Notations

Forcings and Variables

In manual	In code	Description	Units	Prog?	Dims
R_c	`RFco2`	CO2 (effective) radiative forcing	W m^{-2}		
R_x	`ERFx`	Non-CO2 effective radiative forcing	W m^{-2}		
R	`ERF`	Effective radiative forcing	W m^{-2}		
T	`T`	Global surface temperature anomaly	K	*yes*	
T_d	`Td`	Deep ocean temperature anomaly	K	*yes*	
$\mathrm{logit(ff)}$	`logit_ff`	Logit of the climate feedback factor (for calib.)	1		
U_{ohc}	`OHC`	Ocean heat content (anomaly)	W yr m^{-2}		
H_{thx}	`Hthx`	Thermosteric sea level rise	mm		
H_{gla}	`Hgla`	Glaciers' contribution to sea level rise	mm	*yes*	
H_{gis}	`Hgis`	Grenland ice sheet's contribution to sea level rise	mm	*yes*	
$H_{\mathrm{ais,smb}}$	`Hais_smb`	Surface mass balance component of `Hais`	mm		
H_{ais}	`Hais`	Antartica ice sheet's contribution to sea level rise	mm	*yes*	
H_{tot}	`Htot`	Total sea level rise	mm		
H_{lia}	`Hlia`	Sea level rise from relaxation after LIA between 1900 and 2005 (for calib.)	mm		
$C_{o,j}$	`Co_j`	Change in surface ocean carbon subpools	PgC	*yes*	$j \in [1, 5]$
C_o	`Co`	Change in surface ocean carbon pool	PgC		
C_d	`Cd`	Change in deep ocean carbon pool	*yes*		
c_{dic}	`dic`	Change in surface DIC	µmol kg^{-1}		
p_{dic}	`pdic`	Subcomponent of `pCO2`	ppm		
p_{CO2}	`pCO2`	CO2 partial pressure at the ocean surface	ppm		
F_{ocean}	`Focean`	Ocean carbon sink	PgC yr^{-1}		

In manual	In code	Description	Units	Prog?	Dims
r_{npp}	`r_npp`	Relative change in NPP	1		
r_{fire}	`r_fire`	Relative change in wildfire intensity	1		
r_{rh}	`r_rh`	Relative change in heterotrophic respiration rate	1		
F_{npp}	`NPP`	Net primary productivity	PgC yr^{-1}		
E_{fire}	`Efire`	Emissions from wildfire	PgC yr^{-1}		
E_{harv}	`Eharv`	Emissions from harvest and grazing	PgC yr^{-1}		
F_{mort}	`Fmort`	Mortality flux	PgC yr^{-1}		
E_{rh1}	`RH1`	Litter heterotrophic respiration	PgC yr^{-1}		
F_{stab}	`Fstab`	Stabilization flux	PgC yr^{-1}		
E_{rh2}	`RH2`	Active soil heterotrophic respiration	PgC yr^{-1}		
F_{pass}	`Fpass`	Passivization flux	PgC yr^{-1}		
E_{rh3}	`RH3`	Passive soil heterotrophic respiration	PgC yr^{-1}		
F_{land}	`Fland`	Land carbon sink	PgC yr^{-1}		
E_{rh}	`RH`	Heterotrophic respiration	PgC yr^{-1}		
C_v	`Cv`	Vegetation carbon pool	PgC	*yes*	
C_{s1}	`Cs1`	Litter carbon pool	PgC	*yes*	
C_{s2}	`Cs2`	Active soil carbon pool	PgC	*yes*	
C_{s3}	`Cs3`	Passive soil carbon pool	PgC	*yes*	
C_s	`Cs`	Total soil carbon pool	PgC		
r_{rt}	`r_rt`	Relative change in permafrost respiration rate	1		

