Pathfinder

Manual

Run a simulation

Basic instructions to run a simulation with `xarray` from the main folder:

1. open python in a terminal, import numpy and xarray:

```
import numpy as np
import xarray as xr
```

2. import the model from one of the `mod_` files in `core_fct`, for instance the emission-driven mode:

```
from core_fct.mod_Edriven import PF_Edriven as PF
```

3. create or load parameters (`Par`), for instance the saved posterior parameters:

```
Par = xr.open_dataset('internal_data/pyMC_calib/Par_v1.nc').load()
```

4. create or load an initial state (`Ini`), for instance the end of the posterior historical simulation averaged over 11 years: (*Note: this is optional, without a specific initial state, the model will assume it starts from the end of the preindustrial era in 1750.*)

```
Ini = xr.open_dataset('internal_data/pyMC_calib/Var_v1.nc')
yr_ini = int(Ini.year.isel(year=slice(-11, None)).mean('year'))
Ini = Ini.isel(year=slice(-11, None)).mean('year')
```

5. create or load some forcings (`For`), for instance one experiment that keeps forcings constant and another that relaxes them:

```
For = xr.merge([Ini[var] for var in ['Eco2', 'ERFx']])
years = xr.DataArray(yr_ini + np.arange(100), coords={'year': yr_ini + np.arange(100)
For = 0*years + xr.DataArray([1., 0.], coords={'exp': ['const', 'relax']}, dims=['exp
```

6. run the model by calling the `run_xarray` method (turn on `get_Var2` to save more output variables):

```
Out = PF.run_xarray(Par, For, Ini, get_Var2=True)
```

7. display results with whatever package you prefer, for instance matplotlib:

```
import matplotlib.pyplot as plt
plt.figure()
for n, var in enumerate(['T', 'CO2']):
    plt.subplot(1, 2, n+1)
    for exp in Out.exp.values:
        plt.plot(Out.year, Out[var].sel(exp=exp).mean('config'), label=exp)
    plt.legend(loc=0)
plt.show()
```

Check the `run_scripts/run_diagnostics.py` file for more complex examples!

Code structure

The main folder contains the following subfolders:

Folder	Content
`core_fct`	core classes (`cls_`), functions (`fct_`) and objects (`mod_`) that constitute Pathfinder
`input_data`	input data used either for calibration or to run Pathfinder
`internal_data`	internally generated data, notably during calibration
`results`	output data, normally empty in the open-source version
`run_scripts`	scripts used to execute calibration and diagnostic (and possibly more)

The `core_fct` subfolder contains the following files:

File	Content
`cls_calib`	`pymc3` subclasses for use during calibration
`cls_model`	specific class used to wrap models and solve them
`fct_ancillary`	compilation of ancillary functions
`fct_calib`	main functions used for the Bayesian calibration
`fct_default`	functions setting the default (i.e. prior) values of parameters, forcings and constraints

File	Content
`fct_load`	functions loading and formatting `input_data`
`fct_param`	functions doing the OLS calibration of the prior parameters
`fct_traj`	functions to generate future trajectories of `T` and `CO2`
`mod_Cdriven_bgc`	Pathfinder model in concentration-driven mode with only the biogeochemical effect of CO2
`mod_Cdriven_rad`	Pathfinder model in concentration-driven mode with only the radiative effect of CO2
`mod_Cdriven`	Pathfinder model in concentration-driven mode
`mod_Edriven`	Pathfinder model in emission-driven mode
`mod_OBSdriven`	Pathfinder model in observation-driven mode (used for calibration)
`mod_Tdriven`	Pathfinder model in temperature-driven mode

The `input_data` subfolder contains the following data folders: (*Note that data was kept as close to the original format as possible. Sources are detailed in the core functions that call the files.*)

Folder	Content
`AR5`	Data from the IPCC 5th report
`AR6`	Data from the IPCC 6th report
`CMIP5`	Outputs of the CMIP5 models compiled for Pathfinder
`CMIP6`	Outputs of the CMIP6 models compiled for Pathfinder
`Edwards_2021`	Data from Edwards et al. (2021) for the SLR module
`GCB`	Data from the Global Carbon Budget
`obs_CO2`	Observations of atmospheric CO2
`obs_T`	Observations of global mean surface temperature
`RCPs`	RCP scenarios
`SSPs`	SSP scenarios
`TRENDYv7`	Outputs of the TRENDYv7 models compiled for Pathfinder

