Notations

Forcings and Variables

In manual	In code	Description	Units	Prog?	Dims
R_c	`RFco2`	CO2 (effective) radiative forcing	W m^{-2}		
R_x	`ERFx`	Non-CO2 effective radiative forcing	W m^{-2}		
R	`ERF`	Effective radiative forcing	W m^{-2}		
T	`T`	Global surface temperature anomaly	K	*yes*	
T_d	`Td`	Deep ocean temperature anomaly	K	*yes*	
U_ohc	`OHC`	Ocean heat content (anomaly)	W yr m^{-2}		
H_lin	`Hlin`	Linear part of thermosteric sea level rise	mm		
H_thx	`Hthx`	Total thermosteric sea level rise	mm	*yes*	
H_ice	`Hice`	Ice contributions to sea level rise	mm		
H_tot	`Htot`	Total sea level rise	mm	*yes*	
$C_{o,j}$	`Co_j`	Change in surface ocean carbon subpools	PgC	*yes*	$j \in [\![1, 5]\!]$
C_o	`Co`	Change in surface ocean carbon pool	PgC		
C_d	`Cd`	Change in deep ocean carbon pool	*yes*		
c_dic	`dic`	Change in surface DIC	µmol kg^{-1}		
p_dic	`pdic`	Subcomponent of `pCO2`	ppm		
p_CO2	`pCO2`	CO2 partial pressure at the ocean surface	ppm		
F_ocean	`Focean`	Ocean carbon sink	PgC yr^{-1}		
r_npp	`r_npp`	Relative change in NPP	1		
r_fire	`r_fire`	Relative change in wildfire intensity	1		
r_rh	`r_rh`	Relative change in heterotrophic respiration rate	1		
F_npp	`NPP`	Net primary productivity	PgC yr^{-1}		
E_fire	`Efire`	Emissions from wildfire	PgC yr^{-1}		

In manual	In code	Description	Units	Prog?	Dims
E_harv	`Eharv`	Emissions from harvest and grazing	PgC yr^{-1}		
F_mort	`Fmort`	Mortality flux	PgC yr^{-1}		
E_rh1	`RH1`	Litter heterotrophic respiration	PgC yr^{-1}		
F_stab	`Fstab`	Stabilization flux	PgC yr^{-1}		
E_rh2	`RH2`	Active soil heterotrophic respiration	PgC yr^{-1}		
F_pass	`Fpass`	Passivization flux	PgC yr^{-1}		
E_rh3	`RH3`	Passive soil heterotrophic respiration	PgC yr^{-1}		
F_land	`Fland`	Land carbon sink	PgC yr^{-1}		
E_rh	`RH`	Heterotrophic respiration	PgC yr^{-1}		
C_v	`Cv`	Vegetation carbon pool	PgC	*yes*	
C_{s1}	`Cs1`	Litter carbon pool	PgC	*yes*	
C_{s2}	`Cs2`	Active soil carbon pool	PgC	*yes*	
C_{s3}	`Cs3`	Passive soil carbon pool	PgC	*yes*	
C_s	`Cs`	Total soil carbon pool	PgC		
r_rt	`r_rt`	Relative change in permafrost respiration rate	1		
$\bar{a}$	`abar`	Theoretical thawed fraction	1		
a	`a`	Actual thawed fraction	1	*yes*	
E_pf	`Epf`	Emissions from permafrost	PgC yr^{-1}		
$C_{\text{th},j}$	`Cth_j`	Thawed permafrost carbon subpools	PgC	*yes*	$j \in [\![1,3]\!]$
C_fr	`Cfr`	Frozen permafrost carbon pool	PgC		
E_CO2	`Eco2`	Anthropogenic CO2 emissions	PgC yr^{-1}		
C	`CO2`	Atmospheric CO2 concentration	ppm	*yes*	
pH	`pH`	Surface ocean pH	1		