In manual	In code	Description	Units	Prog?	Dims
$\bar{a}$	`abar`	Theoretical thawed fraction	1		
a	`a`	Actual thawed fraction	1	*yes*	
E_{pf}	`Epf`	Emissions from permafrost	PgC yr^{-1}		
$C_{\mathrm{th},j}$	`Cth_j`	Thawed permafrost carbon subpools	PgC	*yes*	$j \in [1,3]$
C_{fr}	`Cfr`	Frozen permafrost carbon pool	PgC		
E_{CO2}	`Eco2`	Anthropogenic CO2 emissions	PgC yr^{-1}		
C	`CO2`	Atmospheric CO2 concentration	ppm	*yes*	
pH	`pH`	Surface ocean pH	1		

Parameters

In manual	In code	Description	Units	Dims
ϕ	`phi`	Radiative parameter of CO2	W m^{-2}	
$T_{2\times}$	`T2x`	Equilibrium climate sensitivity	K	
Θ_s	`THs`	Heat capacity of the surface	W yr m^{-2} K^{-1}	
Θ_d	`THd`	Heat capacity of the deep ocean	W yr m^{-2} K^{-1}	
θ	`th`	Heat exchange coefficient	W m^{-2} K^{-1}	
ϵ_{heat}	`eheat`	Deep ocean heat uptake efficacy	1	
$T_{2\times}^{*}$	`T2x0`	Minimal value of the ECS distribution (for calib.)	K	
α_{ohc}	`aOHC`	Fraction of energy warming the ocean	1	
Λ_{thx}	`Lthx`	Proportionality factor of thermosteric SLR	mm m^{2} W^{-1} yr^{-1}	
λ_{gla}	`lgla0`	Initial imbalance in SLR from Glaciers	mm yr^{-1}	
Λ_{gla}	`Lgla`	Maximum contribution to SLR from Glaciers	mm	

In manual	In code	Description	Units	Dims
Γ_{gla1}	`Ggla1`	Linear sensitivity of steady-state Glaciers SLR to climate	K^{-1}	
Γ_{gla3}	`Ggla3`	Cubic sensitivity of steady-state Glaciers SLR to climate	K^{-3}	
τ_{gla}	`tgla`	Timescale of Glaciers' contribution to SLR	yr	
γ_{gla}	`ggla`	Sensitivity of Glaciers' timescale to climate	K^{-1}	
λ_{gis}	`lgis0`	Initial imbalance in SLR from GIS	mm yr^{-1}	
Λ_{gis1}	`Lgis1`	Linear sensitivity of steady-state GIS SLR to climate	mm K^{-1}	
Λ_{gis3}	`Lgis3`	Cubic sensitivity of steady-state GIS SLR to climate	mm K^{-3}	
τ_{gis}	`tgis`	Timescale of GIS contribution to SLR	yr	
$\Lambda_{\mathrm{ais,smb}}$	`Lais_smb`	Sensitivity of AIS SMB increase due to climate	mm yr^{-1} K^{-1}	
λ_{ais}	`lais`	Initial imbalance in SLR from AIS	mm yr^{-1}	
Λ_{ais}	`Lais`	Sensitivity of steady-state AIS SLR to climate	mm K^{-1}	
τ_{ais}	`tais`	Timescale of AIS contribution to SLR	yr	
α_{ais}	`aais`	Sensitivity of AIS timescale to AIS SLR	mm^{-1}	
α_{dic}	`adic`	Conversion factor for DIC	µmol kg^{-1} PgC^{-1}	
β_{dic}	`bdic`	Inverse-scaling factor for DIC	1	
γ_{dic}	`gdic`	Sensitivity of pCO2 to climate	K^{-1}	
T_o	`To`	Preindustrial surface ocean temperature	°C	
ν_{gx}	`vgx`	Surface ocean gas exchange rate	yr^{-1}	
γ_{gx}	`ggx`	Sensitivity of gas exchange to climate	K^{-1}	
$\alpha_{o,j}$	`aoc_j`	Surface ocean subpools fractions	1	$j \in [\![1, 5]\!]$
$\tau_{o,j}$	`toc_j`	Timescales of surface ocean subpools	yr	$j \in [\![1, 5]\!]$
κ_{τ_o}	`k_toc`	Scaling factor for timescales of surface ocean subpools	1	
β_{npp}	`bnpp`	Sensitivity of NPP to CO2 (= fertilization effect)	1	
α_{npp}	`anpp`	Shape parameter for fertilization effect	1	
γ_{npp}	`gnpp`	Sensitivity of NPP to climate	K^{-1}	