The `internal_data` subfolder contains the following folders:

Folder	Content
`prior_calib`	Results of the OLS calibration for the prior parameters (for faster loading, can be re-calculated on the fly)
`pyMC_calib`	Results of the Bayesian calibration, including historical simulations

The `run_scripts` subfolder contains the following files:

File	Content
`get_best_guess`	script to extract best-guess configuration after calibration
`plot_calib_check`	script to display results of the Bayesian calibration
`run_calib_and_hist`	script to run the Bayesian calibration
`run_diagnostics`	script to run idealized and scenario experiments for model evaluation

Notations

Forcings and Variables

In manual	In code	Description	Units	Prog?	Dims
R_c	`RFco2`	CO2 (effective) radiative forcing	W m^{-2}		
R_x	`ERFx`	Non-CO2 effective radiative forcing	W m^{-2}		
R	`ERF`	Effective radiative forcing	W m^{-2}		
T	`T`	Global surface temperature anomaly	K	*yes*	
T_d	`Td`	Deep ocean temperature anomaly	K	*yes*	
$\mathrm{logit(ff)}$	`logit_ff`	Logit of the climate feedback factor (for calib.)	1		

In manual	In code	Description	Units	Prog?	Dims
U_{ohc}	`OHC`	Ocean heat content (anomaly)	W yr m^{-2}		
H_{thx}	`Hthx`	Thermosteric sea level rise	mm		
H_{gla}	`Hgla`	Glaciers' contribution to sea level rise	mm	*yes*	
H_{gis}	`Hgis`	Grenland ice sheet's contribution to sea level rise	mm	*yes*	
$H_{\mathrm{ais,smb}}$	`Hais_smb`	Surface mass balance component of `Hais`	mm		
H_{ais}	`Hais`	Antartica ice sheet's contribution to sea level rise	mm	*yes*	
H_{tot}	`Htot`	Total sea level rise	mm		
H_{lia}	`Hlia`	Sea level rise from relaxation after LIA between 1900 and 2005 (for calib.)	mm		
$C_{o,j}$	`Co_j`	Change in surface ocean carbon subpools	PgC	*yes*	$j \in [\![1, 5]\!]$
C_o	`Co`	Change in surface ocean carbon pool	PgC		
C_d	`Cd`	Change in deep ocean carbon pool	*yes*		
c_{dic}	`dic`	Change in surface DIC	μmol kg^{-1}		
p_{dic}	`pdic`	Subcomponent of `pCO2`	ppm		
p_{CO2}	`pCO2`	CO2 partial pressure at the ocean surface	ppm		

In manual	In code	Description	Units	Prog?	Dims
F_{ocean}	`Focean`	Ocean carbon sink	PgC yr^{-1}		
r_{npp}	`r_npp`	Relative change in NPP	1		
r_{fire}	`r_fire`	Relative change in wildfire intensity	1		
r_{rh}	`r_rh`	Relative change in heterotrophic respiration rate	1		
F_{npp}	`NPP`	Net primary productivity	PgC yr^{-1}		
E_{fire}	`Efire`	Emissions from wildfire	PgC yr^{-1}		
E_{harv}	`Eharv`	Emissions from harvest and grazing	PgC yr^{-1}		
F_{mort}	`Fmort`	Mortality flux	PgC yr^{-1}		
E_{rh1}	`RH1`	Litter heterotrophic respiration	PgC yr^{-1}		
F_{stab}	`Fstab`	Stabilization flux	PgC yr^{-1}		
E_{rh2}	`RH2`	Active soil heterotrophic respiration	PgC yr^{-1}		
F_{pass}	`Fpass`	Passivization flux	PgC yr^{-1}		
E_{rh3}	`RH3`	Passive soil heterotrophic respiration	PgC yr^{-1}		
F_{land}	`Fland`	Land carbon sink	PgC yr^{-1}		

