Control of Redundant Robots in Presence of External Forces

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Abstract — In the paper we study the influence of generalized external forces, which can act anywhere on the body of the manipulator, on the behavior of the manipulator. These forces are considered as disturbances which are not measurable. First we analyze how these forces act in the task space and in the null space, and define the equivalent generalized forces in both subspaces. Next we propose tracking controller and analyze the influence of external forces on tracking accuracy. We consider also the effects of different psudoinverses on the behavior of the system. In the end some examples of control of a four DOF planar manipulator with revolute joints are given illustrating the influence of the external forces.

I. INTRODUCTION

Some tasks involve contacts with environment and it is necessary to control the interactive forces. For that purpose different control approaches have been proposed like hybrid position/force control [9] or impedance control [2] which have also be applied to redundant manipulators [5, 13, 12, 8]. Usually, it is supposed that the contact occurs between the end-effector or the handling object and the environment. Therefore, these forces act only in the task space. The situation is more complicated if the force can act anywhere on the body of the manipulator. Only some authors consider such forces. Woernle [12] proposes a control algorithm based on complete decoupling of the system. Common to all these approaches is that contact forces are control variables and that the manipulator must have "enough" degrees of freedom to control the desired position and force variables.

In the paper we study situations where external forces, caused by the contact of the manipulator with the environment, are present. As these forces can act anywhere it is questionable if they can be measured. Hence, we assume that these forces are not measurable and they are considered as disturbances. Such situation causes additional problems in control design. Our goal is to design a control algorithm which is stiff in the task space and is compliant to the forces in the null space. In this way tracking properties in the task space are preserved.

II. KINEMATICS

The robotic systems under study are n degrees of freedom serial manipulators. We consider only redundant systems which have more degrees of freedom than needed to accomplish a task, i.e. the dimension of the joint space n exceeds the dimension of the task space m. Let the configuration of the manipulator be represented by the vector $\boldsymbol{q}$ of n joint positions, and the end-effector position (and orientation) by m-dimensional vector $\boldsymbol{x}$ of task positions (and orientations). The joint and task positions are related by the following expression

$$\boldsymbol{x} = \boldsymbol{p}(\boldsymbol{q}) \tag{1}$$

where $\boldsymbol{p}$ is m-dimensional vector function representing the manipulator forward kinematics. Differentiating (1) we obtain the relation between velocities

$$\dot{\boldsymbol{x}} = \mathbf{J}\dot{\boldsymbol{q}} \tag{2}$$

where $\mathbf{J} = \frac{\partial \mathbf{p}}{\partial \boldsymbol{q}}$ is the $m \times n$ manipulator Jacobian matrix. Due to the redundant degrees of freedom any joint space velocity $\dot{\boldsymbol{q}}$ can be decomposed into two parts as follows

$$\dot{\boldsymbol{q}} = \mathbf{J}^{\#}\mathbf{J}\dot{\boldsymbol{q}} + (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\dot{\boldsymbol{q}} \tag{3}$$

where $\mathbf{J}^{\#}$ is the generalized inverse of $\mathbf{J}$ and $(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})$ is $n \times n$ matrix representing the projection of $\dot{\boldsymbol{q}}$ into the null space of $\mathbf{J}$. Note that the decomposition of the system depends on the particular selection of $\mathbf{J}^{\#}$ and that there is an infinite number of generalized inverses $\mathbf{J}^{\#}$. In the following we will assume that the workspace of the manipulator excludes the singular configurations. Hence, $\mathbf{J}(\boldsymbol{q})$ will always have a full rank, $\mathrm{rank}(\mathbf{J}) = m$.

Differentiating (2), we obtain the relation between joint space and task space accelerations

$$\ddot{\boldsymbol{x}} = \mathbf{J}\ddot{\boldsymbol{q}} + \dot{\mathbf{J}}\dot{\boldsymbol{q}} \tag{4}$$

As in (3) we use the generalized inverse of $\mathbf{J}$ to obtain the inverse relation at the acceleration level

$$\ddot{\boldsymbol{q}} = \mathbf{J}^{\#}(\ddot{\boldsymbol{x}} - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) + (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\ddot{\boldsymbol{q}} \tag{5}$$

Equation (3) and (5) form a basis of the inverse kinematics of a redundant manipulator. The first term on the right side corresponds to the motion of the end-effector. The second term is the joint space velocity which belongs to the null space of $\mathbf{J}$ and represents the internal motion of the

