Differential System

1. Climate

diagnostic

$$R_c = \phi \, \ln\left(\frac{C}{C_{\mathrm{pi}}}\right)$$

$$R = R_c + R_x$$

prognostic

$$\Theta_s \, \frac{\mathrm{d}T}{\mathrm{d}t} = R - \frac{\phi \, \ln(2)}{T_{2\times}} \, T - \epsilon_{\mathrm{heat}} \, \theta \, (T - T_d)$$

$$\Theta_d \, \frac{\mathrm{d}T_d}{\mathrm{d}t} = \theta \, (T - T_d)$$

2. Sea level

diagnostic

$$U_{\mathrm{ohc}} = \alpha_{\mathrm{ohc}} \, (\Theta_s \, T + \Theta_d \, T_d)$$

$$\frac{\mathrm{d}U_{\mathrm{ohc}}}{\mathrm{d}t} = \alpha_{\mathrm{ohc}} \, (\Theta_s \, \frac{\mathrm{d}T}{\mathrm{d}t} + \Theta_d \, \frac{\mathrm{d}T_d}{\mathrm{d}t})$$

$$H_{\mathrm{lin}} = \Lambda_{\mathrm{lin}} \, U_{\mathrm{ohc}}$$

$$\frac{\mathrm{d}H_{\mathrm{lin}}}{\mathrm{d}t} = \Lambda_{\mathrm{lin}} \, \frac{\mathrm{d}U_{\mathrm{ohc}}}{\mathrm{d}t}$$

prognostic

$$\frac{\mathrm{d}H_{\mathrm{thx}}}{\mathrm{d}t} = \frac{\mathrm{d}H_{\mathrm{lin}}}{\mathrm{d}t} - \frac{H_{\mathrm{thx}} - H_{\mathrm{lin}}}{\tau_{\mathrm{thx}}} + \frac{\Lambda_{\mathrm{thx}} - \Lambda_{\mathrm{lin}} \, \alpha_{\mathrm{ohc}} \, (\Theta_s + \Theta_d)}{\tau_{\mathrm{thx}}} \, T_d$$

$$\frac{\mathrm{d}H_{\mathrm{tot}}}{\mathrm{d}t} = \frac{\mathrm{d}H_{\mathrm{thx}}}{\mathrm{d}t} - \frac{H_{\mathrm{tot}} - H_{\mathrm{thx}}}{\tau_{\mathrm{ice}}} + \frac{\Lambda_{\mathrm{tot1}} + \Lambda_{\mathrm{tot2}} \, T - \Lambda_{\mathrm{thx}}}{\tau_{\mathrm{ice}}} \, T$$

diagnostic (2nd)

$$H_{\mathrm{ice}} = H_{\mathrm{tot}} - H_{\mathrm{thx}}$$

$$\frac{\mathrm{d}H_{\mathrm{ice}}}{\mathrm{d}t} = \frac{\mathrm{d}H_{\mathrm{tot}}}{\mathrm{d}t} - \frac{\mathrm{d}H_{\mathrm{thx}}}{\mathrm{d}t}$$

3. Ocean carbon

diagnostic

$$C_o = \sum_j C_{o,j}$$

$$c_{\mathrm{dic}} = \frac{\alpha_{\mathrm{dic}}}{\beta_{\mathrm{dic}}}\, C_o$$

$$\begin{aligned} p_{\mathrm{dic}} = {} & (1.5568 - 0.013993\, T_o)\, c_{\mathrm{dic}} \\ & + (7.4706 - 0.20207\, T_o)\, 10^{-3}\, {c_{\mathrm{dic}}}^2 \\ & - (1.2748 - 0.12015\, T_o)\, 10^{-5}\, {c_{\mathrm{dic}}}^3 \\ & + (2.4491 - 0.12639\, T_o)\, 10^{-7}\, {c_{\mathrm{dic}}}^4 \\ & - (1.5768 - 0.15326\, T_o)\, 10^{-10}\, {c_{\mathrm{dic}}}^5 \end{aligned}$$

$$p_{\mathrm{CO2}} = (p_{\mathrm{dic}} + C_{\mathrm{pi}})\ \exp(\gamma_{\mathrm{dic}}\, T)$$

$$F_{\mathrm{ocean}} = \nu_{\mathrm{gx}}\, (1 + \gamma_{\mathrm{gx}}\, T)\, (C - p_{\mathrm{CO2}})$$

prognostic

$$\frac{\mathrm{d}C_{o,j}}{\mathrm{d}t} = -\frac{C_{o,j}}{\kappa_{\tau_o}\, \tau_{o,j}} + \alpha_{o,j}\, F_{\mathrm{ocean}}$$

$$\frac{\mathrm{d}C_d}{\mathrm{d}t} = \sum_j \frac{C_{o,j}}{\kappa_{\tau_o}\, \tau_{o,j}}$$