In manual	In code	Description	Units	Prog?	Dims
E_{rh}	`RH`	Heterotrophic respiration	PgC yr^{-1}		
C_v	`Cv`	Vegetation carbon pool	PgC	*yes*	
C_{s1}	`Cs1`	Litter carbon pool	PgC	*yes*	
C_{s2}	`Cs2`	Active soil carbon pool	PgC	*yes*	
C_{s3}	`Cs3`	Passive soil carbon pool	PgC	*yes*	
C_s	`Cs`	Total soil carbon pool	PgC		
r_{rt}	`r_rt`	Relative change in permafrost respiration rate	1		
$\bar{a}$	`abar`	Theoretical thawed fraction	1		
a	`a`	Actual thawed fraction	1	*yes*	
E_{pf}	`Epf`	Emissions from permafrost	PgC yr^{-1}		
$C_{\mathrm{th},j}$	`Cth_j`	Thawed permafrost carbon subpools	PgC	*yes*	$j \in [\![1, 3]\!]$
C_{fr}	`Cfr`	Frozen permafrost carbon pool	PgC		
E_{CO2}	`Eco2`	Anthropogenic CO2 emissions	PgC yr^{-1}		
C	`CO2`	Atmospheric CO2 concentration	ppm	*yes*	

In manual	In code	Description	Units	Prog?	Dims
pH	`pH`	Surface ocean pH	1		

Parameters

In manual	In code	Description	Units	Dims
ϕ	`phi`	Radiative parameter of CO2	W m^{-2}	
$T_{2\times}$	`T2x`	Equilibrium climate sensitivity	K	
Θ_s	`THs`	Heat capacity of the surface	W yr m^{-2} K^{-1}	
Θ_d	`THd`	Heat capacity of the deep ocean	W yr m^{-2} K^{-1}	
θ	`th`	Heat exchange coefficient	W m^{-2} K^{-1}	
ϵ_{heat}	`eheat`	Deep ocean heat uptake efficacy	1	
$T_{2\times}^{*}$	`T2x0`	Minimal value of the ECS distribution (for calib.)	K	
α_{ohc}	`aOHC`	Fraction of energy warming the ocean	1	
Λ_{thx}	`Lthx`	Proportionality factor of thermosteric SLR	mm m^{2} W^{-1} yr^{-1}	
λ_{gla}	`lgla0`	Initial imbalance in SLR from Glaciers	mm yr^{-1}	
Λ_{gla}	`Lgla`	Maximum contribution to SLR from Glaciers	mm	

In manual	In code	Description	Units	Dims
Γ_{gla1}	`Ggla1`	Linear sensitivity of steady-state Glaciers SLR to climate	K^{-1}	
Γ_{gla3}	`Ggla3`	Cubic sensitivity of steady-state Glaciers SLR to climate	K^{-3}	
τ_{gla}	`tgla`	Timescale of Glaciers' contribution to SLR	yr	
γ_{gla}	`ggla`	Sensitivity of Glaciers' timescale to climate	K^{-1}	
λ_{gis}	`lgis0`	Initial imbalance in SLR from GIS	mm yr^{-1}	
Λ_{gis1}	`Lgis1`	Linear sensitivity of steady-state GIS SLR to climate	mm K^{-1}	
Λ_{gis3}	`Lgis3`	Cubic sensitivity of steady-state GIS SLR to climate	mm K^{-3}	
τ_{gis}	`tgis`	Timescale of GIS contribution to SLR	yr	
$\Lambda_{ais,smb}$	`Lais_smb`	Sensitivity of AIS SMB increase due to climate	mm yr^{-1} K^{-1}	
λ_{ais}	`lais`	Initial imbalance in SLR from AIS	mm yr^{-1}	
Λ_{ais}	`Lais`	Sensitivity of steady-state AIS SLR to climate	mm K^{-1}	
τ_{ais}	`tais`	Timescale of AIS contribution to SLR	yr	
α_{ais}	`aais`	Sensitivity of AIS timescale to AIS SLR	mm^{-1}	