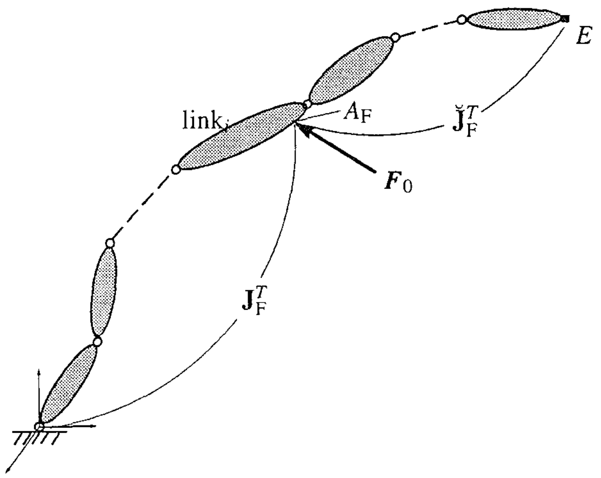

Figure 1: External force acting on the body of the robot

manipulator. As we can see, there is, in general, an infinite number of joint trajectories for a given desired task space trajectory, i.e. $\dot{\boldsymbol{q}}$ or $\ddot{\boldsymbol{q}}$ in the second term of (3) or (5) can be arbitrary joint space velocity $\dot{\boldsymbol{q}}_n$ or acceleration $\ddot{\boldsymbol{q}}_n$. Hence, the redundancy is usually resolved by imposing additional requirements and constraints on the manipulator.

III. STATICS

For redundant manipulators the static relationship between the m-dimensional generalized force in task space $\boldsymbol{F}$ and the corresponding n-dimensional generalized joint space force $\boldsymbol{\tau}$ is[1]

$$\boldsymbol{\tau} = \mathbf{J}^T\boldsymbol{F} + (\mathbf{I} - \mathbf{J}^T\mathbf{J}^{\#T})\boldsymbol{\tau}_n \tag{6}$$

where $(\mathbf{I} - \mathbf{J}^T\mathbf{J}^{\#T})$ is $n \times n$ matrix representing the projection into the null space of $\mathbf{J}^T$ and $\boldsymbol{\tau}_n$ is an arbitrary n-dimensional vector of joint torques. Namely, when the redundant manipulator is in motion, there is an infinite number of joint torques within the null space of $\mathbf{J}^T$ that could be applied to the system without affecting the forces in the task space.

Usually, only generalized forces acting at the end-effector or manipulated object are considered. If these external forces acting at any point on the manipulator body are of interest we can not use (6) directly. Suppose that $\boldsymbol{F}_0$ is acting somewhere on the link i (see Fig. 1), Then the static relation between the external force $\boldsymbol{F}_0$ and joint torques $\boldsymbol{\tau}$ is

$$\boldsymbol{\tau}_{\mathrm{F}} = \mathbf{J}_{\mathrm{F}}^T\boldsymbol{F}_0 \tag{7}$$

where the Jacobian matrix $\mathbf{J}_{\mathrm{F}}$ describes the differential relation between the displacement of joint space positions and the position of the point A_{F} where the force is acting. It is apparent that this force does not influence directly the behavior of the manipulator beyond the link i. Therefore, the Jacobian matrix $\mathbf{J}_F$ has the form

$$\mathbf{J}_{\mathrm{F}} = \left[\breve{\mathbf{J}}_{\mathrm{F}} \,\vdots\, \mathbf{0}_{m\times(n-i)}\right] \tag{8}$$

[1] In the following we denote for convenience the generalized external forces as *external forces* and the generalized joint forces as *torques*.

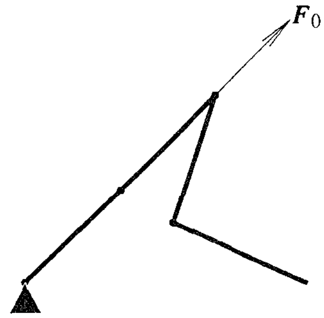

Figure 2: Configuration of the manipulator where $\mathbf{J}_{\mathrm{F}}\boldsymbol{F}_0 = 0$

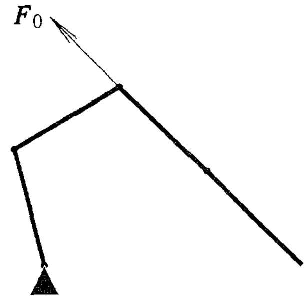

Figure 3: Configuration of the manipulator where $\breve{\mathbf{J}}_{\mathrm{F}}^T\boldsymbol{F}_0 = 0$

where $\breve{\mathbf{J}}_{\mathrm{F}}$ is a $m \times i$ matrix. Consequently, the components $\tau_k, k > i$ are zero. If $\boldsymbol{F}_0$ is not acting at the end-effector it can affect also the torques in the null space of $\mathbf{J}^T$, which is not a case for forces acting at the end-effector. Therefore, $\boldsymbol{F}_0$ can be substituted by an equivalent generalized force acting at the end-effector

