interest, reduces his rate of fatigue, and leaves him with more time to perform any auxiliary tasks that may be required.

Selecting a compensator for a given system requires a detailed knowledge of the dynamics involved plus an understanding of what is needed to minimize the operator's task. This selection is complicated in many systems by parameters that vary over a large range of values with time. Because of such variation, it is often impossible to obtain the desired overall system response through use of a single time-invariant compensator.

The proposed adaptive compensator eliminates the need for a detailed knowledge of system dynamics as its

Manuscript received November 6, 1968; revised February 25, 1969.
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