In manual	In code	Description	Units	Dims
α_{dic}	`adic`	Conversion factor for DIC	µmol kg^{-1} PgC^{-1}	
β_{dic}	`bdic`	Inverse-scaling factor for DIC	1	
γ_{dic}	`gdic`	Sensitivity of pCO2 to climate	K^{-1}	
T_o	`To`	Preindustrial surface ocean temperature	°C	
ν_{gx}	`vgx`	Surface ocean gas exchange rate	yr^{-1}	
γ_{gx}	`ggx`	Sensitivity of gas exchange to climate	K^{-1}	
$\alpha_{o,j}$	`aoc_j`	Surface ocean subpools fractions	1	$j \in [\![1,5]\!]$
$\tau_{o,j}$	`toc_j`	Timescales of surface ocean subpools	yr	$j \in [\![1,5]\!]$
κ_{τ_o}	`k_toc`	Scaling factor for timescales of surface ocean subpools	1	
β_{npp}	`bnpp`	Sensitivity of NPP to CO2 (= fertilization effect)	1	
α_{npp}	`anpp`	Shape parameter for fertilization effect	1	
γ_{npp}	`gnpp`	Sensitivity of NPP to climate	K^{-1}	
β_{fire}	`bfire`	Sensitivity of wildfire intensity to CO2	1	

In manual	In code	Description	Units	Dims
γ_{fire}	`gfire`	Sensitivity of wildfire intensity to climate	K^{-1}	
β_{rh}	`brh`	Sensitivity of heterotrophic respiration to fresh organic matter	1	
γ_{rh}	`grh`	Sensitivity of heterotrophic respiration to climate	K^{-1}	
$F_{\text{npp},0}$	`npp0`	Preindustrial NPP	PgC yr^{-1}	
ν_{fire}	`vfire`	Wildfire intensity	yr^{-1}	
ν_{harv}	`vharv`	Harvest and grazing rate	yr^{-1}	
ν_{mort}	`vmort`	Mortality rate	yr^{-1}	
ν_{stab}	`vstab`	Stabilization rate	yr^{-1}	
ν_{rh1}	`vrh1`	Litter heterotrophic respiration rate	yr^{-1}	
ν_{rh23}	`vrh23`	Soil (active and passive) respiration rate	yr^{-1}	
ν_{rh3}	`vrh3`	Passive soil respiration rate	yr^{-1}	
α_{pass}	`apass`	Fraction of passive soil	1	
α_{lst}	`aLST`	Climate scaling factor over permafrost regions	1	
γ_{rt1}	`grt1`	Sensitivity of (boreal) heterotrophic respiration to climate	K^{-1}	

In manual	In code	Description	Units	Dims
γ_{rt2}	`grt2`	Sensitivity of (boreal) heterotrophic respiration to climate (quadratic)	K^{-2}	
κ_{rt}	`krt`	Scaling factor for sensitivity of permafrost respiration to climate	1	
a_{min}	`amin`	Minimal thawed fraction	1	
κ_a	`ka`	Shape parameter for theoretical thawed fraction	1	
γ_a	`ga`	Sensitivity of theoretical thawed fraction to climate	K^{-1}	
ν_{thaw}	`vthaw`	Thawing rate	yr^{-1}	
ν_{froz}	`vfroz`	Freezing rate	yr^{-1}	
$\alpha_{\mathrm{th},j}$	`ath_j`	Thawed permafrost carbon subpools fractions	1	$j \in [\![1, 3]\!]$
$\tau_{\mathrm{th},j}$	`tth_j`	Timescales of thawed permafrost carbon subpools	yr	$j \in [\![1, 3]\!]$
$\kappa_{\tau_{\mathrm{th}}}$	`k_tth`	Scaling factor for timescales of surface ocean subpools	1	
$C_{\mathrm{fr},0}$	`Cfr0`	Preindustrial frozen permafrost carbon pool	PgC	
α_C	`aCO2`	Conversion factor for atmospheric CO2	PgC ppm^{-1}	
C_{pi}	`CO2pi`	Preindustrial CO2 concentration	ppm	
κ_{pH}	`k_pH`	Scaling factor for surface ocean pH	1	