$$\boldsymbol{F}_{\mathrm{eq}} = \mathbf{J}^{\#T}\mathbf{J}_{\mathrm{F}}^T\boldsymbol{F}_0 = \mathbf{J}^{\#T}\boldsymbol{\tau}_{\mathrm{ext}} \tag{9}$$

and by generalized joint forces within the null space of $\mathbf{J}^T$

$$\boldsymbol{\tau}_{\mathrm{eq}} = (\mathbf{I} - \mathbf{J}^T\mathbf{J}^{\#T})\mathbf{J}_{\mathrm{F}}^T\boldsymbol{F}_0 = (\mathbf{I} - \mathbf{J}^T\mathbf{J}^{\#T})\boldsymbol{\tau}_{\mathrm{ext}} \tag{10}$$

For any generalized external force $\boldsymbol{F}_0 \neq 0$ applied to the redundant manipulator there will always exist equivalent force in task space $\boldsymbol{F}_{\mathrm{eq}} \neq 0$ except when $\mathbf{J}_{\mathrm{F}}$ is singular and the term $\mathbf{J}_{\mathrm{F}}\boldsymbol{F}_0 = 0$ [11]. Fig. 2 show the special configuration of the manipulator where $\mathbf{J}_{\mathrm{F}}\boldsymbol{F}_0 = 0$, and the external forces are compensated by the forces in the base of the manipulator. The other special configuration is when the part of the manipulator between the point A_{f} and the end-effector (E) is in singular configuration. Let $\breve{\mathbf{J}}_{\mathrm{F}}$ denote the Jacobian matrix associated with this part of the manipulator. It can be proved that if $\breve{\mathbf{J}}_{\mathrm{F}}^T\boldsymbol{F}_0 = 0$ then forces $\boldsymbol{F}_{\mathrm{eq}}$ and $\boldsymbol{F}_0$ are equal, $\boldsymbol{F}_{\mathrm{eq}} = \boldsymbol{F}_0$, and $\boldsymbol{\tau}_{\mathrm{eq}} = 0$ [11]. This special configuration is presented in Fig. 3.

IV. DYNAMICS

Assuming the manipulator consists of rigid bodies the joint space equations of motion can be written in a form

$$\boldsymbol{\tau} = \mathbf{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \mathbf{g}(\boldsymbol{q}) - \boldsymbol{\tau}_{\mathrm{ext}} \tag{11}$$

where $\boldsymbol{\tau}$ is n-dimensional vector of control torques, $\mathbf{H}$ is $n \times n$ symmetric positive-definite inertia matrix, $\mathbf{h}$ is n-dimensional vector of Coriolis and centrifugal forces, and $\mathbf{g}$

is n-dimensional vector of gravity forces. Vector $\boldsymbol{\tau}_{\text{ext}}$ summarizes effects of all external forces acting on the manipulator

$$\boldsymbol{\tau}_{\text{ext}} = \sum_i \mathbf{J}_{\text{F},i}^T \boldsymbol{F}_{0,i} \tag{12}$$

where the summation indicates the presence of more external forces.

When the dynamic response of the end-effector is of interest it is of benefit to use the following equation of motion of the end-effector subjected to generalized task forces $\boldsymbol{F}$ [5]

$$\boldsymbol{F} = \mathbf{M}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{\mu}(\boldsymbol{x},\dot{\boldsymbol{x}}) + \boldsymbol{\gamma}(\boldsymbol{x}) - \boldsymbol{F}_{\text{ext}} \tag{13}$$

where $\mathbf{M}$, $\boldsymbol{\mu}$, $\boldsymbol{\gamma}$ and $\boldsymbol{F}_{\text{ext}}$ are, respectively, $m \times m$ symmetric positive-definite matrix describing the inertial properties of the manipulator in the task space, m-dimensional vector of Coriolis and centrifugal forces, m-dimensional vector of gravity forces, and m-dimensional vector of external forces, all acting in task space. The relation between $\mathbf{M}$ and $\mathbf{H}$ is obtained by using the quadratic forms of the kinetic energy and equation (2) [5]

$$\mathbf{M} = (\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)^{-1} \tag{14}$$

The vector $\boldsymbol{F}_{\text{ext}}$ in (13) is obtained as follows

$$\boldsymbol{F}_{\text{ext}} = \mathbf{J}^{\#T} \sum_i \mathbf{J}_{\text{F},i}^T \boldsymbol{F}_{0,i} \tag{15}$$

For the redundant manipulators the equation (13) describes only that part of the whole dynamics of the manipulator which is associated with the end-effector.