Parameters

In manual	In code	Description	Units	Dims

In manual	In code	Description	Units	Dims
ϕ	`phi`	Radiative parameter of CO2	W m^{-2}	
$T_{2\times}$	`T2x`	Equilibrium climate sensitivity	K	
Θ_s	`THs`	Heat capacity of the surface	W yr m^{-2} K^{-1}	
Θ_d	`THd`	Heat capacity of the deep ocean	W yr m^{-2} K^{-1}	
θ	`th`	Heat exchange coefficient	W m^{-2} K^{-1}	
ϵ_{heat}	`eheat`	Deep ocean heat uptake efficacy	1	
α_{ohc}	`aOHC`	Fraction of energy warming the ocean	1	
Λ_{lin}	`Llin`	Linear factor for thermosteric SLR	mm m^{2} W^{-1} yr^{-1}	
Λ_{thx}	`Lthx`	Equilibrium thermosteric SLR	mm K^{-1}	
Λ_{tot1}	`Ltot1`	Linear equilibrium ice-related SLR	mm K^{-1}	
Λ_{tot2}	`Ltot2`	Quadratic equilibrium ice-related SLR	mm K^{-2}	
τ_{thx}	`tthx`	Timescale of thermosteric SLR	yr	
τ_{ice}	`tice`	Timescale of ice-related SLR	yr	
α_{dic}	`adic`	Conversion factor for DIC	µmol kg^{-1} PgC^{-1}	
β_{dic}	`bdic`	Scaling factor for DIC (~ for MLD)	1	
γ_{dic}	`gdic`	Sensitivity of pCO2 to climate	K^{-1}	
T_o	`To`	Preindustrial surface ocean temperature	°C	
ν_{gx}	`vgx`	Surface ocean gas exchange rate	yr^{-1}	
γ_{gx}	`ggx`	Sensitivity of gas exchange to climate	K^{-1}	
$\alpha_{o,j}$	`aoc_j`	Surface ocean subpools fractions	1	$j \in [\![1, 5]\!]$
$\tau_{o,j}$	`toc_j`	Timescales of surface ocean subpools	yr	$j \in [\![1, 5]\!]$
κ_{τ_o}	`k_toc`	Scaling factor for timescales of surface ocean subpools	1	
β_{npp}	`bnpp`	Sensitivity of NPP to CO2 (= fertilization effect)	1	
α_{npp}	`anpp`	Shape parameter for fertilization effect	1	

In manual	In code	Description	Units	Dims
γ_{npp}	`gnpp`	Sensitivity of NPP to climate	K^{-1}	
β_{fire}	`bfire`	Sensitivity of wildfire intensity to CO2	1	
γ_{fire}	`gfire`	Sensitivity of wildfire intensity to climate	K^{-1}	
β_{rh}	`brh`	Sensitivity of heterotrophic respiration to fresh organic matter	1	
γ_{rh}	`grh`	Sensitivity of heterotrophic respiration to climate	K^{-1}	
$F_{\text{npp},0}$	`npp0`	Preindustrial NPP	PgC yr^{-1}	
ν_{fire}	`vfire`	Wildfire intensity	yr^{-1}	
ν_{harv}	`vharv`	Harvest and grazing rate	yr^{-1}	
ν_{mort}	`vmort`	Mortality rate	yr^{-1}	
ν_{stab}	`vstab`	Stabilization rate	yr^{-1}	
ν_{rh1}	`vrh1`	Litter heterotrophic respiration rate	yr^{-1}	
ν_{rh23}	`vrh23`	Soil (active and passive) respiration rate	yr^{-1}	
ν_{rh3}	`vrh3`	Passive soil respiration rate	yr^{-1}	
α_{pass}	`apass`	Fraction of passive soil	1	
α_{lst}	`aLST`	Climate scaling factor over permafrost regions	1	
γ_{rt1}	`grt1`	Sensitivity of (boreal) heterotrophic respiration to climate	K^{-1}	
γ_{rt2}	`grt2`	Sensitivity of (boreal) heterotrophic respiration to climate (quadratic)	K^{-2}	
κ_{rt}	`krt`	Scaling factor for sensitivity of permafrost respiration to climate	1	
a_{min}	`amin`	Minimal thawed fraction	1	
κ_a	`ka`	Shape parameter for theoretical thawed fraction	1	
γ_a	`ga`	Sensitivity of theoretical thawed fraction to climate	K^{-1}	
ν_{thaw}	`vthaw`	Thawing rate	yr^{-1}	
ν_{froz}	`vfroz`	Freezing rate	yr^{-1}	
$\alpha_{\text{th},j}$	`ath_j`	Thawed permafrost carbon subpools fractions	1	$j \in [\![1, 3]\!]$