In manual	In code	Description	Units	Dims
$\widetilde{\sigma}_C$	`std_CO2`	Relative standard deviation of the historical `CO2` time series (for calib.)	1	
ϵ_C	`ampl_CO2`	Noise amplitude of the historical `CO2` time series (for calib.)	ppm	
ρ_C	`corr_CO2`	Autocorrelation of the historical `CO2` time series (for calib.)	1	
$\widetilde{\sigma}_T$	`std_T`	Relative standard deviation of the historical `T` time series (for calib.)	1	
ϵ_T	`ampl_T`	Noise amplitude of the historical `T` time series (for calib.)	K	
ρ_T	`corr_T`	Autocorrelation of the historical `T` time series (for calib.)	1	

Equations

1. Climate

diagnostic

- $$R_c = \phi \ \ln(\frac{C}{C_{\mathrm{pi}}})$$
- $$R = R_c + R_x$$

prognostic

- $$\Theta_s \ \frac{\mathrm{d}T}{\mathrm{d}t} = R - \frac{\phi \ \ln(2)}{T_{2\times}} \ T - \epsilon_{\mathrm{heat}} \ \theta \ (T - T_d)$$
- $$\Theta_d \ \frac{\mathrm{d}T_d}{\mathrm{d}t} = \theta \ (T - T_d)$$

diagnostic (2nd; for calib.)

- $$\mathrm{logit}(\mathrm{ff}) = \ln(\frac{T_{2\times}}{T^*_{2\times}} - 1)$$

2. Sea level

diagnostic

- $$U_{\text{ohc}} = \alpha_{\text{ohc}} \; (\Theta_s \; T + \Theta_d \; T_d)$$
- $$\frac{\mathrm{d}U_{\text{ohc}}}{\mathrm{d}t} = \alpha_{\text{ohc}} \; (\Theta_s \; \frac{\mathrm{d}T}{\mathrm{d}t} + \Theta_d \; \frac{\mathrm{d}T_d}{\mathrm{d}t})$$
- $$H_{\text{thx}} = \Lambda_{\text{thx}} \; U_{\text{ohc}}$$
- $$\frac{\mathrm{d}H_{\text{thx}}}{\mathrm{d}t} = \Lambda_{\text{thx}} \; \frac{\mathrm{d}U_{\text{ohc}}}{\mathrm{d}t}$$

prognostic

- $$\frac{\mathrm{d}H_{\text{gla}}}{\mathrm{d}t} = \lambda_{\text{gla}} + \frac{\exp(\gamma_{\text{gla}} \; T)}{\tau_{\text{gla}}} \; (\Lambda_{\text{gla}} \; (1 - \exp(-\Gamma_{\text{gla1}} \; T - \Gamma_{\text{gla3}} \; T^3)) - H_{\text{gla}})$$
- $$\frac{\mathrm{d}H_{\text{gis}}}{\mathrm{d}t} = \lambda_{\text{gis}} + \frac{1}{\tau_{\text{gis}}} \; (\Lambda_{\text{gis1}} \; T + \Lambda_{\text{gis3}} \; T^3 - H_{\text{gis}})$$
- $$\frac{\mathrm{d}H_{\text{ais,smb}}}{\mathrm{d}t} = -\Lambda_{\text{ais}_\text{s}\text{mb}} \; T$$
- $$\frac{\mathrm{d}H_{\text{ais}}}{\mathrm{d}t} = \frac{\mathrm{d}H_{\text{ais,smb}}}{\mathrm{d}t} + \lambda_{\text{ais}} + \frac{1 + \alpha_{\text{ais}} \; (H_{\text{ais}} - H_{\text{ais,smb}})}{\tau_{\text{ais}}} \; (\Lambda_{\text{ais}} \; T - (H_{\text{ais}} - H_{\text{ais,smb}}))$$

diagnostic (2nd)

- $$H_{\text{tot}} = H_{\text{thx}} + H_{\text{gla}} + H_{\text{gis}} + H_{\text{ais}}$$
- $$\frac{\mathrm{d}H_{\text{tot}}}{\mathrm{d}t} = \frac{\mathrm{d}H_{\text{thx}}}{\mathrm{d}t} + \frac{\mathrm{d}H_{\text{gla}}}{\mathrm{d}t} + \frac{\mathrm{d}H_{\text{gis}}}{\mathrm{d}t} + \frac{\mathrm{d}H_{\text{ais}}}{\mathrm{d}t}$$

diagnostic (3rd; for calib.)