V. CONTROL

To apply a manipulator to a task which involves contact with the environment usually hybrid position/force control [9, 5, 13] or impedance control [2] are used. Common to these control methods is that they control positions and forces, and that the forces are measurable. Now, let the task of the controller be to assure tracking of a given task space trajectory. Further, suppose that there exist external forces (e.g. the manipulator is in the contact with the environment) which are not measurable. Hence, they are considered as disturbances to the position controller. The objective of the paper is to give some insight into the influence of external forces on the tracking accuracy of the task position controller and on the self-motion of the manipulator. We consider control techniques utilizing a formulation of the inverse kinematics at the acceleration level. Using (5) a general formulation of the control law is given in the form

$$\boldsymbol{\tau} = \mathbf{H}(\mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_c - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) + (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\boldsymbol{\phi}) + \mathbf{h} + \mathbf{g} \tag{16}$$

where $\ddot{\boldsymbol{x}}_c$ is the task acceleration and $\boldsymbol{\phi}$ is an arbitrary n-dimensional vector. Combining (11) and (16) yields

$$\begin{aligned} \mathbf{H}\ddot{\boldsymbol{q}} + \mathbf{h} + \mathbf{g} - \boldsymbol{\tau}_{\text{ext}} = {} & \mathbf{H}\mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_c - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) \\ & + \mathbf{H}(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\boldsymbol{\phi}) + \mathbf{h} + \mathbf{g} \end{aligned} \tag{17}$$

which simplifies to

$$\ddot{\boldsymbol{q}} - \mathbf{H}^{-1}\boldsymbol{\tau}_{\text{ext}} = \mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_c - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) + (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\boldsymbol{\phi} \tag{18}$$

Using (5) in (18) and simplifying it yields

$$-\mathbf{H}^{-1}\boldsymbol{\tau}_{\text{ext}} = \mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_c - \ddot{\boldsymbol{x}}) + (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})(-\ddot{\boldsymbol{q}} + \boldsymbol{\phi}) \tag{19}$$

Premultiplying (19) with $\mathbf{J}$ yields

$$\ddot{\boldsymbol{x}}_c - \ddot{\boldsymbol{x}} = -\mathbf{J}\mathbf{H}^{-1}\boldsymbol{\tau}_{\text{ext}} \tag{20}$$

since $\mathbf{J}\mathbf{J}^{\#} = \mathbf{I}$. Using (9) in (20) the following relation is obtained

$$\ddot{\boldsymbol{x}}_c - \ddot{\boldsymbol{x}} = -\mathbf{J}\mathbf{H}^{-1}\mathbf{J}_{\text{F}}^T \boldsymbol{F}_0 \tag{21}$$

where the influence of external force is more evident. Often $\ddot{\boldsymbol{x}}_c$ is selected as

$$\ddot{\boldsymbol{x}}_c = \ddot{\boldsymbol{x}}_d + \mathbf{K}_v \dot{\boldsymbol{e}} + \mathbf{K}_p \boldsymbol{e} \tag{22}$$

where $\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x}$ is the tracking error, $\ddot{\boldsymbol{x}}_d$ is the desired task space acceleration, $\mathbf{K}_v$ and $\mathbf{K}_p$ are $n \times n$ constant gain matrices. Substituting (22) in (20) yields

$$\ddot{\boldsymbol{e}} + \mathbf{K}_v \dot{\boldsymbol{e}} + \mathbf{K}_p \boldsymbol{e} = -\mathbf{J}\mathbf{H}^{-1}\mathbf{J}_{\text{F}}^T \boldsymbol{F}_0 \tag{23}$$

The proper choice of gain matrices $\mathbf{K}_v$ and $\mathbf{K}_p$ assures the asymptotical stability of the homogeneous part of the system $\ddot{\boldsymbol{e}} + \mathbf{K}_v \dot{\boldsymbol{e}} + \mathbf{K}_p \boldsymbol{e} = 0$, and bounded tracking error for bounded external forces [11].

Beside the main task, i.e. the tracking of a trajectory in task space, the redundant system usually performs also some other subtasks by exploiting the redundancy. Premultiplying (19) with $(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})$ and rewriting it yields

$$(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\ddot{\boldsymbol{q}} = (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\boldsymbol{\phi} + (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\mathbf{H}^{-1}\boldsymbol{\tau}_{\text{ext}} \tag{24}$$

since $(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})$ is idempotent. With a careful selection of $\boldsymbol{\phi}$ the desired behavior of the system in the null space of $\mathbf{J}$ can be achieved [5, 3, 4, 8, 11]. As in our case the goal of the controller is task space position tracking we assume that $\boldsymbol{\phi}$ is selected only to stabilize the null-space motion. We can use

$$\boldsymbol{\phi} = -\mathbf{K}_n(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\dot{\boldsymbol{q}} \tag{25}$$

which is a basic method to stabilize the null space motion [5].