In manual	In code	Description	Units	Dims
$\tau_{\mathrm{th},j}$	`tth_j`	Timescales of thawed permafrost carbon subpools	yr	$j \in [\![1,3]\!]$
$\kappa_{\tau_{\mathrm{th}}}$	`k_tth`	Scaling factor for timescales of surface ocean subpools	1	
$C_{\mathrm{fr},0}$	`Cfr0`	Preindustrial frozen permafrost carbon pool	PgC	
α_C	`aCO2`	Conversion factor for atmospheric CO2	PgC ppm^{-1}	
C_{pi}	`CO2pi`	Preindustrial CO2 concentration	ppm	
κ_{pH}	`k_pH`	Scaling factor for surface ocean pH	1	

Equations

1. Climate

diagnostic

- $R_c = \phi \ \ln\left(\dfrac{C}{C_{\mathrm{pi}}}\right)$
- $R = R_c + R_x$

prognostic

- $\Theta_s \ \dfrac{\mathrm{d}T}{\mathrm{d}t} = R - \dfrac{\phi \ \ln(2)}{T_{2\times}} \ T - \epsilon_{\mathrm{heat}} \ \theta \ (T - T_d)$
- $\Theta_d \ \dfrac{\mathrm{d}T_d}{\mathrm{d}t} = \theta \ (T - T_d)$

2. Sea level

diagnostic

- $U_{\mathrm{ohc}} = \alpha_{\mathrm{ohc}} \ (\Theta_s \, T + \Theta_d \, T_d)$
- $\dfrac{\mathrm{d}U_{\mathrm{ohc}}}{\mathrm{d}t} = \alpha_{\mathrm{ohc}} \ (\Theta_s \ \dfrac{\mathrm{d}T}{\mathrm{d}t} + \Theta_d \ \dfrac{\mathrm{d}T_d}{\mathrm{d}t})$
- $H_{\mathrm{lin}} = \Lambda_{\mathrm{lin}} \ U_{\mathrm{ohc}}$
- $\dfrac{\mathrm{d}H_{\mathrm{lin}}}{\mathrm{d}t} = \Lambda_{\mathrm{lin}} \ \dfrac{\mathrm{d}U_{\mathrm{ohc}}}{\mathrm{d}t}$

prognostic

- $\dfrac{\mathrm{d}H_{\mathrm{thx}}}{\mathrm{d}t} = \dfrac{\mathrm{d}H_{\mathrm{lin}}}{\mathrm{d}t} - \dfrac{H_{\mathrm{thx}} - H_{\mathrm{lin}}}{\tau_{\mathrm{thx}}} + \dfrac{\Lambda_{\mathrm{thx}} - \Lambda_{\mathrm{lin}}\, \alpha_{\mathrm{ohc}}\, (\Theta_s + \Theta_d)}{\tau_{\mathrm{thx}}}\, T_d$
- $\dfrac{\mathrm{d}H_{\mathrm{tot}}}{\mathrm{d}t} = \dfrac{\mathrm{d}H_{\mathrm{thx}}}{\mathrm{d}t} - \dfrac{H_{\mathrm{tot}} - H_{\mathrm{thx}}}{\tau_{\mathrm{ice}}} + \dfrac{\Lambda_{\mathrm{tot1}} + \Lambda_{\mathrm{tot2}}\, T - \Lambda_{\mathrm{thx}}}{\tau_{\mathrm{ice}}}\, T$

diagnostic (2nd)