4. Land carbon

diagnostic

$$r_{\mathrm{npp}} = \left(1 + \frac{\beta_{\mathrm{npp}}}{\alpha_{\mathrm{npp}}}\, \left(1 - \left(\frac{C}{C_{\mathrm{pi}}}\right)^{-\alpha_{\mathrm{npp}}}\right)\right)\, (1 + \gamma_{\mathrm{npp}}\, T)$$

$$r_{\mathrm{ef}} = \left(1 + \beta_{\mathrm{ef}}\, \left(\frac{C}{C_{\mathrm{pi}}} - 1\right)\right)\, (1 + \gamma_{\mathrm{ef}}\, T)$$

$$r_{\mathrm{rh}} = \left(1 + \beta_{\mathrm{rh}}\, \left(\frac{C_{s1}}{C_{s1} + C_{s2} + C_{s3}}\, \left(1 + \frac{\nu_{met}}{\nu_{cs2}}\right) - 1\right)\right)\, \exp(\gamma_{\mathrm{rh}}\, T)$$

$$F_{\mathrm{npp}} = F_{\mathrm{npp},0}\, r_{\mathrm{npp}}$$

$$E_{\mathrm{fire}} = \nu_{\mathrm{fire}}\, r_{\mathrm{ef}}\, C_v$$

$$E_{\mathrm{harv}} = \nu_{\mathrm{harv}}\, C_v$$

$$F_{\mathrm{mort}} = \nu_{\mathrm{mort}}\, C_v$$

$$E_{\mathrm{rh1}} = \nu_{\mathrm{rh1}}\, r_{\mathrm{rh}}\, C_{s1}$$

$$F_{\mathrm{met}} = \nu_{\mathrm{met}}\, r_{\mathrm{rh}}\, C_{s1}$$

$$E_{\mathrm{rh2}} = \frac{\nu_{\mathrm{cs2}} - \nu_{\mathrm{rh3}}\, \alpha_{\mathrm{pass}}}{1 - \alpha_{\mathrm{pass}}}\, r_{\mathrm{rh}}\, C_{s2}$$

$$F_{\mathrm{pass}} = \nu_{\mathrm{rh3}}\, \frac{\alpha_{\mathrm{pass}}}{1 - \alpha_{\mathrm{pass}}}\, r_{\mathrm{rh}}\, C_{s2}$$

$$E_{\mathrm{rh3}} = \nu_{\mathrm{rh3}}\, r_{\mathrm{rh}}\, C_{s3}$$

$$F_{\mathrm{land}} = F_{\mathrm{npp}} - E_{\mathrm{fire}} - E_{\mathrm{harv}} - E_{\mathrm{rh1}} - E_{\mathrm{rh2}} - E_{\mathrm{rh3}}$$

prognostic

$$\frac{\mathrm{d}C_v}{\mathrm{d}t} = F_{\mathrm{npp}} - E_{\mathrm{fire}} - E_{\mathrm{harv}} - F_{\mathrm{mort}}$$

$$\frac{\mathrm{d}C_{s1}}{\mathrm{d}t} = F_{\mathrm{mort}} - F_{\mathrm{met}} - E_{\mathrm{rh1}}$$

$$\frac{\mathrm{d}C_{s2}}{\mathrm{d}t} = F_{\mathrm{met}} - F_{\mathrm{pass}} - E_{\mathrm{rh2}}$$

$$\frac{\mathrm{d}C_{s3}}{\mathrm{d}t} = F_{\mathrm{pass}} - E_{\mathrm{rh3}}$$

diagnostic (2nd)