Until now have not posed any limitations concerning external forces except that they are bounded. However, from practical point of view these forces are impact forces caused by the contact between the body of the manipulator and objects in the neighborhood of the manipulator. Suppose that the end-effector has to track a given trajectory, the null space motion is used to optimize some criteria, and that during the task execution a collision with some obstacles occurs. In such situation it is desirable to move the manipulator as fast as possible into a configuration where the body is not in the contact with obstacles regardless of how the execution of the subtask would be influenced. To get a fast reaction in the null space to the external force the inertia matrix should be small. Therefore, we propose the following control law.

$$\begin{aligned} \boldsymbol{\tau} \; = \; & (\mathbf{H} - \mathbf{H}_d)\ddot{\boldsymbol{q}} + \mathbf{H}_d\mathbf{J}^{\#}(\ddot{\boldsymbol{x}}_d + \mathbf{K}_v\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} - \dot{\mathbf{J}}\dot{\boldsymbol{q}}) \\ & - \mathbf{H}_d(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\mathbf{K}_n\dot{\boldsymbol{q}} + \mathbf{h} + \mathbf{g} \end{aligned} \tag{26}$$

where $\mathbf{H}_d$ is the desired inertia matrix. Using (26) in (11) the following task error is obtained

$$\ddot{e}+\mathbf{K}_v\dot{e}+\mathbf{K}_p e = -\mathbf{J}\mathbf{H}_d^{-1}\mathbf{J}_F^T \boldsymbol{F}_0 \tag{27}$$

One possibility to choose is that $\mathbf{H}_d$ represents the inertia matrix of a manipulator with reduced mass of links and enlarged mass of the load. If $\mathbf{H}_d$ is selected as we recommend, then the collision forces will be lower due to faster reaction of the manipulator. The drawback of the proposed control law is that joint accelerations have to be available. Another possibility to make the system compliant to external force is to decrease the null space damping gain $\mathbf{K}_n$ without changing the $\mathbf{H}$. The side effect of this approach is the loss of null space velocity tracking quality.

A Selection of generalized inverse

Analyzing the right side of (21), $\mathbf{J}\mathbf{H}^{-1}\mathbf{J}_F^T\boldsymbol{F}_0$, we can see that the task space acceleration due to the external force acting anywhere at the body of the manipulator is not *directly* dependent on the selection of the generalized inverse of $\mathbf{J}$. Similarly, from (23) we can see that also the steady-state task position error does not depend on $\mathbf{J}^{\#}$. This dependency is introduced by the control law (16). Namely, the null-space motion is directly dependent on the selection of the generalized inverse. As different null-space motions result in different configurations and because the term $\mathbf{J}\mathbf{H}^{-1}\mathbf{J}_F^T$ depends on the configuration of the manipulator, the task space motion is indirectly dependent on the selection of $\mathbf{J}^{\#}$. In summary, the selection of the generalized inverse is a key factor in the design of control systems of redundant robots.

Although there exist many generalized inverses $\mathbf{J}^{\#}$, for robotics systems $\mathbf{J}^{\#}$ can not be selected without imposing some restrictions on it. In the following the behavior of the system will be analyzed and compared for two generalized inverses.

Most authors use the Moore-Penrose pseudoinverse [7] which is defined for $n > m$ as

$$\mathbf{J}^{+} = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1} \tag{28}$$

or its "weighted" counterpart [3] defined as

$$\mathbf{J}_w^{+} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1} \tag{29}$$

where $\mathbf{W}$ is $n \times n$ weighting matrix. A special form of $\mathbf{J}_w^{+}$ is when $\mathbf{W} = \mathbf{H}$. Khatib [6] has proved that

$$\bar{\mathbf{J}} = \mathbf{H}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)^{-1} = \mathbf{H}^{-1}\mathbf{J}^T\mathbf{M} \tag{30}$$

is the only pseudoinverse which is dynamically consistent, i.e. the task space acceleration $\ddot{\boldsymbol{x}}$ is not affected by any arbitrary torques $\boldsymbol{\tau}_n$ applied through the associated null space, $(\mathbf{I}-\mathbf{J}^T\mathbf{J}^{\#T})\boldsymbol{\tau}_n$. Additionally, the dynamically consistent generalized inverse $\bar{\mathbf{J}}$ is the only generalized inverse which assures that an external force does not produce a null space acceleration [1].