- $$H_{\text{lia}} = \sum_{\text{ice} \in \{\text{gla,gis,ais}\}} \lambda_{\text{ice}} \; \tau_{\text{ice}} \; (\exp(-150/\tau_{\text{ice}}) - \exp(-205/\tau_{\text{ice}}))$$

3. Ocean carbon

diagnostic

- $$C_o = \sum_j C_{o,j}$$
- $$c_{\text{dic}} = \frac{\alpha_{\text{dic}}}{\beta_{\text{dic}}} \; C_o$$

- $$\begin{aligned} p_{\mathrm{dic}} = &(1.5568 - 0.013993\ T_o)\ c_{\mathrm{dic}} \\ &+ (7.4706 - 0.20207\ T_o)\ 10^{-3}\ {c_{\mathrm{dic}}}^2 \\ &- (1.2748 - 0.12015\ T_o)\ 10^{-5}\ {c_{\mathrm{dic}}}^3 \\ &+ (2.4491 - 0.12639\ T_o)\ 10^{-7}\ {c_{\mathrm{dic}}}^4 \\ &- (1.5768 - 0.15326\ T_o)\ 10^{-10}\ {c_{\mathrm{dic}}}^5 \end{aligned}$$
- $$p_{\mathrm{CO2}} = (p_{\mathrm{dic}} + C_{\mathrm{pi}})\ \exp(\gamma_{\mathrm{dic}}\ T)$$
- $$F_{\mathrm{ocean}} = v_{\mathrm{gx}}\ (1 + \gamma_{\mathrm{gx}}\ T)\ (C - p_{\mathrm{CO2}})$$

prognostic

- $$\frac{\mathrm{d}C_{o,j}}{\mathrm{d}t} = -\frac{C_{o,j}}{\kappa_{\tau_o}\ \tau_{o,j}} + \alpha_{o,j}\ F_{\mathrm{ocean}}$$
- $$\frac{\mathrm{d}C_d}{\mathrm{d}t} = \sum_j \frac{C_{o,j}}{\kappa_{\tau_o}\ \tau_{o,j}}$$

4. Land carbon

diagnostic

- $$r_{\mathrm{npp}} = (1 + \frac{\beta_{\mathrm{npp}}}{\alpha_{\mathrm{npp}}}\ (1 - (\frac{C}{C_{\mathrm{pi}}})^{-\alpha_{\mathrm{npp}}}))\ (1 + \gamma_{\mathrm{npp}}\ T)$$
- $$r_{\mathrm{fire}} = (1 + \beta_{\mathrm{fire}}\ (\frac{C}{C_{\mathrm{pi}}} - 1))\ (1 + \gamma_{\mathrm{fire}}\ T)$$
- $$r_{\mathrm{rh}} = (1 + \beta_{\mathrm{rh}}\ (\frac{C_{s1}}{C_{s1} + C_{s2} + C_{s3}}\ (1 + \frac{v_{stab}}{v_{rh23}}) - 1))\ \exp(\gamma_{\mathrm{rh}}\ T)$$
- $$F_{\mathrm{npp}} = F_{\mathrm{npp,0}}\ r_{\mathrm{npp}}$$
- $$E_{\mathrm{fire}} = v_{\mathrm{fire}}\ r_{\mathrm{fire}}\ C_v$$
- $$E_{\mathrm{harv}} = v_{\mathrm{harv}}\ C_v$$
- $$F_{\mathrm{mort}} = v_{\mathrm{mort}}\ C_v$$
- $$E_{\mathrm{rh1}} = v_{\mathrm{rh1}}\ r_{\mathrm{rh}}\ C_{s1}$$
- $$F_{\mathrm{stab}} = v_{\mathrm{stab}}\ r_{\mathrm{rh}}\ C_{s1}$$
- $$E_{\mathrm{rh2}} = \frac{v_{\mathrm{rh23}} - v_{\mathrm{rh3}}\ \alpha_{\mathrm{pass}}}{1 - \alpha_{\mathrm{pass}}}\ r_{\mathrm{rh}}\ C_{s2}$$
- $$F_{\mathrm{pass}} = v_{\mathrm{rh3}}\ \frac{\alpha_{\mathrm{pass}}}{1 - \alpha_{\mathrm{pass}}}\ r_{\mathrm{rh}}\ C_{s2}$$