In manual	In code	Description	Units	Dims
β_{fire}	`bfire`	Sensitivity of wildfire intensity to CO2	1	
γ_{fire}	`gfire`	Sensitivity of wildfire intensity to climate	K^{-1}	
β_{rh}	`brh`	Sensitivity of heterotrophic respiration to fresh organic matter	1	
γ_{rh}	`grh`	Sensitivity of heterotrophic respiration to climate	K^{-1}	
$F_{\text{npp},0}$	`npp0`	Preindustrial NPP	PgC yr^{-1}	
ν_{fire}	`vfire`	Wildfire intensity	yr^{-1}	
ν_{harv}	`vharv`	Harvest and grazing rate	yr^{-1}	
ν_{mort}	`vmort`	Mortality rate	yr^{-1}	
ν_{stab}	`vstab`	Stabilization rate	yr^{-1}	
ν_{rh1}	`vrh1`	Litter heterotrophic respiration rate	yr^{-1}	
ν_{rh23}	`vrh23`	Soil (active and passive) respiration rate	yr^{-1}	
ν_{rh3}	`vrh3`	Passive soil respiration rate	yr^{-1}	
α_{pass}	`apass`	Fraction of passive soil	1	
α_{lst}	`aLST`	Climate scaling factor over permafrost regions	1	
γ_{rt1}	`grt1`	Sensitivity of (boreal) heterotrophic respiration to climate	K^{-1}	
γ_{rt2}	`grt2`	Sensitivity of (boreal) heterotrophic respiration to climate (quadratic)	K^{-2}	
κ_{rt}	`krt`	Scaling factor for sensitivity of permafrost respiration to climate	1	
a_{min}	`amin`	Minimal thawed fraction	1	
κ_a	`ka`	Shape parameter for theoretical thawed fraction	1	
γ_a	`ga`	Sensitivity of theoretical thawed fraction to climate	K^{-1}	
ν_{thaw}	`vthaw`	Thawing rate	yr^{-1}	
ν_{froz}	`vfroz`	Freezing rate	yr^{-1}	
$\alpha_{\text{th},j}$	`ath_j`	Thawed permafrost carbon subpools fractions	1	$j \in [\![1,3]\!]$
$\tau_{\text{th},j}$	`tth_j`	Timescales of thawed permafrost carbon subpools	yr	$j \in [\![1,3]\!]$

In manual	In code	Description	Units	Dims
$\kappa_{\tau_{\mathrm{th}}}$	`k_tth`	Scaling factor for timescales of surface ocean subpools	1	
$C_{\mathrm{fr},0}$	`Cfr0`	Preindustrial frozen permafrost carbon pool	PgC	
α_C	`aCO2`	Conversion factor for atmospheric CO2	PgC ppm^{-1}	
C_{pi}	`CO2pi`	Preindustrial CO2 concentration	ppm	
κ_{pH}	`k_pH`	Scaling factor for surface ocean pH	1	
$\tilde{\sigma}_C$	`std_CO2`	Relative standard deviation of the historical `CO2` time series (for calib.)	1	
ϵ_C	`ampl_CO2`	Noise amplitude of the historical `CO2` time series (for calib.)	ppm	
ρ_C	`corr_CO2`	Autocorrelation of the historical `CO2` time series (for calib.)	1	
$\tilde{\sigma}_T$	`std_T`	Relative standard deviation of the historical `T` time series (for calib.)	1	
ϵ_T	`ampl_T`	Noise amplitude of the historical `T` time series (for calib.)	K	
ρ_T	`corr_T`	Autocorrelation of the historical `T` time series (for calib.)	1	

Equations

1. Climate

diagnostic

- $R_c = \phi \ \ln\left(\frac{C}{C_{\mathrm{pi}}}\right)$
- $R = R_c + R_x$

prognostic

- $\Theta_s \ \frac{\mathrm{d}T}{\mathrm{d}t} = R - \frac{\phi \ \ln(2)}{T_{2\times}} \ T - \epsilon_{\mathrm{heat}} \ \theta \ (T - T_d)$
- $\Theta_d \ \frac{\mathrm{d}T_d}{\mathrm{d}t} = \theta \ (T - T_d)$

diagnostic (2nd; for calib.)