- $H_{\mathrm{ice}} = H_{\mathrm{tot}} - H_{\mathrm{thx}}$
- $\dfrac{\mathrm{d}H_{\mathrm{ice}}}{\mathrm{d}t} = \dfrac{\mathrm{d}H_{\mathrm{tot}}}{\mathrm{d}t} - \dfrac{\mathrm{d}H_{\mathrm{thx}}}{\mathrm{d}t}$

3. Ocean carbon

diagnostic

- $C_o = \sum_j C_{o,j}$
- $c_{\mathrm{dic}} = \dfrac{\alpha_{\mathrm{dic}}}{\beta_{\mathrm{dic}}}\, C_o$
- $\begin{aligned} p_{\mathrm{dic}} = &\ (1.5568 - 0.013993\, T_o)\, c_{\mathrm{dic}} \\ &+ (7.4706 - 0.20207\, T_o)\, 10^{-3}\, {c_{\mathrm{dic}}}^2 \\ &- (1.2748 - 0.12015\, T_o)\, 10^{-5}\, {c_{\mathrm{dic}}}^3 \\ &+ (2.4491 - 0.12639\, T_o)\, 10^{-7}\, {c_{\mathrm{dic}}}^4 \\ &- (1.5768 - 0.15326\, T_o)\, 10^{-10}\, {c_{\mathrm{dic}}}^5 \end{aligned}$
- $p_{\mathrm{CO2}} = (p_{\mathrm{dic}} + C_{\mathrm{pi}})\ \exp(\gamma_{\mathrm{dic}}\, T)$
- $F_{\mathrm{ocean}} = \nu_{\mathrm{gx}}\, (1 + \gamma_{\mathrm{gx}}\, T)\, (C - p_{\mathrm{CO2}})$

prognostic

- $\dfrac{\mathrm{d}C_{o,j}}{\mathrm{d}t} = -\dfrac{C_{o,j}}{\kappa_{\tau_o}\, \tau_{o,j}} + \alpha_{o,j}\, F_{\mathrm{ocean}}$
- $\dfrac{\mathrm{d}C_d}{\mathrm{d}t} = \sum_j \dfrac{C_{o,j}}{\kappa_{\tau_o}\, \tau_{o,j}}$

4. Land carbon

diagnostic

- $r_{\mathrm{npp}} = \left(1 + \dfrac{\beta_{\mathrm{npp}}}{\alpha_{\mathrm{npp}}}\, \left(1 - \left(\dfrac{C}{C_{\mathrm{pi}}}\right)^{-\alpha_{\mathrm{npp}}}\right)\right)\, (1 + \gamma_{\mathrm{npp}}\, T)$

- $r_{\text{fire}} = \left(1 + \beta_{\text{fire}} \left(\frac{C}{C_{\text{pi}}} - 1\right)\right) (1 + \gamma_{\text{fire}}\, T)$
- $r_{\text{rh}} = \left(1 + \beta_{\text{rh}} \left(\frac{C_{s1}}{C_{s1} + C_{s2} + C_{s3}} \left(1 + \frac{\nu_{stab}}{\nu_{rh23}}\right) - 1\right)\right) \exp(\gamma_{\text{rh}}\, T)$
- $F_{\text{npp}} = F_{\text{npp,0}}\, r_{\text{npp}}$
- $E_{\text{fire}} = \nu_{\text{fire}}\, r_{\text{fire}}\, C_v$
- $E_{\text{harv}} = \nu_{\text{harv}}\, C_v$
- $F_{\text{mort}} = \nu_{\text{mort}}\, C_v$
- $E_{\text{rh1}} = \nu_{\text{rh1}}\, r_{\text{rh}}\, C_{s1}$
- $F_{\text{stab}} = \nu_{\text{stab}}\, r_{\text{rh}}\, C_{s1}$
- $E_{\text{rh2}} = \frac{\nu_{\text{rh23}} - \nu_{\text{rh3}}\, \alpha_{\text{pass}}}{1 - \alpha_{\text{pass}}}\, r_{\text{rh}}\, C_{s2}$
- $F_{\text{pass}} = \nu_{\text{rh3}}\, \frac{\alpha_{\text{pass}}}{1 - \alpha_{\text{pass}}}\, r_{\text{rh}}\, C_{s2}$
- $E_{\text{rh3}} = \nu_{\text{rh3}}\, r_{\text{rh}}\, C_{s3}$
- $F_{\text{land}} = F_{\text{npp}} - E_{\text{fire}} - E_{\text{harv}} - E_{\text{rh1}} - E_{\text{rh2}} - E_{\text{rh3}}$