- $$E_{\text{rh3}} = v_{\text{rh3}}\ r_{\text{rh}}\ C_{s3}$$
- $$F_{\text{land}} = F_{\text{npp}} - E_{\text{fire}} - E_{\text{harv}} - E_{\text{rh1}} - E_{\text{rh2}} - E_{\text{rh3}}$$

prognostic

- $$\frac{\mathrm{d}C_v}{\mathrm{d}t} = F_{\text{npp}} - E_{\text{fire}} - E_{\text{harv}} - F_{\text{mort}}$$
- $$\frac{\mathrm{d}C_{s1}}{\mathrm{d}t} = F_{\text{mort}} - F_{\text{stab}} - E_{\text{rh1}}$$
- $$\frac{\mathrm{d}C_{s2}}{\mathrm{d}t} = F_{\text{stab}} - F_{\text{pass}} - E_{\text{rh2}}$$
- $$\frac{\mathrm{d}C_{s3}}{\mathrm{d}t} = F_{\text{pass}} - E_{\text{rh3}}$$

diagnostic (2nd)

- $$E_{\text{rh}} = E_{\text{rh1}} + E_{\text{rh2}} + E_{\text{rh3}}$$
- $$C_s = C_{s1} + C_{s2} + C_{s3}$$

5. Permafrost carbon

diagnostic

- $$r_{\text{rt}} = \exp(\kappa_{\text{rt}}\ \gamma_{\text{rt1}}\ \alpha_{\text{lst}}\ T - \kappa_{\text{rt}}\ \gamma_{\text{rt2}}\ (\alpha_{\text{lst}}\ T)^2)$$
- $$\bar{a} = -a_{\text{min}} + \frac{(1 + a_{\text{min}})}{\left(1 + \left(\left(1 + \frac{1}{a_{\text{min}}}\right)^{\kappa_a} - 1\right) \exp(-\gamma_a\ \kappa_a\ \alpha_{\text{lst}}\ T)\right)^{\frac{1}{\kappa_a}}}$$
- $$E_{\text{pf}} = \sum_j \frac{C_{\text{th},j}}{\kappa_{\tau_{\text{th}}}\ \tau_{\text{th},j}}\ r_{\text{rt}}$$

prognostic

- $$\frac{\mathrm{d}a}{\mathrm{d}t} = 0.5\ (v_{\text{thaw}} + v_{\text{froz}})\ (\bar{a} - a) + 0.5\ |(v_{\text{thaw}} - v_{\text{froz}})\ (\bar{a} - a)|$$
- $$\frac{\mathrm{d}C_{\text{th},j}}{\mathrm{d}t} = \alpha_{\text{th},j}\ \frac{\mathrm{d}a}{\mathrm{d}t}\ C_{\text{fr},0} - \frac{C_{\text{th},j}}{\kappa_{\tau_{\text{th}}}\ \tau_{\text{th},j}}\ r_{\text{rt}}$$

diagnostic (2nd)

- $$C_{\text{fr}} = (1 - a)\ C_{\text{fr},0}$$
- $$\frac{\mathrm{d}C_{\text{fr}}}{\mathrm{d}t} = -\frac{\mathrm{d}a}{\mathrm{d}t}\ C_{\text{fr},0}$$

6. Atmospheric CO2

diagnostic

- $$\mathrm{pH} = \kappa_{\mathrm{pH}} \, (8.5541 - 0.00173 \, C + 1.3264 \, 10^{-6} \, C^2 - 4.4943 \, 10^{-10} \, C^3)$$

prognostic

- $$\alpha_C \, \frac{\mathrm{d}C}{\mathrm{d}t} = E_{\mathrm{CO2}} + E_{\mathrm{pf}} - F_{\mathrm{land}} - F_{\mathrm{ocean}}$$