- $\mathrm{logit}(\mathrm{ff}) = \ln\left(\frac{T_{2\times}}{T_{2\times}^*} - 1\right)$

2. Sea level

diagnostic

- $U_\mathrm{ohc} = \alpha_\mathrm{ohc} \; (\Theta_s \, T + \Theta_d \, T_d)$
- $\frac{\mathrm{d}U_\mathrm{ohc}}{\mathrm{d}t} = \alpha_\mathrm{ohc} \; (\Theta_s \, \frac{\mathrm{d}T}{\mathrm{d}t} + \Theta_d \, \frac{\mathrm{d}T_d}{\mathrm{d}t})$
- $H_\mathrm{thx} = \Lambda_\mathrm{thx} \, U_\mathrm{ohc}$
- $\frac{\mathrm{d}H_\mathrm{thx}}{\mathrm{d}t} = \Lambda_\mathrm{thx} \; \frac{\mathrm{d}U_\mathrm{ohc}}{\mathrm{d}t}$

prognostic

- $\frac{\mathrm{d}H_\mathrm{gla}}{\mathrm{d}t} = \lambda_\mathrm{gla} + \frac{\exp(\gamma_\mathrm{gla} \, T)}{\tau_\mathrm{gla}} \; (\Lambda_\mathrm{gla} \; (1 - \exp(-\Gamma_\mathrm{gla1} \, T - \Gamma_\mathrm{gla3} \, T^3)) - H_\mathrm{gla})$
- $\frac{\mathrm{d}H_\mathrm{gis}}{\mathrm{d}t} = \lambda_\mathrm{gis} + \frac{1}{\tau_\mathrm{gis}} \; (\Lambda_\mathrm{gis1} \, T + \Lambda_\mathrm{gis3} \, T^3 - H_\mathrm{gis})$
- $\frac{\mathrm{d}H_\mathrm{ais,smb}}{\mathrm{d}t} = -\Lambda_{\mathrm{ais_smb}} \, T$
- $\frac{\mathrm{d}H_\mathrm{ais}}{\mathrm{d}t} = \frac{\mathrm{d}H_\mathrm{ais,smb}}{\mathrm{d}t} + \lambda_\mathrm{ais} + \frac{1 + \alpha_\mathrm{ais} \; (H_\mathrm{ais} - H_\mathrm{ais,smb})}{\tau_\mathrm{ais}} \; (\Lambda_\mathrm{ais} \; T - (H_\mathrm{ais} - H_\mathrm{ais,smb}))$

diagnostic (2nd)

- $H_\mathrm{tot} = H_\mathrm{thx} + H_\mathrm{gla} + H_\mathrm{gis} + H_\mathrm{ais}$
- $\frac{\mathrm{d}H_\mathrm{tot}}{\mathrm{d}t} = \frac{\mathrm{d}H_\mathrm{thx}}{\mathrm{d}t} + \frac{\mathrm{d}H_\mathrm{gla}}{\mathrm{d}t} + \frac{\mathrm{d}H_\mathrm{gis}}{\mathrm{d}t} + \frac{\mathrm{d}H_\mathrm{ais}}{\mathrm{d}t}$

diagnostic (3rd; for calib.)