If we use $\mathbf{J}^{\#} = \bar{\mathbf{J}}$ in (16), then (23) can be rewritten into the form

$$\begin{aligned}
\ddot{e}+\mathbf{K}_v\dot{e}+\mathbf{K}_p e &= -\mathbf{M}^{-1}\mathbf{M}\mathbf{J}\mathbf{H}^{-1}\mathbf{J}_F^T\boldsymbol{F}_0 \\
&= -\mathbf{M}^{-1}\bar{\mathbf{J}}^T\boldsymbol{\tau}_{\mathrm{ext}} \\
&= -\mathbf{M}^{-1}\boldsymbol{F}_{\mathrm{eq}}
\end{aligned}$$

Table 1: Parameters of the 4-R planar manipulator

Parameter	Value
link length	[0.25, 0.25, 0.25, 0.275]
link center of mass	[0.233, 0.230, 0.205, 0.151]
link mass m	[0.83,0.44,0.18,0.045]
load mass m_l	0.5
link inertia	$[14.6,8.5,7.5,2.9]\cdot 10^{-4}$
motor inertia	$[93, 72, 23, 12]\cdot 10^{-4}$
viscous friction coef.	[0.08, 0.06, 0.05, 0.03]

and finally we obtain

$$\mathbf{M}(\ddot{e}+\mathbf{K}_v\dot{e}+\mathbf{K}_p e) = -\boldsymbol{F}_{\mathrm{eq}} \tag{31}$$

In the same way (24) can be rewritten into the form

$$(\mathbf{I}-\bar{\mathbf{J}}\mathbf{J})\ddot{q} = (\mathbf{I}-\bar{\mathbf{J}}\mathbf{J})\boldsymbol{\phi} + (\mathbf{I}-\bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}\boldsymbol{\tau}_{\mathrm{eq}} \tag{32}$$

As we can see, the system is decoupled; it behaves in the task space as $\boldsymbol{F}_{\mathrm{eq}}$ would act at the end-effector and in the null-space as $\boldsymbol{\tau}_{\mathrm{eq}}$ would be present.

VI. EXAMPLES

To illustrate the behavior of the manipulator with the proposed control algorithms a 4-link planar manipulator with revolute joints is used (4-R). The manipulator is supposed to move in (x,y) plane ($\boldsymbol{x} = [x,y]^T$). The parameters of the manipulator are given in Table 1. The simulation is done in MATLAB [10]. We assume without loss of generality in all examples that the external forces can act only at the end of links, i.e. at any joint.

To show the difference in behavior when $\mathbf{J}^{+}$ or $\bar{\mathbf{J}}$ are used, we assume for a moment that $\boldsymbol{F}_0$ is constant. The controller (16) assures stable motion of the manipulator, and $\boldsymbol{\phi}$ is used only to stabilize the null-space motion, i.e. (25) is used. Then, the steady-state configuration of the manipulator is obtained by using (32). Namely, in the steady-state ($\ddot{q} = 0$, $\dot{q} = 0$, and $\boldsymbol{\phi} = 0$) the last term in (32) must equal zero

$$(\mathbf{I}-\mathbf{J}^{\#}\mathbf{J})\mathbf{H}^{-1}\boldsymbol{\tau}_{\mathrm{ext}} = 0 \tag{33}$$

There are two possibilities: $\boldsymbol{\tau}_{\mathrm{ext}} = 0$ or $\boldsymbol{\tau}_{\mathrm{ext}} \neq 0$. The first case occurs if the when the part of the manipulator between the base and the point A_f is in singular configuration (see Fig. 2. The configuration of this part of the manipulator is independent of the selection of the pseudoinverse. If the rest of the manipulator is still redundant, then the configuration of that part of the manipulator is arbitrary (see Fig. 4). Since $\mathbf{J}_F^T\boldsymbol{F}_0 = 0$ the task error $\boldsymbol{e}$ in steady-state equals zero.

In the second case the steady-state configuration depends on the selection of the pseudoinverse and consequently, also the steady-state task position error. Using the same reasoning as before we would expect that the part of the manipulator between the point A_f and the end-effector (E) would be

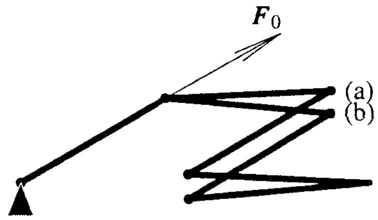

Figure 4: Constant force acting at the second joint of the 4-R planar manipulator; steady-state; (a) $\mathbf{J}^{\#} = \mathbf{J}^{+}$ (b) $\mathbf{J}^{\#} = \bar{\mathbf{J}}$