$$E_{\mathrm{rh}} = E_{\mathrm{rh1}} + E_{\mathrm{rh2}} + E_{\mathrm{rh3}}$$

$$C_s = C_{\mathrm{s1}} + C_{\mathrm{s2}} + C_{\mathrm{s3}}$$

5. Permafrost carbon

diagnostic

$$r_{\mathrm{rt}} = \exp\left(\kappa_{\mathrm{rt}}\, \gamma_{\mathrm{rt1}}\, \alpha_{\mathrm{lst}}\, T - \kappa_{\mathrm{rt}}\, \gamma_{\mathrm{rt2}}\, (\alpha_{\mathrm{lst}}\, T)^2\right)$$

$$\bar{a} = -a_{\mathrm{min}} + \frac{(1 + a_{\mathrm{min}})}{\left(1 + \left(\left(1 + \frac{1}{a_{\mathrm{min}}}\right)^{\kappa_a} - 1\right) \exp(-\gamma_a\, \kappa_a\, \alpha_{\mathrm{lst}}\, T)\right)^{\frac{1}{\kappa_a}}}$$

$$E_{\mathrm{pf}} = \sum_j \frac{C_{\mathrm{th},j}}{\kappa_{\tau_{\mathrm{th}}}\, \tau_{\mathrm{th},j}}\, r_{\mathrm{rt}}$$

prognostic

$$\frac{\mathrm{d}a}{\mathrm{d}t} = 0.5\,(\nu_{\mathrm{thaw}} + \nu_{\mathrm{froz}})\,(\bar{a} - a) + 0.5\,\left|(\nu_{\mathrm{thaw}} - \nu_{\mathrm{froz}})\,(\bar{a} - a)\right|$$

$$\frac{\mathrm{d}C_{\mathrm{th},j}}{\mathrm{d}t} = \alpha_{\mathrm{th},j}\,\frac{\mathrm{d}a}{\mathrm{d}t}\,C_{\mathrm{fr},0} - \frac{C_{\mathrm{th},j}}{\kappa_{\tau_{\mathrm{th}}}\,\tau_{\mathrm{th},j}}\,r_{\mathrm{rt}}$$

diagnostic (2nd)

$$C_{\mathrm{fr}} = (1 - a)\,C_{\mathrm{fr},0}$$

$$\frac{\mathrm{d}C_{\mathrm{fr}}}{\mathrm{d}t} = -\frac{\mathrm{d}a}{\mathrm{d}t}\,C_{\mathrm{fr},0}$$

6. Atmospheric CO2

diagnostic

$$\mathrm{pH}[C] = \kappa_{\mathrm{pH}}\,(8.5541 - 0.00173\,C + 1.3264\,10^{-6}\,C^2 - 4.4943\,10^{-10}\,C^3)$$

prognostic

$$\alpha_C\,\frac{\mathrm{d}C}{\mathrm{d}t} = E_{\mathrm{CO2}} + E_{\mathrm{pf}} - F_{\mathrm{land}} - F_{\mathrm{ocean}}$$

Notations

Drivers

In manual	In code	Units
R_x	`ERFx`	W m^{-2}
E_{CO2}	`Eco2`	PgC yr^{-1}

Variables

In manual	In code	Units	Prog?	Dims
R_c	`RFco2`	W m^{-2}		
R	`ERF`	W m^{-2}		
T	`T`	K	*yes*	
T_d	`Td`	K	*yes*	
U_{ohc}	`OHC`	W yr m^{-2}		
H_{lin}	`Hlin`	mm		

In manual	In code	Units	Prog?	Dims
H_{thx}	`Hthx`	mm	*yes*	
H_{ice}	`Hice`	mm		
H_{tot}	`Htot`	mm	*yes*	
$C_{o,j}$	`Co_j`	PgC	*yes*	$j \in [\![1,5]\!]$
C_o	`Co`	PgC		
C_d	`Cd`	PgC	*yes*	
c_{dic}	`dic`	µmol kg^{-1}		
p_{dic}	`pdic`	ppm		
p_{CO2}	`pCO2`	ppm		
F_{ocean}	`Focean`	PgC yr^{-1}		
r_{npp}	`r_npp`	1		
r_{ef}	`r_ef`	1		
r_{rh}	`r_rh`	1		
F_{npp}	`NPP`	PgC yr^{-1}		
E_{fire}	`Efire`	PgC yr^{-1}		
E_{harv}	`Eharv`	PgC yr^{-1}		
F_{mort}	`Fmort`	PgC yr^{-1}		
E_{rh1}	`RH1`	PgC yr^{-1}		
F_{met}	`Fmet`	PgC yr^{-1}		
E_{rh2}	`RH2`	PgC yr^{-1}		
F_{pass}	`Fpass`	PgC yr^{-1}		
E_{rh3}	`RH3`	PgC yr^{-1}		
F_{land}	`Fland`	PgC yr^{-1}		
E_{rh}	`RH`	PgC yr^{-1}		
C_v	`Cv`	PgC	*yes*	
C_{s1}	`Cs1`	PgC	*yes*	
C_{s2}	`Cs2`	PgC	*yes*	