- $H_\mathrm{lia} = \sum_{\mathrm{ice} \in \{\mathrm{gla,gis,ais}\}} \lambda_\mathrm{ice} \; \tau_\mathrm{ice} \, (\exp(-150 / \tau_\mathrm{ice}) - \exp(-205 / \tau_\mathrm{ice}))$

3. Ocean carbon

diagnostic

- $C_o = \sum_j C_{o,j}$

- $c_{\text{dic}} = \dfrac{\alpha_{\text{dic}}}{\beta_{\text{dic}}} \, C_o$
- $$\begin{aligned} p_{\text{dic}} = & \; (1.5568 - 0.013993 \, T_o) \, c_{\text{dic}} \\ & + (7.4706 - 0.20207 \, T_o) \, 10^{-3} \, {c_{\text{dic}}}^2 \\ & - (1.2748 - 0.12015 \, T_o) \, 10^{-5} \, {c_{\text{dic}}}^3 \\ & + (2.4491 - 0.12639 \, T_o) \, 10^{-7} \, {c_{\text{dic}}}^4 \\ & - (1.5768 - 0.15326 \, T_o) \, 10^{-10} \, {c_{\text{dic}}}^5 \end{aligned}$$
- $p_{\text{CO2}} = (p_{\text{dic}} + C_{\text{pi}}) \; \exp(\gamma_{\text{dic}} \, T)$
- $F_{\text{ocean}} = \nu_{\text{gx}} \, (1 + \gamma_{\text{gx}} \, T) \, (C - p_{\text{CO2}})$

prognostic

- $\dfrac{\mathrm{d}C_{o,j}}{\mathrm{d}t} = -\dfrac{C_{o,j}}{\kappa_{\tau_o} \, \tau_{o,j}} + \alpha_{o,j} \, F_{\text{ocean}}$
- $\dfrac{\mathrm{d}C_d}{\mathrm{d}t} = \sum_j \dfrac{C_{o,j}}{\kappa_{\tau_o} \, \tau_{o,j}}$

4. Land carbon

diagnostic

- $r_{\text{npp}} = \left(1 + \dfrac{\beta_{\text{npp}}}{\alpha_{\text{npp}}} \, \left(1 - \left(\dfrac{C}{C_{\text{pi}}}\right)^{-\alpha_{\text{npp}}}\right)\right) (1 + \gamma_{\text{npp}} \, T)$
- $r_{\text{fire}} = \left(1 + \beta_{\text{fire}} \left(\dfrac{C}{C_{\text{pi}}} - 1\right)\right) (1 + \gamma_{\text{fire}} \, T)$
- $r_{\text{rh}} = \left(1 + \beta_{\text{rh}} \left(\dfrac{C_{s1}}{C_{s1} + C_{s2} + C_{s3}} \left(1 + \dfrac{\nu_{stab}}{\nu_{rh23}}\right) - 1\right)\right) \exp(\gamma_{\text{rh}} \, T)$
- $F_{\text{npp}} = F_{\text{npp,0}} \, r_{\text{npp}}$
- $E_{\text{fire}} = \nu_{\text{fire}} \, r_{\text{fire}} \, C_v$
- $E_{\text{harv}} = \nu_{\text{harv}} \, C_v$
- $F_{\text{mort}} = \nu_{\text{mort}} \, C_v$
- $E_{\text{rh1}} = \nu_{\text{rh1}} \, r_{\text{rh}} \, C_{s1}$
- $F_{\text{stab}} = \nu_{\text{stab}} \, r_{\text{rh}} \, C_{s1}$
- $E_{\text{rh2}} = \dfrac{\nu_{\text{rh23}} - \nu_{\text{rh3}} \, \alpha_{\text{pass}}}{1 - \alpha_{\text{pass}}} \, r_{\text{rh}} \, C_{s2}$
- $F_{\text{pass}} = \nu_{\text{rh3}} \; \dfrac{\alpha_{\text{pass}}}{1 - \alpha_{\text{pass}}} \, r_{\text{rh}} \, C_{s2}$