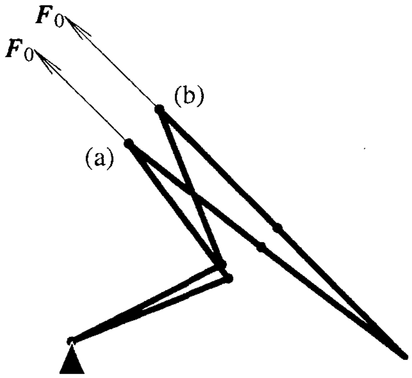

Figure 5: Constant force acting at the third joint of the 4-R planar manipulator; steady-state; (a) $\mathbf{J}^{\#} = \mathbf{J}^{+}$ (b) $\mathbf{J}^{\#} = \bar{\mathbf{J}}$

in singular configuration, i.e. $\check{\mathbf{J}}_{\mathrm{F}}^{T}\boldsymbol{F}_0 = 0$ and $\boldsymbol{F}_0 = \boldsymbol{F}_{\mathrm{eq}}$, as it is shown in Fig. 3. Unfortunately, the situation in general this is not so. Namely, expanding (33) by using $\boldsymbol{F}_{\mathrm{eq}}$ and $\boldsymbol{\tau}_{\mathrm{eq}}$ results in

$$(\mathbf{I}-\mathbf{J}^{\#}\mathbf{J})\mathbf{H}^{-1}\mathbf{J}^{T}\boldsymbol{F}_{\mathrm{eq}} + (\mathbf{I}-\mathbf{J}^{\#}\mathbf{J})\mathbf{H}^{-1}\boldsymbol{\tau}_{\mathrm{eq}} = 0 \tag{34}$$

and although $\check{\mathbf{J}}_{\mathrm{F}}^{T}\boldsymbol{F}_0 = 0$ implies that the null-space torques are zero, $\boldsymbol{\tau}_{\mathrm{eq}} = 0$, the first term is zero only if $\mathbf{J}^{\#} = \bar{\mathbf{J}}$. In Fig. 5 two examples are presented: one with $\mathbf{J}^{\#} = \mathbf{J}^{+}$ and the other with $\mathbf{J}^{\#} = \bar{\mathbf{J}}$.

In the last example the dynamic behavior of the manipulator is presented when an external force impuse is acting on the manipulator. The controller is supposed to hold a position in the task space. The initial configuration of the manipulator is $\boldsymbol{q} = [0, \pi/2, 0, pi/2]^T$ and a force $F_0 = [\cos(3\pi/4), \sin(3\pi/4)]^T$ is acting for 0.5s at joint 3. The control algorithm is (26) with gains $\mathbf{K}_p = 8000$ and $\mathbf{K}_v = 160$. We compare three situations regarding the selection of desired inertial properties $\mathbf{H}_d$ (by selecting the link masses and the load mass) and the null space damping gain $\mathbf{K}_n$. The first case (a) is used as the reference for comparison. In the next case (b) the damping gain $\mathbf{K}_n$ is decreased to achieve better compliance in null space. In the last case (c) desired masses of the links are decreased and load mass is increased for the same reason. In cases (b) and (c) the controller parameters are tuned to reach a similar end configuration after the force impulse. The particular null space controller parameters are:

Case	m_d	$m_{l,d}$	$\mathbf{K}_n$
(a)	m	m_l	25
(b)	m	m_l	5
(c)	$m/5$	$3m_l$	25

Fig. 6 shows time response for task space error $\mathbf{e}$, null-space velocity $(\mathbf{I}-\bar{\mathbf{J}}\mathbf{J})\dot{\boldsymbol{q}}$, and initial and final configuration of the manipulator. As we can see there is almost no null space motion in the case (a) and the manipulator reach the same configuration in the case (b) and (c). The obvious difference between (b) and (c) can be seen in the velocity time response. On one side (c) exhibits fast reaction (high accelerations) and on the other side (b) reacts slowly and the settling time is much larger. The tracking error is almost equal for all cases because the task controller parameters are equal for all cases and the manipulator configuration is not changed significantly. The little higher error in case (c) is because the change of the inertia matrix. The changed values of m and m_l have reduced the effecive inertia $\mathbf{M}$ (see (27)). .