In manual	In code	Units	Prog?	Dims
C_{s3}	`Cs3`	PgC	*yes*	
C_s	`Cs`	PgC		
r_{rt}	`r_rt`	1		
$\bar{a}$	`abar`	1		
a	`a`	1	*yes*	
E_{pf}	`Epf`	PgC yr^{-1}		
$C_{\mathrm{th},j}$	`Cth_j`	PgC	*yes*	$j \in [\![1,3]\!]$
C_{fr}	`Cfr`	PgC		
C	`CO2`	ppm	*yes*	
pH	`pH`	1		

Parameters

In manual	In code	Units	Dims
ϕ	`phi`	W m^{-2}	
$T_{2\times}$	`T2x`	K	
Θ_s	`THs`	W yr m^{-2} K^{-1}	
Θ_d	`THd`	W yr m^{-2} K^{-1}	
θ	`th`	W m^{-2} K^{-1}	
ϵ_{heat}	`eheat`	1	
α_{ohc}	`aOHC`	1	
Λ_{lin}	`Llin`	mm m^2 W^{-1} yr^{-1}	
Λ_{thx}	`Lthx`	mm K^{-1}	
Λ_{tot1}	`Ltot1`	mm K^{-1}	
Λ_{tot2}	`Ltot2`	mm K^{-2}	
τ_{thx}	`tthx`	yr	
τ_{ice}	`tice`	yr	
α_{dic}	`adic`	µmol kg^{-1} PgC^{-1}	

In manual	In code	Units	Dims
β_{dic}	bdic	1	
γ_{dic}	gdic	K^{-1}	
T_o	To	°C	
ν_{gx}	vgx	yr^{-1}	
γ_{gx}	ggx	K^{-1}	
$\alpha_{o,j}$	aoc_j	1	$j \in [\![1,5]\!]$
$\tau_{o,j}$	toc_j	yr	$j \in [\![1,5]\!]$
κ_{τ_o}	k_toc	1	
β_{npp}	bnpp	1	
α_{npp}	anpp	1	
γ_{npp}	gnpp	K^{-1}	
β_{ef}	bef	1	
γ_{ef}	gef	K^{-1}	
β_{rh}	brh	1	
γ_{rh}	grh	K^{-1}	
$F_{\text{npp},0}$	npp0	PgC yr^{-1}	
ν_{fire}	vfire	yr^{-1}	
ν_{harv}	vharv	yr^{-1}	
ν_{mort}	vmort	yr^{-1}	
ν_{met}	vmet	yr^{-1}	
ν_{rh1}	vrh1	yr^{-1}	
ν_{cs2}	vcs2	yr^{-1}	
ν_{rh3}	vrh3	yr^{-1}	
α_{pass}	apass	1	
α_{lst}	aLST	1	
γ_{rt1}	grt1	K^{-1}	

In manual	In code	Units	Dims
γ_{rt2}	grt2	K^{-2}	
κ_{rt}	krt	1	
a_{min}	amin	1	
κ_a	ka	1	
γ_a	ga	K^{-1}	
ν_{thaw}	vthaw	yr^{-1}	
ν_{froz}	vfroz	yr^{-1}	
ν_{froz}	vfroz	yr^{-1}	
$\alpha_{\mathrm{th},j}$	ath_j	1	$j \in [\![1,3]\!]$
$\tau_{\mathrm{th},j}$	tth_j	yr	$j \in [\![1,3]\!]$
$\kappa_{\tau_{\mathrm{th}}}$	k_tth	1	
$C_{\mathrm{fr},0}$	Cfr0	PgC	
α_C	aCO2	PgC ppm^{-1}	
C_{pi}	CO2pi	ppm	
κ_{pH}	k_pH	1	