- $E_{\mathrm{rh3}} = \nu_{\mathrm{rh3}} \, r_{\mathrm{rh}} \, C_{s3}$
- $F_{\mathrm{land}} = F_{\mathrm{npp}} - E_{\mathrm{fire}} - E_{\mathrm{harv}} - E_{\mathrm{rh1}} - E_{\mathrm{rh2}} - E_{\mathrm{rh3}}$

prognostic

- $\frac{\mathrm{d}C_v}{\mathrm{d}t} = F_{\mathrm{npp}} - E_{\mathrm{fire}} - E_{\mathrm{harv}} - F_{\mathrm{mort}}$
- $\frac{\mathrm{d}C_{s1}}{\mathrm{d}t} = F_{\mathrm{mort}} - F_{\mathrm{stab}} - E_{\mathrm{rh1}}$
- $\frac{\mathrm{d}C_{s2}}{\mathrm{d}t} = F_{\mathrm{stab}} - F_{\mathrm{pass}} - E_{\mathrm{rh2}}$
- $\frac{\mathrm{d}C_{s3}}{\mathrm{d}t} = F_{\mathrm{pass}} - E_{\mathrm{rh3}}$

diagnostic (2nd)

- $E_{\mathrm{rh}} = E_{\mathrm{rh1}} + E_{\mathrm{rh2}} + E_{\mathrm{rh3}}$
- $C_s = C_{\mathrm{s1}} + C_{\mathrm{s2}} + C_{\mathrm{s3}}$

5. Permafrost carbon

diagnostic

- $r_{\mathrm{rt}} = \exp\big(\kappa_{\mathrm{rt}} \; \gamma_{\mathrm{rt1}} \; \alpha_{\mathrm{lst}} \; T - \kappa_{\mathrm{rt}} \; \gamma_{\mathrm{rt2}} \; (\alpha_{\mathrm{lst}} \; T)^2\big)$
- $\bar{a} = -a_{\mathrm{min}} + \frac{(1 + a_{\mathrm{min}})}{\left(1 + \left(\left(1 + \frac{1}{a_{\mathrm{min}}}\right)^{\kappa_a} - 1\right) \exp(-\gamma_a \; \kappa_a \; \alpha_{\mathrm{lst}} \; T)\right)^{\frac{1}{\kappa_a}}}$
- $E_{\mathrm{pf}} = \sum_j \frac{C_{\mathrm{th},j}}{\kappa_{\tau_{\mathrm{th}}} \; \tau_{\mathrm{th},j}} \, r_{\mathrm{rt}}$

prognostic

- $\frac{\mathrm{d}a}{\mathrm{d}t} = 0.5 \; (\nu_{\mathrm{thaw}} + \nu_{\mathrm{froz}}) \; (\bar{a} - a) + 0.5 \; |(\nu_{\mathrm{thaw}} - \nu_{\mathrm{froz}}) \; (\bar{a} - a)|$
- $\frac{\mathrm{d}C_{\mathrm{th},j}}{\mathrm{d}t} = \alpha_{\mathrm{th},j} \, \frac{\mathrm{d}a}{\mathrm{d}t} \, C_{\mathrm{fr},0} - \frac{C_{\mathrm{th},j}}{\kappa_{\tau_{\mathrm{th}}} \; \tau_{\mathrm{th},j}} \, r_{\mathrm{rt}}$

diagnostic (2nd)

- $C_{\mathrm{fr}} = (1 - a) \, C_{\mathrm{fr},0}$
- $\frac{\mathrm{d}C_{\mathrm{fr}}}{\mathrm{d}t} = -\frac{\mathrm{d}a}{\mathrm{d}t} \, C_{\mathrm{fr},0}$

6. Atmospheric CO2

diagnostic

- $\mathrm{pH} = \kappa_{\mathrm{pH}} \, (8.5541 - 0.00173 \, C + 1.3264 \, 10^{-6} \, C^2 - 4.4943 \, 10^{-10} \, C^3)$

prognostic

- $\alpha_C \, \dfrac{\mathrm{d}C}{\mathrm{d}t} = E_{\mathrm{CO2}} + E_{\mathrm{pf}} - F_{\mathrm{land}} - F_{\mathrm{ocean}}$