VII. CONCLUSION

In the paper we consider the influence of external forces on the behavior of the manipulator. The system under study is a redundant manipulator with n degrees of freedom. The external forces, which can act anywhere on the body of the manipulator, are analyzed. We define the equivalent generalized forces in the task space and in the null space and enlighten some if their properties. The aim of the control we propose is to track a given task space trajectory in the presence of external forces with minimal tracking error. For that purpose we design a compliant motion controller which ensures stiff behavior in the task space and is compliant to external forces in the null space. The external forces are considered as disturbances which are not measurable. Two possible guide lines for the selection of the controller parameters are given: one is a reduction of the effective link masses and the other is a low null space damping. The first one assures better performances but the control algorithm includes the accelerations which may cause some problems in the application of the control algorithm on real systems. The main drawback of the second choice is bad velocity tracking in the null space also when external forces are zero. A special attention is given to the selection of the pseudoinverse in the control algorithm. We recommend the use of the inertia weighted pseudoinverse. Namely, only this pseudoinverse ensures no interaction between the null space and the task space motion.

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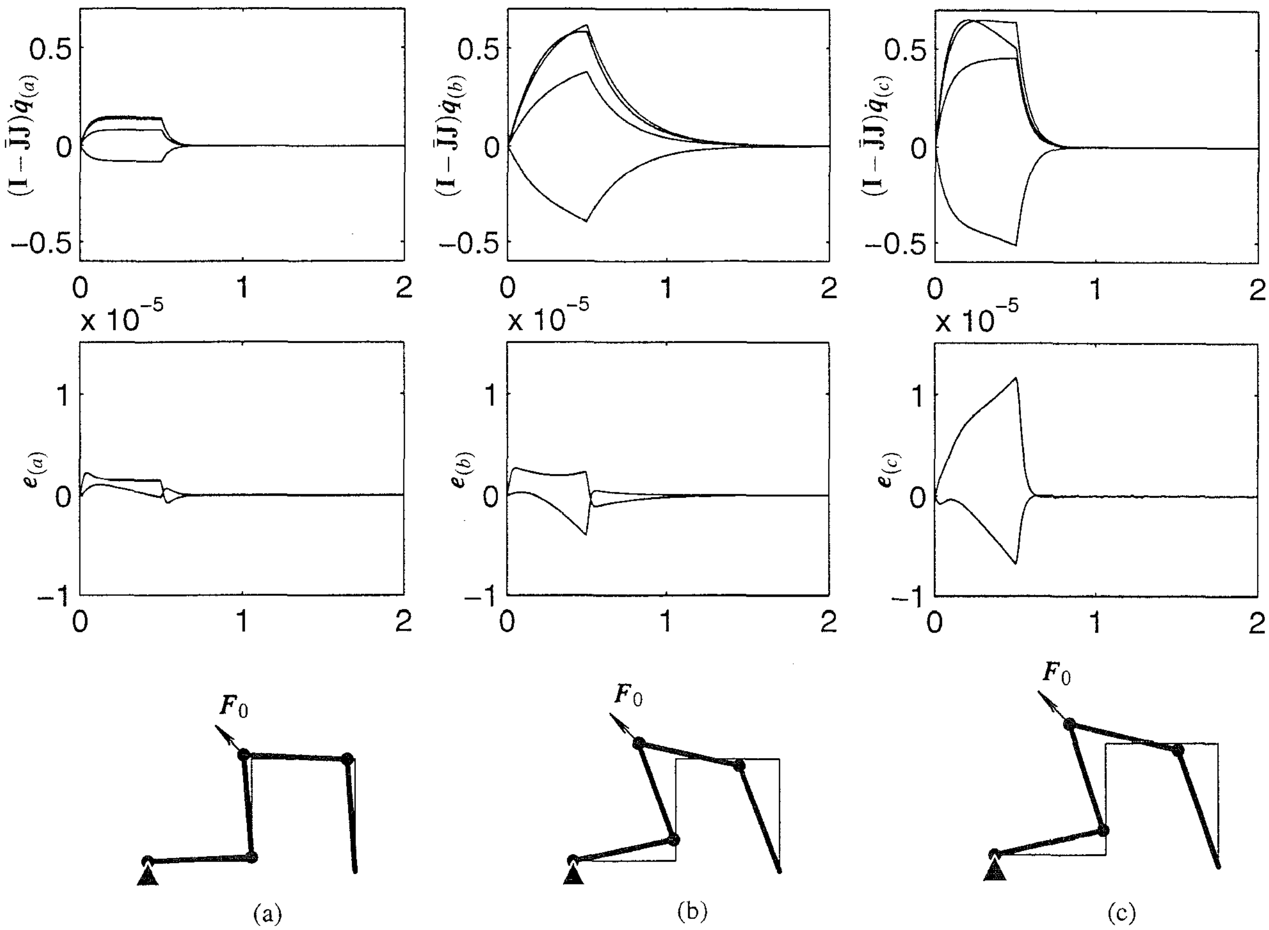

Figure 6: Force impulse acting at the third joint of the 4-R planar manipulator

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