prognostic

- $\frac{\mathrm{d}C_v}{\mathrm{d}t} = F_{\text{npp}} - E_{\text{fire}} - E_{\text{harv}} - F_{\text{mort}}$
- $\frac{\mathrm{d}C_{s1}}{\mathrm{d}t} = F_{\text{mort}} - F_{\text{stab}} - E_{\text{rh1}}$
- $\frac{\mathrm{d}C_{s2}}{\mathrm{d}t} = F_{\text{stab}} - F_{\text{pass}} - E_{\text{rh2}}$
- $\frac{\mathrm{d}C_{s3}}{\mathrm{d}t} = F_{\text{pass}} - E_{\text{rh3}}$

diagnostic (2nd)

- $E_{\text{rh}} = E_{\text{rh1}} + E_{\text{rh2}} + E_{\text{rh3}}$
- $C_s = C_{\text{s1}} + C_{\text{s2}} + C_{\text{s3}}$

5. Permafrost carbon

diagnostic

- $r_{\text{rt}} = \exp\left(\kappa_{\text{rt}}\, \gamma_{\text{rt1}}\, \alpha_{\text{lst}}\, T - \kappa_{\text{rt}}\, \gamma_{\text{rt2}}\, (\alpha_{\text{lst}}\, T)^2\right)$

- $\bar{a} = -a_{\mathrm{min}} + \dfrac{(1 + a_{\mathrm{min}})}{\left(1 + \left(\left(1 + \frac{1}{a_{\mathrm{min}}}\right)^{\kappa_a} - 1\right) \exp(-\gamma_a \; \kappa_a \; \alpha_{\mathrm{lst}} \; T)\right)^{\frac{1}{\kappa_a}}}$
- $E_{\mathrm{pf}} = \sum_j \dfrac{C_{\mathrm{th},j}}{\kappa_{\tau_{\mathrm{th}}} \; \tau_{\mathrm{th},j}} \, r_{\mathrm{rt}}$

prognostic

- $\dfrac{\mathrm{d}a}{\mathrm{d}t} = 0.5 \; (\nu_{\mathrm{thaw}} + \nu_{\mathrm{froz}}) \; (\bar{a} - a) + 0.5 \; |(\nu_{\mathrm{thaw}} - \nu_{\mathrm{froz}}) \; (\bar{a} - a)|$
- $\dfrac{\mathrm{d}C_{\mathrm{th},j}}{\mathrm{d}t} = \alpha_{\mathrm{th},j} \; \dfrac{\mathrm{d}a}{\mathrm{d}t} \, C_{\mathrm{fr},0} - \dfrac{C_{\mathrm{th},j}}{\kappa_{\tau_{\mathrm{th}}} \; \tau_{\mathrm{th},j}} \, r_{\mathrm{rt}}$

diagnostic (2nd)

- $C_{\mathrm{fr}} = (1 - a) \; C_{\mathrm{fr},0}$
- $\dfrac{\mathrm{d}C_{\mathrm{fr}}}{\mathrm{d}t} = -\dfrac{\mathrm{d}a}{\mathrm{d}t} \; C_{\mathrm{fr},0}$

6. Atmospheric CO2

diagnostic

- $\mathrm{pH} = \kappa_{\mathrm{pH}} \; (8.5541 - 0.00173 \, C + 1.3264 \; 10^{-6} \; C^2 - 4.4943 \; 10^{-10} \; C^3)$

prognostic

- $\alpha_C \; \dfrac{\mathrm{d}C}{\mathrm{d}t} = E_{\mathrm{CO2}} + E_{\mathrm{pf}} - F_{\mathrm{land}} - F_{\mathrm{ocean}}$